Electromagnetic properties of neutrinos

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Abstract

The theoretical and experimental investigation of neutrino electromagnetic properties can provide a window to

- *Physics beyond the Standard Model*
- *New Physics*

Introduction

To help us put the issues in the proper perspective we will review some of the developments of the e.m. properties of ν and related subjects throughout the recent(??) years, Pre-SM, SM, GUTs, Matter effects, ...

- Intrinsic electromagnetic properties (in vacuum)
- Effects on the propagation in matter
- Electromagnetic properties in a medium (matter/thermal backgrounds)
- Macroscopic effects - effects on the e.m. properties of the medium

$\sim$ SM - 70's

- Chiral (left-handed) neutrinos
- Maybe Dirac neutrinos
- Majorana neutrinos were not popular [*The kinetic energy of a Majorana neutrino is a strange piece indeed*!]

The SM provided a consistent framework for

- doing many calculations
- considering variations and/or extensions

SM++ and GUTs - 80's

- Majorana neutrinos started to become popular due to
  - GUTs, $B - L$
  - The problem of neutrino masses (see-saw mechanisms)
  - Supersymmetry
  - ...
- Interest in the calculations of radiative neutrino decays: $\nu_1 \to \nu_2 + (\gamma \text{ or } \gamma\gamma)$ in the SM and extensions.
- The Majorana case was (is) confusing
- Restrictions due to the discrete symmetries (CP), in particular in the Majorana case

Matter effects - 80-90's-today

- Neutrino interactions influence their propagation in matter (Wolfenstein effect, MSW mechanism).
- The neutrino e.m. properties (e.g, magnetic moments) can contribute to that effect
- The neutrino e.m. interactions are also modified in matter/thermal backgrounds
- The neutrino e.m. interactions also modify the macroscopic e.m. properties of the matter/thermal backgrounds

Vertex function

The off-shell electromagnetic vertex function $\Gamma(k, k')$ is defined such that the on-shell matrix element of the electromagnetic current is

$$\langle \nu_2(k') | j_\mu^{(em)}(0) | \nu_1(k) \rangle = \bar{u}_2(k') \Gamma_\mu(k, k') u_1(k)$$

Current conservation implies ($q \equiv k - k'$)

$$q^\mu \Gamma_\mu(k, k') = 0$$

and for neutrinos (zero charge)

$$\Gamma_\mu(k, k) = 0 \qquad (q \to 0)$$

These two imply (in vacuum)

- General

$$\Gamma_\mu(k,k') = (q^2\gamma_\mu - q_\mu \not{q})(F + f\gamma_5) + i\sigma_{\mu\nu}q^\nu(G + g\gamma_5)$$

- Chiral neutrinos

$$\Gamma_\mu(k,k') = (q^2\gamma_\mu - q_\mu \not{q})LF$$

- F charge radius
- f axial charge radius (*anapole moment*)
- G, g magnetic and electric dipole moments

Notes

- F, f, G, g are functions of q^2
- For real photons only G, g contribute ($\Rightarrow G(0)$ and $g(0)$ are gauge invariant.
- If the neutrino electric charge is not zero $\Gamma_\mu(k, k')$ contains a term

$$F_0 \gamma_\mu \qquad \text{(a neutrino charge)}$$

in the diagonal $\nu_1 = \nu_2$ case.

Neutrino charge radius

$$\begin{aligned}\langle r^2 \rangle &\equiv \int d^3x\, r^2 \rho(r) \\ &\sim 6F(0)\end{aligned}$$

The neutrino charge radius contributes to the $\nu - e$ elastic scattering cross section (Grifols, 1986; Degrassi et al., 1989; Vogel and Engel, 1989; Hagiwara et al., 1994):

$$g_V^{\nu_\ell} \to g_V^{\nu_\ell} + \frac{2}{3} m_W^2 \langle r_{\nu_\ell}^2 \rangle \sin^2\theta_W$$

Numerical estimates are $\sim 10^{-33}$ cm^2, and experimental limits are $< 10^{-32}$ cm^2.

The Standard Model calculations of the neutrino charge radius have a long history (and controversies) (Lee-Shrock 1977; Lucio et al 1985; Bernabeu et al 2000,2002,2004, Fujikawa-Shrock 2003,2004; Papavassiliou 2004) related to gauge invariance, definition as a physical quantity, etc.

“If the experimental value of a neutrino charge radius is found to be different from the Standard Model prediction in Eqs. (7.36)-(7.38) it will be necessary to clarify the precision of the theoretical calculation in order to understand if the difference is due to new physics beyond the Standard Model.”

Neutrino charge

In the SM (no ν_R), the $B-L$ current is not anomaly-free.

$$\Downarrow$$

$B-L$ cannot gauged

$$\Downarrow$$

Only Y_{SM} can be gauged $\Rightarrow$ the neutrino has zero charge

$$Q = I_3 + Y_{SM}$$

With ν_R

$$L(\nu_R) \;=\; 1$$
$$\Downarrow$$

$B - l$ current is anomaly-free

$$\Downarrow$$

Any combination of Y_{SM} and $B - L$ can be gauged!

$$\Downarrow$$

$$Y \equiv Y_{SM} + \epsilon(B - L)$$

ϵ arbitrary $\Rightarrow$ Charge quantization is lost with ν_R

$$
\begin{aligned}
Q_\nu = \epsilon \quad &, \quad Q_e = -1 + \epsilon \\
Q_u = \frac{2}{3} - \frac{\epsilon}{3} \quad &, \quad Q_d = -\frac{1}{3} - \frac{\epsilon}{3} \\
Q_p = 1 - \epsilon \quad &, \quad Q_n = -\epsilon
\end{aligned}
$$

(E.g., the Hydrogen atom is neutral, but atoms with neutrons are charged)
Neutrinos may be electrically millicharged particles (Minahan et al (1990); Foot et al (1990b); Babu and Mohapatra (1989); Okun et al (1984); Shrock (1996)

With a Majorana mass term ($\bar{\nu}_L^c \nu_R$) there is no additional $U(1)$.

$$\Downarrow$$

- $Q_\nu = 0$
- Charge quantization is recovered

Or in GUTs, e.g., SO(10), ...

Approximate experimental limits on q_ν

Limit	Method	Reference
$\lvert q_{\nu_\tau} \rvert \lesssim 3 \times 10^{-4}\, e$	SLAC e^- beam dump	Davidson:1991si
$\lvert q_{\nu_\tau} \rvert \lesssim 4 \times 10^{-4}\, e$	BEBC beam dump	Babu:1993yh
$\lvert q_{\nu} \rvert \lesssim 6 \times 10^{-14}\, e$	Solar cooling (plasmon decay)	Raffelt:1999gv
$\lvert q_{\nu} \rvert \lesssim 2 \times 10^{-14}\, e$	Red giant cooling (plasmon decay)	Raffelt:1999gv
$\lvert q_{\nu_e} \rvert \lesssim 3 \times 10^{-21}\, e$	Neutrality of matter	Raffelt:1999gv
$\lvert q_{\nu_e} \rvert \lesssim 3.7 \times 10^{-12}\, e$	Nuclear reactor	Gninenko:2006fi
$\lvert q_{\nu_e} \rvert \lesssim 1.5 \times 10^{-12}\, e$	Nuclear reactor	Studenikin:2013my

The limits on q_ν apply to all flavors

(From Giunti & Studenikin [RMP 87 (2015)])

EM properties of Majorana ν

The possible restrictions on the form-factors follow from

- The discrete symmetries C, P, T and possible combinations. In practice only CP (in vacuum).
- Hermiticity
- Majorana condition, if applicable

Consider

$$M(\nu_1(k) \to \nu_2(k')+\gamma(q)) = \langle \nu_2(k)|j_\mu^{(em)}(0)|\nu_1(k)\rangle \epsilon^{\mu *}$$

with

$$\langle \nu_2(k')|j_\mu^{(em)}(0)|\nu_1(k)\rangle = \bar{u}_2(k')\Gamma_\mu(k,k')u_1(k)$$

The idea is to consider the amplitude $M^{(X)}$ ($X = C, P, T$) calculated with a Lagrangian ($j.A$) transformed by X. If the Lagrangian is invariant under X, then the two amplitudes must be equal.

