Towards an LHC optics model in PyHEADTAIL

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Introduction:

- Limited modelling of optics in PyHEADTAIL.

- Amplitude detuning provided from detuning coefficients in order to recreate similar tune footprints to MAD-X.

- May not be accurate when including other optics effects (i.e. linear coupling).

- Objective is to move towards a mini LHC lattice in PyHEADTAIL consisting of thin octupole kicks (for amplitude detuning), skew quadrupoles (for coupling) and dispersion (for Q'').

- Will compare tune footprints as a function of coupling for three cases
  - Detuning model in PyHEADTAIL vs full lattice in MAD-X
  - Single octupole kick in PyHEADTAIL vs single octupole kick in MAD-X
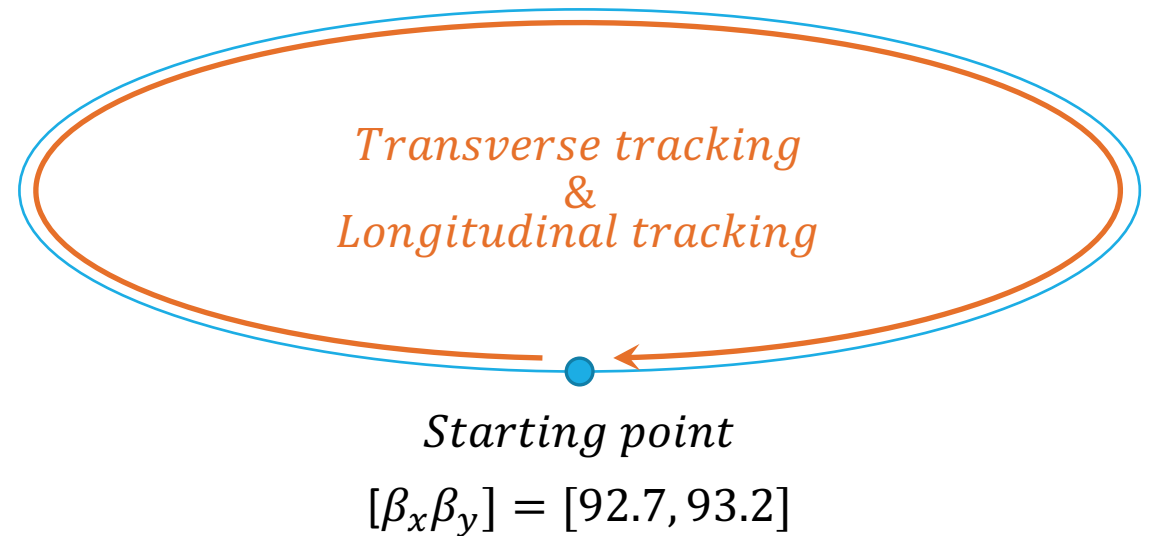 - Double octupole kick in PyHEADTAIL vs full lattice in MAD-X

Theory reminder

- Detuning coefficients in PyHEADTAIL are calculated from action variables.

- Measuring these coefficients is done by setting few particles to a specific action, and compare their tune shift to a reference particle.

- Linear coupling is introduced with a skew quadrupole.
  - Powering calibration is done with MAD-X to make sure the same coupling is introduced in both simulations.
  - Tune shift from linear coupling is compensated with the help of a 2D minimisation function.

- For details, see backup slides.

Detuning model vs Full MAD-X Lattice

- **Setup**

  - MAD-X: LHC beam 1 optics, powering all octupoles.

  - PyHT: detuning model, provided with different LOF and LOD currents (100A – 500A).

  - Any additionnal perturbing effect is kept out of the setup:
    - No dispersion
    - No wakefield
    - No damper
    - No chromaticity

Transverse tracking
&
Longitudinal tracking

Starting point

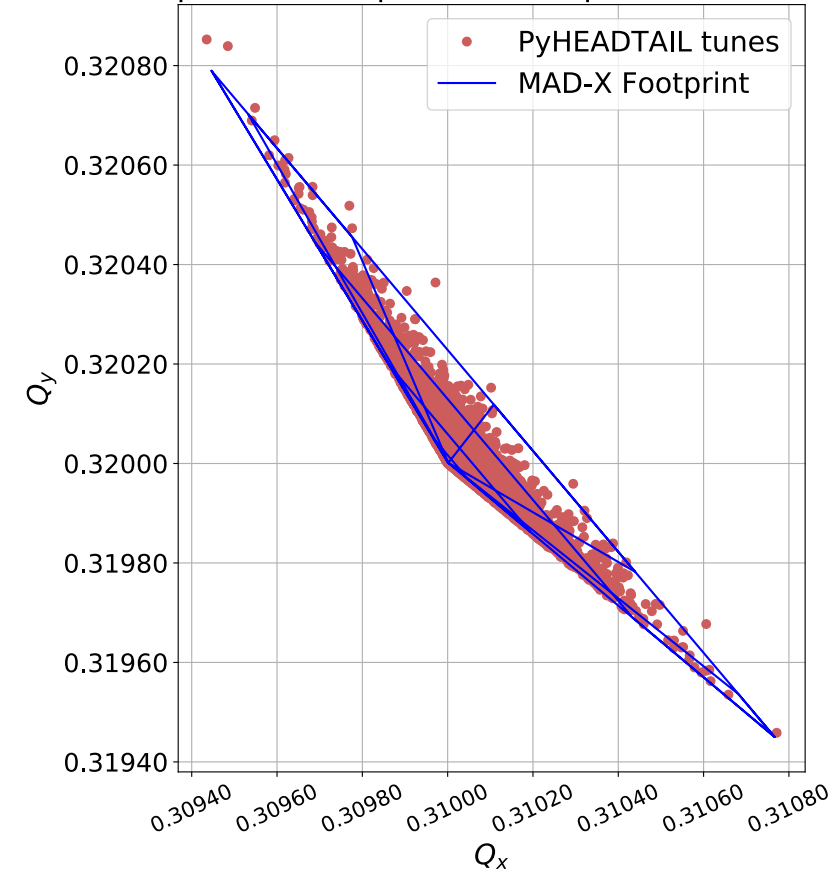$[\beta_x \beta_y] = [92.7, 93.2]$

Detuning model vs Full MAD-X Lattice

- **Comparing Footprints**

- Good overlapping → good agreement.

- Crossing of the outline → different a_{xy} ?

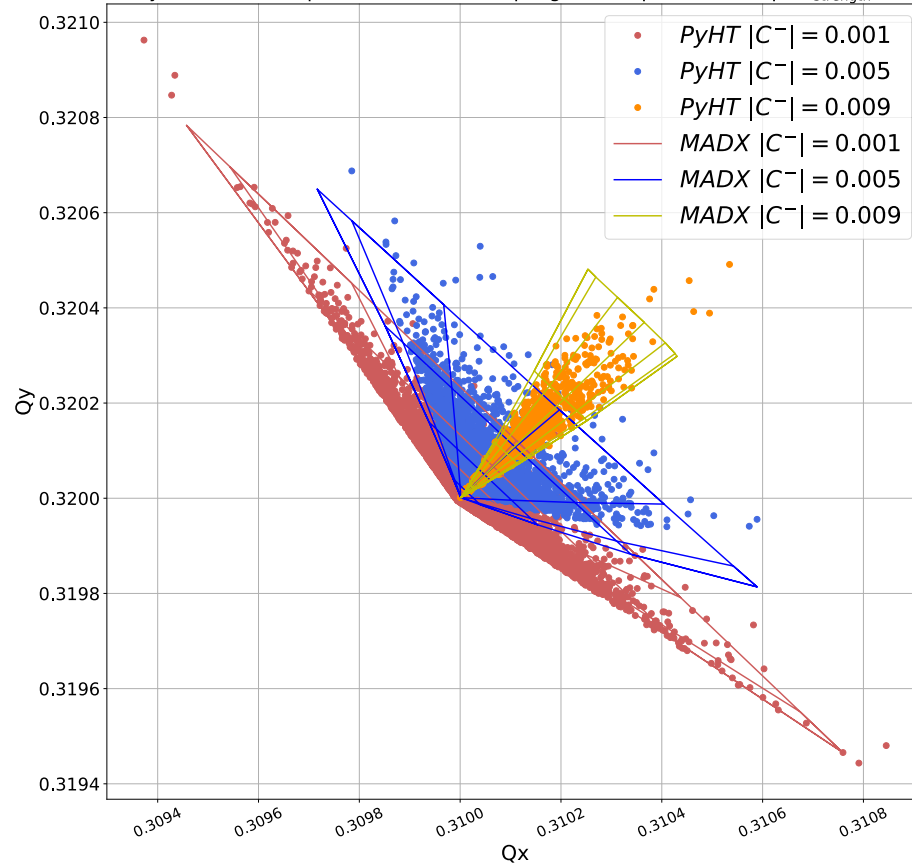|	PyHT detuning model	MAD sequence*	Relative error
a_x	269153	308202	0.126699372
a_y	280971	316423	0.112039896
a_{xy}	-191488	-223434	0.142977345

* Needs verification.

Tunes Footprint, 4σ Amplitude, Octupole current = 500Amps
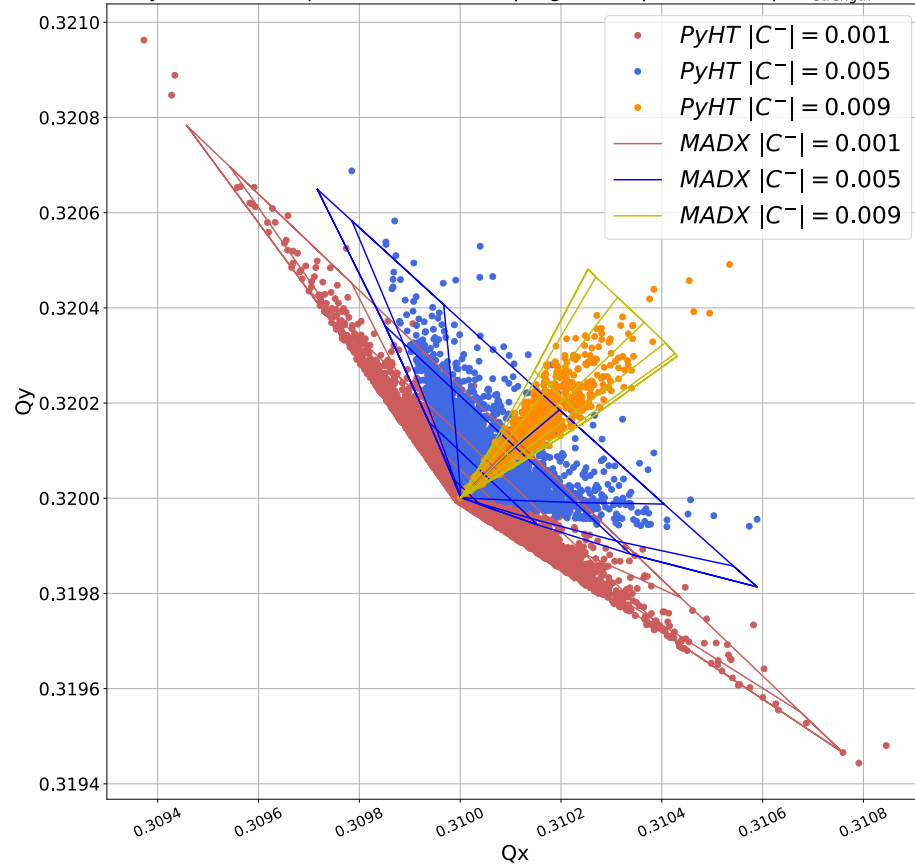
Detuning model vs Full MAD-X Lattice

$MAD-X$ and PyHEADTAIL Footprints with linear coupling, 4σ Amplitude, Octupole$_{strength}$ = $-500[m^{-3}]$


- Overall good agreement between the codes.

