Structure and beta decay properties of medium-heavy nuclei from the relativistic nuclear field theory

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• **Motivation:** to build a consistent and predictive approach to describe the entire nuclear chart (ideally, an arbitrary strongly-correlated many-body system)

• **Challenges:** the nuclear hierarchy problem, complexity of NN-interaction

• **Approximate, but quite accurate non-perturbative solutions:** Relativistic Nuclear Field Theory (RNFT). Emerged as a synthesis of Landau-Migdal Fermi-liquid theory, Copenhagen-Milano NFT and Quantum Hadrodynamics; now put in the context of a systematic equation of motion (EOM) method (Coll: P. Schuck)

• **Technique:** Green function formalism, EOM, time blocking method

• **Applications:** single-particle states (states in odd nuclei), excitation spectra of even-even and odd-odd nuclear systems, beta decay, astrophysics (Coll: C. Robin, A. Afanasjev, P. Ring, T. Marketin, D. Vretenar, …)

• **Conclusions and perspectives**
- **Nuclear scales: Hierarchy problem**

\[ H = K + V \]

center of mass  
internal DOF’s:  
next energy scale

- No connection between the scales in the traditional NS models
- Effective theories (most often) lose the energy dependence of the “interaction”
The first direct detection of gravitational waves and gamma-ray bursts from the same source of merging neutron stars (kilonova)

B. P. Abbott et al. (LIGO Scientific Collaboration and Virgo Collaboration)
Phys. Rev. Lett. 119, 161101

NSF/LIGO/Sonoma State University/A. Simonnet

Astrophysical origins of chemical elements:

The major part of the r-process path: neutron-rich medium-heavy nuclei
A strongly-correlated many-nucleon system: interaction, single-fermion propagator and related observables

Quantum Hadrodynamics (QHD):

\[ H = \sum_{12} t_{12} \psi_{1}^\dagger \psi_{2} + \frac{1}{4} \sum_{1234} \bar{v}_{1234} \psi_{1}^\dagger \psi_{2}^\dagger \psi_{4} \psi_{3} \]

\[ G_{11'}(t - t') = -i \langle T \psi(1) \psi^\dagger(1') \rangle \]

\[ G(\xi, \xi'; \varepsilon) = \sum_{n} \frac{(\Psi(\xi))_{0n}(\Psi^\dagger(\xi'))_{n0}}{\varepsilon - (E_{n}^{(N+1)} - E_{0}^{(N)}) + i\delta} + \sum_{m} \frac{(\Psi^\dagger(\xi'))_{0m}(\Psi(\xi))_{m0}}{\varepsilon + (E_{m}^{(N-1)} - E_{0}^{(N)}) - i\delta}, \]

\[ (\Psi^\dagger(\xi))_{n0} = \langle \Phi_{n}^{(N+1)} | \Psi^\dagger(\xi) | \Phi_{0}^{(N)} \rangle, \]

\[ (\Psi(\xi))_{n0} = \langle \Phi_{n}^{(N-1)} | \Psi(\xi) | \Phi_{0}^{(N)} \rangle, \]

\[ \chi_{EFT} \]

Hamiltonian

Single-particle propagator

Fourier transform:

Spectral expansion (Lehmann)

Observables:

Residues - spectroscopic (occupation) factors (basis-dependent)

Poles - single-particle energies
Exact equations of motion (EOM) for binary instantaneous interactions:

One-body problem

Instantaneous term (Hartree-Fock incl. “tadpole”)

\[
\Sigma_{11'}^{(0)} = -\delta(t - t') \langle [[[\psi_1, V], \psi_1^\dagger]] \rangle = -\sum_{j} \overline{v}_{1j} \rho_{jj'} = \]

Mean field, where \( \rho_{ij} = -i \lim_{t\to t'} G_{ij}(t-t') \) is the full solution of (1): includes the dynamical term!

Dynamical self-energy

\[
\Sigma_{11'}^{(r)}(t - t') = -i\langle T[\psi_1, V](t)[V, \psi_1^\dagger](t')\rangle^{irr}
\]

\[
= -\frac{1}{4} \sum_{234} \sum_{2'3'4'} \overline{v}_{1234} G^{irr}(432', 23'4')\overline{v}_{4'3'2'1'}
\]

EOM method:

P. Schuck and M. Tohyama, PRB 93, 165117 (2016). etc.
**Exact mapping to the particle-vibration coupling**

- **Model-independent mapping to the QVC-TBA:**
  \[
  \sum_{343'4'} \tilde{V}^*_{12,34} R_{34,3'4'}(\omega) \tilde{V}_{3'4'1'2'} = \sum_m g_{12}^m D_m(\omega) g_1^m
  \]

  \[R_{12,1'2'}(\omega) = \sum_m \left( \frac{\rho_{12}^m \rho_{1'2'}^m}{\omega - \Omega_m + i\delta} - \frac{\rho_{21}^m \rho_{2'1'}^m}{\omega + \Omega_m - i\delta} \right)\]

- **“phonon” vertex:**
  \[g_{12}^m = \sum_{34} \tilde{V}_{12,34} \rho_{34}^m\]