For example parity -

$$U_P^{-1}\psi(x)U_P = \eta P\psi(\Lambda^{-1}x)$$

$$O^{(X)} = U^{-1}OU$$

$$\Downarrow$$

$$\langle \nu_2\gamma|O^{(X)}|\nu_1\rangle = \langle U\nu_2\gamma|O|U\nu_1\rangle$$

$$\Downarrow$$

Calculate $M^{(X)}$ using the original action but with the transformed wavefunctions: $|U\nu_1\rangle \to Pu_1(k)$

$$M^{(P)}(\nu_1(k) \to \nu_2(k') + \gamma(q)) = \eta_1^P \eta_2^{P*} \bar{u}_2(k') \Gamma_\mu^{(P)}(k, k') u_1(k) \epsilon^{\mu *}$$

$\Gamma_\mu^{(P)}(k, k')$ is obtained from $\Gamma_\mu(k, k')$ by multiplying each quantity in $\Gamma_\mu(k, k')$ by its parity-phase.

	η_P	η_T	η_C	η_{CP}	η_{CPT}
1	+	+	+	+	+
i	+	−	+	+	−
γ_5	−	+	+	−	−
γ_μ	+	−	−	−	+
$\gamma_\mu \gamma_5$	−	−	+	−	+
$\sigma_{\mu\nu}$	+	−	−	−	+
$\sigma_{\mu\nu}\gamma_5$	−	−	−	+	−
$\epsilon_{\alpha\beta\lambda\rho}$	−	−	+	−	+

Similarly (using crossing in C)

$$M^{(T)}(\nu_1(k) \to \nu_2(k')+\gamma(q)) = -\eta_1^T \eta_2^{T*} \bar{u}_2(k')\Gamma_\mu^{(T)}(-k,-k')u_1(k)\epsilon^{\mu*}$$

$$M^{(C)}(\nu_2(k) \to \nu_1(k')+\gamma(q)) = -\eta_1^c \eta_2^{c*} \bar{u}_1(k')\Gamma_\mu^{c}(-k',-k)u_2(k)\epsilon^{\mu*}$$

Hermiticity -

$$M(\nu_2(k) \to \nu_1(k')+\gamma(q)) = \bar{u}_1(k')\bar{\Gamma}_\mu(k',k)u_2(k)\epsilon^{\mu*} \quad (\bar{\Gamma} = \gamma_0\Gamma^\dagger\gamma_0)$$

Majorana (+ crossing) -

$$M(\nu_2(k) \to \nu_1(k') + \gamma(q)) = \bar{u}_1(k')\Gamma_\mu^c(-k',-k)u_2(k)\epsilon^{\mu*}$$

Schematically

- Hermiticity: amplitude for $\nu_2 \to \nu_1 + \gamma$

$$\Gamma_{2\to 1} = \bar{\Gamma}_{1\to 2}(q \to -q)$$

- CP (+ crossing): amplitude for $\nu_2 \to \nu_1 + \gamma$
- Majorana (+ crossing): amplitude for $\nu_2 \to \nu_1 + \gamma$

$$\Gamma_{2\to 1} = \Gamma^c_{1\to 2}$$

- Dirac off-diagonal: Hermiticity by itself gives no restrictions (gives $2 \to 1$ in terms of $1 \to 2$,
  $[\Gamma_{2\to 1} = \bar{\Gamma}_{1\to 2}(q \to -q)]$
  With CP,

  F, f, G, g have the same phase; are relatively real

- Dirac diagonal: Hermiticity and CP give independent conditions

$$\begin{aligned} \text{Hermiticity} \quad &\Rightarrow \quad F, f, G \text{ real; } g \text{ imaginary} \\ \text{CP} \quad &\Rightarrow \quad g = 0 \end{aligned}$$

- Majorana off-diagonal
  1. Majorana cond: the form factors for $\nu_2 \to \nu_1$ are the negatives of those for $\nu_1 \to \nu_2$, except for $\gamma_\mu\gamma_5$ $[\Gamma_{2\to1} = \Gamma^c_{1\to2}]$
  2. $+$ Hermiticity: $\gamma_\mu, \sigma_{\mu\nu}$ imaginary; $\gamma_\mu\gamma_5, \sigma_{\mu\nu}\gamma_5$ real $[\Gamma_{2\to1} = \bar{\Gamma}_{1\to2}(q \to -q)]$
  3. CP: $\gamma_\mu\gamma_5, \sigma_{\mu\nu}\gamma_5$ if relative CP even, $\gamma_\mu, \sigma_{\mu\nu}$ if relative CP odd. (Opposite to Dirac case when P is conserved)
- Majorana diagonal: (1) $\Rightarrow$ all form factors except $\gamma_\mu\gamma_5$ are zero. (2) $\Rightarrow$ it is real

The Mechanics -
Given a diagram D that contributes $\Gamma_\mu^{(D)}$ to $\nu_1 \to \nu_2\gamma$ there is a corresponding diagram $\bar{D}$ for $\nu_2 \to \nu_1\gamma$, with

$$\Gamma_\mu^{(\bar{D})} = \bar{\Gamma}_\mu^{(D)}$$

and by crossing

$$\begin{aligned} M^{(\bar{D})}(\bar{\nu}_1(k) \to \bar{\nu}_2(k') + \gamma(q)) &= -\bar{v}_1\Gamma_\mu^{(\bar{D})}(-k,-k')v_2\epsilon^{\mu*} \\ &= \bar{u}_2\Gamma_\mu^{(\bar{D})c}(-k,-k')u_1\epsilon^{\mu*} \end{aligned}$$

For Majorana neutrinos, this contributes to the same amplitude

$$\Gamma^{(M)}(k,k') = \Gamma_\mu^{(D)}(k,k') + \Gamma_\mu^{(\bar{D})c}(-k,-k')$$

$$\Downarrow$$

$$\begin{aligned} F_M = F_D - F_D^* \quad &; \quad f_M = f_D + f_D^* \\ G_M = G_D - G_D^* \quad &; \quad g_M = g_D + g_D^* \end{aligned}$$

Consistent with what we saw earlier, in all cases.
all from Majorana condition + crossing.

Another approach is to use CPT invariance to show that only $\gamma_\mu \gamma_5$ is non-zero for a Majorana neutrino. However it seems that "CPT is not needed".

But there are some assumptions:

- The form factors are functions only of q^2 (or $k \cdot k'$) and not k, k' separately (Lorentz invariance)
- Substitution rule (crossing) holds
- Others (??)

which probably imply CPT invariance ("we get CPT for free").

- Phys. Rev. D 26, 3152 (1982)
- B. H. J. McKeller, Los Alamos Report No. LA-UR-82-1197 (unpublished)
- B. Kayser, Phys. Rev. D 26, 1662 (1982)
- P. Pal and L. Wolfenstein, Phys. Rev. D 25, 766 (1982)
- R. Shrock, Nucl. Phys. B206, 359 (1982)

It now seems to me that the arguments based on CPT are straight forward. We can go to the non-relativistic limit (ν rest frame) for the various operators

$$\sigma_{\mu\nu} F^{\mu\nu} \to \vec{\sigma} \cdot \vec{B}$$

$$\gamma_5 \sigma_{\mu\nu} F^{\mu\nu} \to \vec{\sigma} \cdot \vec{E}$$

$$\gamma_\mu A^\mu \to \vec{p} \cdot \vec{A} \quad \text{or} \quad \rho \varphi$$

$$\gamma_\mu \gamma_5 A^\mu \to \vec{\sigma} \cdot \vec{A} \quad \text{or} \quad \vec{\sigma} \cdot \vec{J} \qquad (q^2 \vec{A} \to \Box \vec{A} \to \vec{J})$$

Now for C, P, T we have

	$\vec{\sigma}$	$\vec{p}$	$\vec{B}$	$\vec{E}$	$\vec{A}$
P	+	−	+	−	−
T	−	−	−	+	−
C	+	+	−	−	−

The crucial point is that for a Majorana (whether CP even or odd) $\vec{\sigma}$ and $\vec{p}$ are even under C. Only the last ($\vec{\sigma} \cdot \vec{A}$) of the four terms is invariant under CPT. Probably these arguments could be made more formal.