- A discrepancy is seen in at high coupling strengths.

  ➢ Detuning model starts collapsing against MAD.
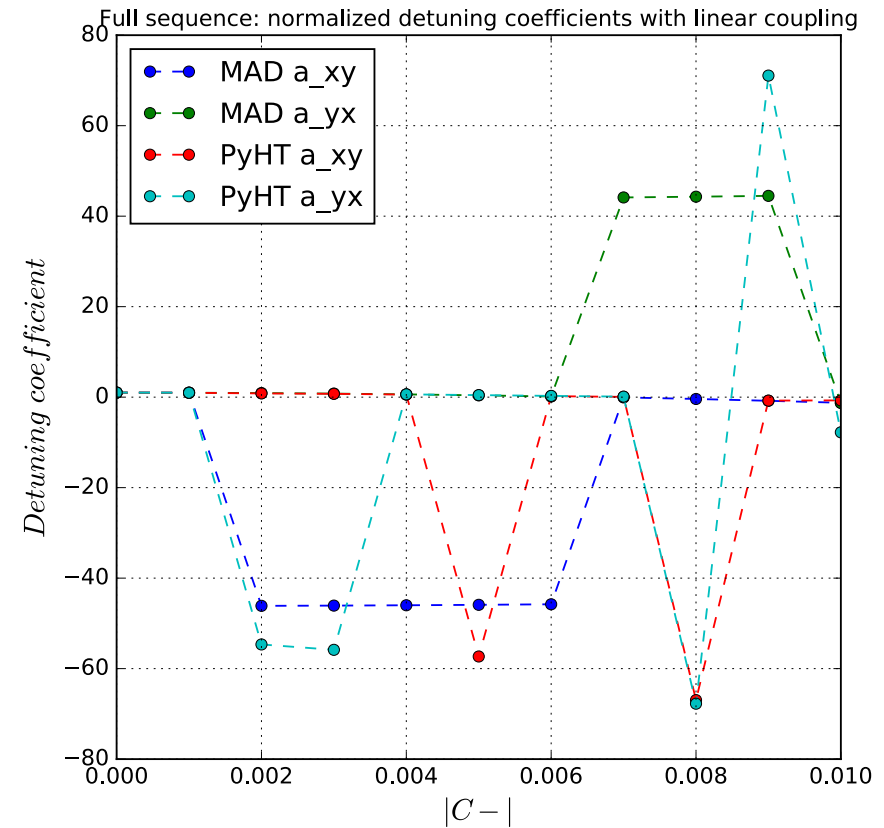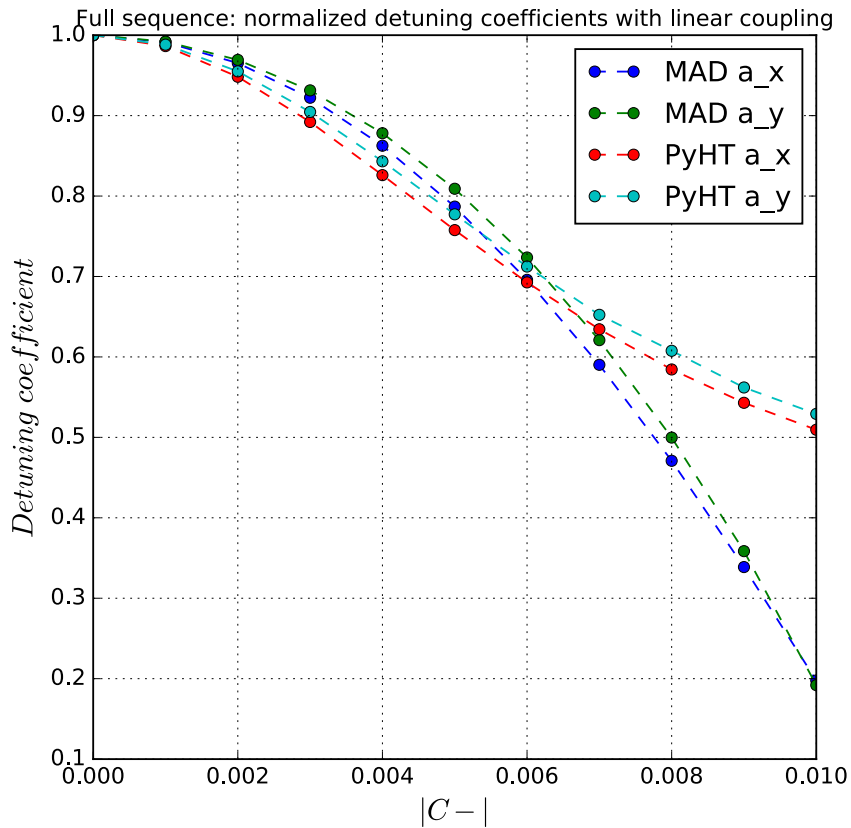
Detuning model vs Full MAD-X Lattice

MAD − X and PyHEADTAIL Footprints with linear coupling, 4σ Amplitude, Octupole$_{strength}$ = −500[m^{-3}]

PyHEADTAIL detuning model and MAD Footprints with linear coupling, 4σ Amplitude, Octupole$_{current}$ = 500A
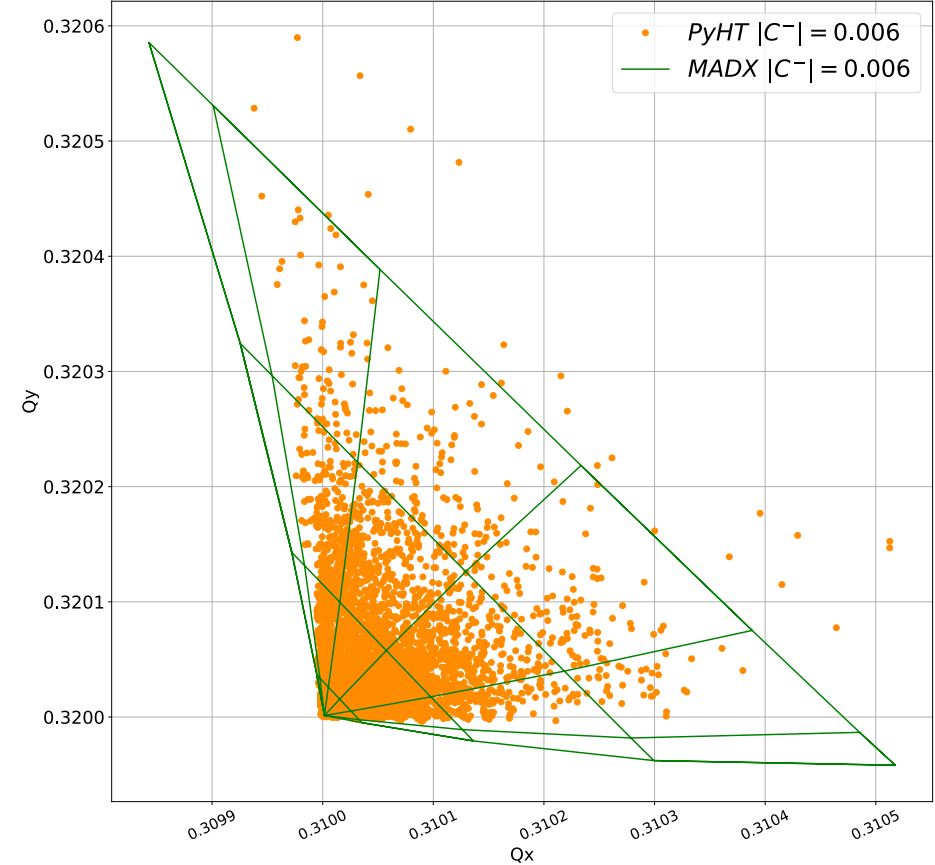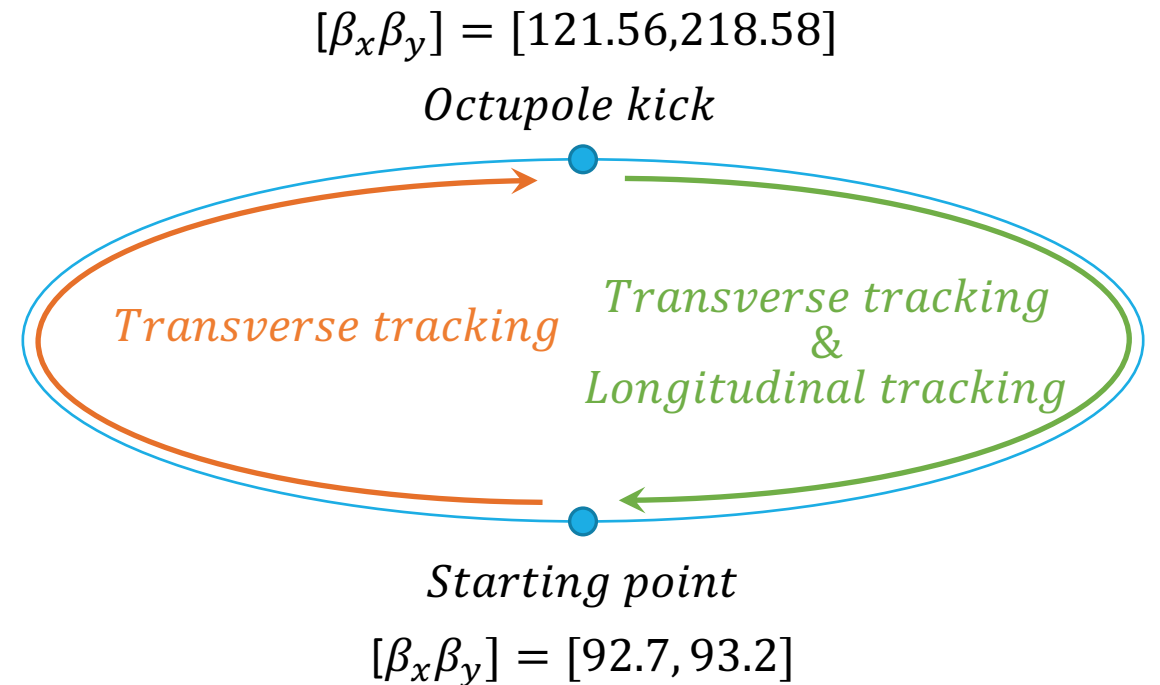
Detuning model vs Full MAD-X Lattice

Direct terms don't appear to vary that much. Main discrepancy comes from cross-terms. Plan to test in PTC.