- **“phonon” propagator:**
  \[D_m(\omega) = \frac{1}{\omega - \Omega_m + i\delta} - \frac{1}{\omega + \Omega_m - i\delta}\]

**Graphical Diagrams:**

- **ph correlator:** coupling to normal phonons
- **pp correlator:** coupling to pairing phonons

**Equations:**

- **Dropping the uncorrelated term:** “factor 1/2” problem (Ring & Schuck, Book)  
  “sign problem” (Danielewicz & Schuck, NPA 567 (1994) 78)

**But:** The lowest order is not the leading order in the strong coupling regime!  
=> the uncorrelated term can be safely neglected
Fragmentation of single-particle states and particle-hole excitations

Single-particle structure

Energy

No correlations (instantaneous interaction only)

Correlations (QVC)

Dominant level

Strong fragmentation

Response

No correlations (instantaneous interaction only)

Correlations (QVC)

Spectroscopic factors $S_k^{(\nu)}$

$$E_k = \sum_k E_k^{(\nu)} S_k^{(\nu)}$$

$$\sum_\nu S_k^{(\nu)} = 1$$
(Quasi)particle-vibration coupling (QVC, PVC): Pairing correlations of the superfluid type + coupling to phonons

**Dominant states and spectroscopic factors in $^{120}$Sn:**

<table>
<thead>
<tr>
<th>(nlj) $\nu$</th>
<th>$S^\text{th}$</th>
<th>$S^\text{exp}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2$d_{5/2}$</td>
<td>0.32</td>
<td>0.43</td>
</tr>
<tr>
<td>1$g_{7/2}$</td>
<td>0.40</td>
<td>0.60</td>
</tr>
<tr>
<td>2$d_{3/2}$</td>
<td>0.53</td>
<td>0.45</td>
</tr>
<tr>
<td>3$s_{1/2}$</td>
<td>0.43</td>
<td>0.32</td>
</tr>
<tr>
<td>1$h_{11/2}$</td>
<td>0.58</td>
<td>0.49</td>
</tr>
<tr>
<td>2$f_{7/2}$</td>
<td>0.31</td>
<td>0.35</td>
</tr>
<tr>
<td>3$p_{3/2}$</td>
<td>0.58</td>
<td>0.54</td>
</tr>
</tbody>
</table>

E.L., PRC 85, 021303(R) (2012)

**Spin-orbit splittings in $^{36}$S vs a bubble nucleus $^{34}$Si; neutron states:**

Exp: Burgunder et al., PRL 112, 042502 (2014)
Th: K. Karakatsanis et al., PRC 95, 034901 (2017)
Single-particle states in $^{100}$Sn: pion dynamics included

Truncation scheme: phonons below 20 MeV
Phonon basis: $\Delta T=0$ phonons: 2+, 3-, 4+, 5-, 6+
$\Delta T=1$ phonons: 0, ±1±, 2±, 3±, 4±, 5±, 6±

Backward-going terms neglected for isospin-flip phonons

Exact equations of motion for binary interactions: two-body problem

\[ R_{12,1',2'}^{(ph)}(t-t') = -i \langle T(\psi_1^\dagger \psi_2)(t)(\psi_2^\dagger \psi_1')(t') \rangle \]

\[ R(\omega) = R^{(0)}(\omega) + R^{(0)}(\omega)W(\omega)R(\omega) \]

\[ R(12',21') = \tilde{R}(12',21') - G(1,1')G(2',2) \]

\[ W(t-t') = W^{(0)}(t-t') + W^{(r)}(t-t') \]

\[ W = F^{irr} \]

**Instantaneous term (“bosonic” mean field):**

\[ \rho_{\alpha\beta,\alpha'\beta'} = \langle \psi_{\alpha'}^{\dagger} \psi_{\beta'}^{\dagger} \psi_{\beta} \psi_{\alpha} \rangle \]

\[ \begin{align*}
F^{(0)}(t-t') &= -\frac{i}{2} \langle \psi_{1'}^{\dagger} \psi_2 \psi_1^\dagger \psi_2' \rangle \\
&+ \frac{i}{2} \langle \psi_{1'}^{\dagger} \psi_2 \psi_1^\dagger \psi_2' \rangle \\
&+ \frac{i}{2} \langle \psi_{1'}^{\dagger} \psi_2 \psi_1^\dagger \psi_2' \rangle
\end{align*} \]

**T-dependent (retarded & advanced) term**

\[ F_{12,1';2'}^{(r)}(t-t') = F_{12,1';2'}^{(r;11)}(t-t') + F_{12,1';2'}^{(r;12)}(t-t') + F_{12,1';2'}^{(r;21)}(t-t') + F_{12,1';2'}^{(r;22)}(t-t') \]