Properties of Majorana Neutrinos," I received a copy of the revised version of the paper. I was most interested to see the appendix on CPT properties, as reviewing your paper was one of the stimuli which led me to construct a similar discussion on CPT properties.

Following receipt of your revised paper I decided there was no point in my publishing the result in the form you had also obtained. I therefore extended the work somewhat, and have now written it up

The idea of using CPT to obtain the whole matrix element $\langle \nu^{Maj} | J^{EM} | \nu^{Maj} \rangle$, as opposed to showing that the dipole moments vanish as I had done, is due, to my knowledge, to you, José, and to Bruce McKellar. (I'll bet there were also important discussions with Lincoln.) But José and Bruce believed that one can set $\eta^{c}_{CPT} = 1$, and this turns out not to be right.

Dipole moments

Hermiticity $\Rightarrow \{G, g\}_{2\to 1} = \{G^*, -g^*\}_{1\to 2}$

$$G(0) = \mu,\ g(0) = id \qquad (\mu, d \text{ Hermitian})$$

Majorana cond $\Rightarrow \{G, g\}_{2\to 1} = -\{G, g\}_{1\to 2}$

μ, d anti-symmetric and imaginary

Using $1, \gamma_5 = R \pm L$ in $\bar{\nu}\sigma\{1, \gamma_5\}\nu$
(Schechter and Valle 1981; Grimus and Schwetz 2000)

$$L_{eff} = \bar{\nu}_R \sigma \cdot F(\mu - id)\nu_L + h.c.$$

(1/2 in Majorana case)

In the simplest case (e.g., radiative decay)

$$\mu_{eff}^2 = |\mu|^2 + |d|^2$$

In general processes (νe scattering) μ_{eff}^2 is a combination also involving initial flux, mixing and oscillation effects.

In the SM ($+$ right-handed ν_R) for the diagonal case (Fujikawa and Shrock (1980); Pal and Wolfenstein (1982); Shrock (1982)),

$$\begin{aligned} \mu_{\nu_\ell} &\sim eG_F m_{\nu_\ell} \simeq 10^{-19}\left(\frac{m_{\nu_\ell}}{eV}\right)\mu_B \\ d_{\nu_\ell} &= 0 \end{aligned}$$

The transition moments are suppressed by the GIM mechanism ($\sim 10^{-4}$ smaller):

$$\mu_{\nu_\ell,\nu_{\ell'}} \sim -eG_F(m_{\nu_\ell} + m_{\nu_{\ell'}}) \times \left(\frac{m_{\ell''}}{m_W}\right)^2$$

$$d_{\nu_\ell,\nu_{\ell'}} \sim -eG_F(m_{\nu_\ell} - m_{\nu_{\ell'}}) \times \left(\frac{m_{\ell''}}{m_W}\right)^2$$

An effect (.e.g, in $\nu + e \to \nu + e$) observing a μ much larger than these values would be a clear indication of physics beyond the SM $+ \nu_R$ model.

- Left-Right symmetric models
- Supersymmetric models, $R-$ parity breaking
- Voloshin symmetry
- Zee model
- Extra dimensions, Kaluza Klein towers, etc

Experimental limits on neutrino effective magnetic moments (from Giunti and Studenikin)

Method	Experiment	Limit	CL	Reference
Reactor $\bar{\nu}_e$-e^-	Krasnoyarsk	$\hat{\mu}_{\nu_e} < 2.4 \times 10^{-10}\,\mu_B$	90%	Vidyakin:1992nf
	Rovno	$\hat{\mu}_{\nu_e} < 1.9 \times 10^{-10}\,\mu_B$	95%	Derbin:1993wy
	MUNU	$\hat{\mu}_{\nu_e} < 9 \times 10^{-11}\,\mu_B$	90%	Daraktchieva:2005kn
	TEXONO	$\hat{\mu}_{\nu_e} < 7.4 \times 10^{-11}\,\mu_B$	90%	Wong:2006nx
	GEMMA	$\hat{\mu}_{\nu_e} < 2.9 \times 10^{-11}\,\mu_B$	90%	Beda:2012zz
Accelerator ν_e-e^-	LAMPF	$\hat{\mu}_{\nu_e} < 1.1 \times 10^{-9}\,\mu_B$	90%	Allen:1992qe
Accelerator $(\nu_\mu, \bar{\nu}_\mu)$-e^-	BNL-E734	$\hat{\mu}_{\nu_\mu} < 8.5 \times 10^{-10}\,\mu_B$	90%	Ahrens:1990fp
	LAMPF	$\hat{\mu}_{\nu_\mu} < 7.4 \times 10^{-10}\,\mu_B$	90%	Allen:1992qe
	LSND	$\hat{\mu}_{\nu_\mu} < 6.8 \times 10^{-10}\,\mu_B$	90%	Auerbach:2001wg
Accelerator $(\nu_\tau, \bar{\nu}_\tau)$-$e^-$	DONUT	$\hat{\mu}_{\nu_\tau} < 3.9 \times 10^{-7}\,\mu_B$	90%	Schwienhorst:2001sj

There are also (similar) limits from

- Solar neutrino data, Joshipura and Mohanty 2002; Grimus et al 2003; Tortola 2003
- Supernova, Dar 1987; Nussinov and Rephaeli 1987; Goldman et al 1988; Lattimer and Cooperstein 1988; Barbieri and Mohapatra 1988; Notzold 1988; Voloshin 1988b; Ayala et al 1999, 2000; Balantekin et al 2007
- Miranda, Kosmas, Valle et al (COHERENT, etc)

Radiative decay

$$\Gamma(\nu_1 \to \nu_2\gamma) \;=\; \frac{1}{8\pi}\left(\frac{m_1^2 - m_2^2}{m_1}\right)^3 \left(|\mu|^2 + |d|^2\right)$$

$$\tau \;\sim\; 0.2\left(\frac{eV}{m_1}\right)^3 \left(\frac{\mu_B}{\mu_{eff}}\right)^2 s$$

$$\Downarrow \textbf{SM} + \nu_R$$

$$\tau \sim \left(\frac{eV}{m_1}\right)^5 (10^{43} s)$$

Astrophysical/cosmological limits (Cowsik 1977; Sato and Kobayashi 1977; Dicus et al. 1978; De Rujula and Glashow 1980b; Stecker 1980; Kimble et al. 1981; Dolgov and Zeldovich 1981; Ressell and Turner 1990; Biller et al. 1998; Raffelt 1998,1999b; Masso and Toldra 1999; Mirizzi et al. 2007):

$$\frac{\mu_{eff}}{\mu_B} < \begin{cases} 0.5\times 10^{-5}\,(\mathrm{eV}/m_\nu)^2 & \text{Sun } (\nu_e), \\ 1.5\times 10^{-8}\,(\mathrm{eV}/m_\nu)^2 & \text{SN 1987A (all flavors)}, \\ 1.0\times 10^{-11}\,(\mathrm{eV}/m_\nu)^{9/4} & \text{Cosmic background (all flavors).} \end{cases}$$

Effects on propagation in matter

Precession $\nu_{eL} \to \nu_{eR}$ in the magnetic field of the Sun could solve the solar neutrino puzzle (Arturo Cisneros 1971) (if neutrinos are Dirac particles, ν_{eR} is sterile and the conversion explains the disappearance of solar ν_{eL}'s).