Single Octupole Kick: PyHEADTAIL vs MAD-X

- **Setup**

  - MAD-X: LHC beam 1 optics.
  - Octupole element inserted at IP3 (in MAD-X).
  - PyHT: octupole kick element inserted.
  - IP3 position reproduced by giving the same beta values in PyHT's map.

  - Footprints generated for different powering values.
    - $Oct_{strength} = Normalised_{strength} \cdot Oct_{length}$

$[\beta_x \beta_y] = [121.56, 218.58]$

Octupole kick

Transverse tracking

Transverse tracking
&
Longitudinal tracking

Starting point

$[\beta_x \beta_y] = [92.7, 93.2]$

Single Octupole Kick: PyHEADTAIL vs MAD-X

- Good agreement in the behavior (especially angle).
  - The MAD element seems to provide a stronger spread.

Single Kick Footprint, 4σ Amplitude, Octupole strength $= -500[m^{-3}]$

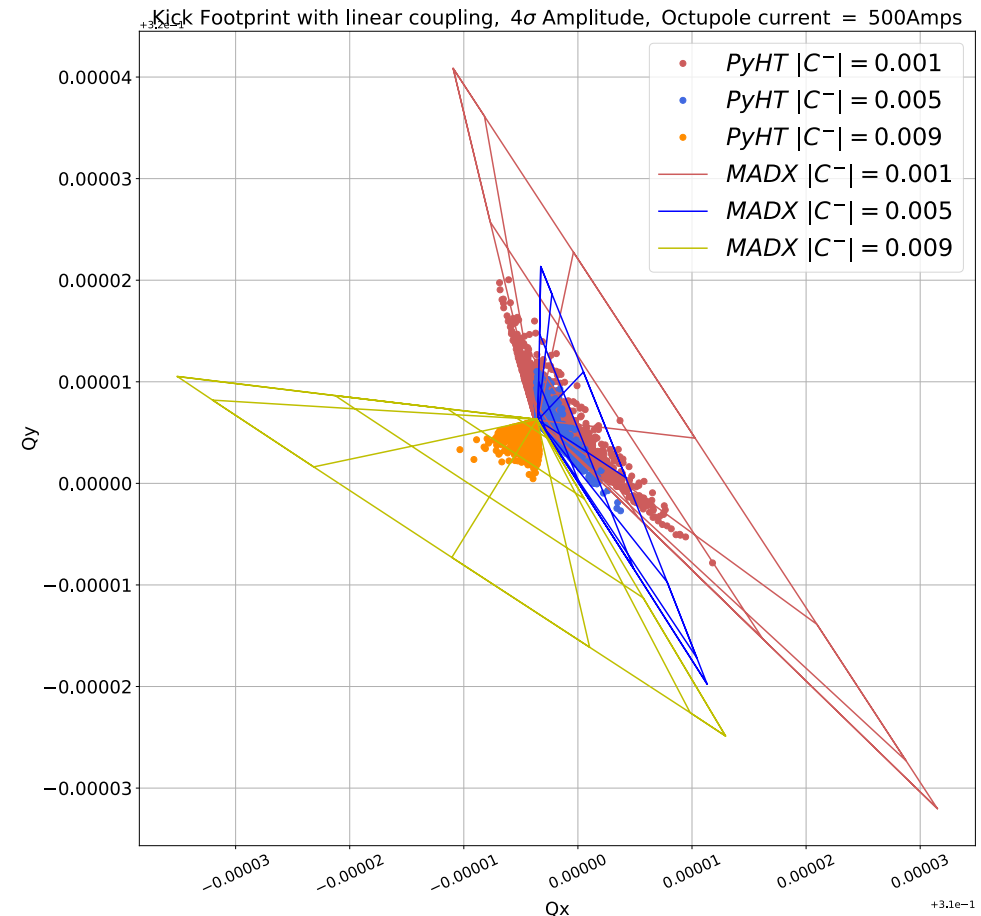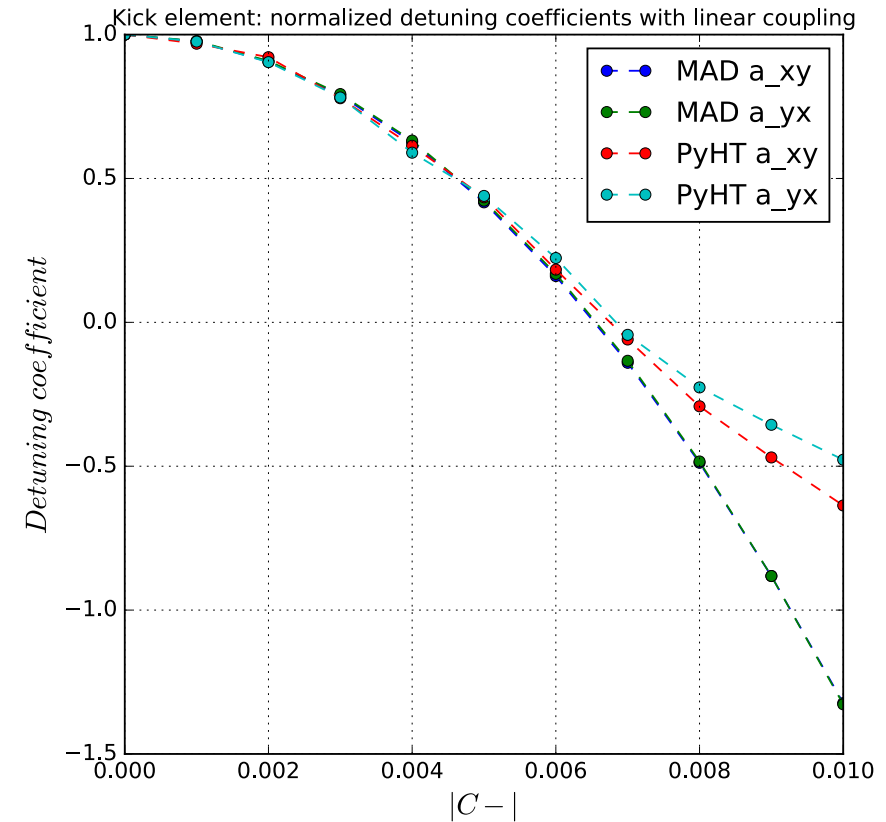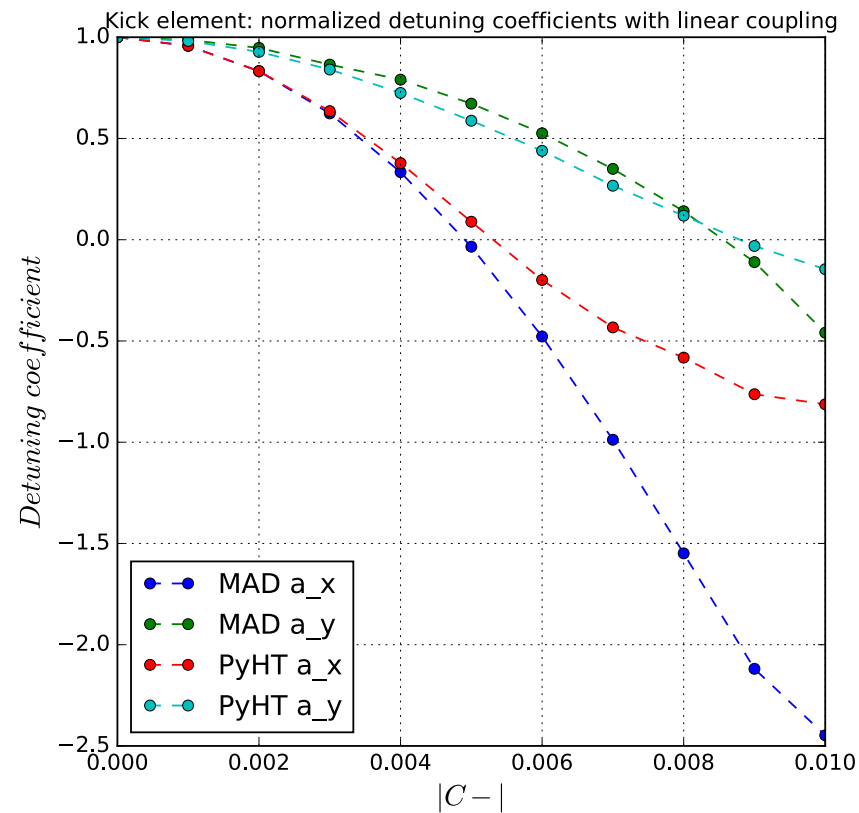|	PyHT octupole	MAD octupole*	Relative error
a_x	451070	313558	0.438553633
a_y	455970	333868	0.365719368
a_{xy}	-918317	-242863	2.7812141
a_{yx}	-897609	-241699	2.71374726
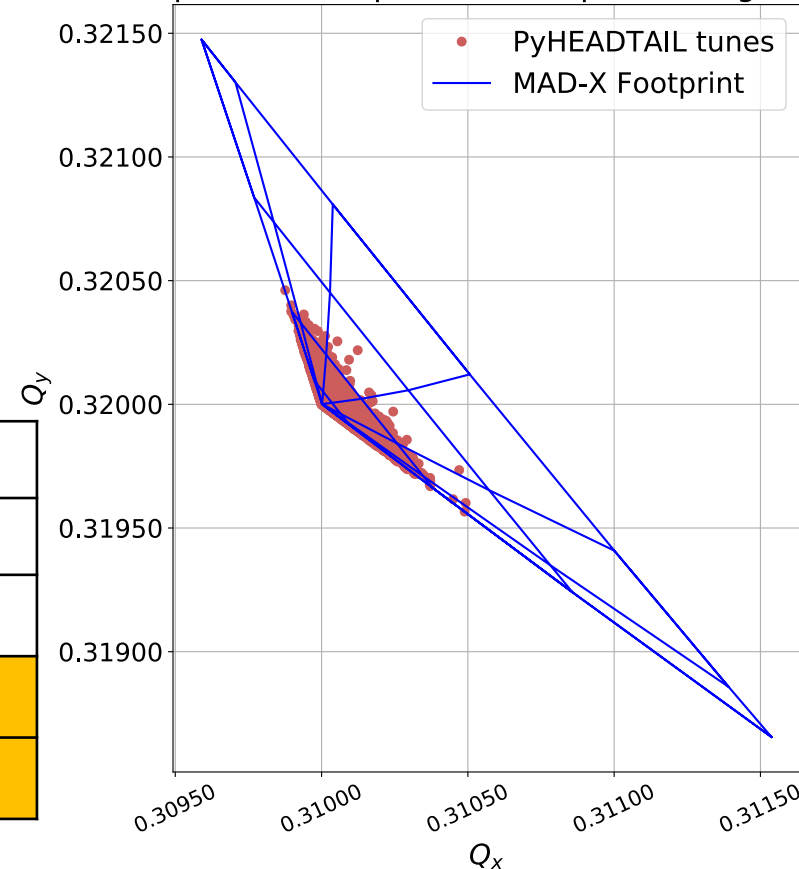
Single Octupole Kick: PyHEADTAIL vs MAD-X – linear coupling

