ELECTROMAGNETIC SHOWERS

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WAPP 2014
Ooty 21th december 2014

Lecture 1

1. Fundamental processes that determine an electromagnetic shower

2. Mathematical methods to study the average development of a shower

An electromagnetic shower
is formed by 3 particles

$$e^{-} \qquad e^{+} \qquad \gamma$$

Published: march 15, 1933

Great Discovery in Cosmic Ray

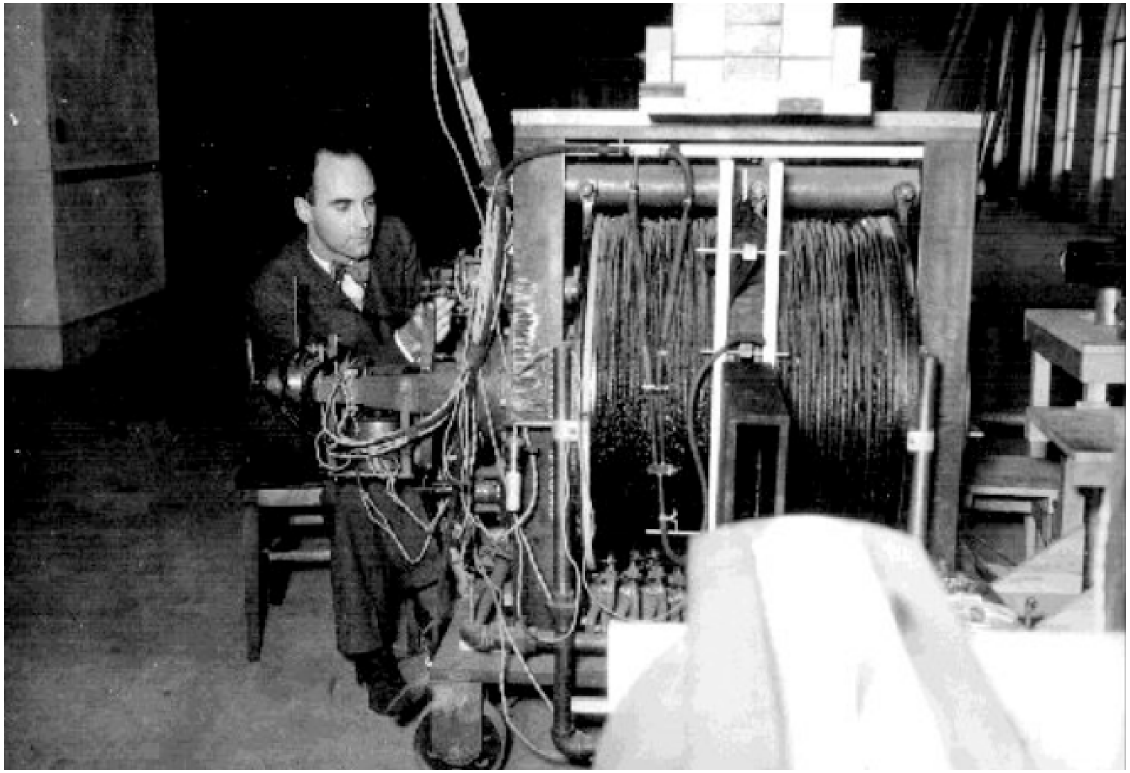
The Positive Electron

CARL D. ANDERSON, *California Institute of Technology, Pasadena, California*

(Received February 28, 1933)

Out of a group of 1300 photographs of cosmic-ray tracks in a vertical Wilson chamber 15 tracks were of positive particles which could not have a mass as great as that of the proton. From an examination of the energy-loss and ionization produced it is concluded that the charge is less than twice, and is probably exactly equal to, that of the proton. If these particles carry unit positive charge the curvatures and ionizations produced require the mass to be less than twenty times the electron mass. These particles will be called positrons. Because they occur in groups associated with other tracks it is concluded that they must be secondary particles ejected from atomic nuclei.

Editor



Carl Anderson
“Wilson chamber”

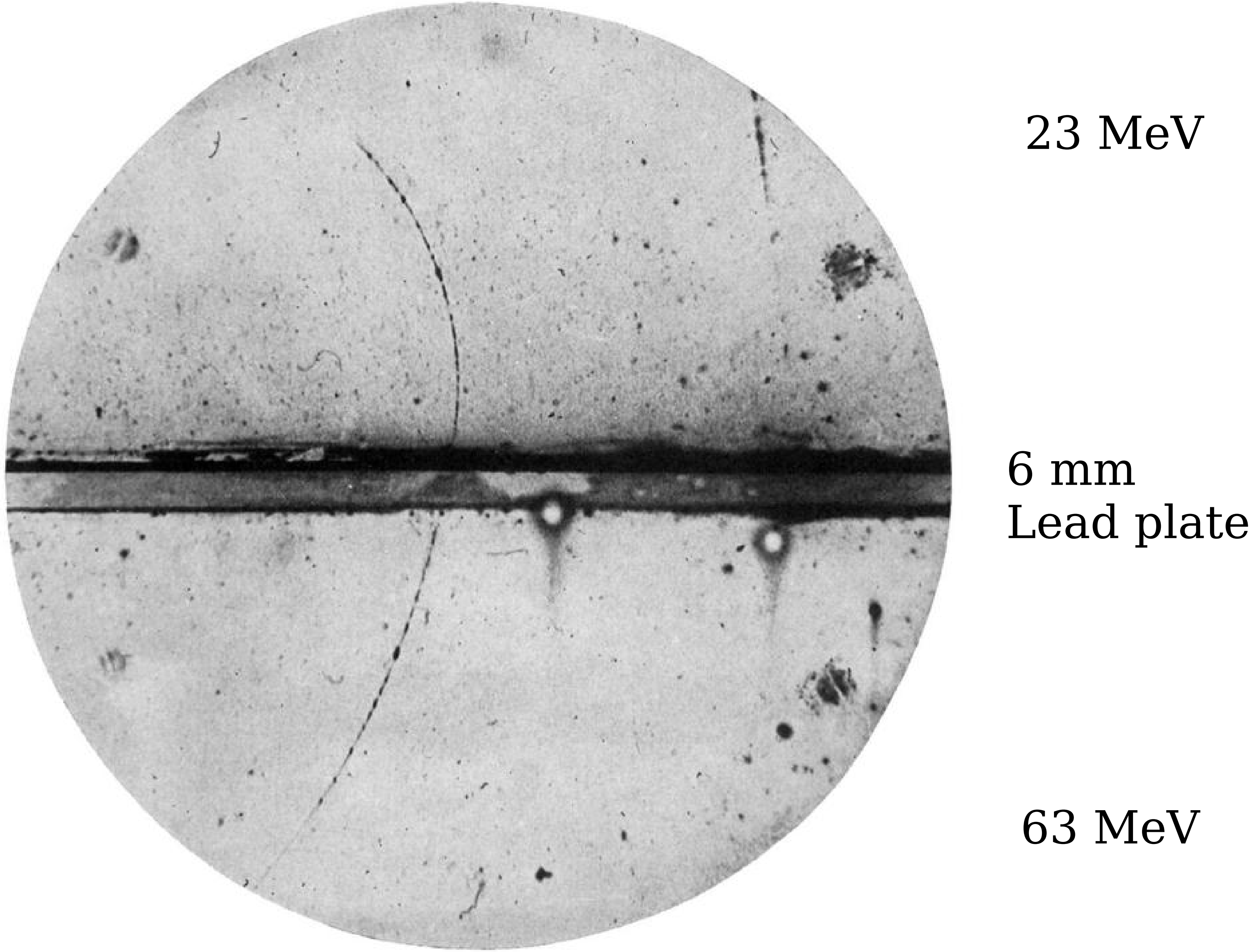


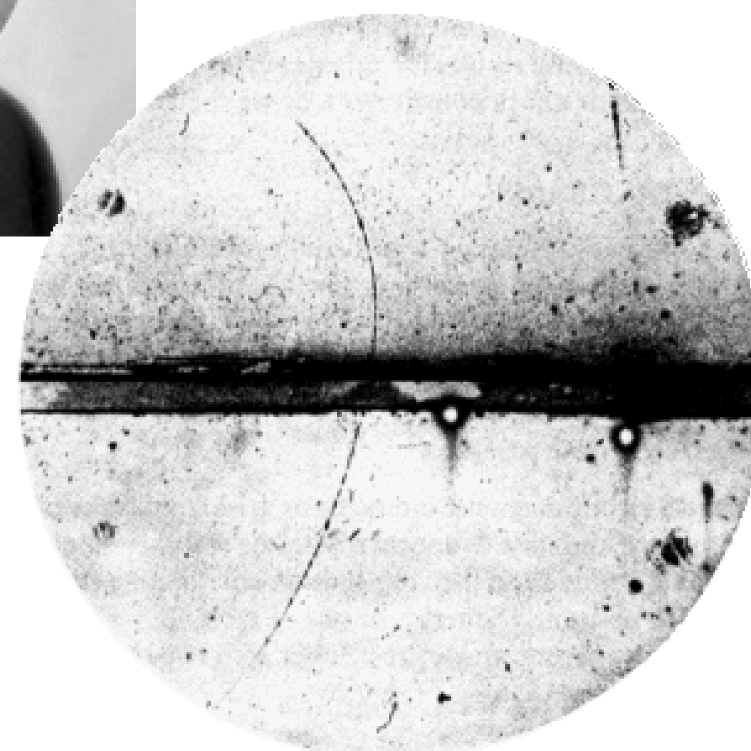
FIG. 1. A 63 million volt positron ($H\rho = 2.1 \times 10^5$ gauss-cm) passing through a 6 mm lead plate and emerging as a 23 million volt positron ($H\rho = 7.5 \times 10^4$ gauss-cm). The length of this latter path is at least ten times greater than the possible length of a proton path of this curvature.

Nobel Prize in Physics 1936

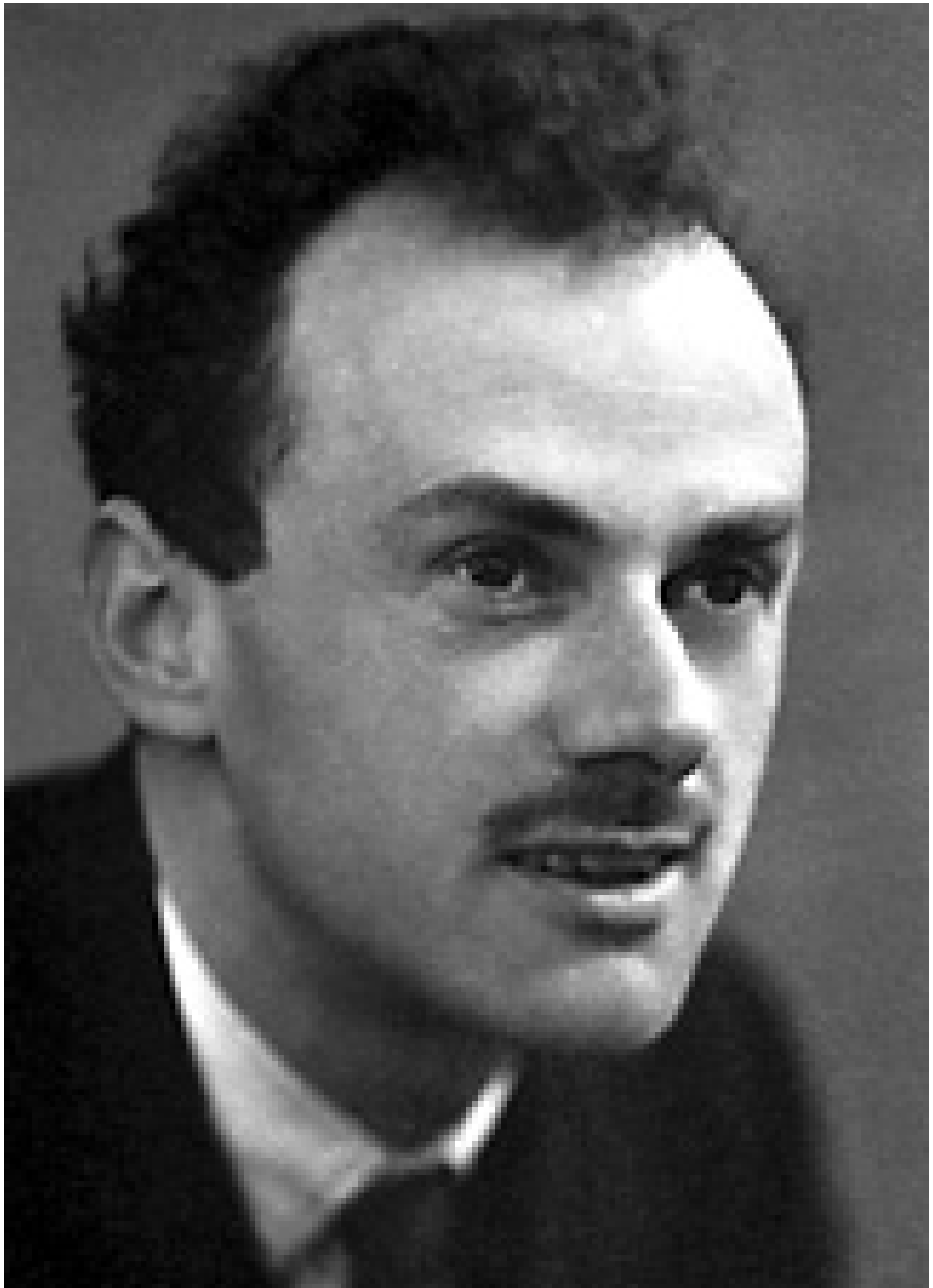
shared between:

Victor Hess
Discovery of Cosmic Rays

Carl Anderson
Discovery of the positron



Balloon Flights
of 1911/1912
h > 5000 meters.



Paul M. DIRAC

had *predicted*
in 1930

the existence
of the positron

The “Dirac equation”

[The most
beautiful equation
in Physics ?]

Matter

Anti-Matter

Paul Dirac

For each

particle

there is an

anti-particle

With the same mass
but opposite charger

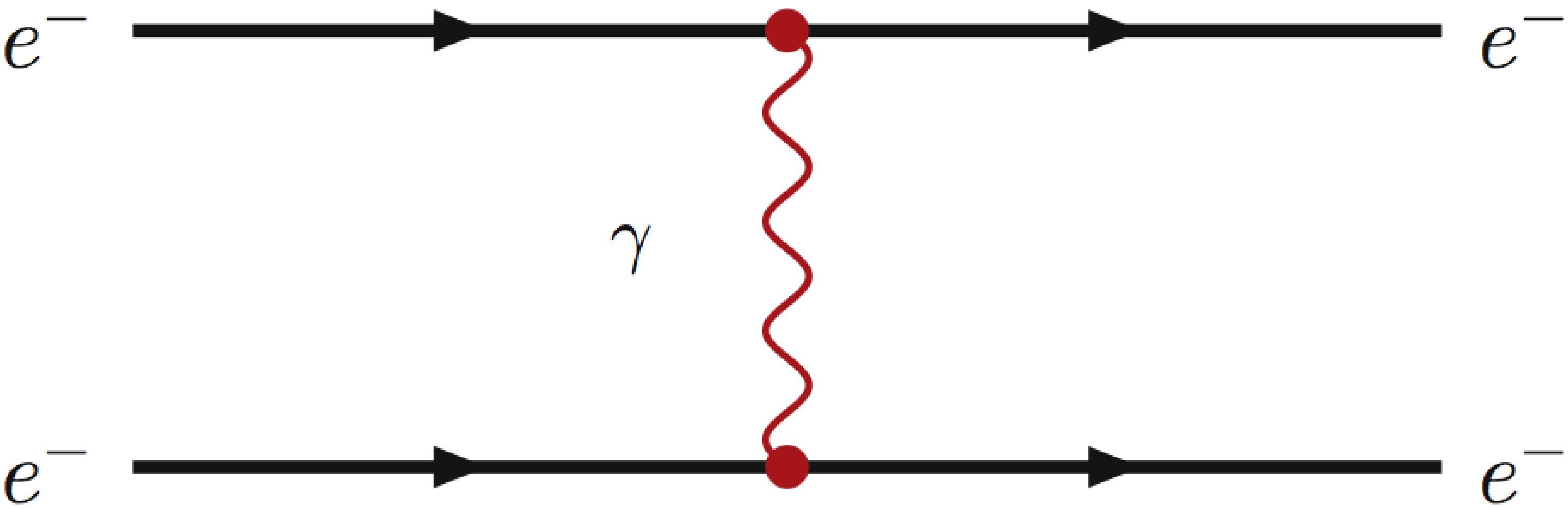
$$e^{-} \quad e^{+}$$

$$p \quad \overline{p}$$

$$n \quad \overline{n}$$

....

$$\boxed{e^- + e^- \longrightarrow e^- + e^-}$$

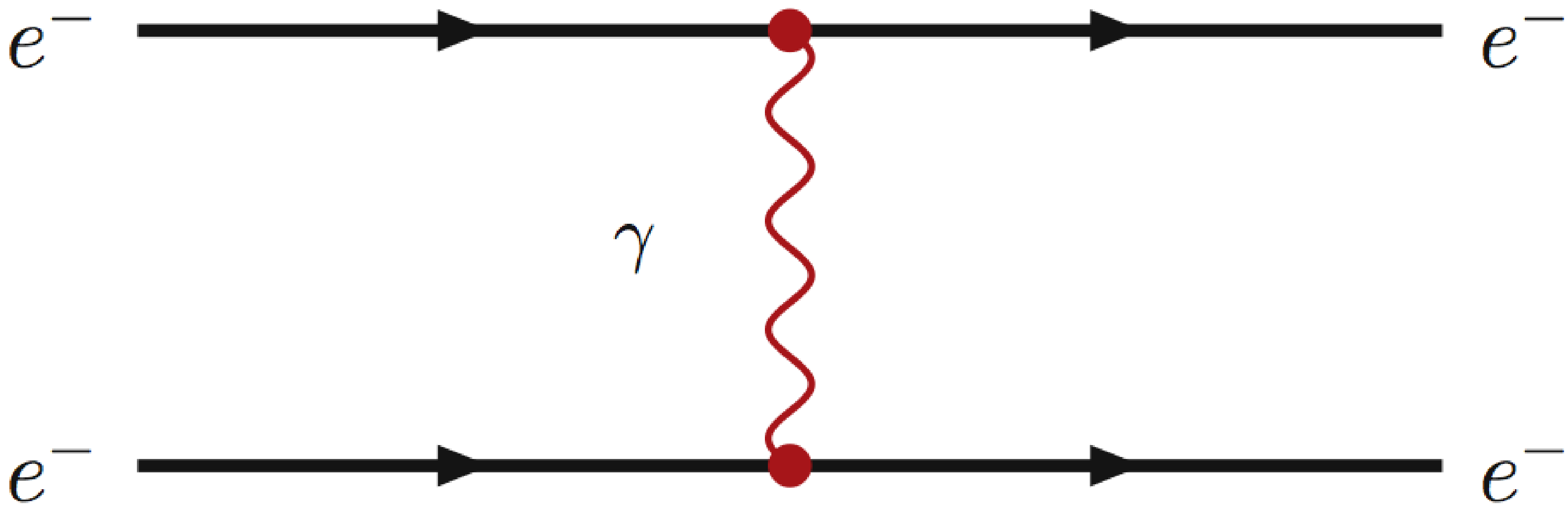


Development of $\boxed{\text{“Quantum electrodynamics”}}$

Electric Force is due to the
exchange of “virtual photons”