The matter potential (Wolfenstein term) breaks the degeneracy of left-handed and right-handed states and suppresses the $\nu_{eL} \to \nu_{eR}$ transition (Voloshin and Vysotsky 1986; Okun et al. 1986).

$\Rightarrow$ The matter potentials can cause *resonant* spin-flavor precession if different flavor neutrinos have transition magnetic moments (Akhmedov 1988; Lim and Marciano 1988).

For Majorana neutrinos, the interplay of spin precession and flavor oscillations can generate $\nu_{eL} \to \nu_{eR}^c$ transitions.

It has been studied in

- Solar neutrinos (many)
- Explosion of core-collapse supernovae: Fujikawa and Shrock (1980); Dar (1987); Nussinov and Rephaeli (1987); Goldman et al. (1988); Lattimer and Cooper-stein (1988); Barbieri and Mohapatra (1988); Voloshin (1988b)
- Terrestrial measurements of the neutrino flux of a core-collapse supernova: (Ando and Sato (2003) Akhmedov and Fukuyama (2003); Cuesta and Lambiase (2008); Yoshida et al. (2009, 2011))

- The effects of spin-flavor precession on the collective neutrino oscillations due to neutrino-neutrino interactions (de Gouvea and Shalgar 2012, 2013). There can be collective spin-flavor oscillations (in addition to the usual mass-generated collective neutrino oscillations), sensitive to (Majorana transition) moments $\sim\sim 10^{-21}\mu_B$ (in the SM they are $\sim 10^{-19} \times 10^{-4}$).

$$\Downarrow$$

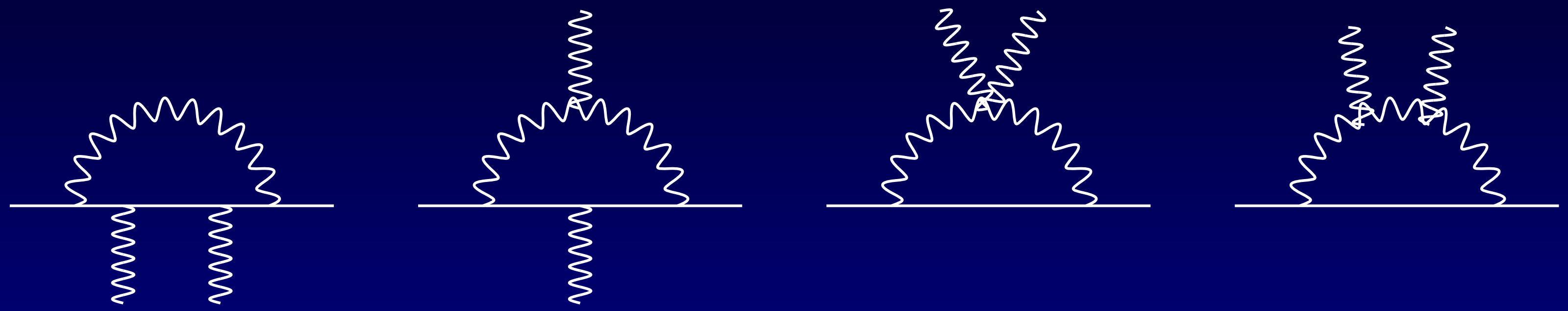
May be the most sensitive context

Two-photon decay

With $V - A$ coupling the $\nu\gamma \to \nu\gamma$ amplitude vanishes (Gell-Mann, Michael Levine) by Yang's theorem (a spin-1 particle cannot decay into two photons). With ν_R and $m_\nu \neq 0$ the theorem is avoided.

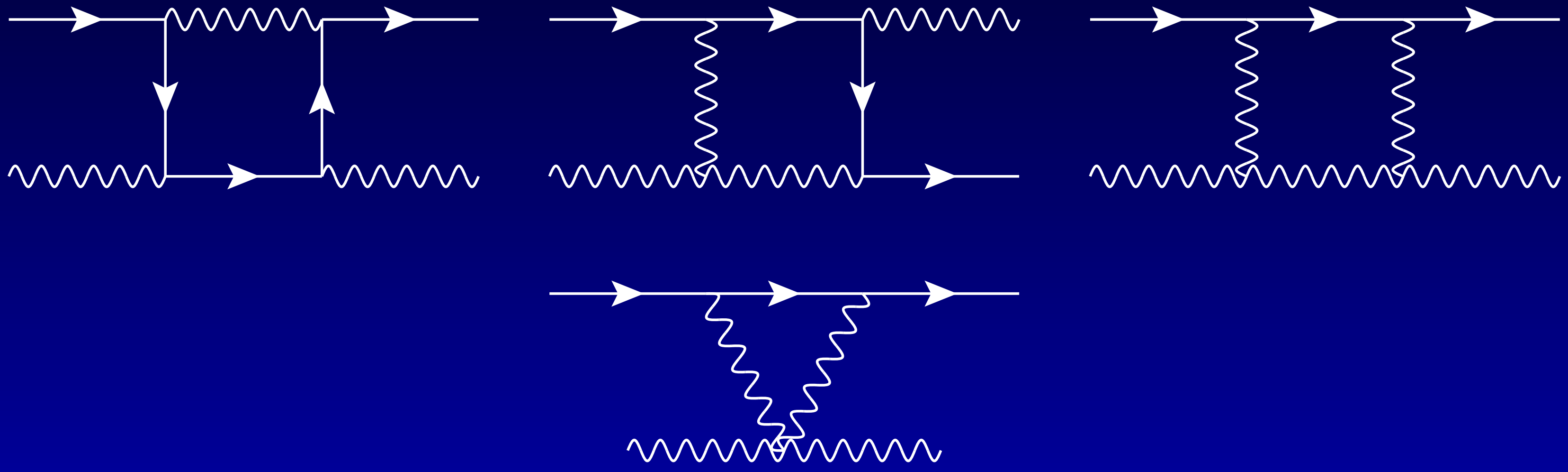
Instead of the GIM suppression factor $(m_\ell/m_W)^4$ for the one-photon mode, the rate for the two-photon mode is suppressed by $(m_\nu/m_\ell)^4$.

Diagrams for $\nu\gamma \to \nu\gamma$



In principle, diagrams in which each W line is replaced by an unphysical Higgs scalar should also be included. In addition, there is a set of diagrams with the photon lines crossed.

The same diagrams for $\nu\gamma \to \nu\gamma$



The leading contribution to the amplitude (in powers of $1/m_W$) is of order $1/m_W^2$ and comes from the first diagram. The remaining diagrams contribute to order $1/m_W^4$ at most.

$$M(\nu_1 \to \nu_2\gamma\gamma) = \epsilon^{\mu*}(k')\epsilon^{\nu*}(k)\bar{u}_2(p_2)\Gamma_{\mu\nu}u_1(p_1)$$

$$\Gamma_{\mu\nu} = -2i\epsilon_{\mu\nu\alpha\beta}k'^{\alpha}k^{\beta}\gamma_5(F_5 + f_5\gamma_5)$$

$$F_5, f_5 \sim \alpha G_F(m_{\nu_1} \pm m_{\nu_2}) \sum_i \frac{U_{i2}^* U_{i1} I_i(s)}{m_{\ell_i}^2}$$

(The details are similar to $s \to d\gamma\gamma$ [Gaillard and Lee 1974], triangle anomaly, etc)

The two-photon amplitude is controlled by the coupling to the lightest charged lepton (“e”) while the one-photon amplitude is controlled by the coupling to the heaviest charged lepton (“τ”).