- **Introducting coupling:**

  - Coupling ~ rotate reference frame.
  - In the beam frame ~ rotate elements.

  - Possibly due to the following: increase coupling, beams sees:
    ➢ More of opposite polarity octupole (reverting)
    ➢ Less oct. and more skew oct, then oct. again (spread decreases then increases again).
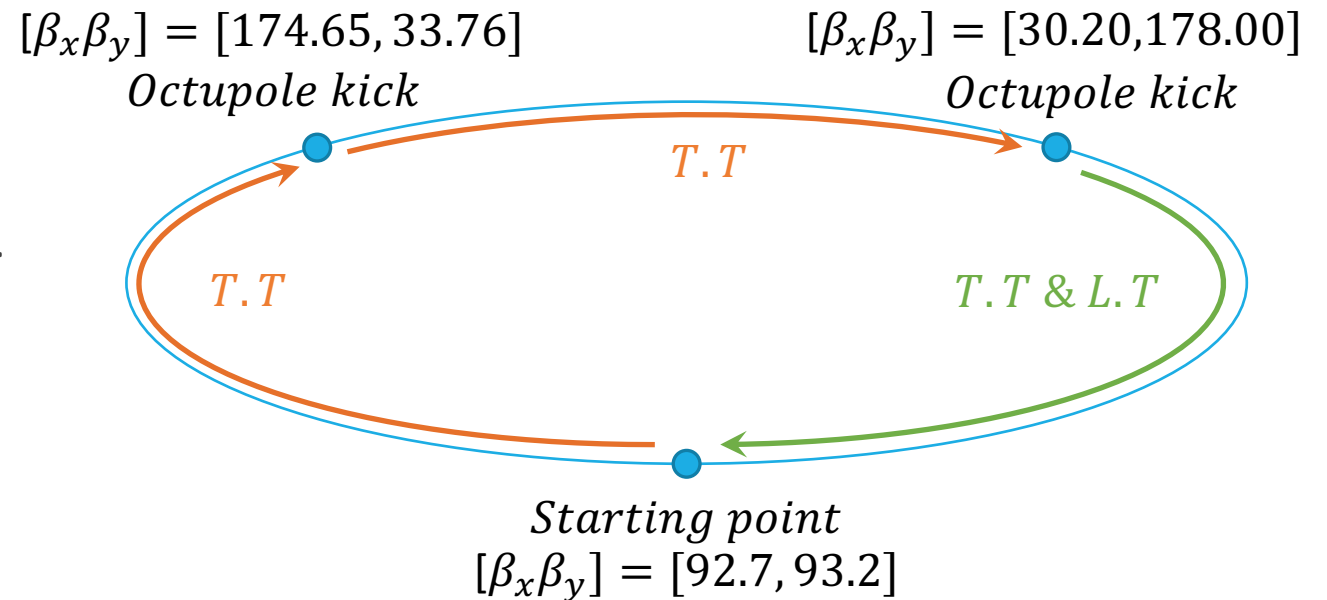

Kick Footprint with linear coupling, 4σ Amplitude, Octupole current = 500Amps

Legend:
- $PyHT\ |C^-| = 0.001$
- $PyHT\ |C^-| = 0.005$
- $PyHT\ |C^-| = 0.009$
- $MADX\ |C^-| = 0.001$
- $MADX\ |C^-| = 0.005$
- $MADX\ |C^-| = 0.009$

Single Octupole Kick: PyHEADTAIL vs MAD-X — linear coupling

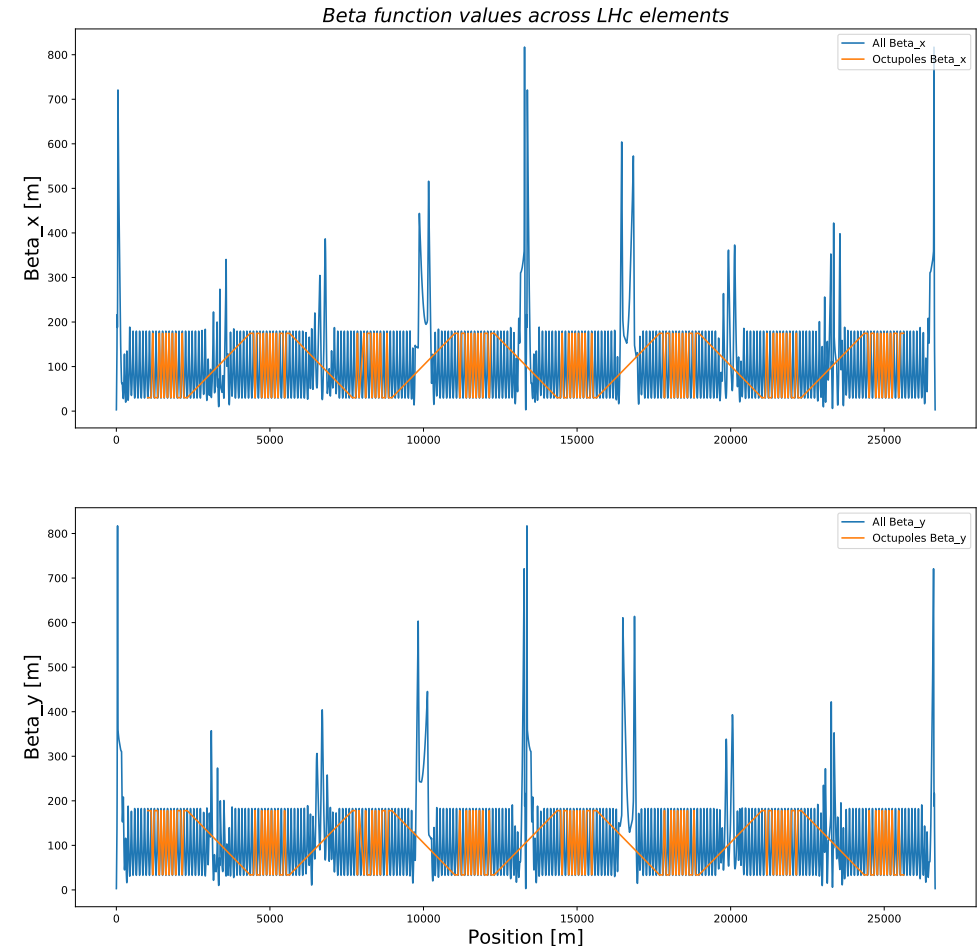Kick element: normalized detuning coefficients with linear coupling

Double Octupole Kick vs Full MAD-X Lattice

- **An LHC lattice ?**

- Can we do an LHC simplified map?

- Octupole has the advantage of allowing to include more effects (from dispertion...).
  - Ultimately, goal is to move to a short series of kicks and have it reproduce relevant properties.
  - Need to keep a simple model.
  - ➤ Start with a double kick map.

- Compare results to MAD-X with LHC lattice, all octupoles powered.
  - ➤ Not comparing element models, but the overall effect.

$[\beta_x \beta_y] = [174.65, 33.76]$
Octupole kick

$[\beta_x \beta_y] = [30.20, 178.00]$
Octupole kick

$T.T$

$T.T$

$T.T \,\&\, L.T$

Starting point
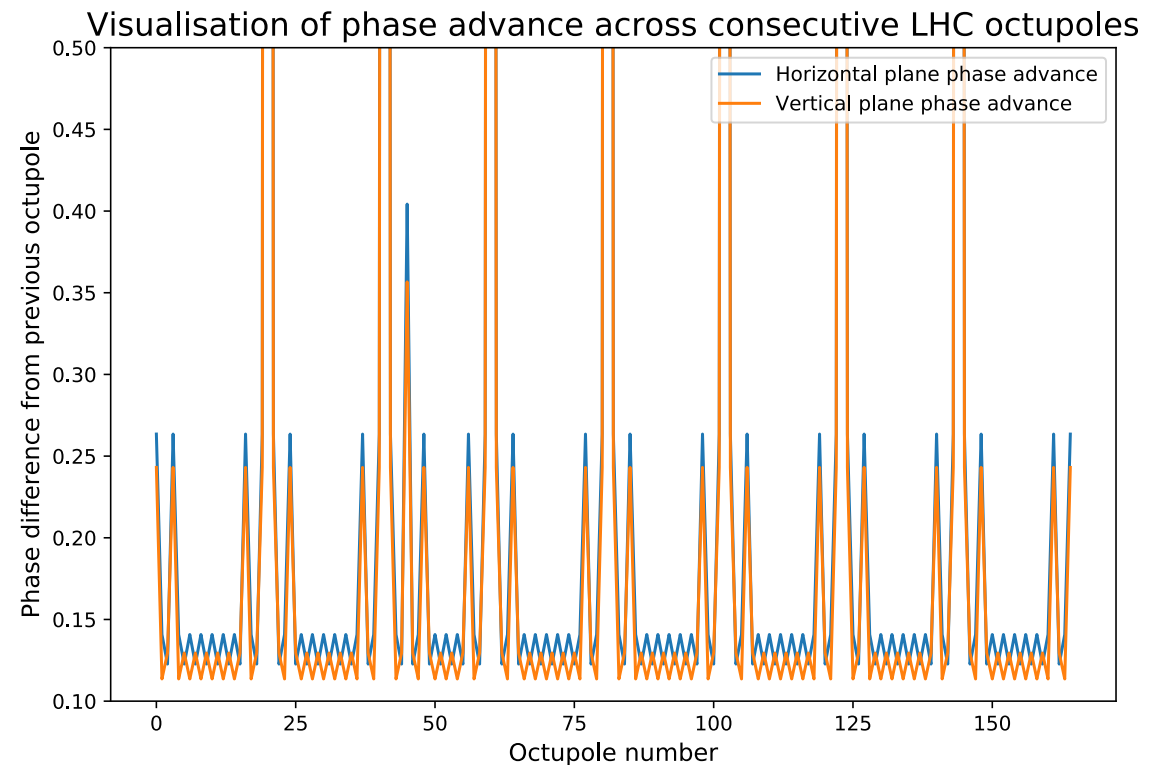$[\beta_x \beta_y] = [92.7, 93.2]$

Double Octupole Kick vs Full MAD-X Lattice

- Two octupole families: LOF & LOD.
  - ➤ One kick to represent each family.