$$\boxed{e^- + e^- \rightarrow e^- + e^-}$$

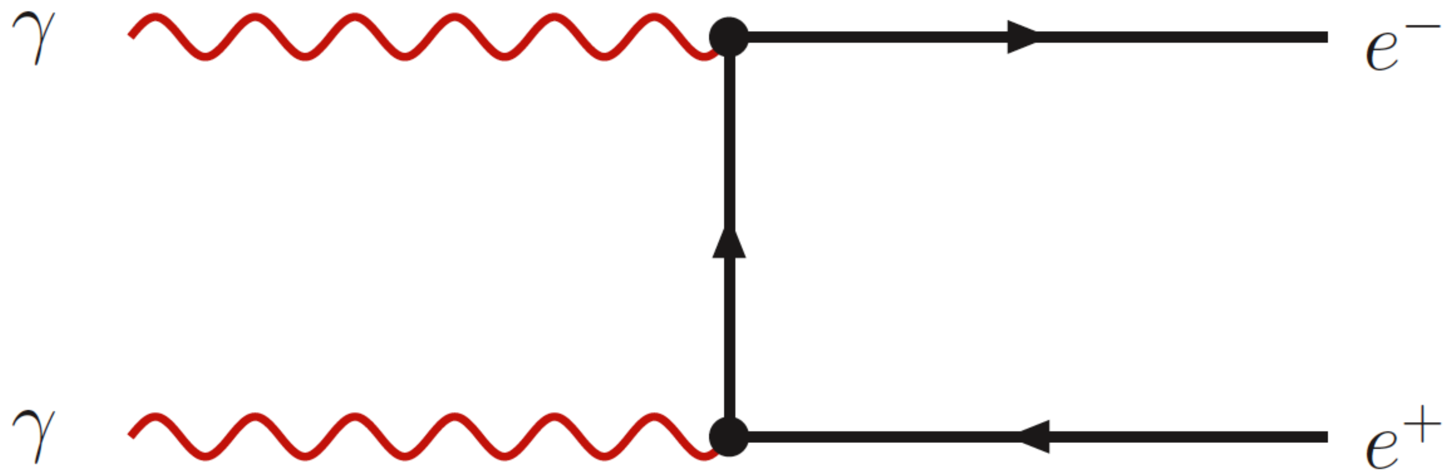
Two new processes discovered by DIRAC



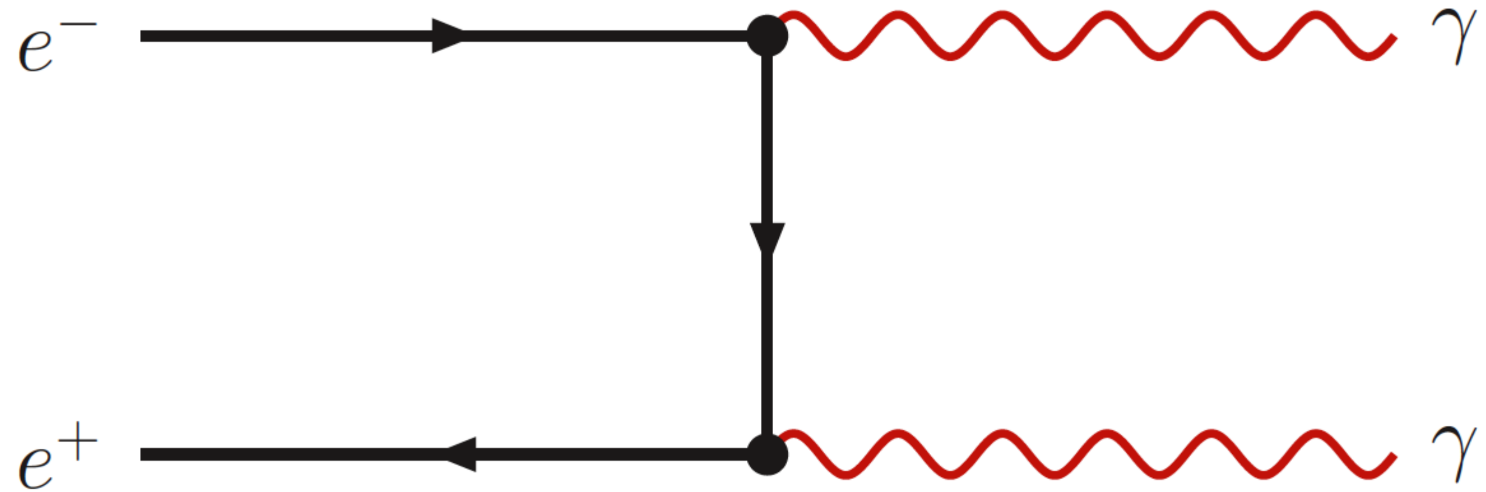
Development of "Quantum electrodynamics"

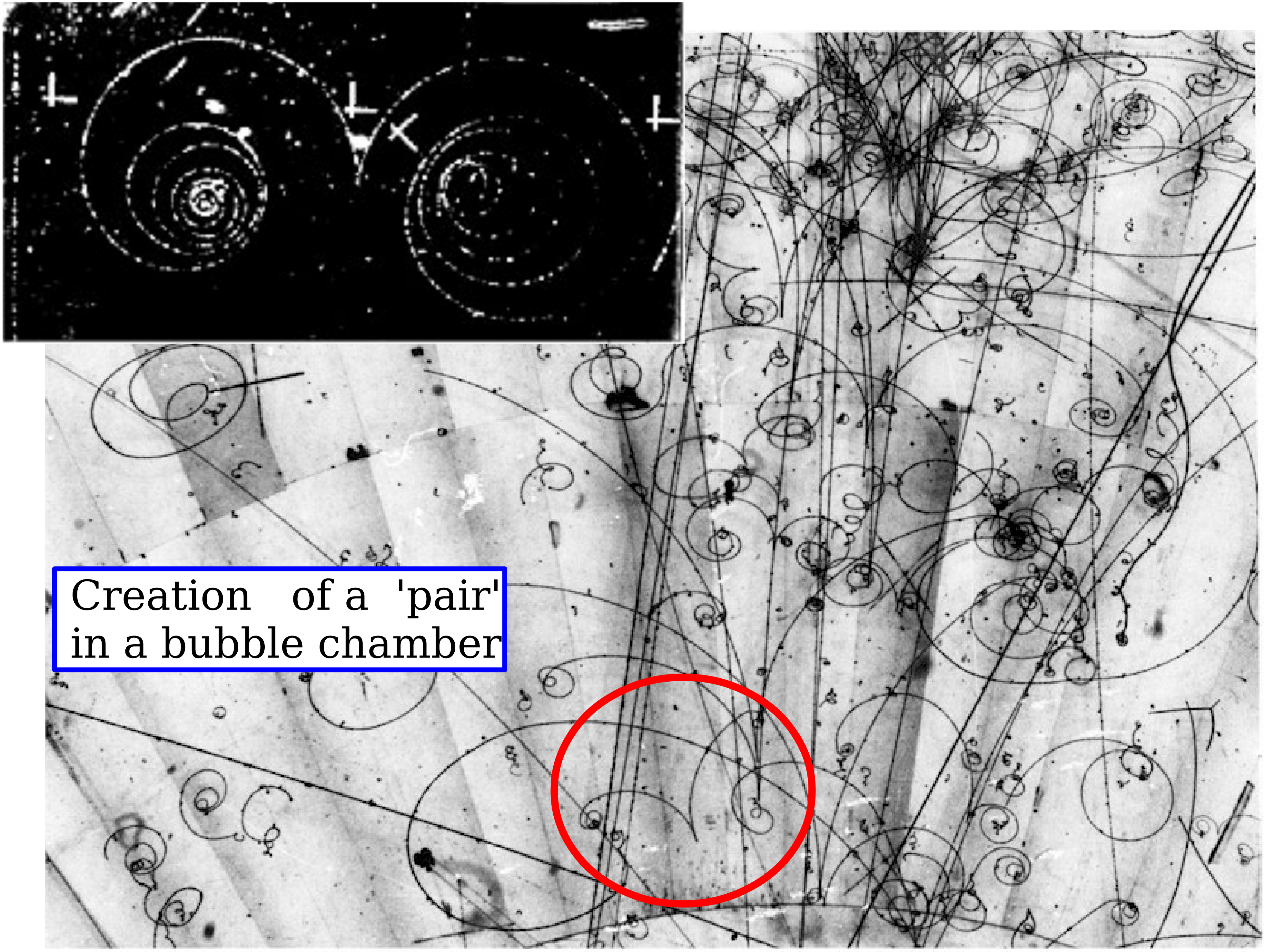
Electric Force is due to the
exchange of "virtual photons"

“Creation” of an electron-positron pair

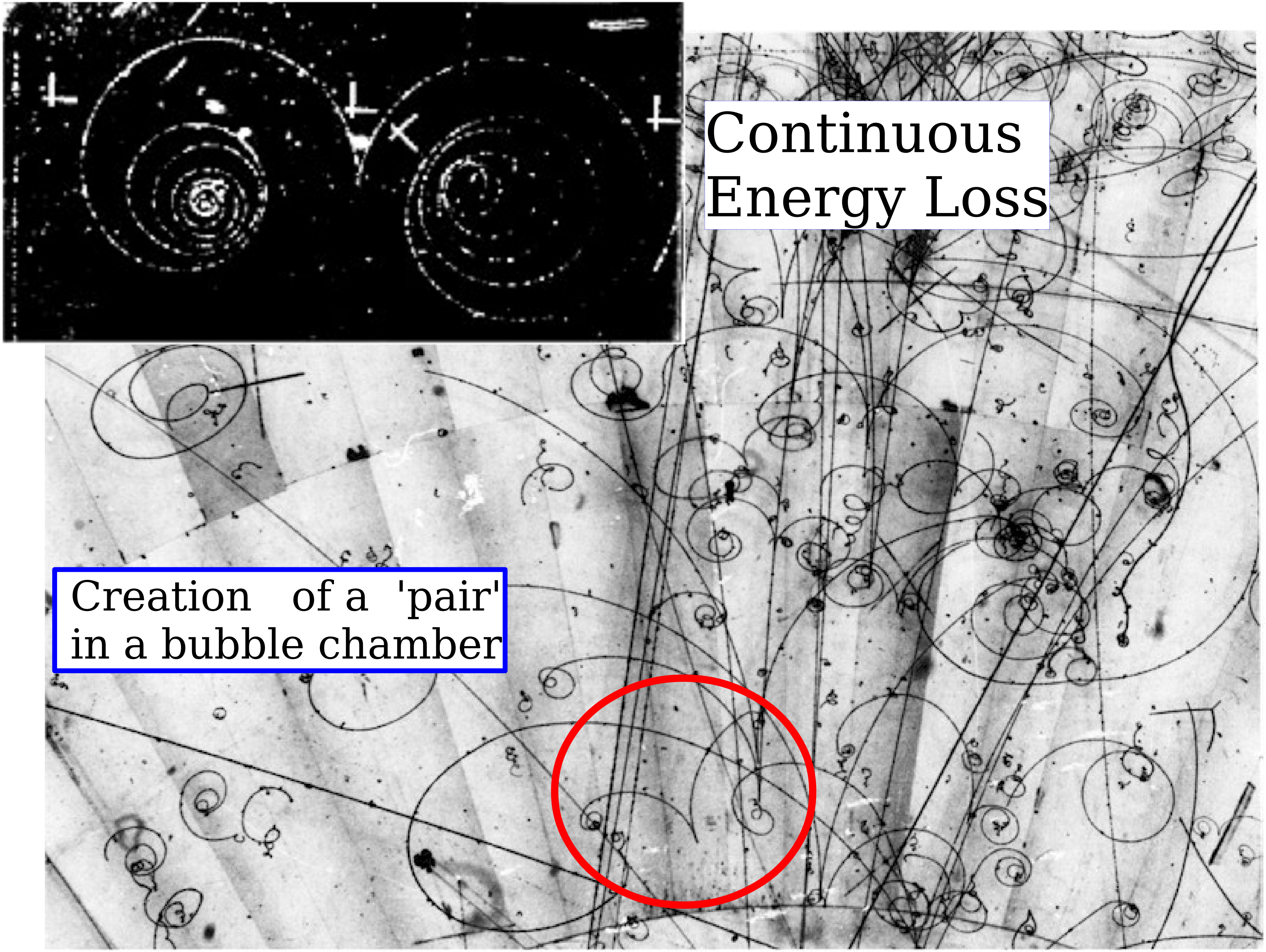


“Annihilation” of an electron-positron pair





Creation of a 'pair'
in a bubble chamber


Continuous Energy Loss
Creation of a 'pair' in a bubble chamber

THE

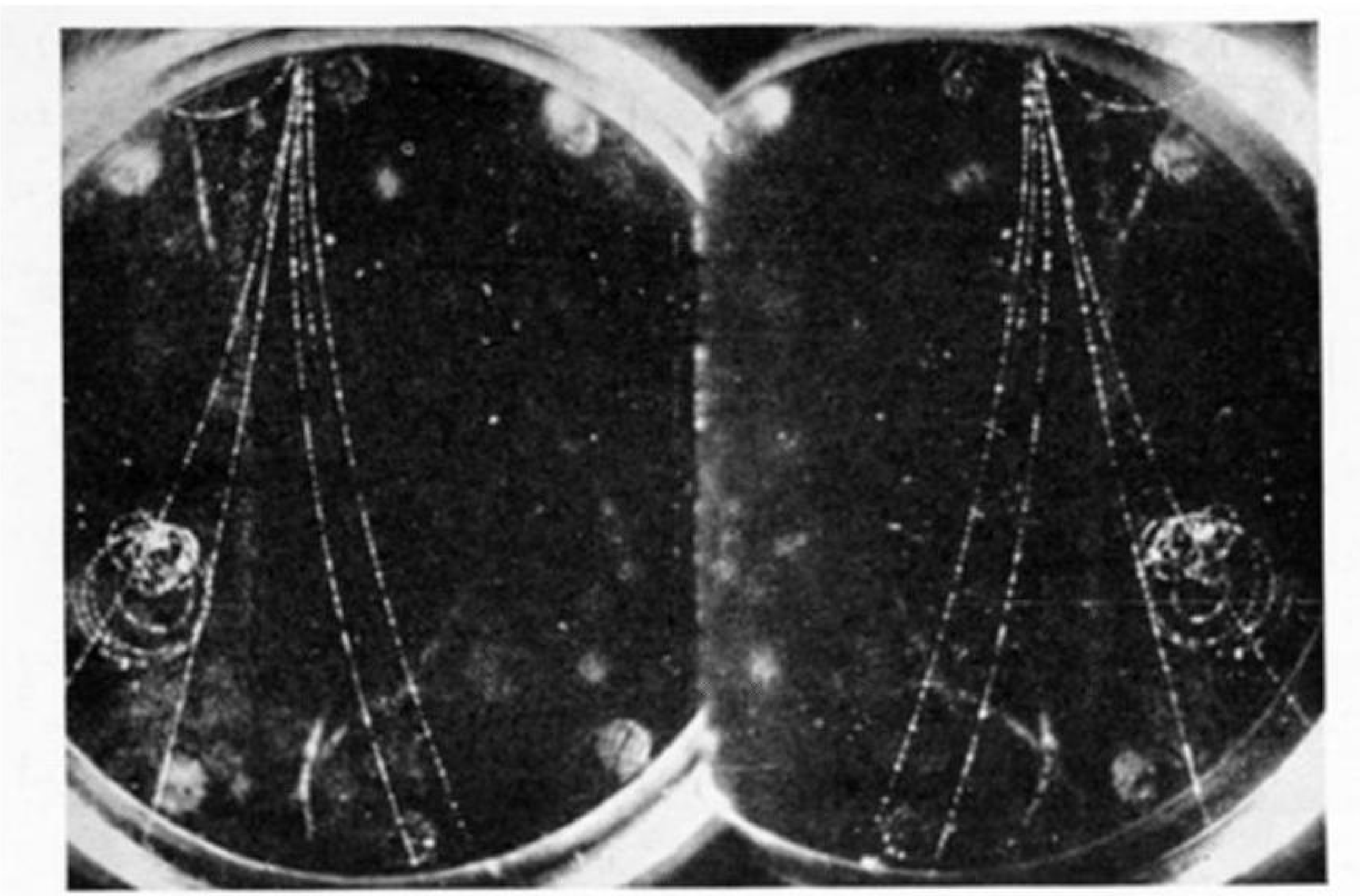
PHYSICAL REVIEW

A Journal of Experimental and Theoretical Physics Established by E. L. Nichols in 1893

VOL. 50, NO. 4 AUGUST 15, 1936 SECOND SERIES

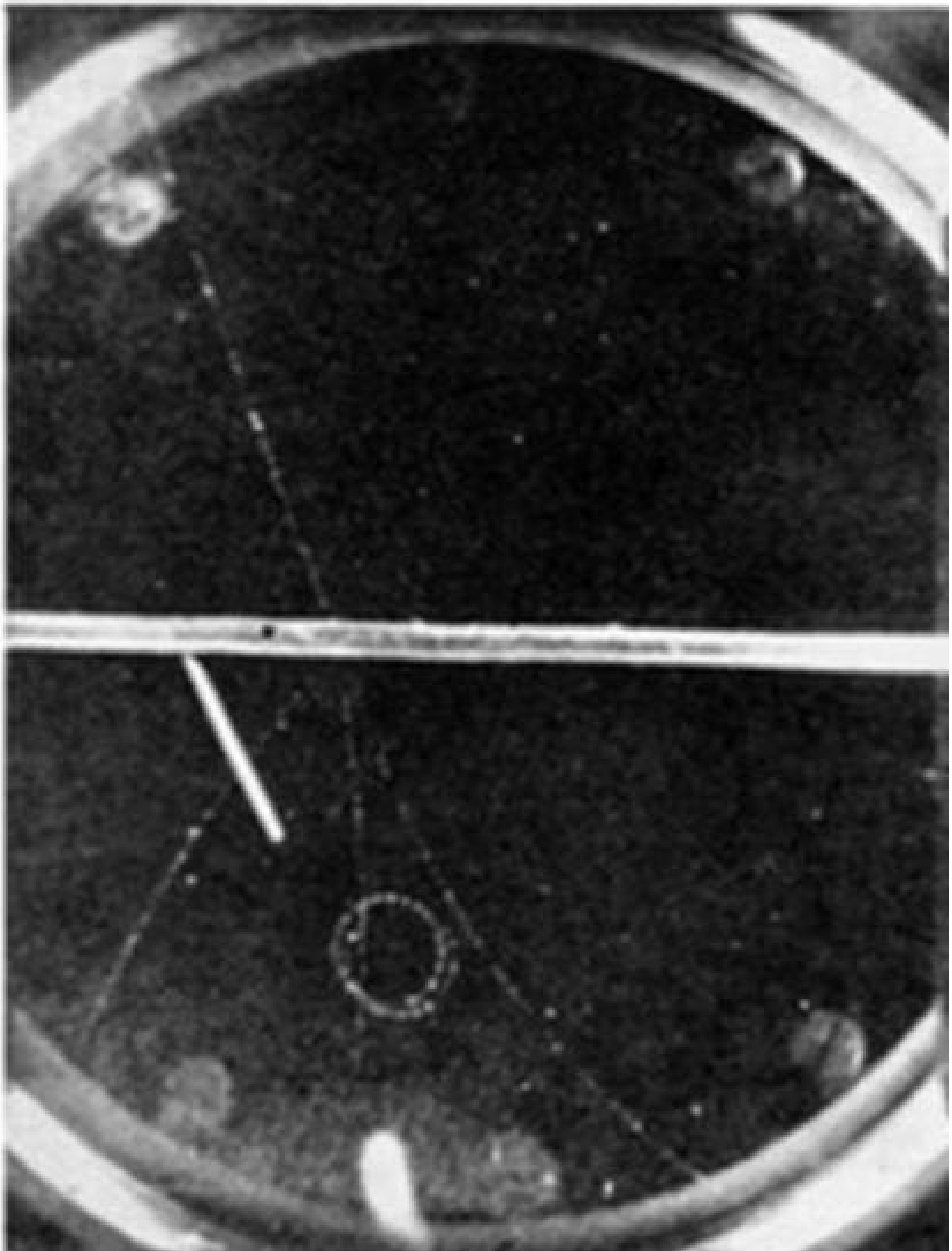
Cloud Chamber Observations of Cosmic Rays at 4300 Meters Elevation and Near Sea-Level

CARL D. ANDERSON AND SETH H. NEDDERMEYER, *Norman Bridge Laboratory of Physics, California Institute of Technology*