$$\Gamma^{(\gamma\gamma)} = \left(\frac{\alpha}{288\pi}\right)\left(\frac{\alpha G_F^2 m_{\nu_1}^5}{128\pi^4}\right)\left(\frac{m_{\nu_1}}{m_e}\right)^4 \times \text{mixing}$$

$$\Gamma_0^{(\gamma)} = \frac{9}{16}\left(\frac{\alpha G_F^2 m_{\nu_1}^5}{128\pi^4}\right)\left(\frac{m_\tau}{m_W}\right)^4 \times \text{mixing}$$

Related

- “Circular polarization: a new probe of dark matter and neutrinos in the sky”, Bœhm, Mattelaer, Vincent, JCAP 05, 043 (2017)
- “Radiative decay of keV-mass sterile neutrinos in a strongly magnetized plasma”, Dobrynina, Mikheev, Raffelt, Phys. Rev. D 90, 113015 (2014)

Generalization to Majorana DM fermions

- “Gamma lines from Majorana dark matter”, Duerr, Fileviez Pérez, Smirnov, Phys. Rev. D 93, 023509 (2016)
- “Two-photon interactions with Majorana fermions”, Latimer, Phys. Rev. D 94, 093010 (2016)
- “Anapole dark matter annihilation into photons”, Latimer, Phys. Rev. D 95, 095023 (2017)

EM properties in a medium

The calculations (e.g. $\langle \nu_2(k')|J_\mu(0)|\nu_1(k)\rangle$) involve

$$Z = e^{-\beta P\cdot u + \sum_A \alpha_A Q_A}$$

$\Rightarrow \Gamma_\mu$ (and Σ) depends on u^μ and α

- More tensor structures involving u^μ
- $F_i \to F_i(k, k', u, \alpha)$

In the *static* limit ($k' \to k$, $q \to 0$) the F_i are functions of k and u; i.e.,

$$\omega = k\cdot u \qquad \kappa = \sqrt{\omega^2 - k^2}$$

$$u^\mu = (1, \vec{0}) \qquad k^\mu = (\omega, \vec{\kappa})$$

In vacuum

$$\Gamma_\mu = i\sigma_{\mu\nu}q^\nu(D_M + D_E\gamma_5) + ...$$

now for example (omitting k, k', u, α arguments)

$$\Gamma'_\mu = iD'_E(\gamma_\mu u_\nu - \gamma_\nu u_\mu)q^\nu\gamma_5 + iD'_M\epsilon_{\mu\nu\lambda\rho}\gamma^\nu\gamma_5 q^\lambda u^\rho$$

$$\Gamma \cdot A \to O_E, O_M \ , \qquad \Gamma' \cdot A \to O'_E, O'_M$$

$$O_E = d_E\bar{\nu}\sigma_{\mu\nu}\nu\tilde{F}^{\mu\nu} \ , \ \ O_M = d_M\bar{\nu}\sigma_{\mu\nu}\nu F^{\mu\nu}$$

$$O'_E = d'_E\bar{\nu}\gamma_\mu\gamma_5\nu F^{\mu\nu}u_\nu \ , \ \ O'_M = d'_M\bar{\nu}\gamma_\mu\gamma_5\nu\tilde{F}^{\mu\nu}$$

$$O_E + O_M \stackrel{NR}{\rightarrow} d_E(\mathbf{s} - \bar{\mathbf{s}}) \cdot E + d_M(\mathbf{s} - \bar{\mathbf{s}}) \cdot B$$

$$O'_E + O'_M \stackrel{NR}{\rightarrow} d'_E(\mathbf{s} + \bar{\mathbf{s}}) \cdot E + d'_M(\mathbf{s} + \bar{\mathbf{s}}) \cdot B$$

$$(\mathrm{s}, \bar{\mathrm{s}} \quad = \quad \text{particle,antiparticle (NR) spin op})$$

- $O'_{E,M}$ can be non-zero for Majorana ν
- For Dirac ν, particle and antiparticle differ: $\pm d + d'$.

- NR reduction, or

	$\hat{O}_E$	$\hat{O}_M$	$\hat{O}'_E$	$\hat{O}'_M$
P	$-$	$+$	$-$	$+$
T	$-$	$+$	$-$	$+$
C	$+$	$+$	$-$	$-$

$\hat{O}'_{E,M}$ are CPT-odd

$\Rightarrow$ $D'_{E,M}$ at zero momentum are:

- $\propto (n - \bar{n})$
- zero in a CPT symmetric background

$n + \bar{n}$ terms in $D'_{E,M}$ are momentum (ω, κ) dependent.

Imagine calculating the amplitude ($M^{(X)}$) with the transformed action. We can calculate $M^{(X)}$ using the original action but using the transformed wavefunctions ($|U\nu_1\rangle \to Pu_1(k)$) as before (in vac), but now also using the transformed u^μ and α:

$$u^\mu \sim P^\mu \qquad \alpha_A \sim Q_A$$

For example, T, C

$$M^{(T)}(\nu_1(k) \to \nu_2(k')+\gamma(q)) = -\eta_1^T \eta_2^{T*} \bar{u}_2(k')\Gamma_\mu^{(T)}(-k, -k', -u, \alpha)u_1(k)\epsilon^{\mu*}$$

$$M^{(C)}(\nu_2(k) \to \nu_1(k')+\gamma(q)) = -\eta_1^c \eta_2^{c*} \bar{u}_1(k')\Gamma_\mu^c(-k', -k, u, -\alpha)u_2(k)\epsilon^{\mu*}$$

Diagonal case, in the *static* limit ($q \to 0$):

$$\text{Hermiticity} \;\Rightarrow\; d'_{E,M} \qquad \text{real}$$

$$\text{Majorana} \;\Rightarrow\; d'_{E,M}(-\omega,\kappa) = d'_{E,M}(\omega,\kappa)$$

$$P: \qquad d'_E = 0$$

$$T: \qquad d'_E \text{ imaginary}, \; d'_M \text{ real}$$

$$C: \qquad d'_{E,M}(-\omega,\kappa) = -\, d'_{E,M}(\omega,\kappa)\Big|_{\alpha\to-\alpha}$$

$$CP: \qquad d'_{E,M}(-\omega,\kappa) = \pm\, d'_{E,M}(\omega,\kappa)\Big|_{\alpha\to-\alpha}$$

$$CPT: \qquad d'^{*}_{E,M}(-\omega,\kappa) = -\, d'_{E,M}(\omega,\kappa)\Big|_{\alpha\to-\alpha}$$

Assuming that the action is CPT invariant (!), Hermiticity + CPT give

$$d'_{E,M}(-\omega,\kappa) = -\, d'_{E,M}(\omega,\kappa)\Big|_{\alpha\to-\alpha}$$

If $\alpha = 0$ (CPT-symmetric background) then $d'_{E,M}$ vanish for $\omega = 0$. But in general can be non-zero even in a CPT-symmetric background; they would be odd in the variable ω. More generally, expect

$$\begin{aligned} \text{odd terms in } \omega &\quad\propto\quad n + \bar{n} \\ \text{even terms in } \omega &\quad\propto\quad n - \bar{n} \end{aligned}$$

Note

Something similar happens with the index of refraction in matter. The Wolfenstein term (the leading, momentum-independent, term in the self-energy expanded in k/m_W)

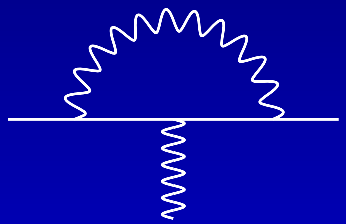
$$\Sigma \sim G_F(n_e - n_{\bar{e}})$$

vanishes in a CPT-symmetric background (e.g., Early Universe). The next term in powers of ω is

$$\sim G_F \frac{T\omega}{m_W^2}(n_e + n_{\bar{e}})$$

$\Downarrow$

Calculation of Γ_μ in matter (e.g., electron gas): chiral neutrinos, SM interactions, no helicity flip, independent of m_ν. The $\sim O(1/m_W^2)$ term is given by