- Octupole betas are set to be representative of these families:
  - One octupole is at high β_x and low β_y.
  - The other is at low β_x and high β_y.
  - Powering scaled to the number of octs in LHC.

Beta function values across LHc elements
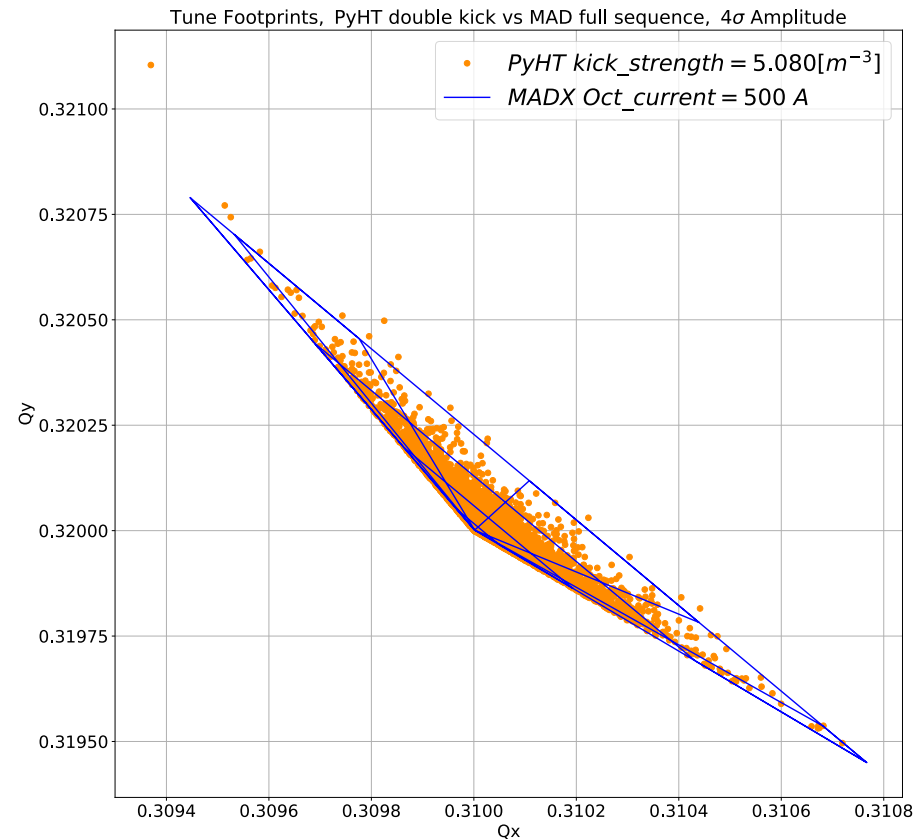
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Double Octupole Kick vs Full MAD-X Lattice

- Two octupole families: LOF & LOD.
  - ➤ One kick to represent each family.

- Octupole betas are set to be representative of these families:
  - One octupole is at high β_x and low β_y.
  - The other is at low β_x and high β_y.
  - Powering scaled to the number of octs in LHC.

- Phase advance is set between the two octupoles according to an average value:
  - ➤ $\begin{cases} \mu_{x,oct2} = \mu_{x,oct1} + avg(phase\_advance\_x) \\ \mu_{y,oct2} = \mu_{y,oct1} + avg(phase\_advance\_y) \end{cases}$

Visualisation of phase advance across consecutive LHC octupoles

Double Octupole Kick vs Full MAD-X Lattice

Tune Footprints, PyHT double kick vs MAD full sequence, 4σ Amplitude

Legend:
- PyHT $kick\_strength = 5.080\,[m^{-3}]$
- MADX $Oct\_current = 500\,A$

- Overall good agreement between the codes.

- The two kicks model gives a reasonable tune spread.

- Detuning coefficients still need to be computed.

# Double Octupole Kick vs Full MAD-X Lattice – linear coupling

Double kick Footprint with linear coupling, 4σ Amplitude, Octupole current = 500Amps
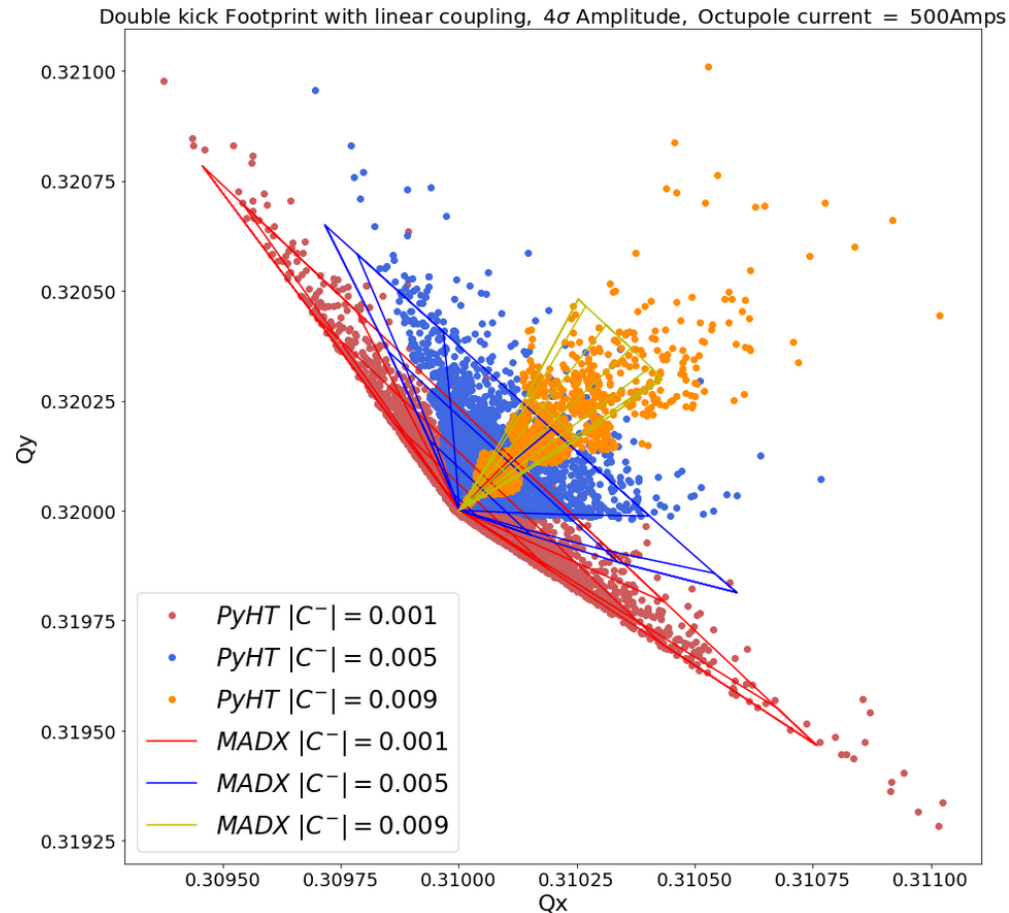
- Double kick model seems to give a good spread.

- Discrepancy region is still present at high coupling. To be figured out.

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Double Octupole Kick vs Full MAD-X Lattice – linear coupling