3 electrons
3.5, 55, 190 MeV

3 positrons
78, 70, 90 MeV



Positron of 480 MeV

0.35 cm of Lead

Positron 45 MeV
Positron 31 MeV
“Negatron” 45 MeV

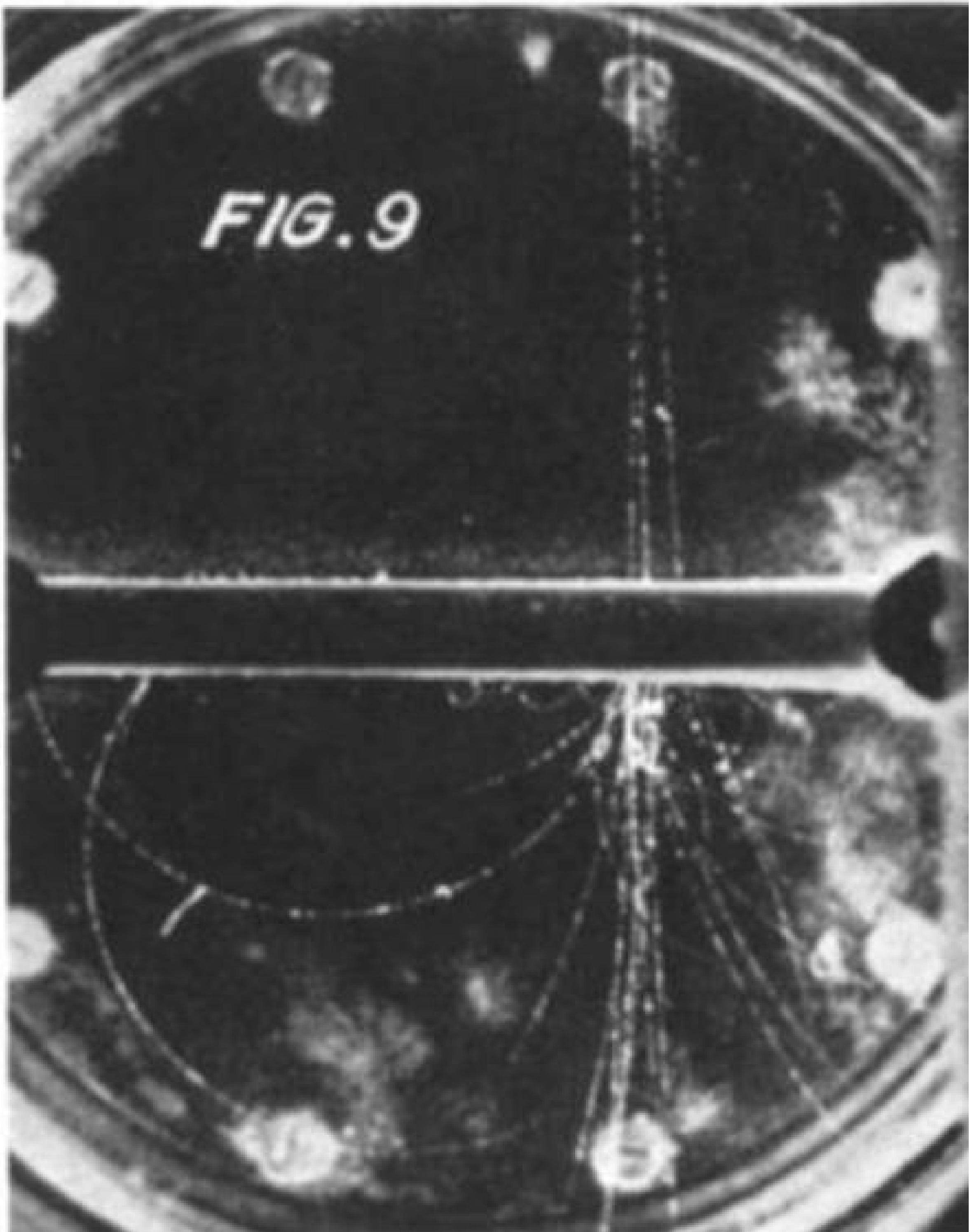


FIG. 9

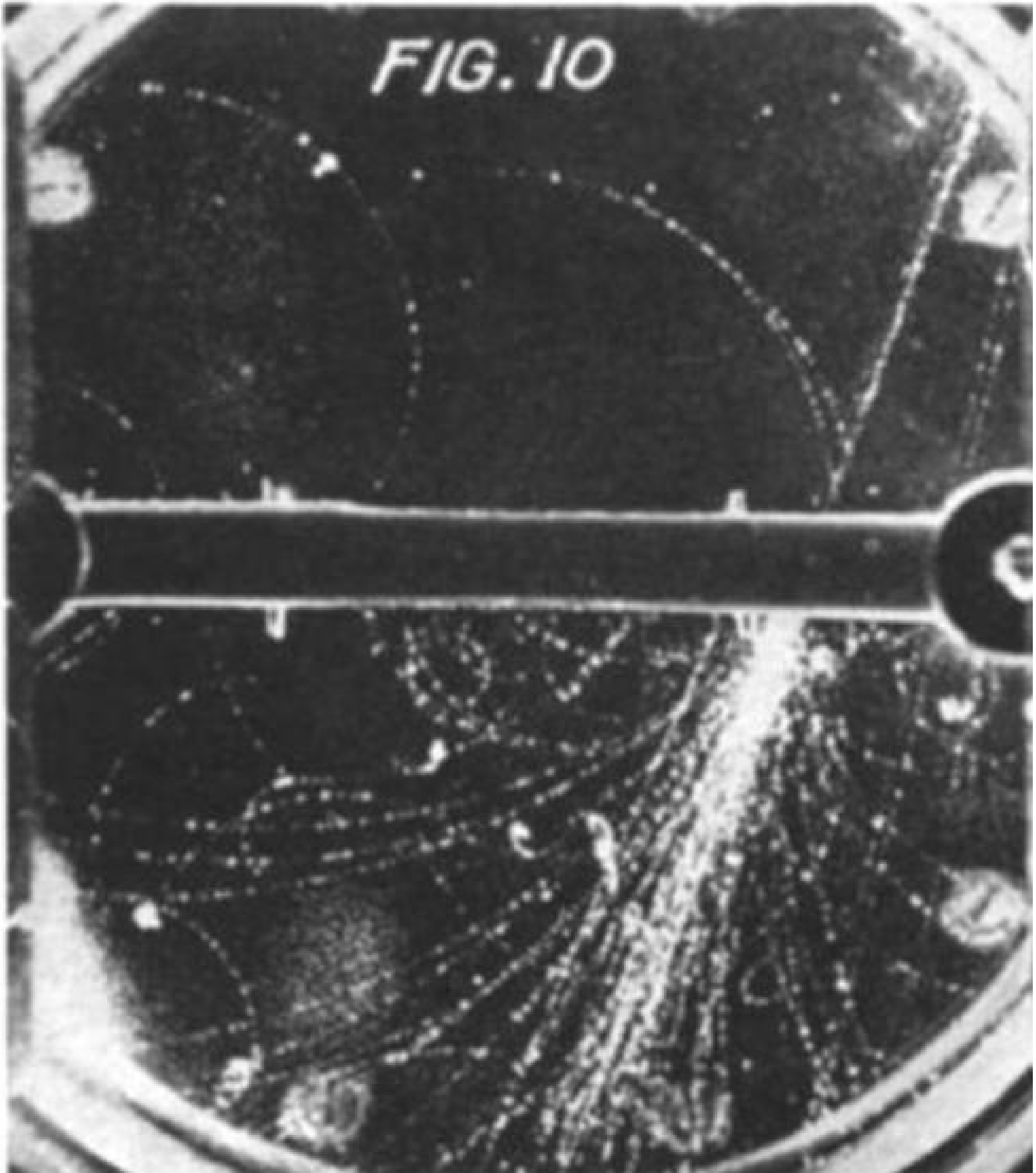


FIG. 10

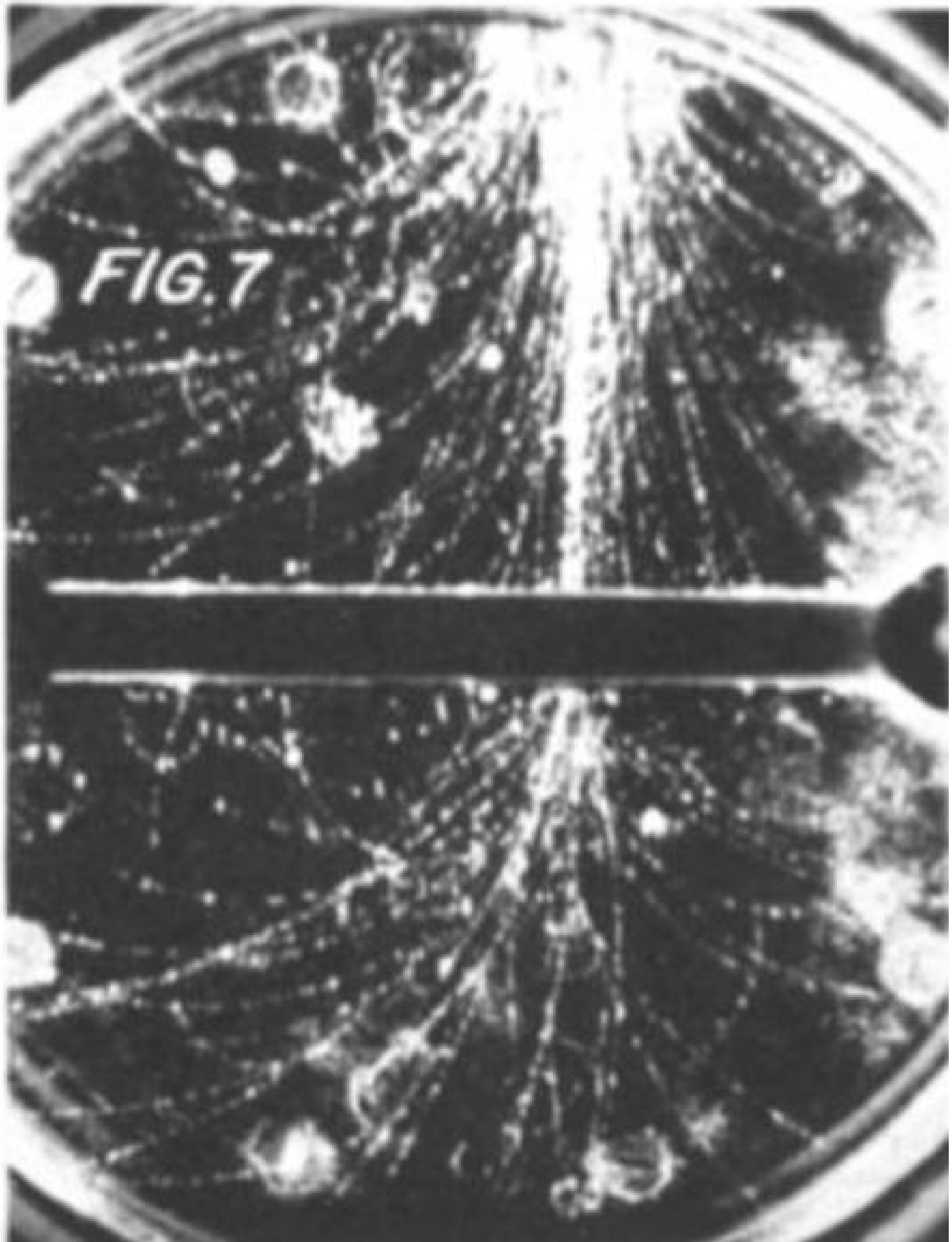
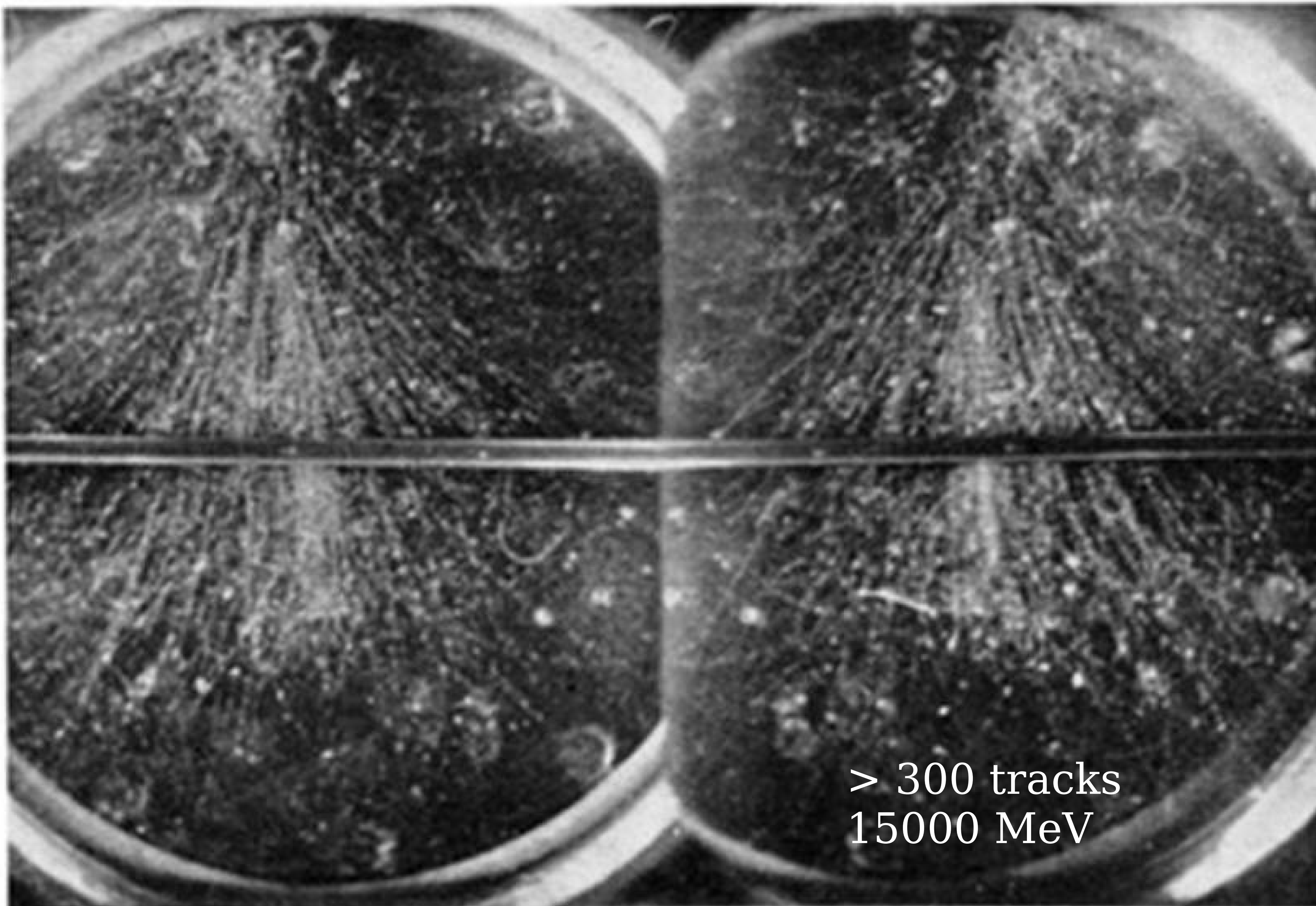


FIG. 7


> 300 tracks
15000 MeV

FUNDAMENTAL PROCESSES in QUANTUM ELECTRODYNAMICS

COMPTON SCATTERING

$$e + \gamma \longrightarrow e + \gamma$$

ELECTRON-POSITRON ANNIHILATION

$$e^+ + e^- \longrightarrow \gamma + \gamma$$

ELECTRON-POSITRON CREATION

$$\gamma + \gamma \longrightarrow e^+ + e^-$$

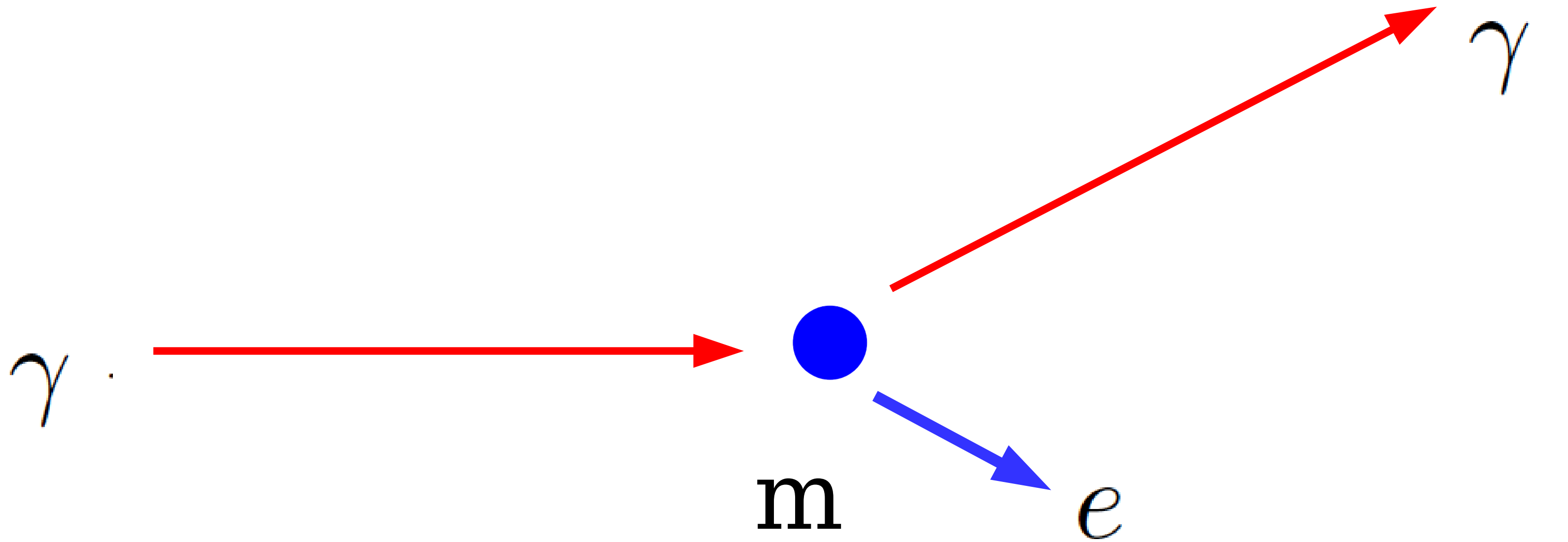
Compton scattering

$$e^{\pm} + \gamma \rightarrow e^{\pm} + \gamma$$

scattering between an electron (or positron) [or more in general with a charged particle] and a photon.