Some effects:

Index of refraction in a B field

In the presence of a B field, the $\nu\nu\gamma$ interaction vertex gives a contribution to the neutrino self-energy $\sim \Gamma \cdot A$.

$$\Sigma^{(m)} = b\gamma \cdot u + c\gamma \cdot B \Rightarrow \omega(\vec{k}) = |\vec{k} - c\vec{B}| + b$$

Dispersion relation is not isotropic

- b is the Wolfenstein term $\sim G_F(n_e - n_{\bar{e}})$
- c is essentially D'_M evaluated in the *static* $(q \to 0)$ limit

$$\sim eG_F \int \frac{d^3p}{2E}(f'_e - f'_{\bar{e}})$$

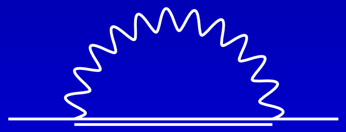
For a NR electron gas

$$\begin{aligned} c &\sim \mu_B \beta n_e \\ \omega(\vec{k}) &\approx |\vec{k}| + b(1 + r\mu_B \beta \hat{k} \cdot \vec{B}) \\ r &= \text{ratio of SM coupling constants} \end{aligned}$$

“For the Sun, $\mu_B \beta B \sim 10^{-8}$. Although the application of these results to the solar neutrinos appears to be of no consequences, their possible application in other physical contexts remains an open question and their implications should be kept in mind.”

Generalized

- Beyond linear approximation using the propagators in the B field, Elmfors et al. (1996); Erdas et al. (1998); Elizalde et al. (2002, 2004).
- Finite temperature corrections $O(1/m_W^4)$, Zhukovsky et al. (1993); Esposito and Capone (1996)



Others

- Radiative decay in matter; not suppressed by the GIM mechanism (D'Olivo et al)
- Neutrino effective charge; "Cherenkov radiation" (Oraevsky, Sawyer, D'Olivo et al)
- Transition radiation (D'Olivo and Loza, Ioannisian et al)

EM effects of a ν background

Macroscopic effects

- Optical activity (with Mohanty, Pal)
- Instabilities, growth of EM fields (Semikoz et al)

Optical activity of a ν gas

Maxwell's equations (in momentum space):

$$\vec{k} \cdot \vec{B} = 0 \qquad \vec{k} \times \vec{E} = \omega \vec{B}$$

$$i\vec{k} \cdot \vec{E} = \rho \qquad i\vec{k} \times \vec{B} + i\omega \vec{E} = \vec{j}$$

Conventionally,

$$\rho = \rho_{ext} + \rho_{ind} \qquad \vec{j} = \vec{j}_{ext} + \vec{j}_{ind}$$

with

$$\vec{j}_{ind} = i\vec{k} \times \vec{M} - i\omega \vec{P} \qquad \rho_{ind} = -i\vec{k} \cdot \vec{P}$$

$$\vec{P} = (\epsilon - 1)\vec{E} \qquad \vec{M} = (1 - \frac{1}{\mu})\vec{B}$$

In other words,

$$\begin{aligned} \vec{j}_{ind} &= i\left(1 - \frac{1}{\mu}\right)\vec{k} \times \vec{B} - i(\epsilon - 1)\omega\vec{E} \\ \rho_{ind} &= -i(\epsilon - 1)\vec{k} \cdot \vec{E} \end{aligned}$$

We are trying to write ρ_{ind} and $\vec{j}_{ind}$ as linear functions of $\vec{E}$ and $\vec{B}$. *Are these combinations the most general ones?*

"Yes" for ρ_{ind} $\quad$ $(\vec{k} \cdot \vec{B} = 0)$.

For $\vec{j}_{ind}$ we can use $\omega\vec{E},\ \vec{k} \times \vec{B},\ \omega\vec{B},\ \vec{k} \times \vec{E} (= \omega\vec{B})$.

$$\vec{j}_{ind} = i\left(1 - \frac{1}{\mu}\right)\vec{k} \times \vec{B} - i(\epsilon - 1)\omega\vec{E} - i\zeta\omega\vec{B}$$

Since $\vec{j}_{ind}$ is a vector in general we need three independent vectors to express it.

The inhomogeneous equations become

$$
\begin{aligned}
i\epsilon\vec{k}\cdot\vec{E} &= \rho_{ext} \\
i\epsilon\omega\vec{E} + \frac{i}{\mu}\vec{k}\times\vec{B} + i\zeta\omega\vec{B} &= \vec{j}_{ext}
\end{aligned}
$$

The solutions for $\rho_{ind}, \vec{j}_{ind} = 0$ give the photon dispersion relations.

- ζ breaks the degeneracy of the transverse modes; the two circularly polarized modes have a unique $\omega_{\pm}(k)$
- $\zeta \neq 0 \Rightarrow P, CP, CPT$ must be broken (e.g., a gas of chiral neutrinos with $n_\nu \neq n_{\bar{\nu}}$)

Another way: eliminate $\vec{B}$ in terms of $\vec{E}$ and ρ in terms $\vec{j}$, and the only independent equation is

$$\frac{i}{\omega}\vec{k} \times (\vec{k} \times \vec{E}) + i\omega\vec{E} = \vec{j}$$

Conventional:

$$\vec{j}_{ind} = -i\omega[(\epsilon_t - 1)\vec{E}_t + (\epsilon_l - 1)\vec{E}_l]$$

with

$$\vec{E}_l = \hat{k}(\hat{k} \cdot \vec{E}) \qquad\qquad \vec{E}_t = \vec{E} - \vec{E}_l$$

In other words, $\vec{j}$ is written as a longitudinal and a transverse part, with respect to $\vec{k}$.

$$\begin{aligned}\epsilon_l &= \epsilon \\ \epsilon_t &= (\epsilon - 1) + \left(\frac{\vec{k}^2}{\omega^2}\right)\left(1 - \frac{1}{\mu}\right)\end{aligned}$$

This is equivalent to write

$$\epsilon_{ij} = \hat{k}_i\hat{k}_j\epsilon_l + (\delta_{ij} - \hat{k}_i\hat{k}_j)\epsilon_t$$

Is this the most general form of ε_{ij}, consistent with isotropy and linearity? No, $\vec{j}$ can have an additional term:

$$\vec{j}_{ind} = -i\omega[(\epsilon_t - 1)\vec{E}_t + (\epsilon_l - 1)\vec{E}_l] - i\epsilon_p \hat{k} \times \vec{E}$$

Equivalent to

$$\epsilon_{ij} = \hat{k}_i \hat{k}_j \epsilon_l + (\delta_{ij} - \hat{k}_i \hat{k}_j)\epsilon_t + i\varepsilon_{ijk}\hat{k}_k \epsilon_p$$

ϵ_p is related to the π_P in the photon self-energy, and

$$\zeta = \frac{\omega \epsilon_p}{|\vec{k}|}$$

In vacuum

$$\text{gauge invariance} \Rightarrow \pi_{\mu\nu}(k) = \pi(k)\tilde{g}_{\mu\nu}$$

$$\tilde{g}_{\mu\nu} \equiv g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}$$

Automatically satisfies Bose symmetry

$$(i) \quad \pi_{\nu\mu}(-k) = \pi_{\mu\nu}(k)$$

but in addition satisfies

$$\text{(ii)} \quad \pi_{\mu\nu}(k) = \pi_{\nu\mu}(k)$$
$$\text{(iii)} \quad \pi_{\mu\nu}(-k) = \pi_{\mu\nu}(k)$$

separately.