PyHEADTAIL detuning model and MAD Footprints with linear coupling, 4σ Amplitude, $\text{Octupole}_{current} = 500A$
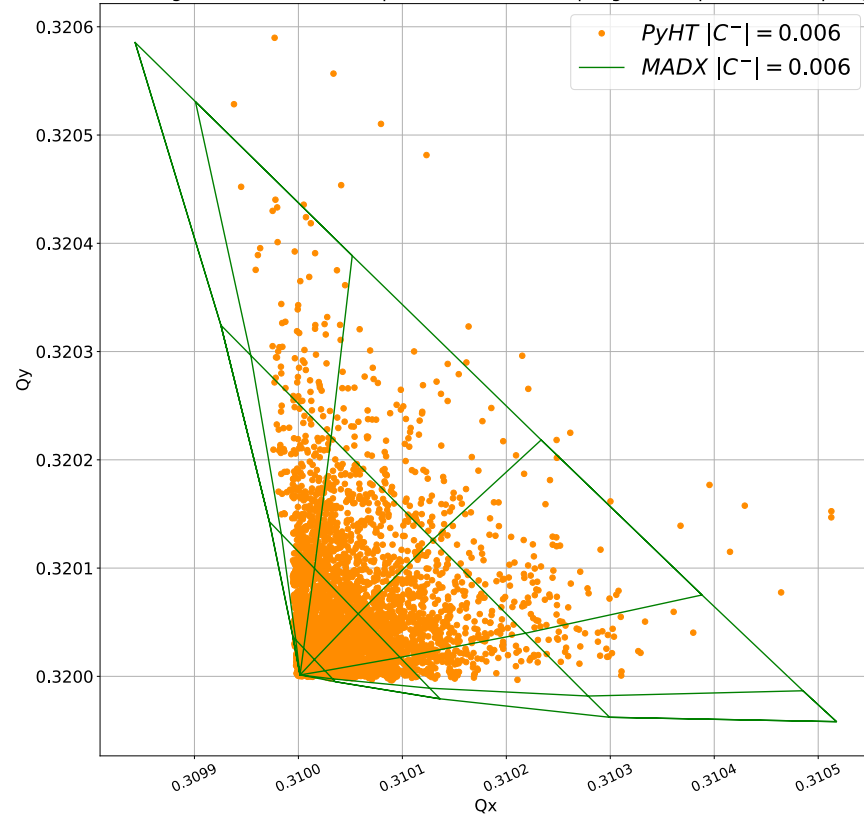
PyHT vs MAD double kick footprints with linear coupling, 4σ Amplitude, $I_{octupole} = 500A$
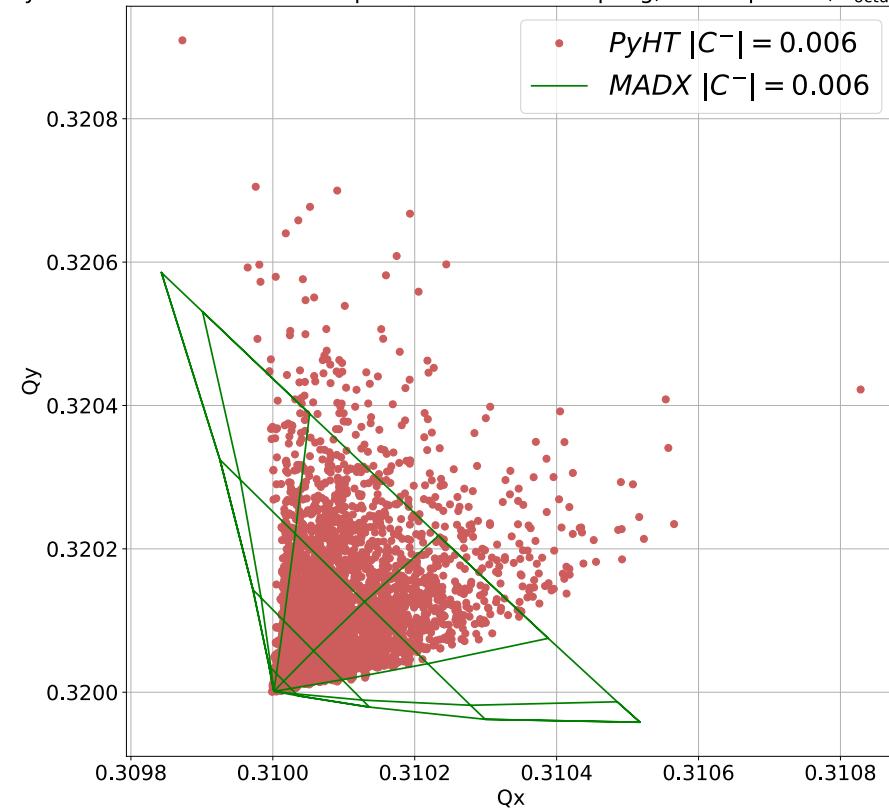
Possible future work

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Next Steps

- **Measuring chromaticity**
  - Q' and Q''. Some difficulties in PyHT.

- **Including dispersion for Q''**
  - Calibrate dispersion against Q'' at octupoles in PyHT with MADX.

- Increase number of octupoles to see if it improves agreement. How many are needed? What if 84 kicks were used, exactly replicating LHC lattice?

Conclusion

- Octupole kick element does doesn't provide as much spread as its MAD counterpart.

- Two octupole kicks compensate each other's defaults.
  - ➤ Possibility to use (and create a complex) LHC representative map ?

- Linear coupling can be correctly implemented with all setups that have been simulated.
  - ➤ Allows for study on an important mechanism.
  - ➤ BUT: discrepancy from the detuning model is found with the two octupole kicks. To be figured out.

- For now: double kick map ~ same as detuning model, but:
  - ➤ Brings possibility for optics considerations ☺

Backup

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PyHEADTAIL's detuning model

- Detuning is applied at the end of each turn.

- Transverse amplitude of a particle dictates the detuning:

$$\begin{cases} \Delta Q_x = a_x J_x + a_{xy} J_y \\ \Delta Q_y = a_{xy} J_x + a_y J_y \end{cases}$$

- J_x, J_y are the action variables.

- a_x, a_y and a_{xy} are the detuning coefficients:

$$a_x = \frac{7000}{p_0 [GeV/c]} \left(267065 \frac{I_{oct}^F [A]}{550} - 7856 \frac{I_{oct}^D [A]}{550} \right),$$

$$a_y = \frac{7000}{p_0 [GeV/c]} \left(9789 \frac{I_{oct}^F [A]}{550} - 277203 \frac{I_{oct}^D [A]}{550} \right),$$

$$a_{xy} = \frac{7000}{p_0 [GeV/c]} \left(-102261 \frac{I_{oct}^F [A]}{550} + 93331 \frac{I_{oct}^D [A]}{550} \right)$$

Measuring detuning coefficients

Simulation setup :

- What about quantification of the agreement ?

- From the equations shown previously,

- For an action variable of 1 and only in one plane, one gets: $a_u = \Delta Q_u$, $u = (x, y)$.

- Set 3 particles:
  - Particle 0: no offsets.
  - Particle 1: $J_x = 10^{-9}$, $J_y = 0$.
  - Particle 2: $J_x = 0$, $J_y = 10^{-9}$.

- Many possible phase space locations possible, easier to make it for $x' = y' = 0$.
  - Actions set to 10^{-9} to get a reasonable offset.

$$\begin{cases} \Delta Q_x = a_x J_x + a_{xy} J_y \\ \Delta Q_y = a_{xy} J_x + a_y J_y \end{cases}$$

$$\Delta Q_{u,i} = Q_{u,i} - Q_{u,0}$$

$$\begin{cases} J_x = \frac{1}{2}(\frac{(1 + \alpha_x x^2)}{\beta_x x^2} + 2\alpha_x x x' + \beta_x x'^2) \\ J_y = \frac{1}{2}(\frac{(1 + \alpha_y y^2)}{\beta_y y^2} + 2\alpha_y y y' + \beta_y y'^2) \end{cases}$$
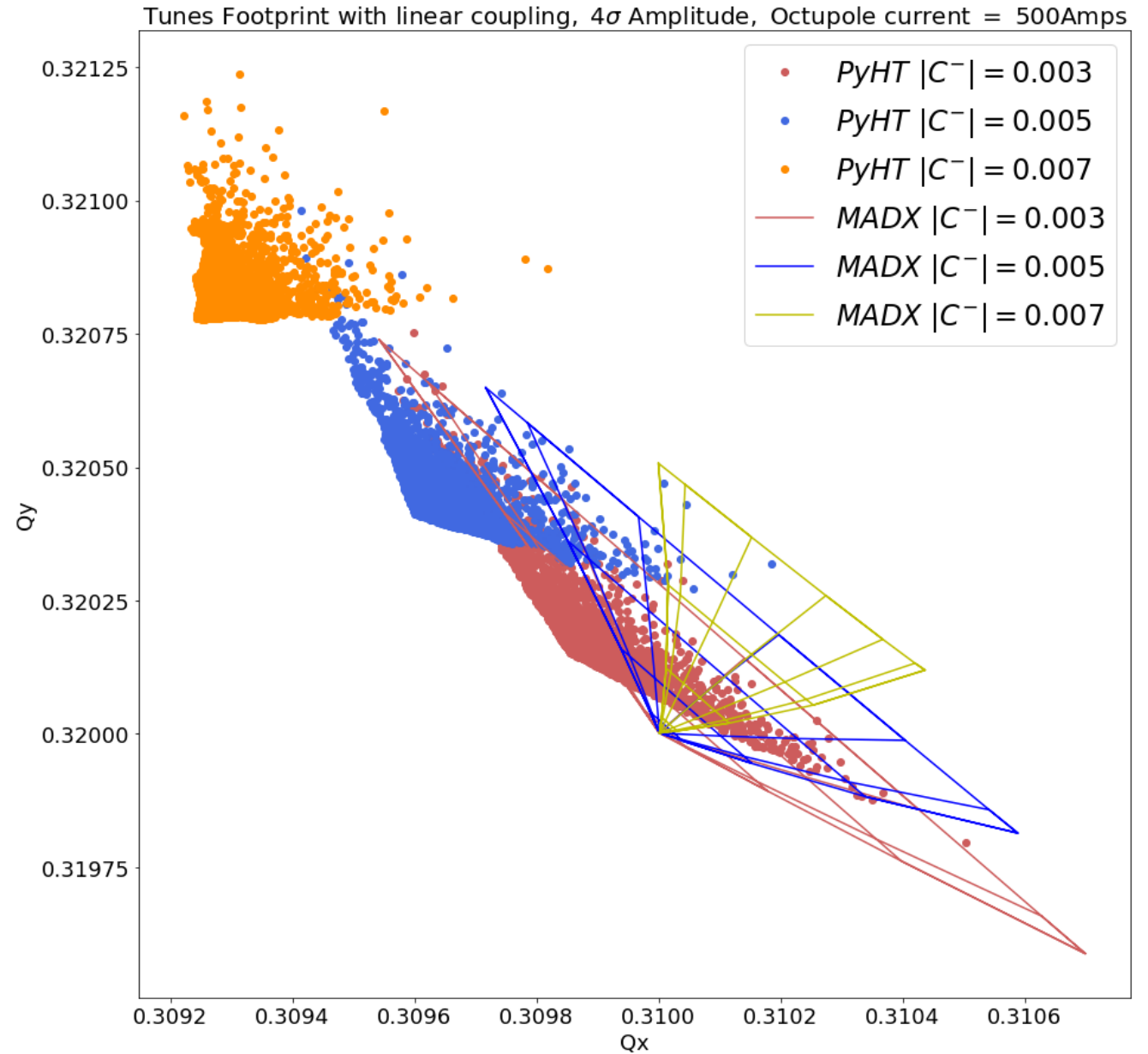
Linear coupling in PyHEADTAIL

- **Introducting coupling:**

  - Linear coupling is introduced with a skew quadrupole and calibrated against MAD-X.

  - Equivalent values are found that provide the same $|C^-|$ (coupling strength) in each case.