“Normally” (and historically) it is studied experimentally in the

electron rest frame



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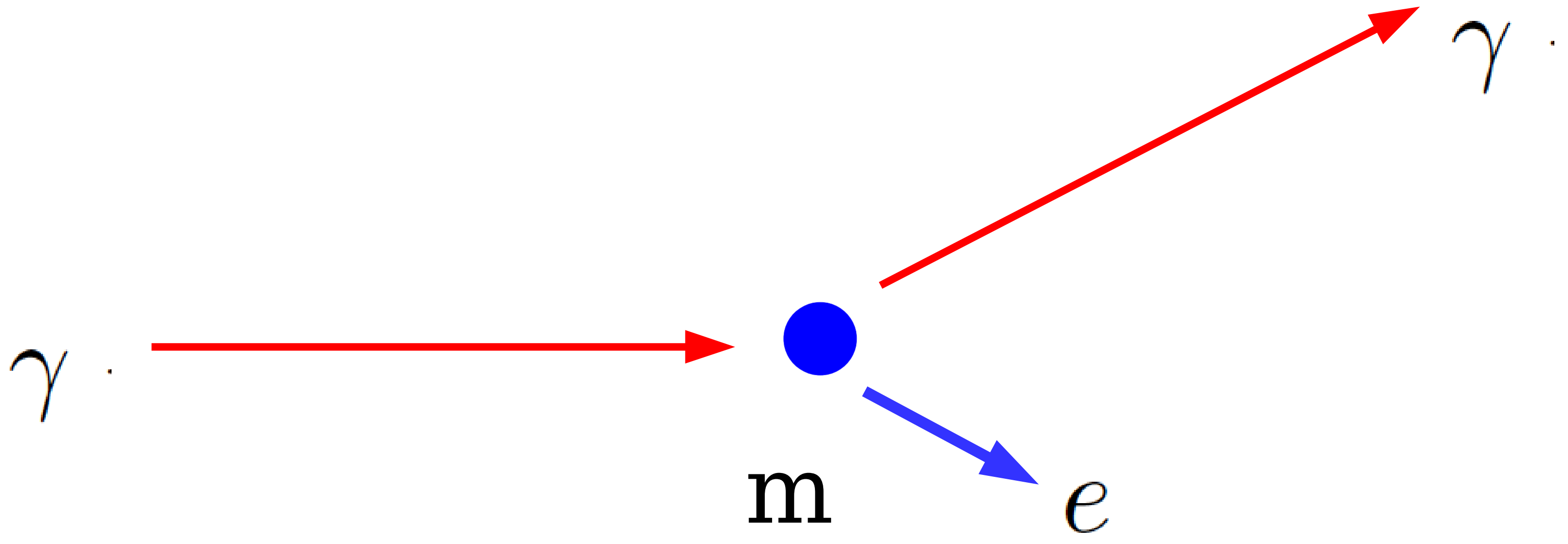
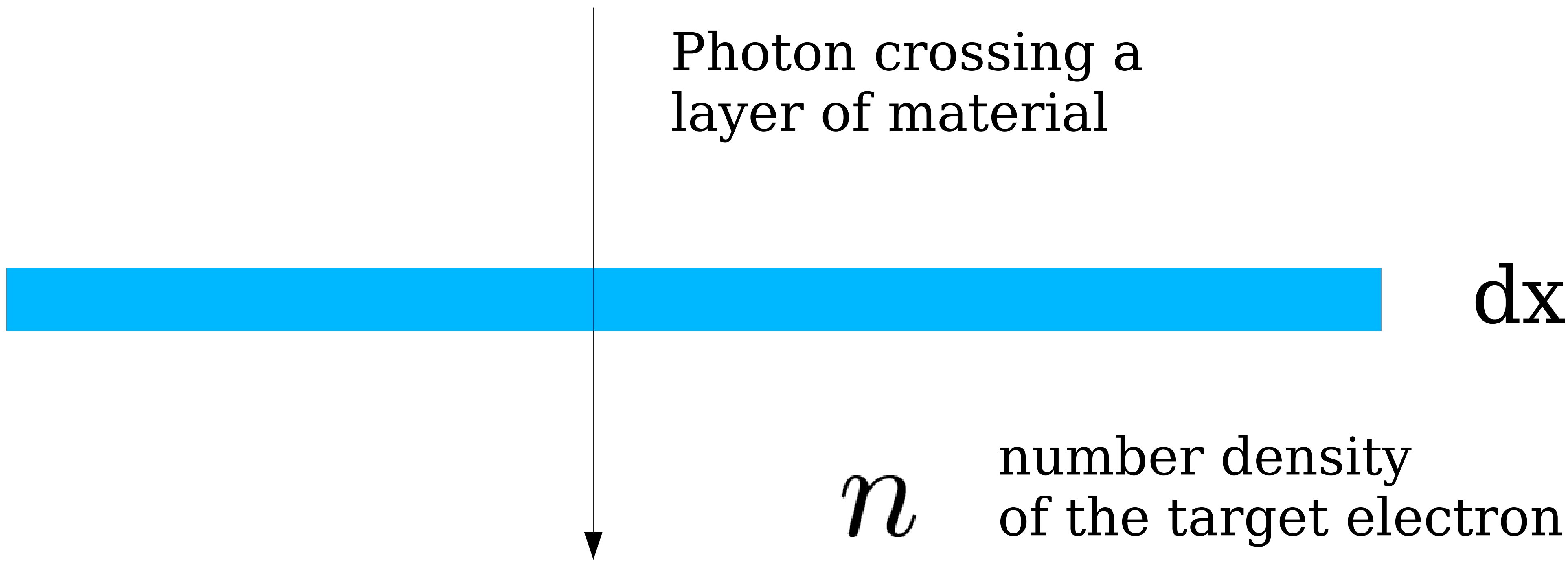
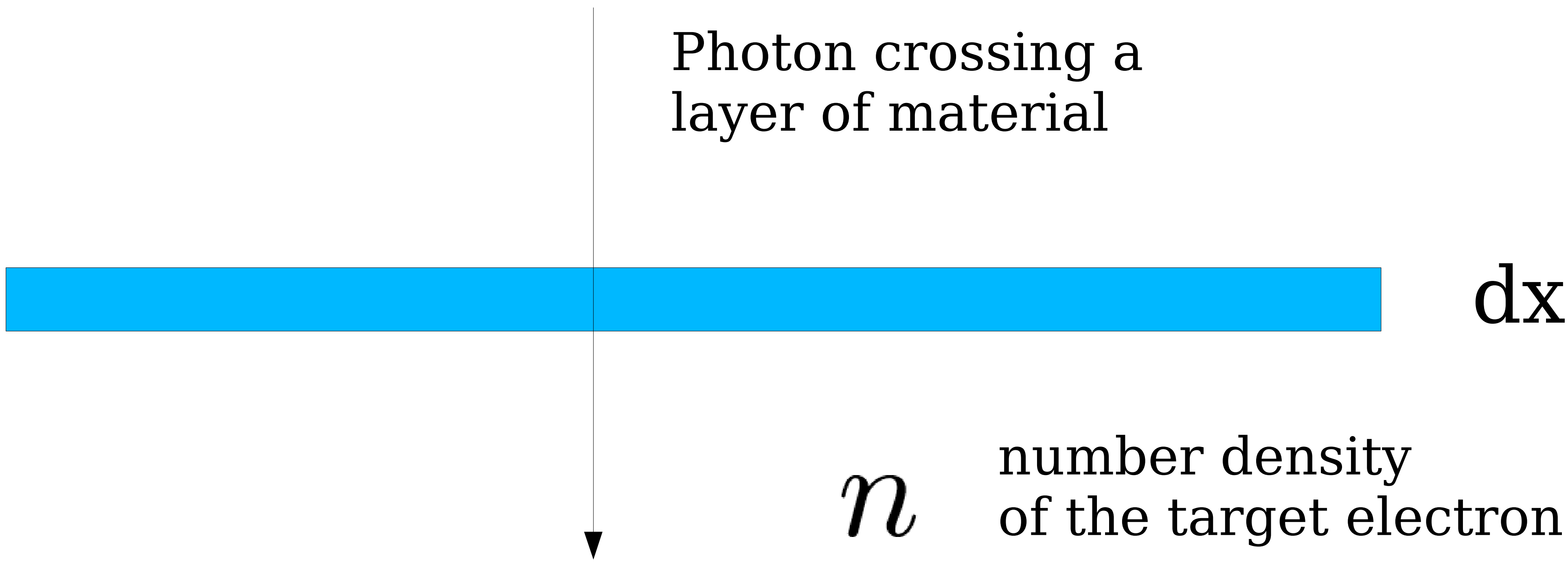


Cross section for Compton Scattering
(as a function of the photon energy)


Photon crossing a
layer of material
dx
n
number density
of the target electron

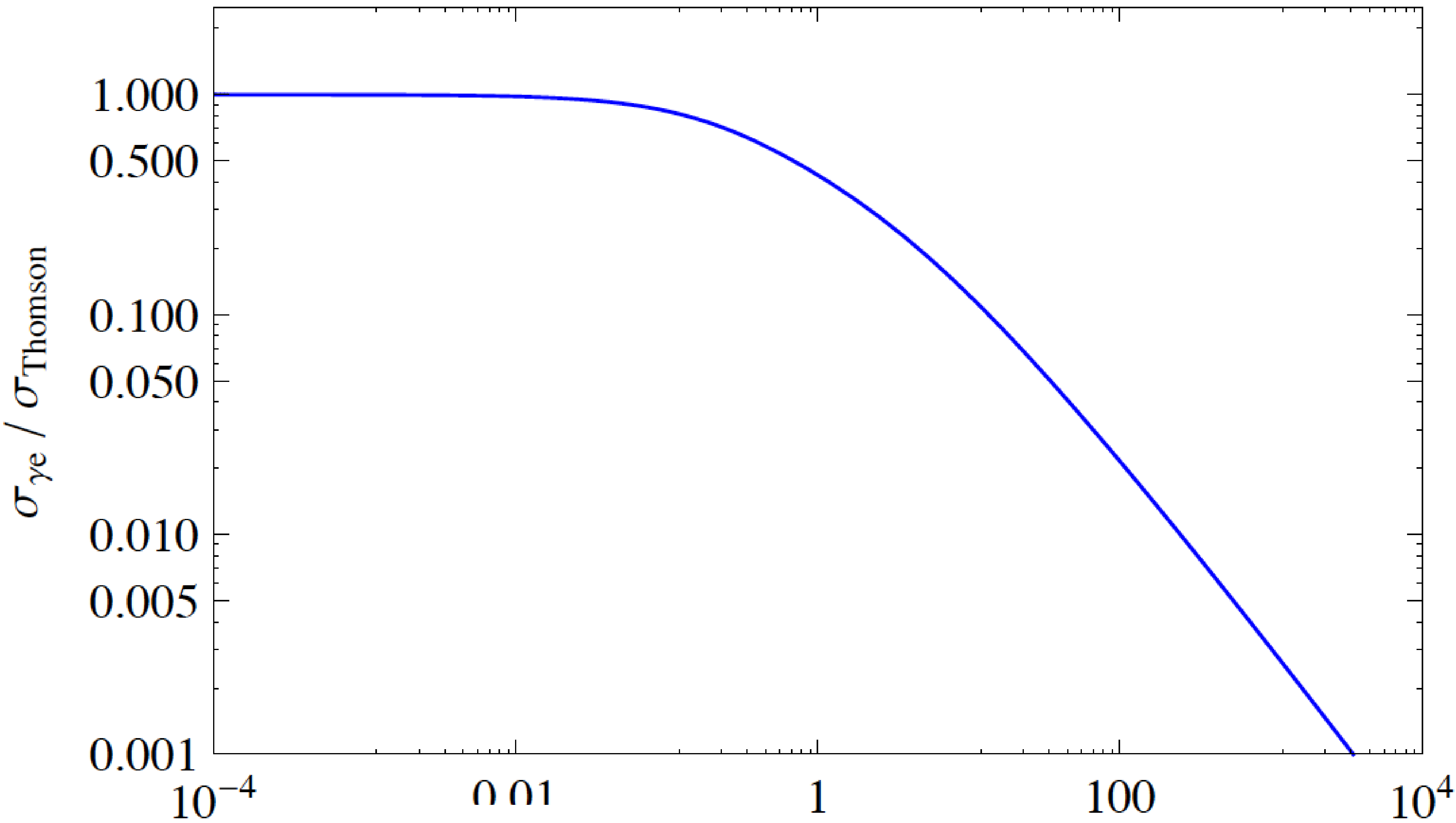


$$dN_{\rm scattering} = n \; dx \; \sigma$$

$$\text{Pure number} = [\text{cm}^{-3}] \times [\text{cm}] \times [\text{cm}^2]$$

Cross section for electron-photon Compton scattering

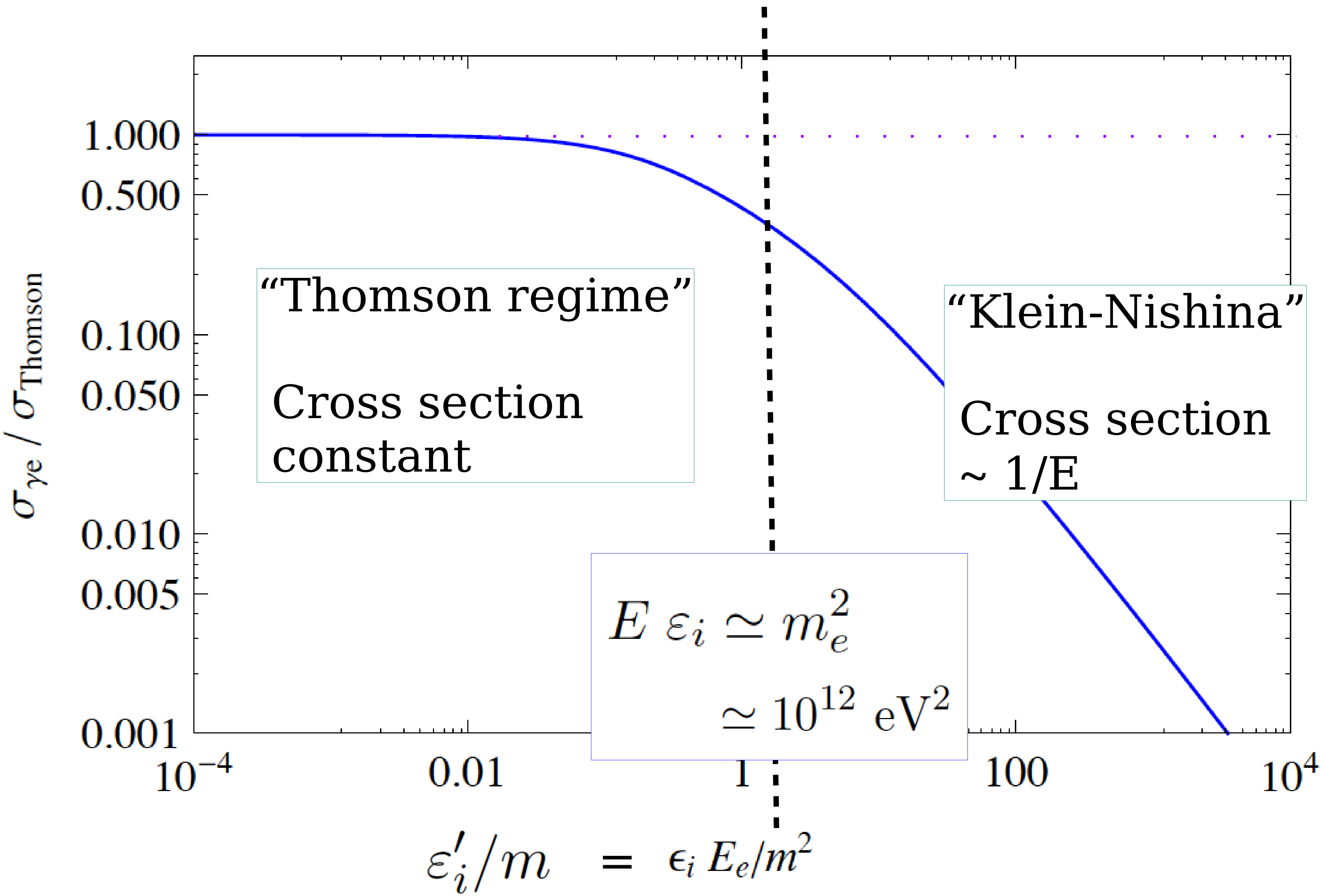
$$e + \gamma \rightarrow e + \gamma$$



$\varepsilon_i'/m \;=\; \epsilon_i\, E_e/m^2$

Cross section for
electron-photon scattering

$$e + \gamma \rightarrow e + \gamma$$



$$\sigma_{\mathrm{Thomson}} = \frac{8\pi}{3}\left(\frac{e^2}{m_e\,c^2}\right)^2 = \frac{8\pi}{3}\,r_0^2$$
$$\simeq 6.65 \times 10^{-25}\ \mathrm{cm}^2$$

$$\sigma \propto \frac{1}{m^2}$$

Scattering process
important only for
electrons/positrons

$$a_{\mathrm{Bohr}} = \frac{\hbar^2}{m_e^2\ e^2}$$

Bohr Radius

(dimension of the hydrogen atom)

$$a_{\mathrm{Bohr}} = \frac{r_0}{\alpha^2}$$

$$r_0 = 2.81 \times 10^{-13}\ \mathrm{cm}$$

$$a_{\mathrm{Bohr}} = 0.53 \times 10^{-8}\ \mathrm{cm}$$

$$r_0 = \left(\frac{e^2}{m_e\, c^2} \right)$$

"classical radius of the electron"

$$\simeq 2.81 \times 10^{-13} \text{ cm}$$

$$a_{\mathrm{Bohr}} = \frac{\hbar^2}{m_e^2\, e^2}$$

Bohr Radius

(dimension of the hydrogen atom)

$$= \frac{r_0}{\alpha^2}$$

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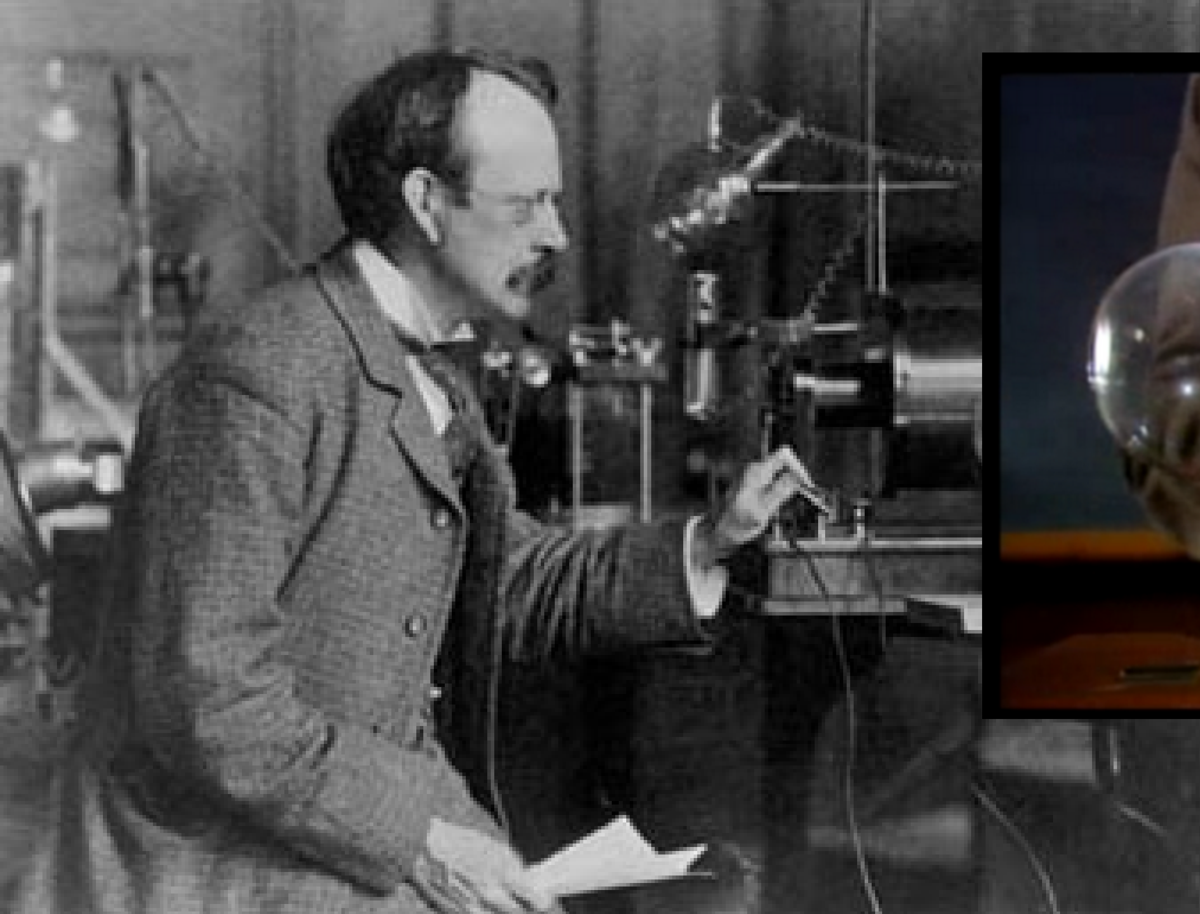
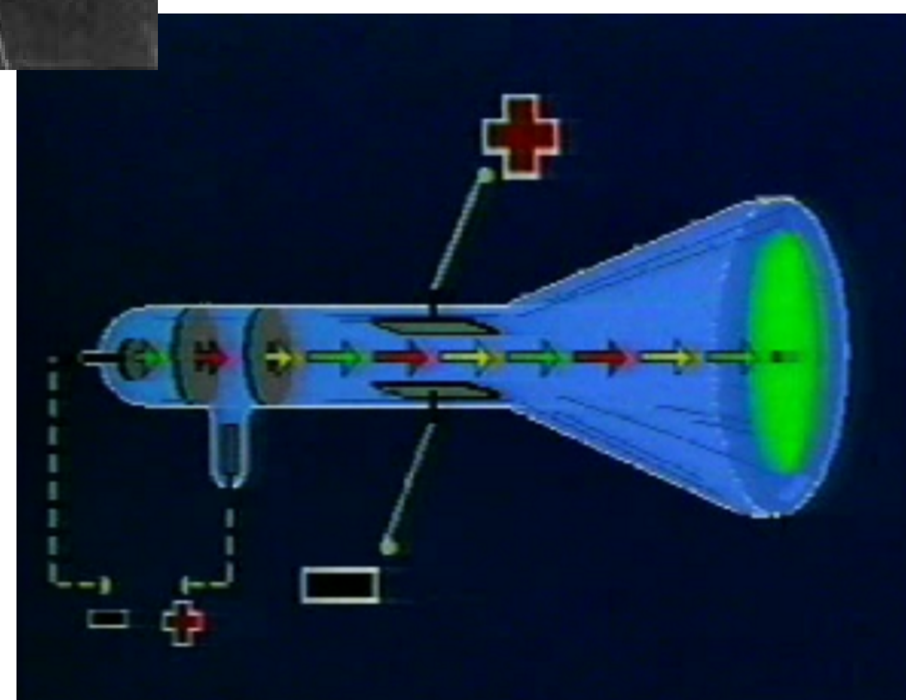