In a medium, $\pi_{\mu\nu}$ is a function of k^μ and u^μ. The most general (gauge invariant) form consistent with (ii) and (iii) is

$$\pi_{\mu\nu} = \pi_T R_{\mu\nu} + \pi_L Q_{\mu\nu}$$

where

$$\begin{aligned}
\tilde{u}_\mu &\equiv \tilde{g}_{\mu\nu} u^\nu \\
Q_{\mu\nu} &\equiv \frac{\tilde{u}_\mu \tilde{u}_\nu}{\tilde{u}^2} \\
R_{\mu\nu} &\equiv \tilde{g}_{\mu\nu} - Q_{\mu\nu}
\end{aligned}$$

But nothing demands (ii) and (iii) separately; only (i).

$$\Downarrow$$

$\pi_{\mu\nu}$ can have an additional term:

$$\pi_{\mu\nu} = \pi_T R_{\mu\nu} + \pi_L Q_{\mu\nu} + \pi_P P_{\mu\nu}$$

where

$$P_{\mu\nu} \equiv \frac{i}{|\vec{k}|} \epsilon_{\mu\nu\lambda\rho} k^\lambda u^\rho$$

- $\pi_{T,L}$ are related to ϵ, μ
- π_P is related to ζ. It is non-zero only as a result of P, CP and CPT asymmetric effects (in the Lagrangian and/or in the background).

Effective Lagrangian

$$L = \frac{1}{2} A^*_\mu \left[(\Delta^{-1})^{\mu\nu} + \pi^{\mu\nu} \right] A_\nu - A^* \cdot j_{ext}$$

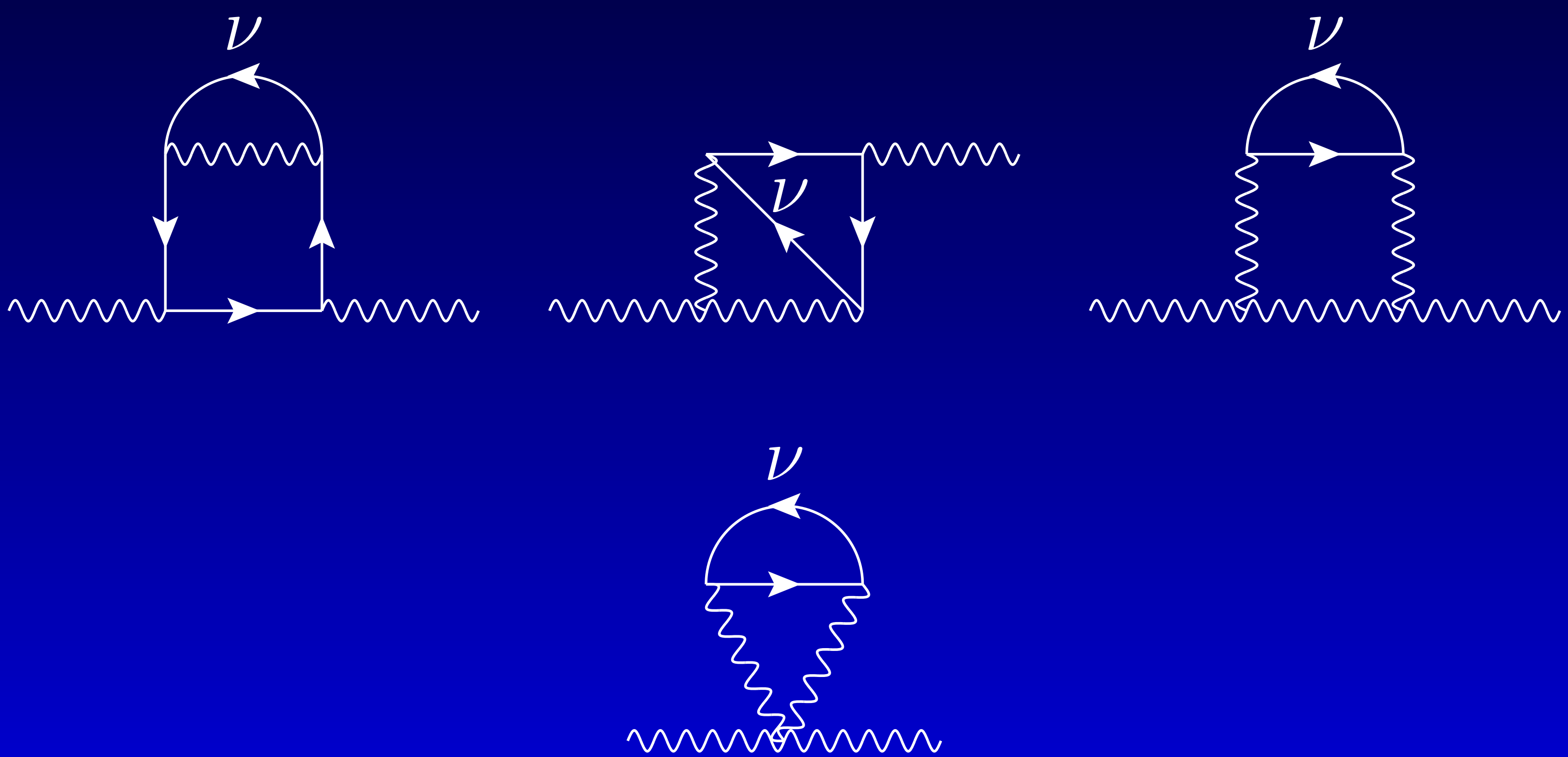
$$= -\frac{1}{4} F^*_{\mu\nu} F^{\mu\nu} + A^*_\mu \pi^{\mu\nu} A_\nu - A^* \cdot j_{ext}$$

$$\Rightarrow \qquad j^\mu_{ind} = -\pi^{\mu\nu} A_\nu$$

This gives

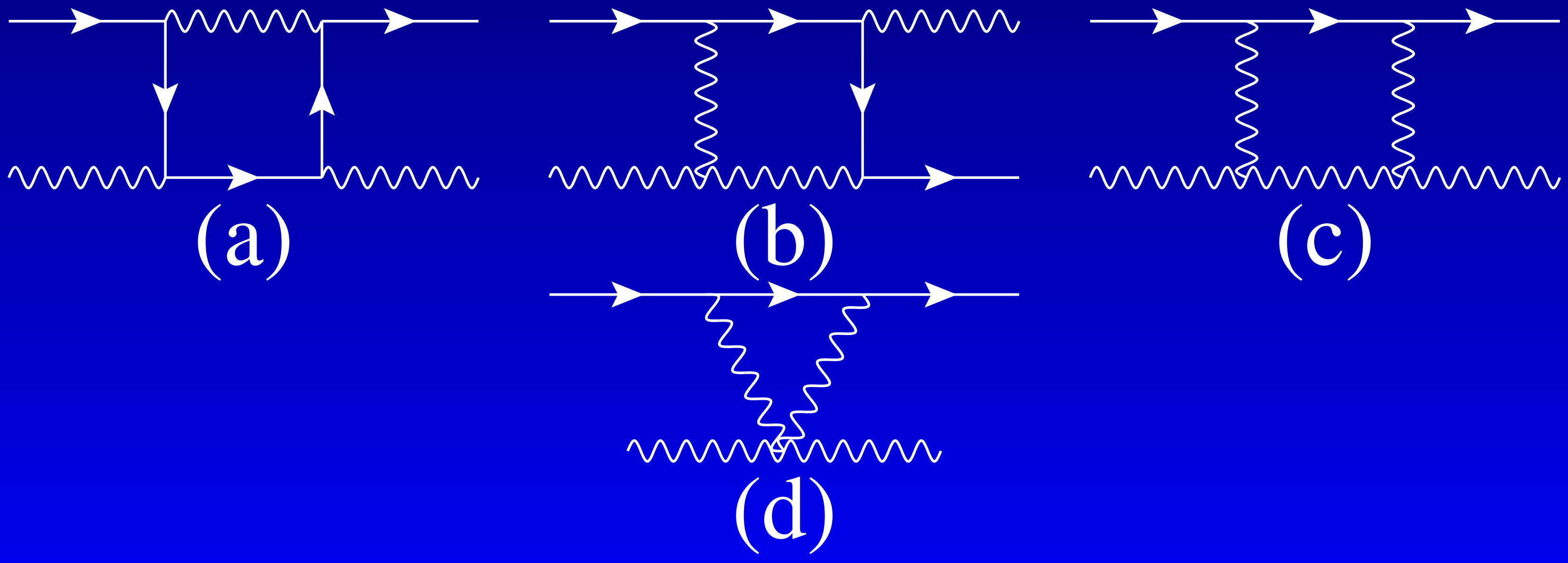
$$\begin{aligned}
\epsilon_l &= 1 - \frac{\pi_L}{k^2} \\
\epsilon_t &= 1 - \frac{\pi_T}{\omega^2} \\
\epsilon_p &= \frac{\pi_P}{\omega^2}
\end{aligned}$$