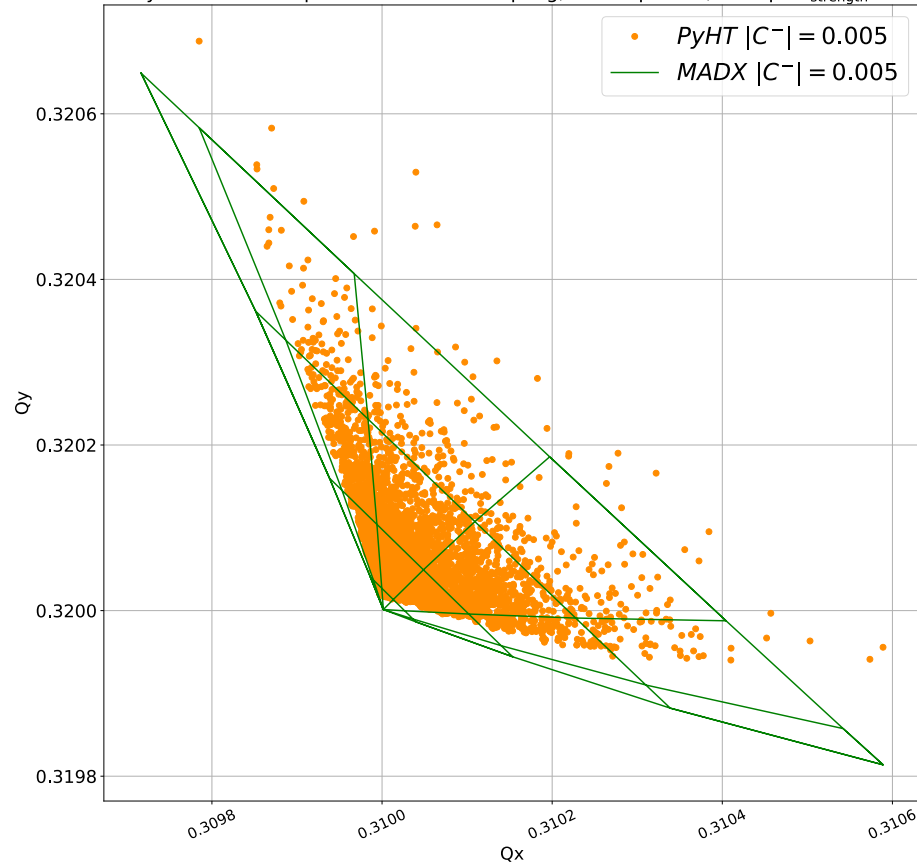 - A 2D minimization function is needed to find correct initial tunes.

$Q_u = 0.31, Q_v = 0.32$

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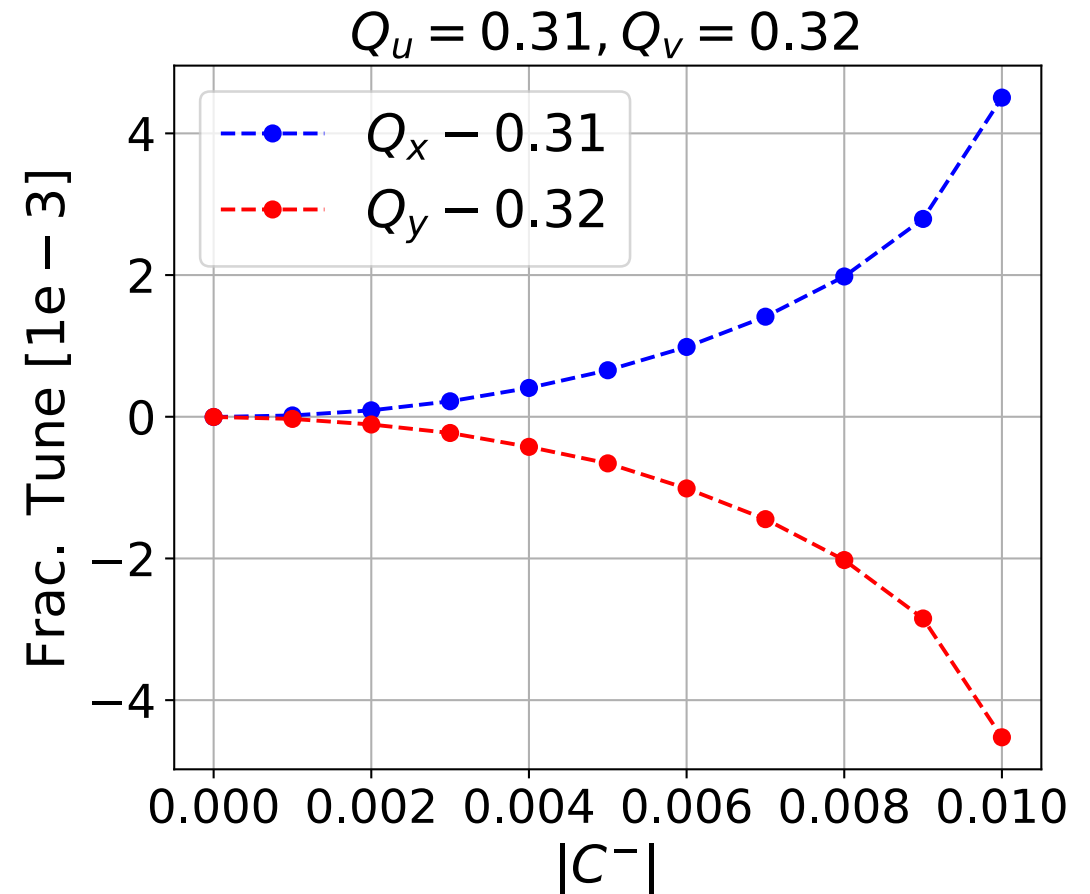
Coupling optimizer

Effect on footprints without optimizer

Tunes Footprint with linear coupling, 4σ Amplitude, Octupole current = 500Amps

Legend:
- PyHT $|C^-| = 0.003$
- PyHT $|C^-| = 0.005$
- PyHT $|C^-| = 0.007$
- MADX $|C^-| = 0.003$
- MADX $|C^-| = 0.005$
- MADX $|C^-| = 0.007$

Qy (vertical axis), Qx (horizontal axis)

Linear coupling with PyHEADTAIL's detuning model

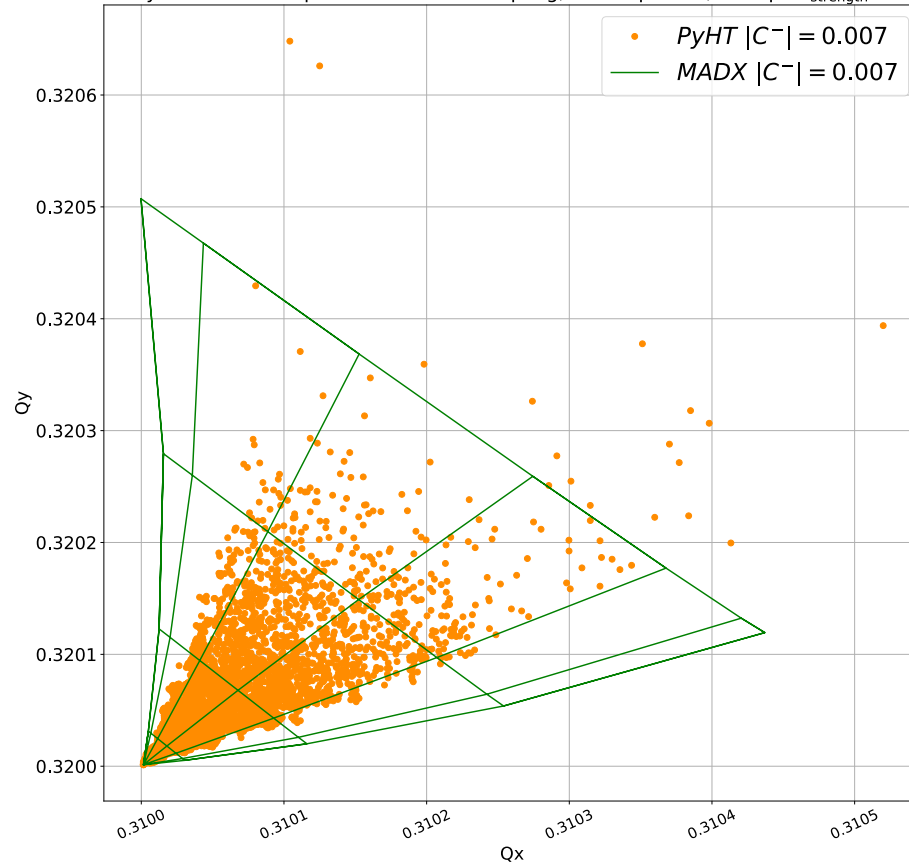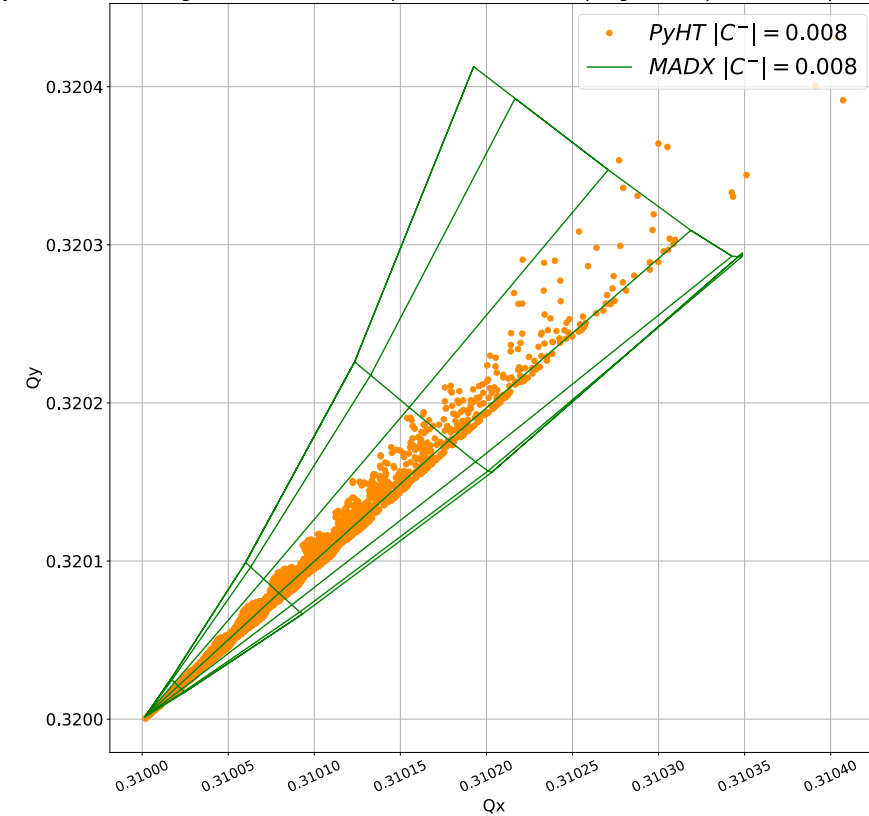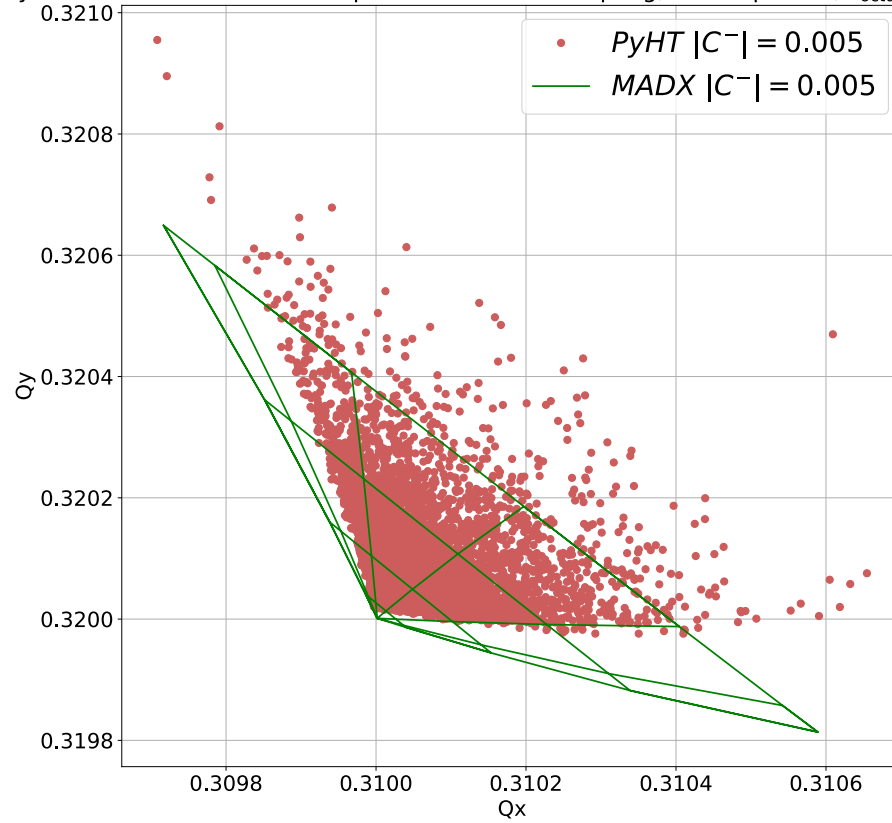MAD − X and PyHEADTAIL Footprints with linear coupling, 4σ Amplitude, Octupole$_{strength}$ = −500[m^{-3}]