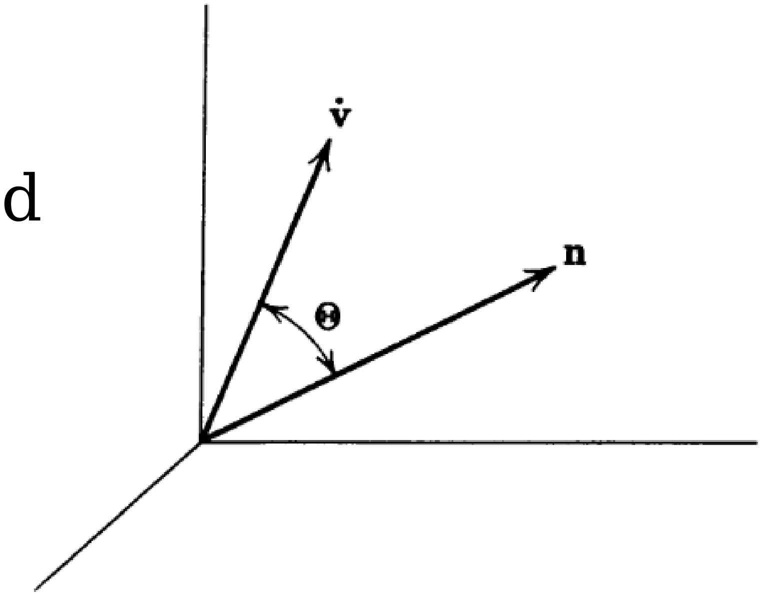
J.J Thomson (1897)

Discovery of the electron



Thomson "classical" analysis

Classical results
of radiation from an accelerated
electric charge that has
A small velocity (v/c << 1)



$$\frac{dP}{d\Omega} = \frac{e^2}{4\pi c^3} |\dot{\mathbf{v}}|^2 \sin^2\Theta$$

Power emitted
per unit solid angle

$$P = \frac{2}{3}\frac{e^2}{c^3} |\dot{\mathbf{v}}|^2$$

Total emitted power

Larmor's formula

Jackson, Classical Electrodynamics, sec.14.8

Thomson Scattering of Radiation

If a plane wave of monochromatic electromagnetic radiation is incident on a free particle of charge e and mass m, the particle will be accelerated and so emit radiation. This radiation will be emitted in directions other than that of the incident plane wave, but for nonrelativistic motion of the particle it will have the same frequency as the incident radiation. The whole process may be described as scattering of the incident radiation.

$$\frac{d\sigma}{d\Omega} = \frac{\text{Energy radiated/unit time/unit solid angle}}{\text{Incident energy flux in energy/unit area/unit time}}$$

$$\frac{d\sigma}{d\Omega} = \left(\frac{e^2}{mc^2}\right)^2 \cdot \frac{1}{2}(1 + \cos^2\theta)$$

$$\sigma_T = \frac{8\pi}{3}\left(\frac{e^2}{mc^2}\right)^2$$

$$\sigma_{\text{Thomson}} = 6.65 \times 10^{-25}\ \text{cm}^2$$

Average over polarization
of incident wave,
sum over polarization
of scattered wave

$$\frac{dP}{d\Omega} = \frac{e^2}{4\pi c^3} \dot{v}^2 \sin^2 \Theta$$

Electromagnetic Plane wave

$$\mathbf{E}(\mathbf{x}, t) = \boldsymbol{\epsilon} E_0 e^{i\mathbf{k}\cdot\mathbf{x} - i\omega t}$$ Electric field

$$\dot{\mathbf{v}}(t) = \boldsymbol{\epsilon} \frac{e}{m} E_0 e^{i\mathbf{k}\cdot\mathbf{x} - i\omega t}$$ Acceleration

$$\left\langle \frac{dP}{d\Omega} \right\rangle = \frac{c}{8\pi} |E_0|^2 \left(\frac{e^2}{mc^2} \right)^2 \sin^2 \Theta$$

$$\sigma_T = \frac{8\pi}{3}\left(\frac{e^2}{mc^2}\right)^2$$

$$\sigma_{KN} = \left(\frac{e^2}{mc^2}\right)^2 \begin{cases} \dfrac{8\pi}{3}\left(1 - \dfrac{2\hbar\omega}{mc^2} + \cdots\right), & \hbar\omega \ll mc^2 \\ \pi\,\dfrac{mc^2}{\hbar\omega}\left[\ln\left(\dfrac{2\hbar\omega}{mc^2}\right) + \frac{1}{2}\right], & \hbar\omega \gg mc^2 \end{cases}$$

Classically, the scattered radiation has the same frequency of the incident wave.

Quantum mechanically,
in the scattering between particles
this cannot be exactly true, because
of conservation of energy and momentum.

$$\boxed{\varepsilon_f(\cos\theta) = \frac{\varepsilon_i}{1 + \frac{\varepsilon_i}{m}(1 - \cos\theta)}}$$

Variables in electron rest frame

Classically, the scattered radiation has the same frequency of the incident wave.

Quantum mechanically,
in the scattering between particles this cannot be exactly true, because of conservation of energy and momentum.

$$\boxed{\varepsilon_f(\cos\theta) = \frac{\varepsilon_i}{1 + \frac{\varepsilon_i}{m}(1-\cos\theta)}}$$

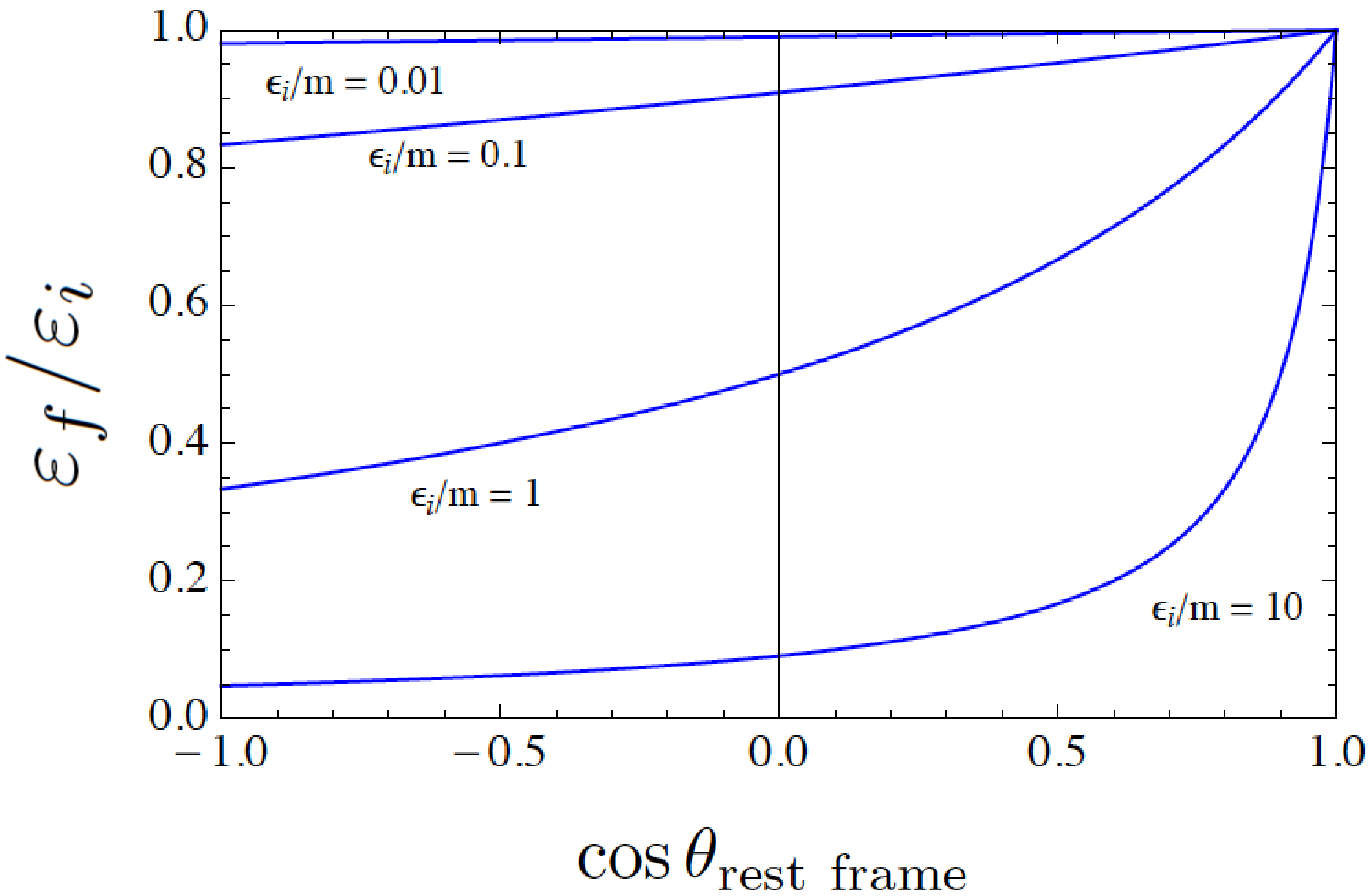
$$\boxed{E_\gamma = h\,\nu = \frac{h\,c}{\lambda}}$$

Variables in electron rest frame

COMPTON experiment confirms Einstein/Planck theory !
Scattered light changes its wavelength

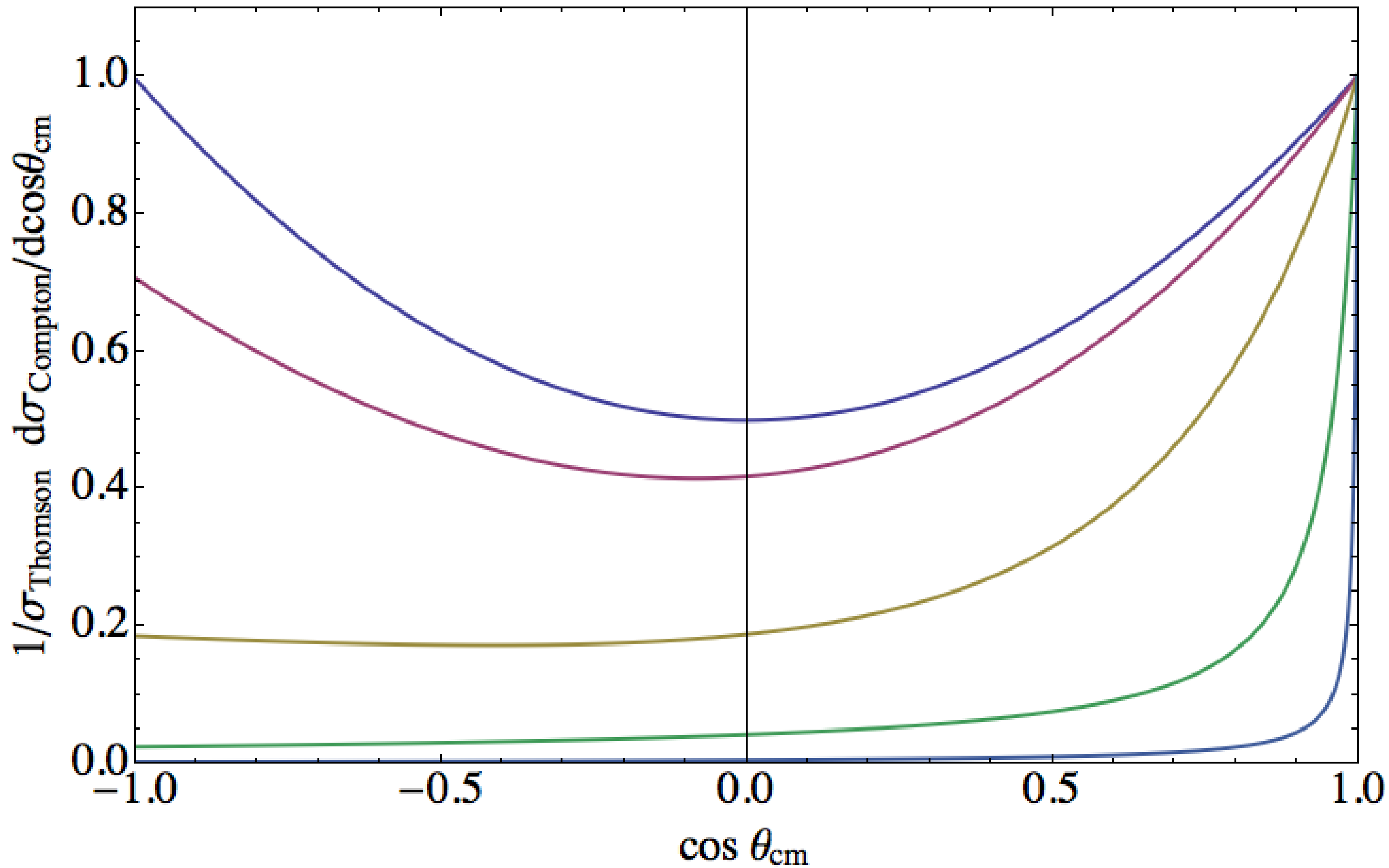
Kinematics of Compton Scattering in the electron rest frame

$$\varepsilon_f(\cos\theta) = \frac{\varepsilon_i}{1 + \frac{\varepsilon_i}{m}(1 - \cos\theta)}$$



COMPTON scattering $\cos\theta_{\text{rest frame}}$ distribution

```
Plot [{dsigcompt[ct, 0.001], dsigcompt[ct, 0.1],
  dsigcompt[ct, 1], dsigcompt[ct, 10], dsigcompt[ct, 100]},
 {ct , -1, 1 }, PlotStyle → col1, Frame → True]
```



$$\frac{\text{alfa}^2\,(1+\text{ct}^2)}{2\,\text{m}^2}+\frac{\text{alfa}^2\,(-1+\text{ct}-\text{ct}^2+\text{ct}^3)\,\epsilon}{\text{m}^2}+O[\epsilon]^2$$

```
dsigcompt [ct, ϵ ] // TraditionalForm
```

ditionalForm=

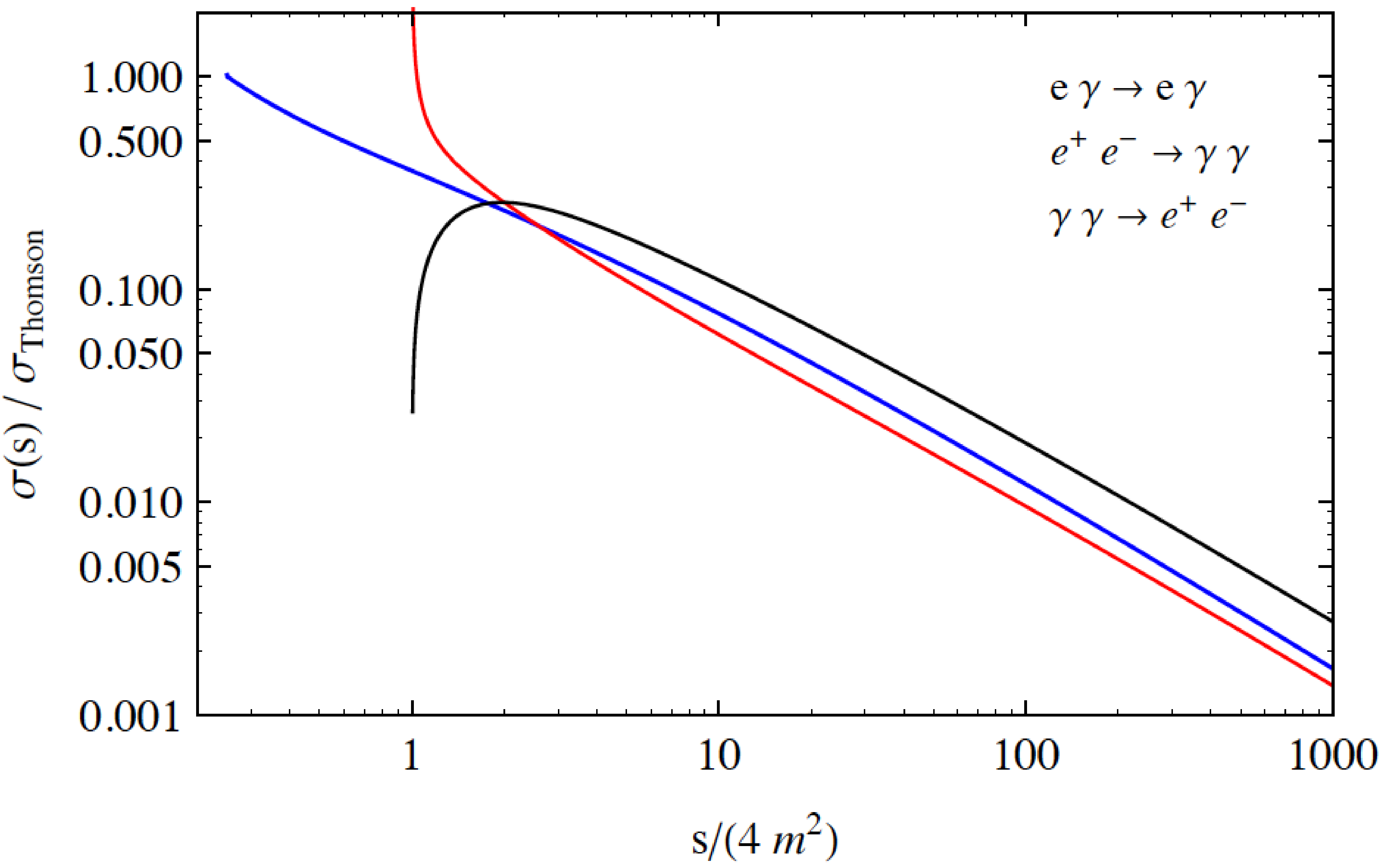
$$-\frac{-\epsilon\,\mathrm{ct}^3+(\epsilon^2+\epsilon+1)\,\mathrm{ct}^2-\epsilon(2\,\epsilon+1)\,\mathrm{ct}+\epsilon^2+\epsilon+1}{2\,((\mathrm{ct}-1)\,\epsilon-1)^3}$$

Total COMPTON scattering cross section

$$-\frac{1}{m^2\,\epsilon^3}\left(\mathrm{alfa}^2\,\pi\left(-\frac{2\,\epsilon\,(2+\epsilon\,(1+\epsilon)\,(8+\epsilon))}{(1+2\,\epsilon)^2}+(2-(-2+\epsilon)\,\epsilon)\,\mathrm{Log}[1+2\,\epsilon]\right)\right)$$

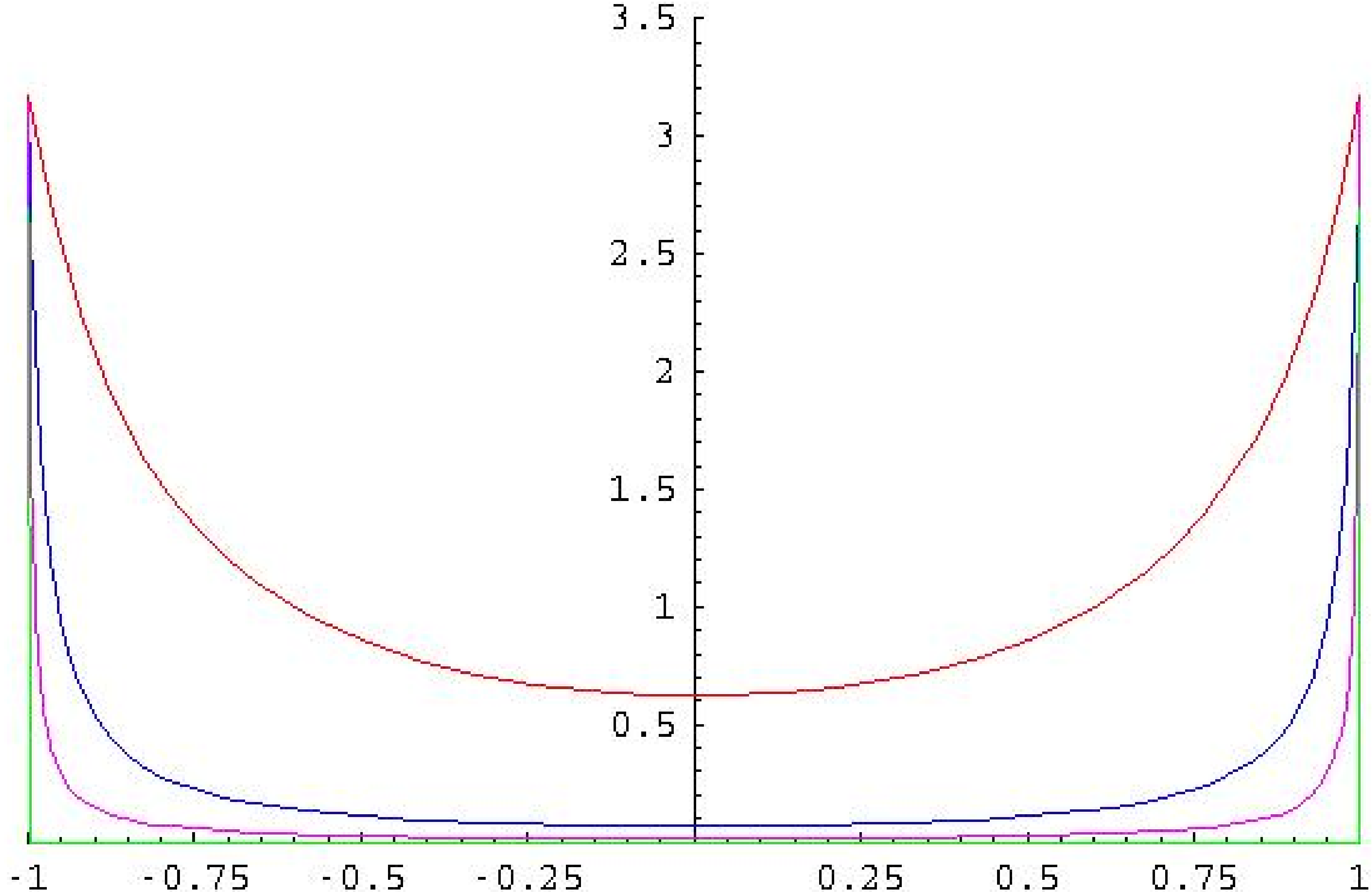
Cross sections
for the 3 processes

$$s = (p_1 + p_2)^2 = E_{\mathrm{tot}}^2 - |\vec{p}_{\mathrm{tot}}|^2$$
$$= E_{\mathrm{c.m.}}^2$$