Some details of the calculation (π_P in a ν gas)



$$i\pi'_{\mu\nu}(k) \;=\; -\int \frac{d^4p}{(2\pi)^4}\,\mathrm{Tr}\,[S_F(p)M_{\mu\nu}(p,k)]$$

$$M_{\mu\nu}(p,k) \;=\; \gamma\nu \text{ forward scattering amplitude}$$



To determine the leading contribution (in powers of m_W^2) to the gauge invariant part of the amplitude we need to calculate only the diagram (a), retaining from that only the terms of order $1/m_W^2$ that contain at least two powers of k. The result so obtained is proportional to $1/(m_W^2 m_\ell^2)$ and has significance in the kinematic regime $|k^2| < m_\ell^2$. For the kinematic regime $|k^2| > m_\ell^2$, we need to calculate all the diagrams.

The result for the amplitude is (with $\epsilon_{0123} = -1$)

$$M_{\mu\nu} = \frac{-ie^2g^2}{8\pi^2 m_W^2 m_e^2}\gamma_\lambda L\,(ik^2\varepsilon_{\mu\nu\lambda\rho}k^\rho)J(k^2)$$

$$J(k^2) = \int_0^1 dx\,\frac{x(1-x)}{1-x(1-x)(k^2/m_e^2)}$$

and for the self-energy

$$\pi_P^{(\nu)}(k) \;=\; \frac{e^2 g^2 k^2 |\vec{k}|}{8\pi^2 m_W^2 m_e^2} \left(n_{\nu_e} - n_{\bar{\nu}_e}\right) J(k^2)$$

$$\simeq \; \frac{\sqrt{2}\, G_F \alpha}{3\pi} \left(\frac{k^2}{m_e^2}\right) \left(n_{\nu_e} - n_{\bar{\nu}_e}\right) |\vec{k}|$$

$\pi_{T,L}$ get their normal contribution from the e background.

Dispersion relations

Equation of motion:

$$D^{-1}_{\mu\nu} A^{\nu} \;=\; 0$$

$$\begin{aligned} D^{-1}_{\mu\nu} \;&=\; (-k^2 + \pi_T) R_{\mu\nu} \\ &\quad + (-k^2 + \pi_L) Q_{\mu\nu} + \pi_P P_{\mu\nu} \end{aligned}$$

Transversality $\Rightarrow A_\mu = A_+ e^{(+)}_\mu + A_- e^{(-)}_\mu + A_L e_{3\mu}$

$A_\pm$ represent the amplitudes of the left($-$) and right($+$) circularly polarized modes, and A_L the longitudinal mode.

$$e_\mu^{(\pm)} \;=\; \frac{1}{\sqrt{2}}\left(e_{1\mu} \pm i e_{2\mu}\right)\;, \qquad k^2 = \pi_T \pm \pi_P$$

$$e_3^\mu \;=\; \frac{\tilde{u}^\mu}{\sqrt{-\tilde{u}^2}}\,, \qquad k^2 = \pi_L$$

The two transverse modes have different dispersion relations. This optical activity in the present case is induced by the chiral nature of the neutrino interactions.

Writing $K = |\vec{k}|$, and with

$$\pi_T \simeq \omega_P^2 \simeq \frac{4\pi\alpha n_e}{m_e}$$

$$\omega_\pm(K) \;=\; \sqrt{K^2 + \omega_P^2} \pm \frac{1}{2}(a_\nu \omega_P^2)\frac{K}{\sqrt{K^2 + \omega_P^2}}$$

$$a_\nu \;\equiv\; \frac{\sqrt{2}\,G_F \alpha}{3\pi}\left(\frac{n_{\nu_e} - n_{\bar{\nu}_e}}{m_e^2}\right)$$

An alternate form is

$$K_{\pm} = (\omega^2 - \omega_P^2)^{1/2} \mp \frac{1}{2}\, a_\nu \omega_P^2\,.$$

Which one should be used depends on the initial conditions of the particular situation.

Example: Consider a monochromatic wave propagating along the z-axis. If the wave is linearly polarized at $z = 0$ (that is, it contains an equal admixture of the two circular polarizations), then the amplitude is of the form

$$
\begin{aligned}
\vec{A}(z,t) &= A_\omega e^{-i\omega t}\left(e^{izK_+}\hat{e}^{(+)} + e^{izK_-}\hat{e}^{(-)}\right) \\
&= A_\omega e^{-i\omega t} e^{i\frac{1}{2}(K_+ + K_-)z} \\
&\quad \times \left(e^{-i\frac{1}{2}(K_- - K_+)z}\hat{e}^{(+)} + e^{i\frac{1}{2}(K_- - K_+)z}\hat{e}^{(-)}\right)
\end{aligned}
$$

Convention: at $z = 0$, the linear polarization vector of the wave points along $\hat{e}_1$, or the x-direction. Using $\hat{e}^{(\pm)} = (\hat{e}_1 \pm i\hat{e}_2)/\sqrt{2}$, then at any given distance $z = \ell$ the polarization vector of the wave points at an angle, relative to the x-axis,

$$
\begin{aligned}
\phi(\ell) &= \frac{1}{2}(K_- - K_+)\ell = \frac{1}{2}a_\nu \omega_P^2 \ell \\
&= 10^{-11} R_\nu \left(\frac{\omega_P}{\mathrm{MeV}}\right)^2 \left(\frac{\ell}{\mathrm{cm}}\right)
\end{aligned}
$$

$$
R_\nu \equiv \frac{n_{\nu_e} - n_{\overline{\nu}_e}}{10^{24}\mathrm{cm}^{-3}}
$$

Related

- “Circular polarization: a new probe of dark matter and neutrinos in the sky”, Bœhm, Mattelaer, Vincent, JCAP 05, 043 (2017)

Instabilities

- “Large-scale magnetic field generation by alpha-effect driven by collective neutrino-plasma interaction”, Semikoz and Sokoloff, Phys. Rev. Lett. 92, 131301 (2004).
- “Electromagnetic effects of neutrinos in an electron gas”, with Sarira Sahu, Phys.Rev. D71 (2005) 073006
- “Chiral transport of neutrinos in supernovae: Neutrino-induced fluid helicity and helical plasma instability”, Yamamoto, Phys. Rev. D 93, 065017 (2016)

End

It seems that most accessible e.m. properties of ν are

- Charge radius. The SM model gives a value about one order of magnitude smaller than the experimental limit
- (Majorana) transition moments. Supernova neutrino fluxes are sensitive to the effects of collective spin-flavor oscillations due to Majorana transition magnetic moments $\sim 10^{-21}\mu_B$ (de Gouvea and Shalgar, 2012, 2013)

$\Rightarrow$ Another confirmation of the SM or

- *Physics beyond the Standard Model*
- *New Physics*

- *Exotic and/or unexpected*
  The *direct* method using Hermiticity + substitution rule + Majorana condition can give useful results in cases in which CPT, Lorentz symmetry, ..., are broken
  (Example, dependence on ω, κ, chemical potentials, in matter/thermal backgrounds).