PyHEADTAIL detuning model and MAD Footprints with linear coupling, 4σ Amplitude, Octupole$_{current}$ = 500A

Linear coupling with PyHEADTAIL's detuning model

MAD $-$ X and PyHEADTAIL Footprints with linear coupling, 4σ Amplitude, Octupole$_{strength}$ $=$ $-500[m^{-3}]$

PyHEADTAIL detuning model and MAD Footprints with linear coupling, 4σ Amplitude, Octupole$_{current}$ $=$ $500A$
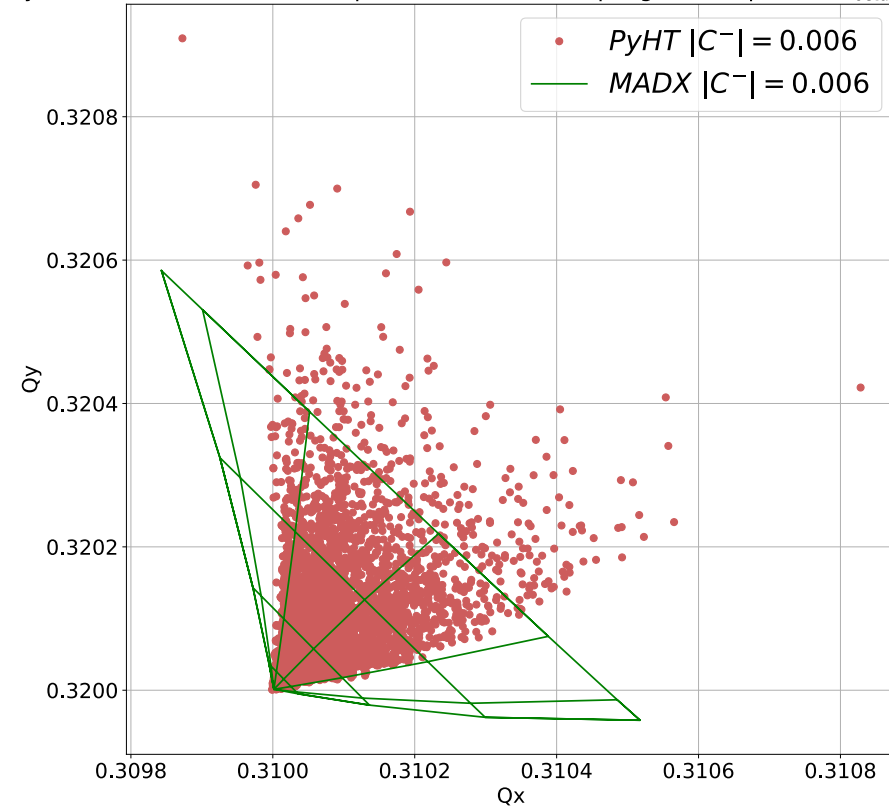
Linear coupling with a double kick

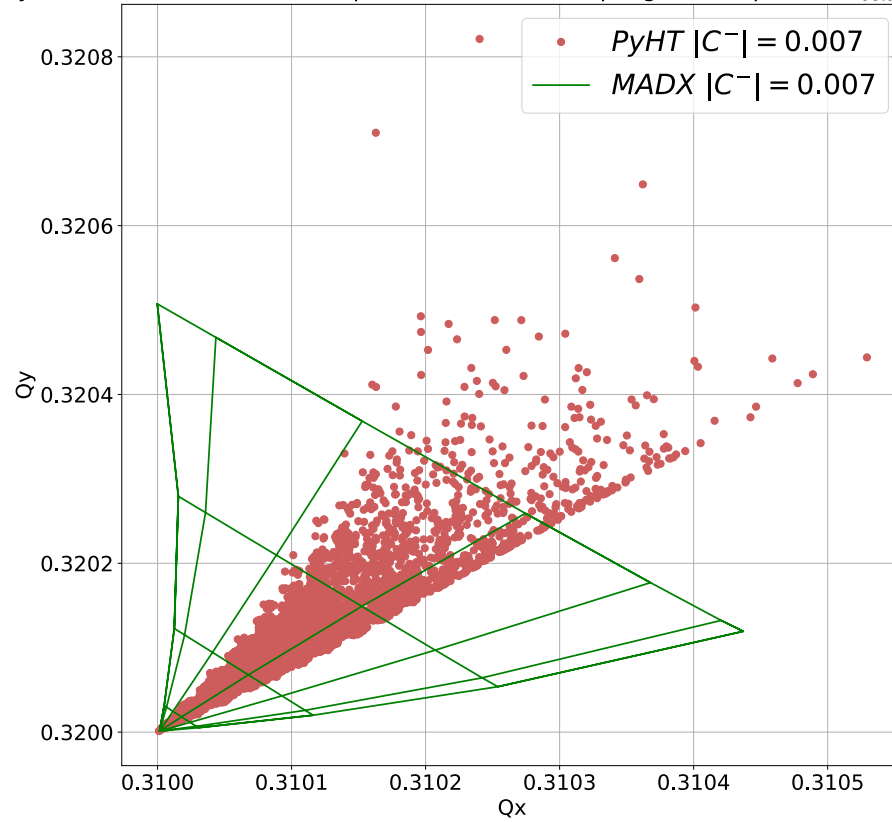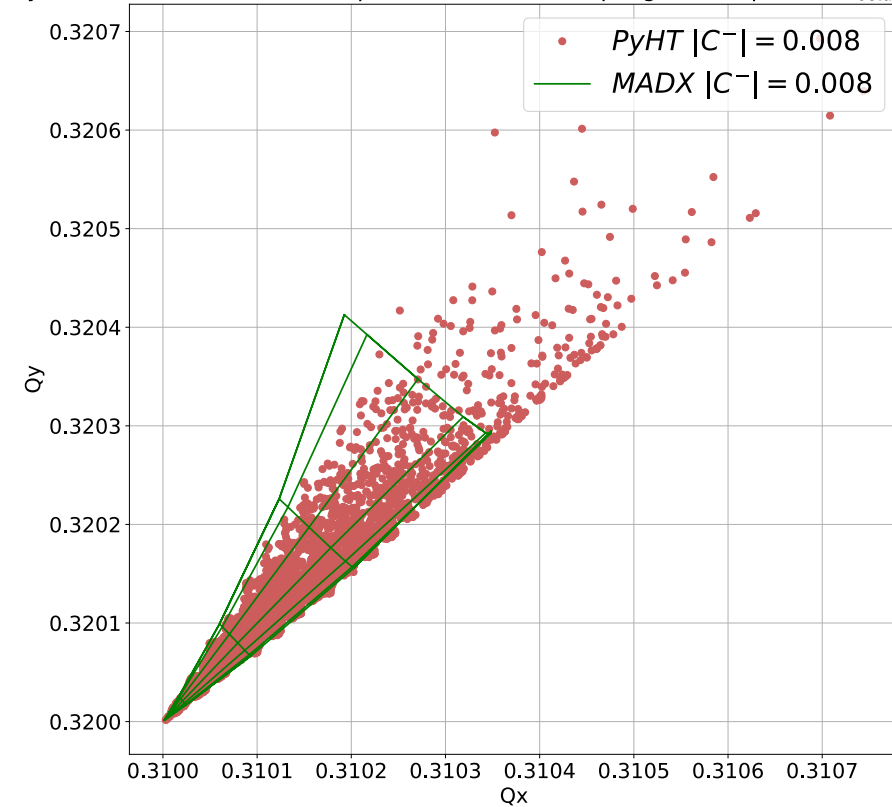PyHT vs MAD double kick footprints with linear coupling, 4σ Amplitude, $I_{octupole} = 500A$

- *PyHT* $|C^-| = 0.005$
- *MADX* $|C^-| = 0.005$

PyHT vs MAD double kick footprints with linear coupling, 4σ Amplitude, $I_{octupole} = 500A$

- *PyHT* $|C^-| = 0.006$
- *MADX* $|C^-| = 0.006$

Linear coupling with a double kick

PyHT vs MAD double kick footprints with linear coupling, 4σ Amplitude, $I_{octupole} = 500A$

Very good agreement is back for |C-| = 0,009, strange beharior for |C-| = 0,01.

Very good agreement is back for |C-| = 0,009, strange behavior for |C-| = 0,01.