Pair Creation (or Annihilation) $\cos\theta_{cm}$ distribution

```
In[54]:= Plot [{ddsig[ct, 1.005], ddsig[ct, 2], ddsig[ct, 5],
    ddsig[ct, 10], ddsig[ct, 1000]}, {ct, -1, 1},
   PlotStyle → col, PlotRange → {{-1, 1}, {0, 3.5}}]
```



Fundamental Processes in an electromagnetic Shower:

Bremsstrahlung

Pair Production

[Collision of electrons and positrons
with the electrons of the medium]

Bremsstrahlung

$$e + Z \rightarrow e + \gamma + Z$$

Pair Creation

$$\gamma + Z \rightarrow e^{+} + e^{-} + Z$$

$\boxed{\text{Bremsstrahlung}}$

$$e + Z \rightarrow e + \gamma + Z$$

$$e + \gamma_{\text{virtual}} \rightarrow e + \gamma$$

$\boxed{\text{Pair Creation}}$

$$\gamma + Z \rightarrow e^{+} + e^{-} + Z$$

$$\gamma + \gamma_{\text{virtual}} \rightarrow e^{+} + e^{-}$$

An electric field can be considered as an ensemble of “Virtual Photons”

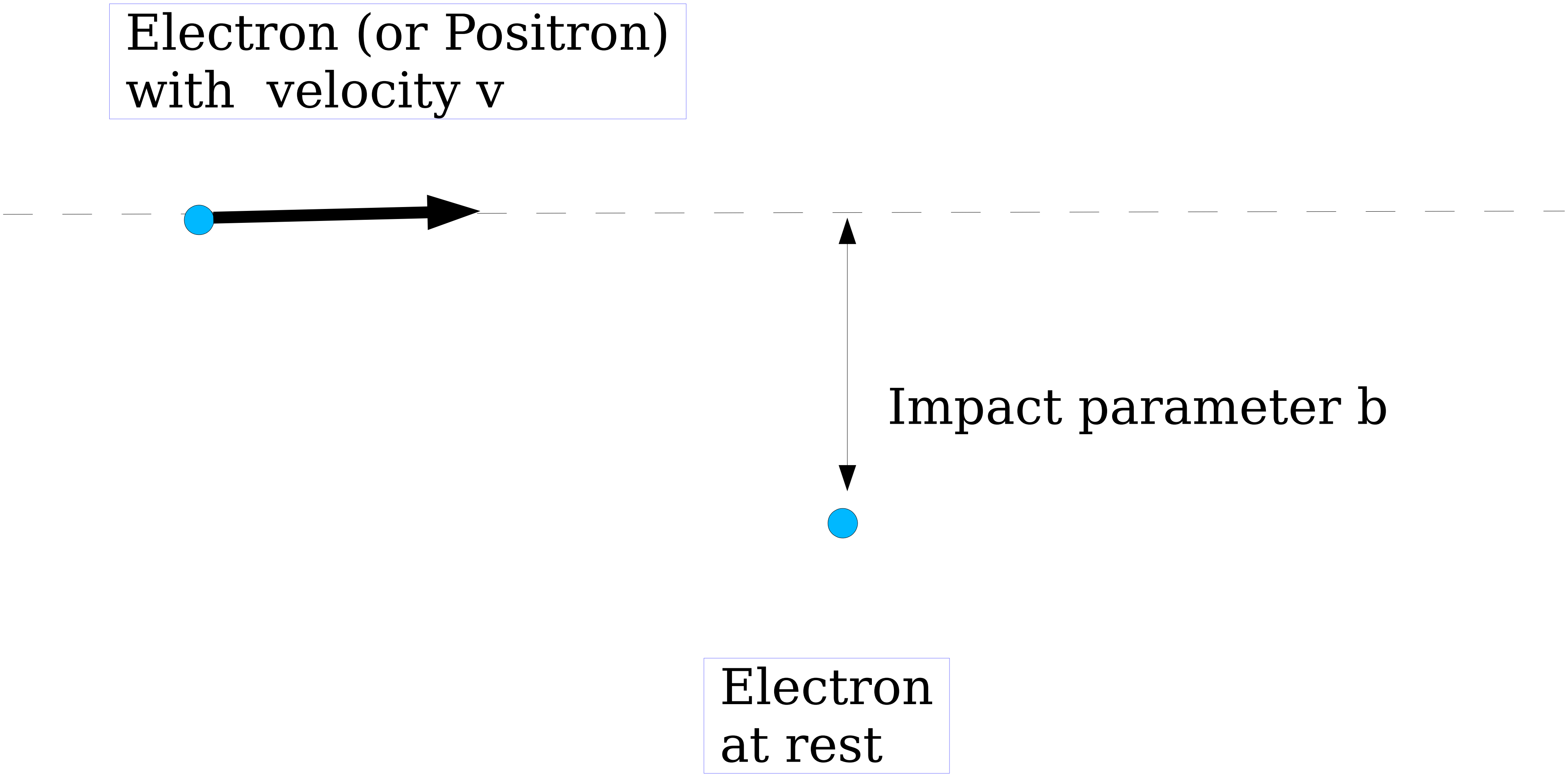
An electric field can be considered as an ensemble of "Virtual Photons"

Bremsstrahlung

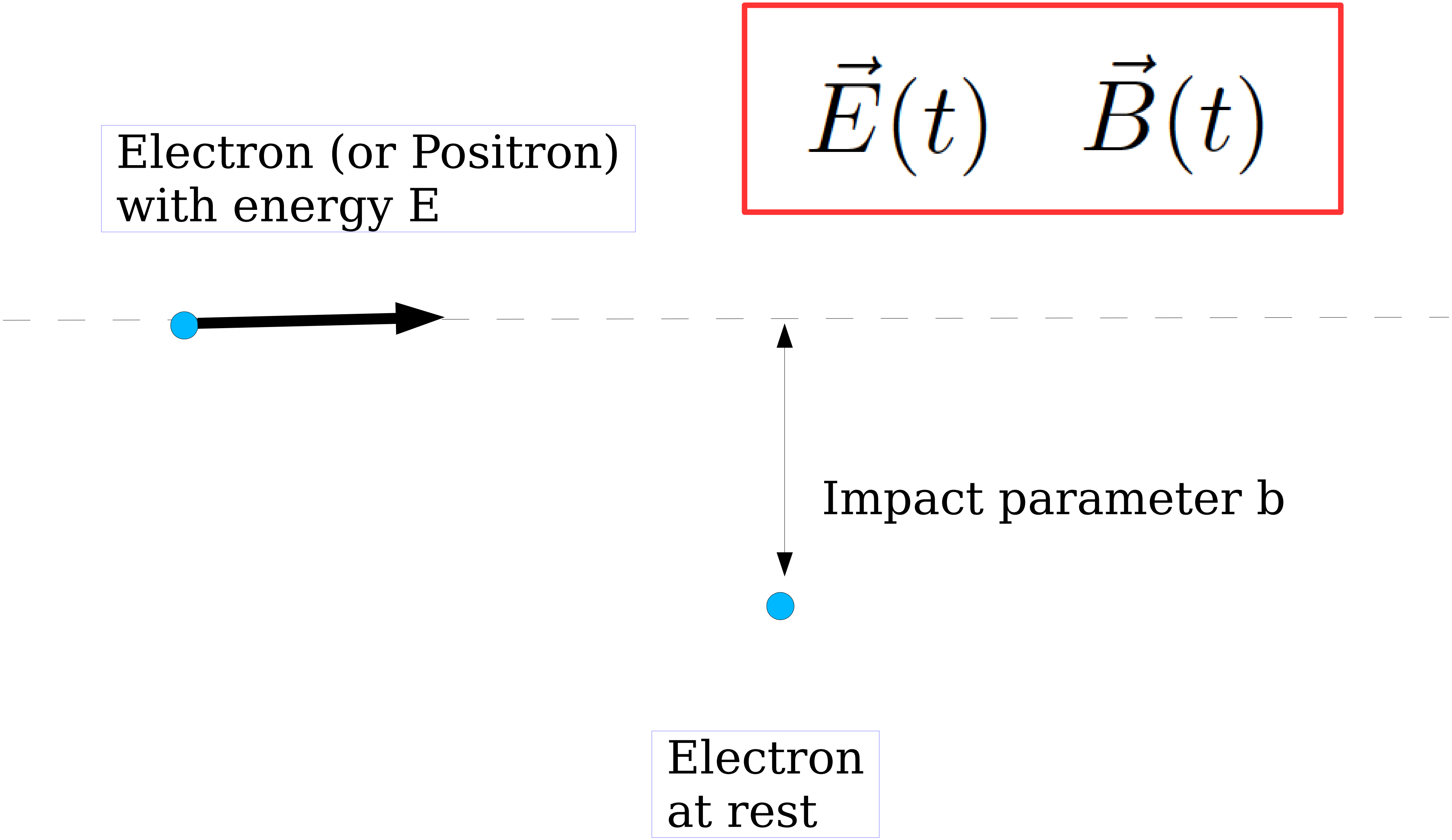
A relativistic electron traveling near an atomic nucleus sees a field of virtual photons and can undergo Compton scattering with these virtual photons

Pair Production

A photon traveling near an atomic nucleus sees a field of virtual photons and can undergo process of pair production "fusing" with a virtual phtoton



What field of photons sees the moving particle ?



What field of photons sees the moving particle ?

The particle sees time-varying electric + magnetic fields

Method of Virtual Photons

$$\vec{E} = q\,\frac{\vec{r}}{r^3}, \qquad \vec{B} = 0$$

Charged particle at rest
Particle at Rest

,

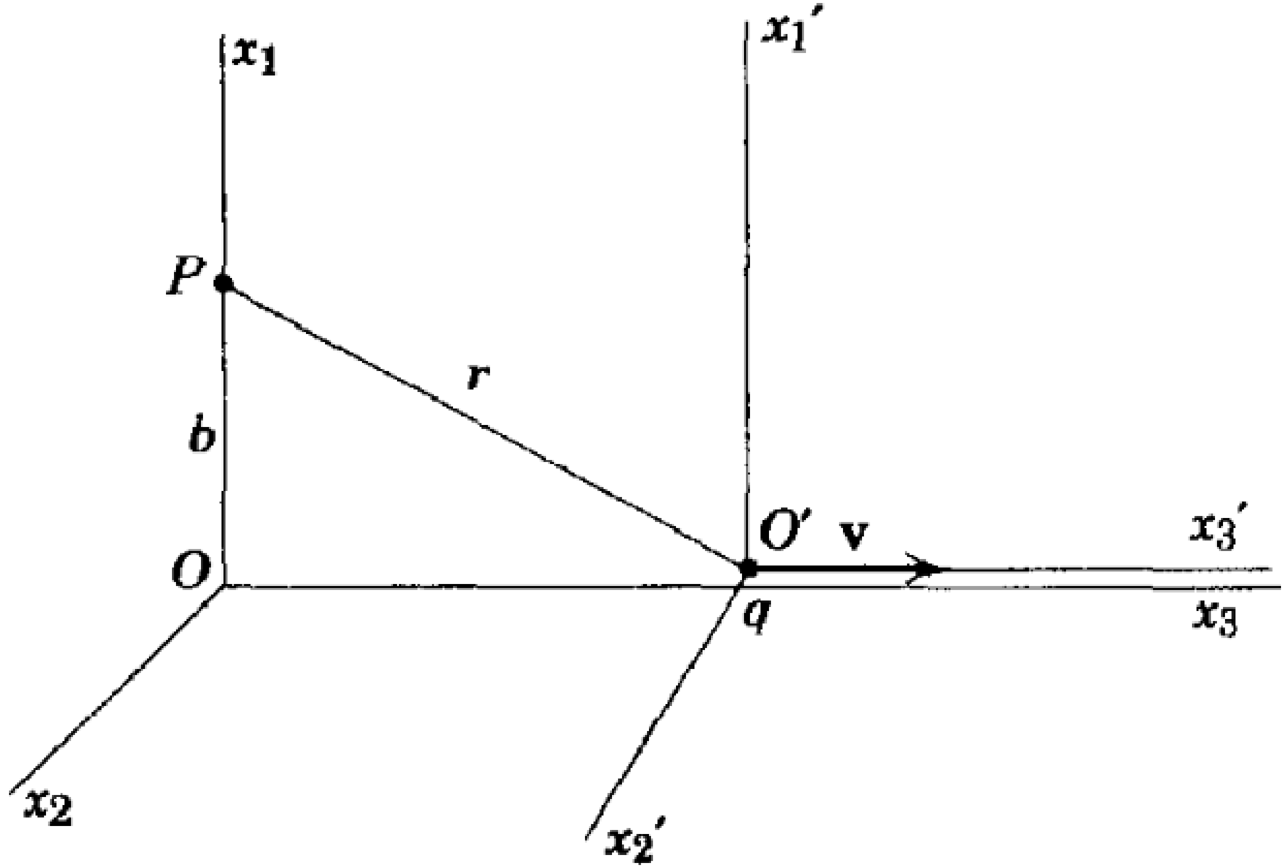
The particle moves with a constant velocity β (Lorentz factor γ) , passing by an observer placed at the observation point P with a minimum distance (impact parameter) b.

Question: What are the electric and magnetic field $(\vec{E}(t),\ \vec{B}(t))$ that the observer at P measures as a function of observer time t?

$$\boxed{(\vec{E}(t),\ \vec{B}(t))}$$

Method of Virtual Photons

E(t)
B(t)
At point P



Particle of charge q
Velocity v

Method of Virtual Photons

$$\vec{E} = q\,\frac{\vec{r}}{r^3}, \qquad \vec{B} = 0$$

Fields created by a charged particle at rest

(primed reference frame = rest frame system)

$$P' = \{x', y', z'\} = \{b, 0, -\beta\, t'\}$$

$$\vec{E}' = \{\frac{q\,b}{(r')^3}, \quad ,0, \quad -\frac{q\,\beta\,t}{(r')^3}\} \qquad \vec{B}' = 0$$

$$\vec{E}' = \{\frac{q\,b}{(b^2+\beta\,t')^{3/2}}, \quad 0, \quad -\frac{q\,\beta\,t}{(b^2+\beta\,t')^{3/2}}\}$$

Lorentz Transformation of electromagnetic Field

Lorentz Transformation of electromagnetic Field

$(E'_x,\, E'_y \neq 0)$

$$\boxed{\begin{aligned}\vec{E}' &= \{\gamma\, E_x, \quad 0 \quad E_z\} \\ \vec{B}' &= \{0, \quad -\beta\,\gamma\, E_x, \quad 0 \quad \}\end{aligned}}$$

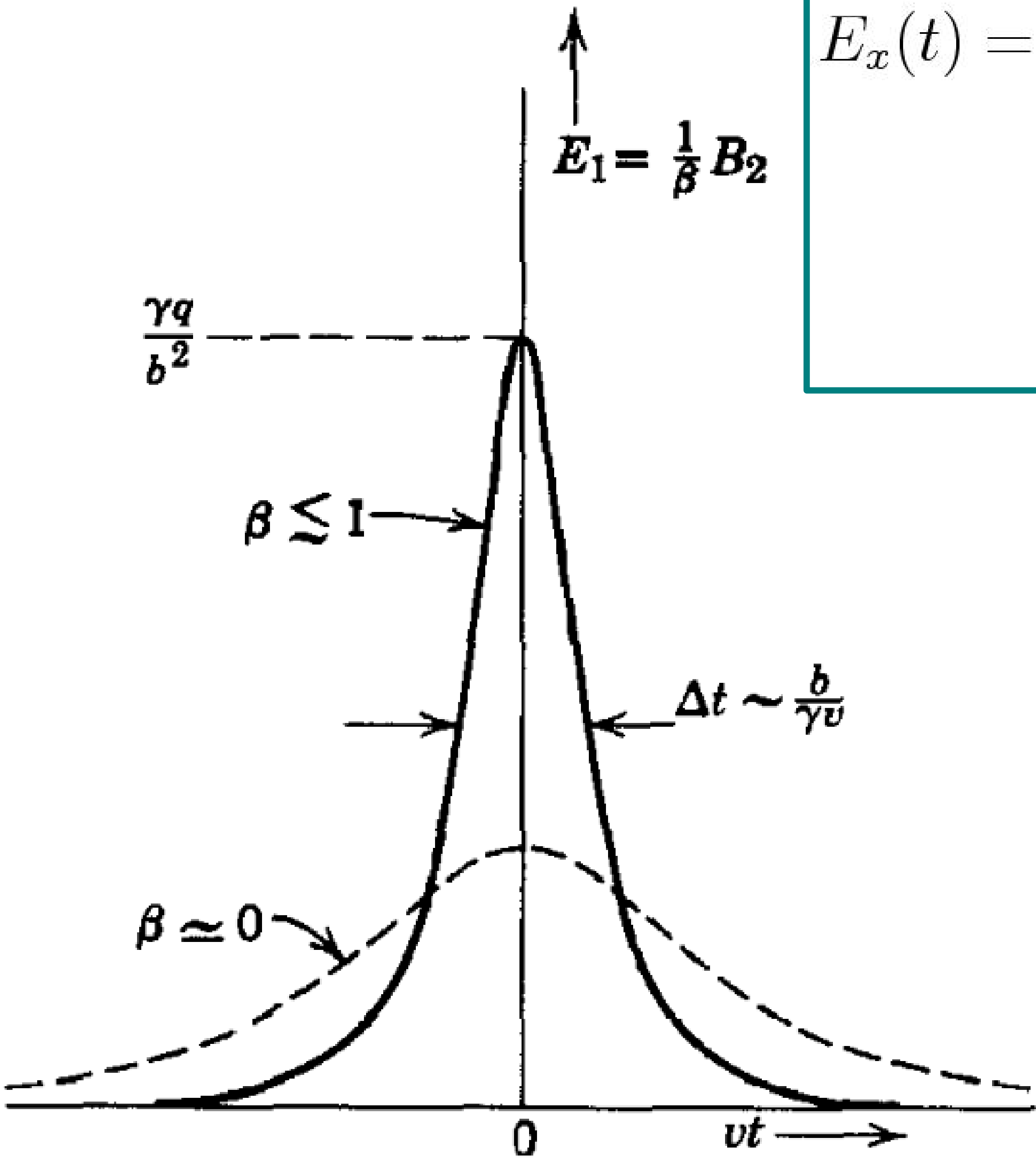
$$\vec{E} = \{\frac{q\, b\, \gamma}{[b^2 + (\beta t')^2]^{3/2}} \quad , 0, \qquad - \frac{q\, (\beta\, t)}{(b^2 + [\beta t')^2]^{3/2}}\}$$

$$\vec{B} = \{0, \qquad \frac{q\, b\, \beta\, \gamma}{[b^2 + (\beta t')^2]^{3/2}} \quad , 0\}$$

$$t' = \gamma\, t$$

$$\boxed{E_x(t) = \frac{q\, b\, \gamma}{[b^2 + (\beta\, \gamma\, t)^2]^{3/2}}}$$

$$\vec{E} \perp \vec{B}$$
$$\vec{E} \perp \hat{v}$$
$$\vec{B} \perp \hat{v}$$



$$E_x(t) = \frac{q\,b\,\gamma}{[b^2 + (\beta\,\gamma\,t)^2]^{3/2}}$$

The observer at P, that sees a relativistic particle "zipping by" at relativistic speeed ($\beta \simeq 1, \quad \gamma \gg 1$) sees an electromagnetic field that is undistinguishable from an electromagnetic plane wave propagating along the z direction. $\vec{E}$, $\vec{B}$, $\hat{v}$ mutually perpendicular.