In the absence of 3-body and 4-body forces, \( G^{(4)} \) (approximately) factorizes into \( R^{(ph)} \), \( R^{(hp)} \), \( R^{(pp)} \), and \( R^{(hh)} \) (see below). This leads to:

\[ \hat{R}(\omega) = \hat{R}^{(0)}(\omega) + \hat{R}^{(0)}(\omega)W[\hat{R}(\omega)]\hat{R}(\omega) \]

with

\[ \hat{R} = \{ \hat{R}^{(ph)}, \hat{R}^{(hp)}, \hat{R}^{(pp)}, \hat{R}^{(hh)} \} \]
Nuclear response with QVC in time blocking approximation. Higher orders: toward a complete theory

Bethe-Salpeter Equation:

\[ R(\omega) = A(\omega) + A(\omega) [V + W(\omega)] R(\omega) \]

Time blocking approximation (TBA):
V.I. Tselyaev, Yad. Fiz. 50, 1252 (1989)

Generalized TBA for correlated propagator:
2-phonon: V. Tselyaev, PRC 75, 024306 (2007)
Excitation modes in medium-mass and heavy nuclei within Relativistic Quasiparticle Time-Blocking Approximation (RQTBA)

**Giant dipole resonance (GDR) in stable nuclei:**

- \(^{120}\text{Sn}\), \(^{116}\text{Sn}\), \(^{90}\text{Zr}\), \(^{88}\text{Sr}\)

**Pygmy dipole resonances**

- \(^{208}\text{Pb}\)

**GDR in neutron-rich \(\text{Sn}\):**

- \(^{132}\text{Sn}\), \(^{130}\text{Sn}\)

**Gamow-Teller (with IV spin-monopole) resonance:**

- \(E.L., B.A.\ Brown, D.-L.\ Fang, T.\ Marketin, R.G.T.\ Zegers, PLB 730, 307 (2014)\)

**Spin-Dipole resonance:**

Gamow-Teller resonance in open-shell nuclei: spectra of odd-odd nuclei. Superfluid pairing and phonon coupling (pn-RQTBA)

Overall strength

\[ S_{GT}(E^*) \text{ (MeV)}^{-1} \]

Low-energy part

\[ S_{GT}(E^*) \text{ (MeV)}^{-1} \]

Beta decay half-lives

\[ T_{1/2} \text{ (s)} \]

No artificial proton-neutron pairing

Gamow-Teller resonance in open-shell nuclei: superfluid pairing and phonon coupling (pn-RQTBA)

Neutron-rich Sn isotopes

Beta decay half-lives

GTR calculated in pn-RQTBA gives reasonable predictions on the strength distributions and beta decay half-lives in neutron-rich nuclei.

In $\beta^+$ branch the importance of ground state correlations associated with particle-vibration coupling is revealed: work in progress.

C. Robin and E. Litvinova, arXiv:1709.0360
Exotic spin-isospin excitations vs RNFT calculations

Recent measurements at MSU

$^{100}$Mo ($t$, $^3$He)$^{100}$Nb

$^{28}$Si ($^{10}$Be, $^{10}$B)$^{28}$Al

Isovector monopole

Isovector dipole

Isovector spin monopole resonance

K. Miki, R.G.T. Zegers,...
E.L., ... , C. Robin et al.,

M. Scott, R.G.T. Zegers,...
E.L., ... , C. Robin et al.,
The lowest 1+ solutions in the addition channel become unstable indicating the onset of the triplet deuteron condensate. The particle-vibration coupling provides an overall attractive interaction and, thus, reinforces the condensate formation.

In the odd-odd $N=Z$ nuclei around closed shells the lowest 0+ and 1+ states are accurately described.

The pairing interaction in the proton-neutron channel is a delicate interplay of the $\rho$-meson and $\pi$ exchanges, and the exchange by core vibrations.

Response in proton-neutron particle-particle (deuteron transfer) channel: quest for deuteron condensate and pn-pairing

Response in proton-neutron particle-particle (deuteron transfer) channel: quest for deuteron condensate and pn-pairing

\[ J^\pi = 0^+ \quad J^{\pi} = 1^+ \]

- The pairing interaction in the proton-neutron channel is a delicate interplay of the $p$-meson and $\pi$ exchanges, and the exchange by core vibrations.

- In the odd-odd $N=Z$ nuclei around closed shells the lowest $0^+$ and $1^+$ states are accurately described.

- The lowest $1^+$ solutions in the addition channel become unstable indicating the onset of the triplet deuteron condensate.

- The particle-vibration coupling provides an overall attractive interaction and, thus, reinforces the condensate formation.

Summary:

- Relativistic NFT offers a powerful framework for a high-precision solution of the nuclear many-body problem.

- The self-consistent Green function formalism and the non-perturbative response theory based on QHD and including high-order correlations are available for a large class of nuclear excited states in even-odd, even-even and odd-odd nuclei.

- RNFT allows for a wide range of applications to nuclear structure, astrophysics and nuclear data.

Current and future developments:

- Dynamical like-particle and proton-neutron pairing; higher-order and complex ground-state correlations; finite temperature.

- Covariant response theory for deformed nuclear systems.

- Toward a completely “ab initio” description: realization of the approach based on the bare relativistic meson-exchange potential.

- Applications of RNFT to double beta-decay, neutron stars and EOS.
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