Fourier analysis of the electromagnetic wave

$$E_\omega = \frac{1}{2\pi} \int dt\; e^{i\omega t}\; E(t)$$

$$E_x(t) = \frac{q\, b\, \gamma}{[b^2 + (\beta\, \gamma\, t)^2]^{3/2}}$$

$$E_\omega = \frac{1}{2\pi} \int dt\; e^{i\omega t}\; \frac{q\, \gamma\, b}{(b^2 + \gamma^2\, t^2)^{3/2}}$$

$$E_\omega = \frac{q}{2\pi}\, \frac{b}{b^2} \int dt\; e^{i\omega t}\; \frac{1}{[1 + ((\gamma/b)\, t)^2]^{3/2}}$$

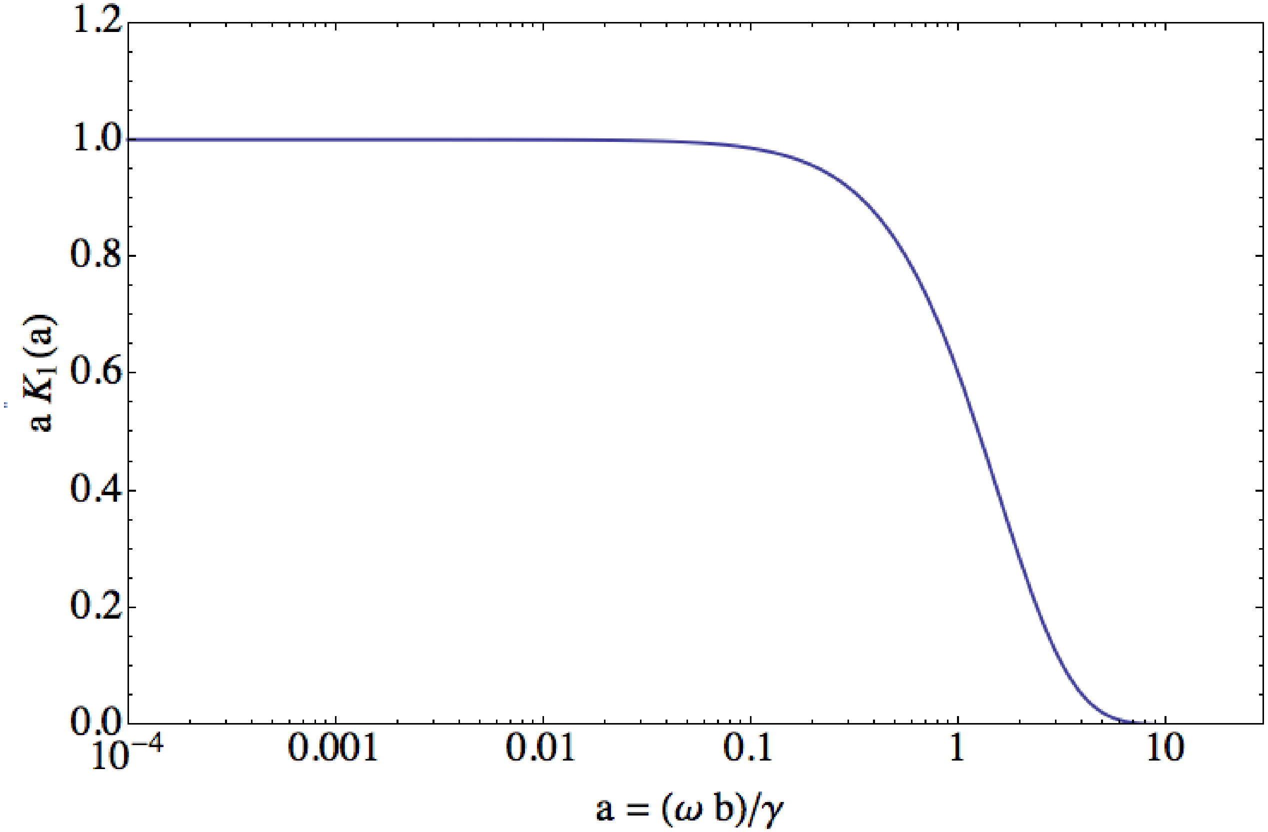
$$x = \frac{\gamma}{b}\, t$$

$$\boxed{E_\omega = \frac{q}{2\pi\, b} \int dx\; e^{i\,(\omega\, b/\gamma)\, x}\; \frac{1}{(1 + x^2)^{3/2}}}$$

$$\boxed{E_\omega = \frac{q}{2\,\pi\,b}\int dx\; e^{i\,(\omega\,b/\gamma)\;x}\;\frac{1}{(1+x^2)^{3/2}}}$$

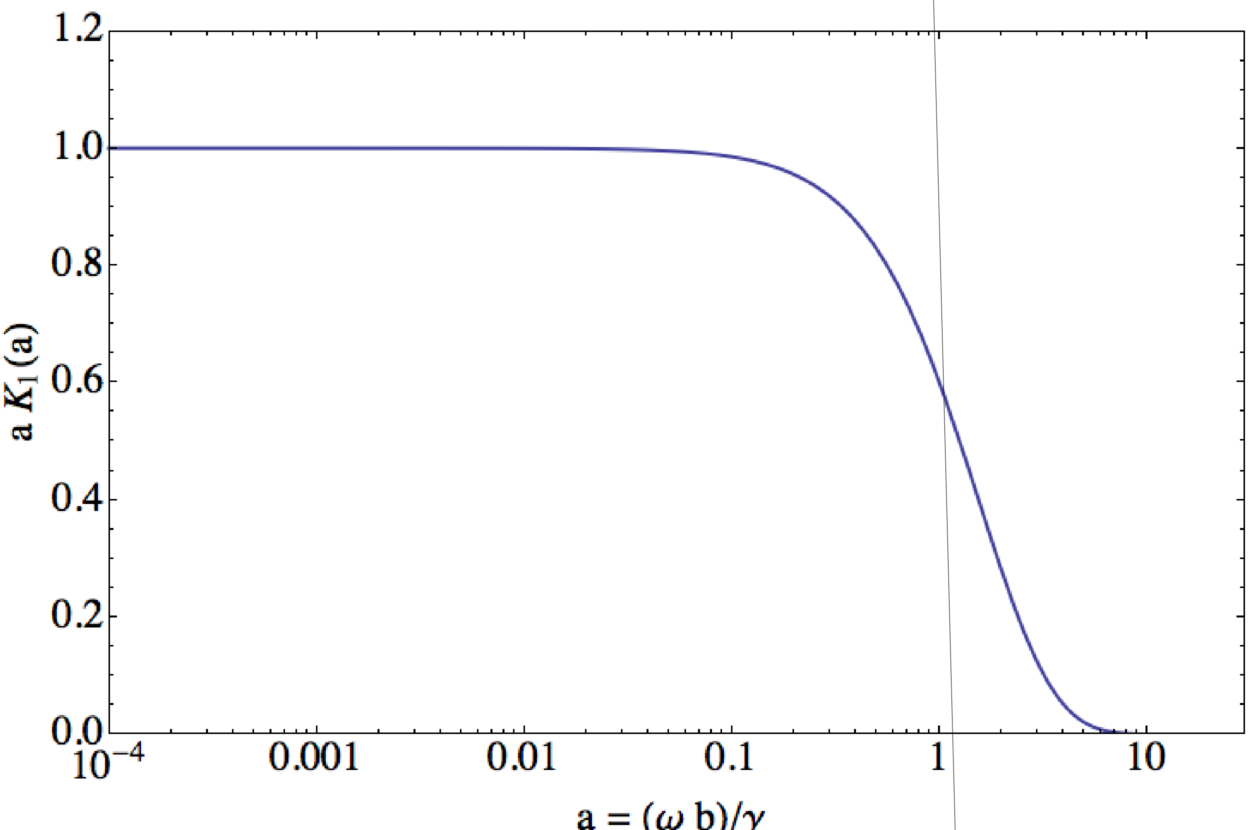
$$\int dt\; e^{i\,a\,x}\;\frac{1}{(1+x^2)^{3/2}} = 2\;a\;K_1(a)$$

$$\int dx\;\frac{1}{(1+x^2)^{3/2}} = 2$$



$$E_\omega \simeq \frac{q}{\pi \ b}$$

$$E_\omega \simeq 0$$



Energy Fluence : Energy/(unit Area)

$$\int d\omega \ |E(\omega)|^2 = \int dt \ |E(t)|^2$$

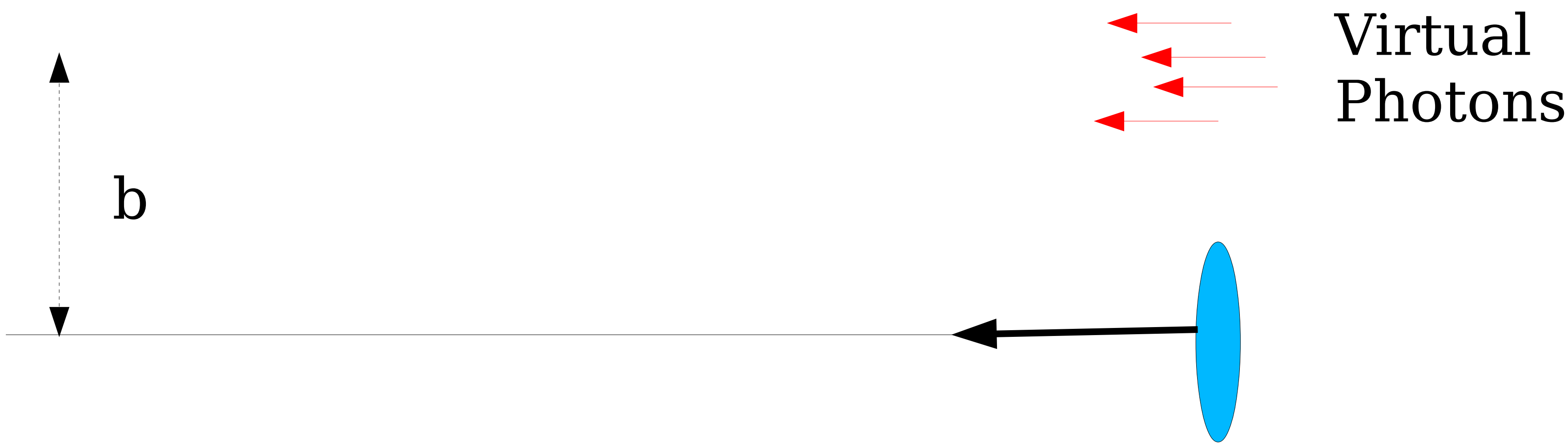
$$|E(\omega)|^2 = \frac{d\mathcal{E}}{d\omega \ dA} = \frac{dN_\gamma^{\rm virtual}}{d\omega \ dA} \ (\hbar \ \omega)$$

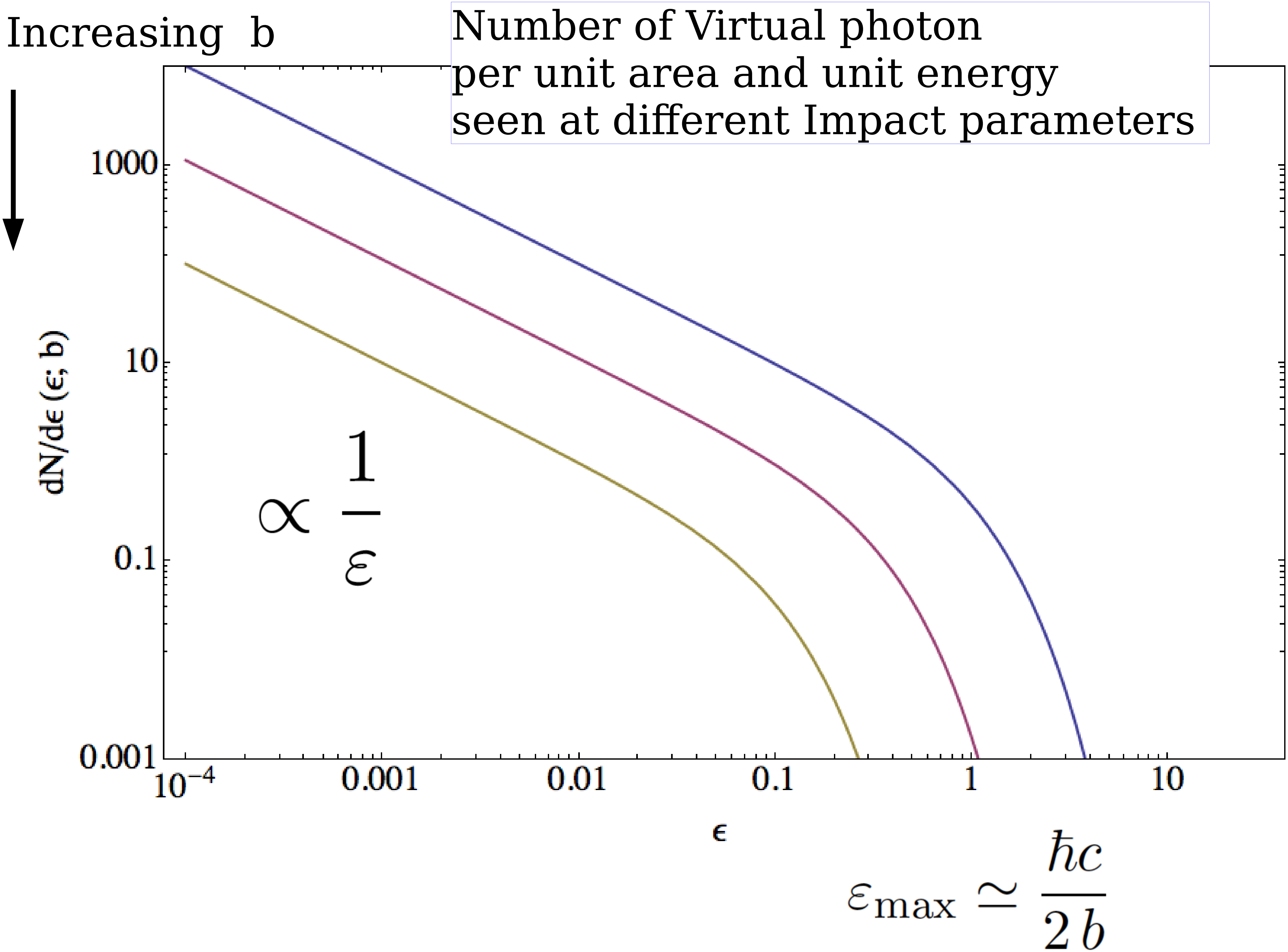
$$\boxed{\frac{dN_\gamma^{\rm virtual}}{d\omega \ dA} = \frac{q^2}{\pi^2} \frac{1}{b^2} \ \frac{1}{\omega}}$$

$$\frac{dN_\gamma^{\rm virtual}}{d\omega \ dA} \simeq 0$$

Electron/Photon interaction with a nucleus.

Can be seen as an interaction with a virtual photon of energy ε





Increasing b
Number of Virtual photon
per unit area and unit energy
seen at different Impact parameters
dN/dε (ε; b)
1000
10
0.1
0.001
10^{-4}
0.001
0.01
0.1
1
10
ε
$\propto \frac{1}{\varepsilon}$
$\varepsilon_{\max} \simeq \frac{\hbar c}{2\,b}$

$$\frac{dN_{\gamma}^{\text{virtual}}}{d\omega\ dA} = \frac{q^2}{\pi^2}\frac{1}{b^2}\ \frac{1}{\omega}$$

Extends only up to a maximum impact parameter

$$E_\omega = \frac{q}{2\,\pi\,b}\int dx\; e^{i\,(\omega\,b/\gamma)\;x}\;\frac{1}{(1+x^2)^{3/2}}$$

$$e^{i\,(\omega\,b/\gamma)\;x}$$

$$b_{\max} = \frac{\gamma}{\omega}$$

Considering the “Target Rest Frame

$$\varepsilon = \frac{\omega}{2\,\gamma}$$

$$b_{\max} = \frac{1}{2\,\varepsilon}$$

$$\boxed{\langle r_{\mathrm{atomo}}\rangle \propto \frac{\xi}{m\,\alpha}\;Z^{-1/3} \propto \underbrace{183\;\;Z^{-1/3}}_{\mathrm{m}}}$$

Total Pair Production Cross section

$$\sigma_{\gamma\to e^+e^-}(K) = \int d^2b \int d\varepsilon \; \frac{dN_\gamma^{\mathrm{virtual}}}{dA\; d\omega} \; \sigma_{\gamma\gamma\to e^+e^-}(\hat{s})$$

$$\hat{s} = 4\,K\,\varepsilon$$

Total Pair Production Cross section

$$\sigma_{\gamma\to e^+e^-}(K) = \int d^2b \int d\varepsilon \, \frac{dN_\gamma^{\mathrm{virtual}}}{dA\, d\omega} \, \sigma_{\gamma\gamma\to e^+e^-}(\hat{s})$$

$$(2\,\pi) \int db\; b \int d\varepsilon \left\{ \frac{\alpha\, Z^2}{\pi^2\, b^2}\, \frac{1}{\varepsilon}\, \Theta[b_{\max}(\varepsilon) - b] \right\} \quad \sigma_{\gamma\gamma}(4\,K\,\varepsilon)$$

$$b_{\max}(\varepsilon) \simeq \frac{1}{2\,\varepsilon}$$

$$b_{\max} \simeq R_{\mathrm{atom}}$$

$$\sigma_{\gamma\to e^+e^-}(K) = \frac{\alpha\ Z^2}{\pi^2}\ \left[(2\,\pi)\ \int_{b_{\min}}^{b_{\max}} \frac{db}{b}\right]$$

$$\int_{4\,m^2/K}^{\infty} \frac{d\varepsilon}{\varepsilon}\ \sigma_{\gamma\gamma}(4\,K\,\varepsilon)$$

$$\sigma_{\gamma\to e^+e^-}(K) = \frac{\alpha\ Z^2}{\pi^2}\ \left[(2\ \pi)\ \int_{b_{\min}}^{b_{\max}} \frac{db}{b}\right]$$

$$\int_{4\,m^2/K}^{\infty} \frac{d\varepsilon}{\varepsilon}\ \sigma_{\gamma\gamma}(4\,K\,\varepsilon)$$

$$\frac{2\,\alpha\ Z^2}{\pi}\ \log\left[\frac{b_{\max}}{b_{\min}}\right]\ \int_{4\,m^2}^{\infty} \frac{ds}{s}\ \sigma_{\gamma\gamma}(s)$$

$$\boxed{\int_{4\,m^2}^{\infty} \frac{ds}{s}\sigma_{\gamma\gamma\to e^+e^-}(s) = (2\pi\ r_0^2)\ \frac{7}{9}}$$

$$\sigma_{\gamma \to e^+ e^-}(K) = \frac{2\,\alpha\ Z^2}{\pi}\ \log\left[\frac{b_{\rm max}}{b_{\rm min}}\right] \quad (2\pi\ r_0^2)\ \frac{7}{9}$$

Estimate of bmax and bmin

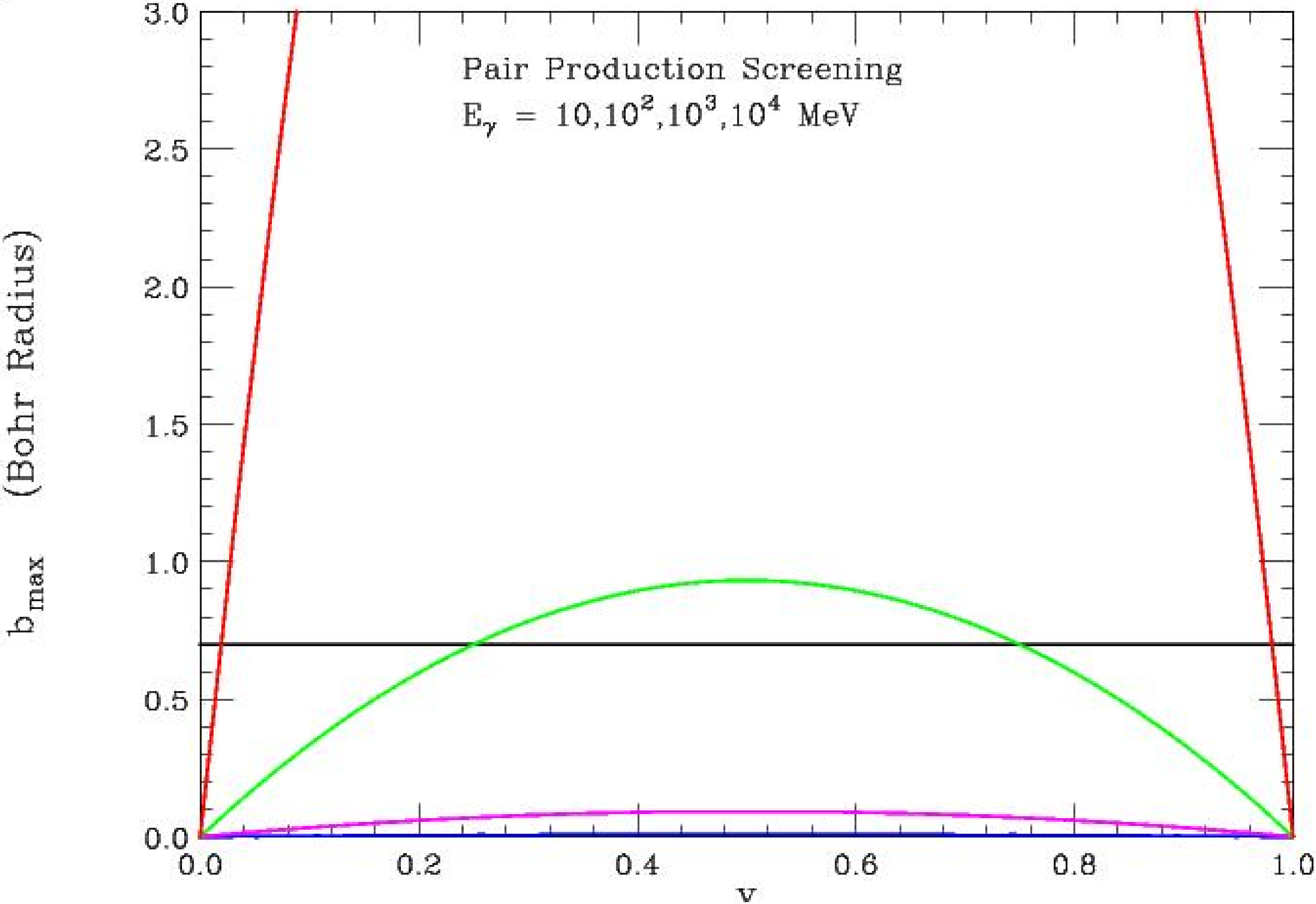
$$b_{\rm min} \simeq \frac{\hbar}{m_e\, c}$$

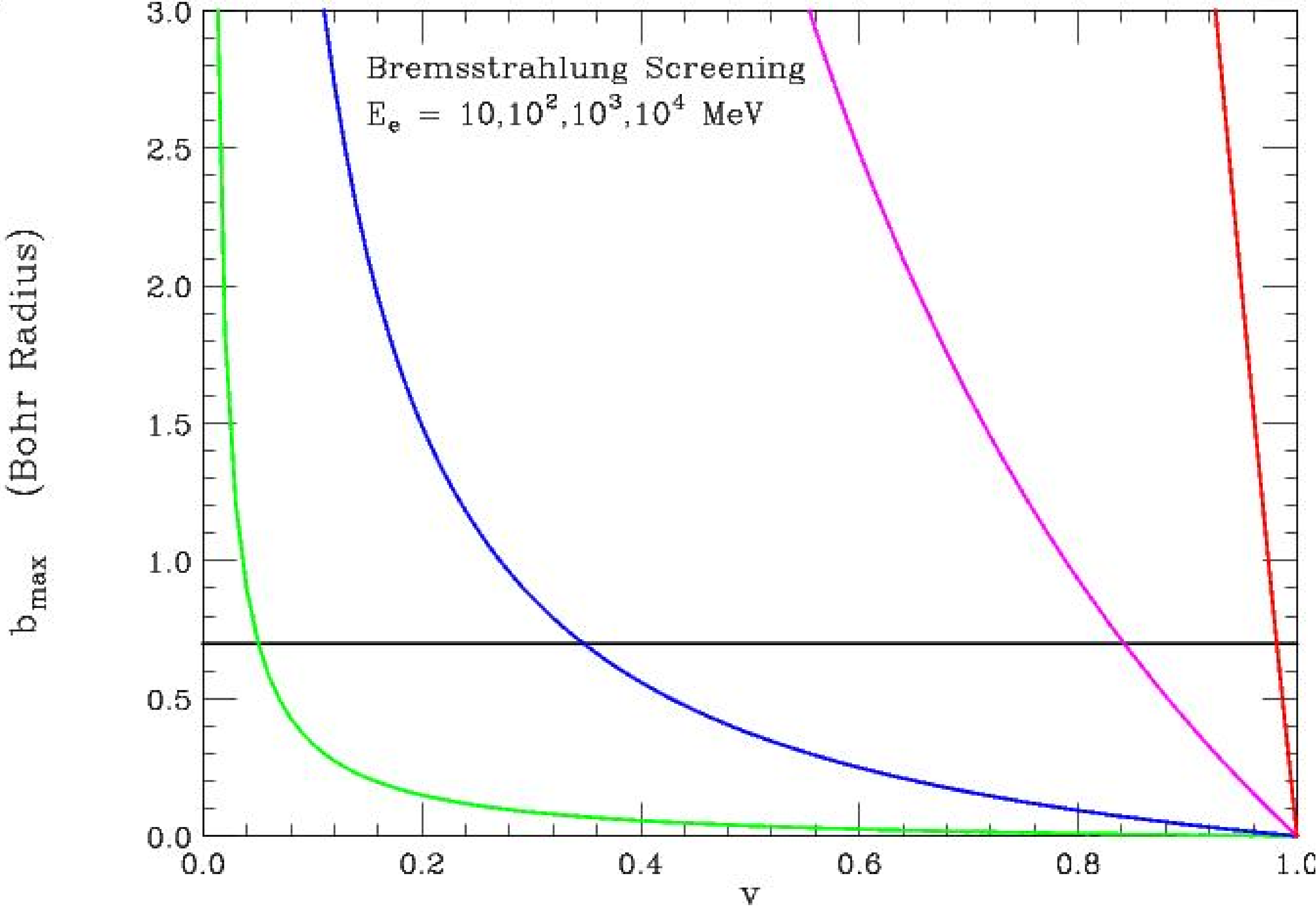
Quantum mechanical
Effect
(uncertainty principle)

$$\frac{d\sigma_{\text{brems}}}{dv}(v, E_e) = \int_{\varepsilon_{\min}(v,E_e)}^{\infty} d\varepsilon \; [\ldots]$$

$$\frac{d\sigma_{\text{pair}}}{du}(u, E_\gamma) = \int_{\varepsilon_{\min}(u,E_e)}^{\infty} d\varepsilon \; [\ldots]$$

$$\boxed{\begin{aligned} \varepsilon_{\min}(v, E_e) &= \frac{m^2}{4\,E_e}\,\frac{v}{1-v} \\[2ex] \varepsilon_{\min}(u, E_\gamma) &= \frac{m^2}{4\,E_e\,u(1-u)} \end{aligned}}$$


Pair Production Screening
E_γ = 10,10^2,10^3,10^4 MeV
b_{max} (Bohr Radius)
v
0.0
0.5
1.0
1.5
2.0
2.5
3.0
0.0
0.2
0.4
0.6
0.8
1.0


Bremsstrahlung Screening
$E_e = 10, 10^2, 10^3, 10^4$ MeV
b_{max} (Bohr Radius)
v
3.0
2.5
2.0
1.5
1.0
0.5
0.0
0.0
0.2
0.4
0.6
0.8
1.0

$$\sigma^{\mathrm{air}}_{\gamma\to e^+e^-} \simeq 5.0 \times 10^{-25}\ \mathrm{cm}^2$$

$$\lambda_{\mathrm{pair}} \simeq \frac{\langle m \rangle}{\sigma^{\mathrm{air}}_{\gamma\to e^+e^-}} \simeq 47\ \frac{\mathrm{g}}{\mathrm{cm}^{-2}}$$

$$N_\gamma(X) = N(0)\ e^{-X/\lambda_{\mathrm{pair}}}$$

$$\sigma[e \to e\gamma] \to \infty$$

$$\left.\frac{d\sigma}{dE_\gamma}\right|_{e\to e\gamma}(E_\gamma, E_e) = \text{finite}$$

diverges only
for $E_\gamma \to 0$

$\propto E_\gamma^{-1}$

$$\left.\frac{d\sigma}{dv}\right|_{e\to e\gamma}(v) = \text{finite}$$

$$v = \frac{E_\gamma}{E_v}$$

Independent from
the electron energy !

$$\int_0^{E_e} dE_\gamma \ \frac{d\sigma}{dE_\gamma}(E_\gamma; E_e) \to \infty$$

Divergent cross section

$$-\frac{dE}{dX} = \frac{1}{m} \int_0^{E_e} dE_\gamma \ E_\gamma \ \frac{d\sigma}{dE_\gamma}(E_\gamma; E_e) = \frac{E_e}{\lambda_{\rm rad}}$$

Finite energy Loss]

$$\frac{E_e}{m} \int_0^{E_e} dv \ v \ \frac{d\sigma}{dv}(v) = \frac{E_e}{\lambda_{\rm rad}}$$

$$\left.\frac{dE}{dX}\right|_{\mathrm{brems}} = \frac{N_A}{A} \int dE_\gamma \; E_\gamma \; \frac{d\sigma}{dE_\gamma}(E_\gamma, E_e)$$

$$\frac{dE}{dX} = \left\{ \frac{N_A}{A} \int_0^1 dv \; v \; \frac{d\sigma}{dv}(v) \right\} \; E = \frac{E}{\lambda_{\mathrm{rad}}}$$

$$\lambda_{\mathrm{rad}} = \frac{N_A}{A} \, 4\,\alpha \; r_0^2 \; \left\{ \ln[183 \; Z^{-1/3}] + \frac{1}{18} \right\}$$

$$\boxed{\lambda_{\mathrm{rad}} = \frac{N_A}{A} \, 4\,\alpha \; r_0^2 \; \ln[183 \; Z^{-1/3}]}$$

Radiation
Length

Physical Meaning of the "Radiation Length"

$$\langle E_e(X) \rangle = E_e(0)\ e^{-X/\lambda_{\rm rad}}$$

Physical Meaning of the "Pair Length"

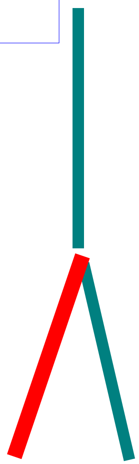
$$N_\gamma(X) = N(0)\ e^{-X/\lambda_{\rm pair}}$$

$$\lambda_{\rm rad}^{\rm air} \simeq 37\ \frac{\rm g}{\rm cm^2}$$

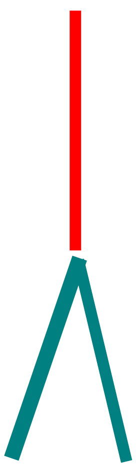
$$\lambda_{\rm pair} \simeq \frac{9}{7}\ \lambda_{\rm rad}$$

The "SPLITTING FUNCTIONS"

$$\varphi(v) = \left[\frac{d\sigma}{dv}(v)\right]_{\text{brems}} \left(\frac{N_A}{A}\,\lambda_{\text{rad}}\right)$$



$$\psi(u) = \left[\frac{d\sigma}{du}(u)\right]_{\text{pair}} \left(\frac{N_A}{A}\,\lambda_{\text{rad}}\right)$$



BREMSSTRAHLUNG

Fully ionized free nucleus (approximation of infinite mass)

$$\left.\frac{d\sigma}{d\varepsilon}\right|_{e\to e+\gamma}(v;E) = 4\,Z^2\,\alpha\,r0^2$$

$$\frac{1}{v}\;\left[1+(1-v^2)-\frac{2}{3}(1-v)\right]\;\left[\ln\left(\frac{2\,E}{m}\;\frac{v}{1-v}\right)-\frac{1}{2}\right]$$

High Energy Limit (Full screening)

$$v = \frac{E_\gamma}{E_e}$$

$$\left.\frac{d\sigma}{d\varepsilon}\right|_{e\to e+\gamma}(v;E) = 4\,Z^2\,\alpha\,r0^2$$

$$\frac{1}{v}\;\left\{\left[1+(1-v^2)-\frac{2}{3}(1-v)\right]\;\ln\left(183\,Z^{-1/3}\right)+\frac{1}{9}(1-v)\right\}$$

PAIR PRODUCTION

Fully ionized free nucleus (approximation of infinite mass)

$$\left.\frac{d\sigma}{du}\right|_{\gamma\to e^+e^-}(u;K) = 4\ Z^2\ \alpha\ r0^2$$

$$\left[u^2 + (1-u)^2 + \frac{2}{3}u\,(1-v)\right]\ \left[\ln\left(\frac{2\ K}{m}\ u(1-u)\right) - \frac{1}{2}\right]$$

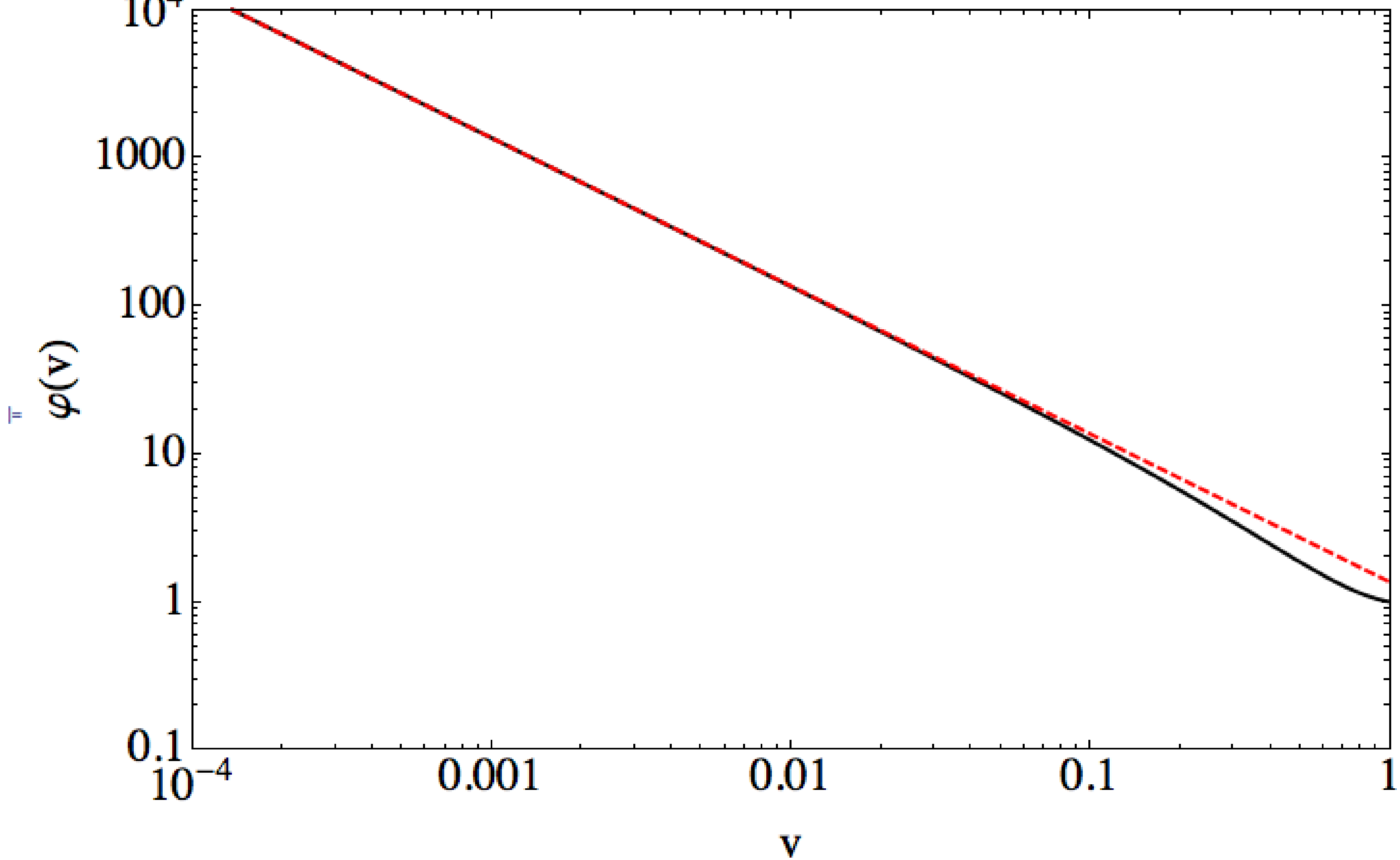
High Energy Limit (Full screening)

$$u = \frac{E_{e^+}}{E_\gamma}$$

$$\left.\frac{d\sigma}{du}\right|_{\gamma\to e^+e^-}(u;K) = 4\ Z^2\ \alpha\ r0^2$$

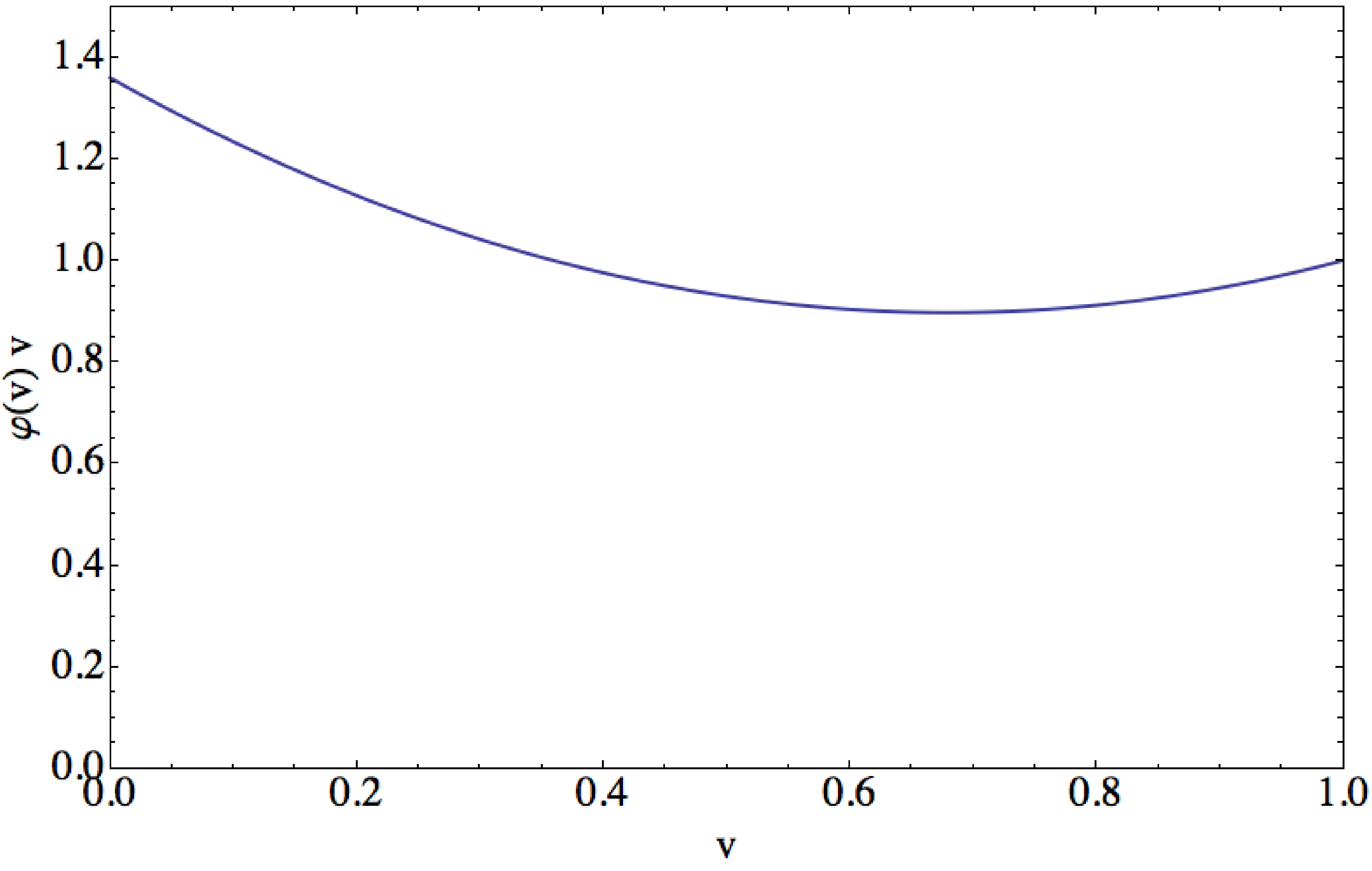
$$\left\{\ \left[u^2 + (1-u)^2 + \frac{2}{3}u\,(1-v)\right]\ \ln\left(183\,Z^{-1/3}\right) - \frac{1}{9}u(1-u)\right\}$$

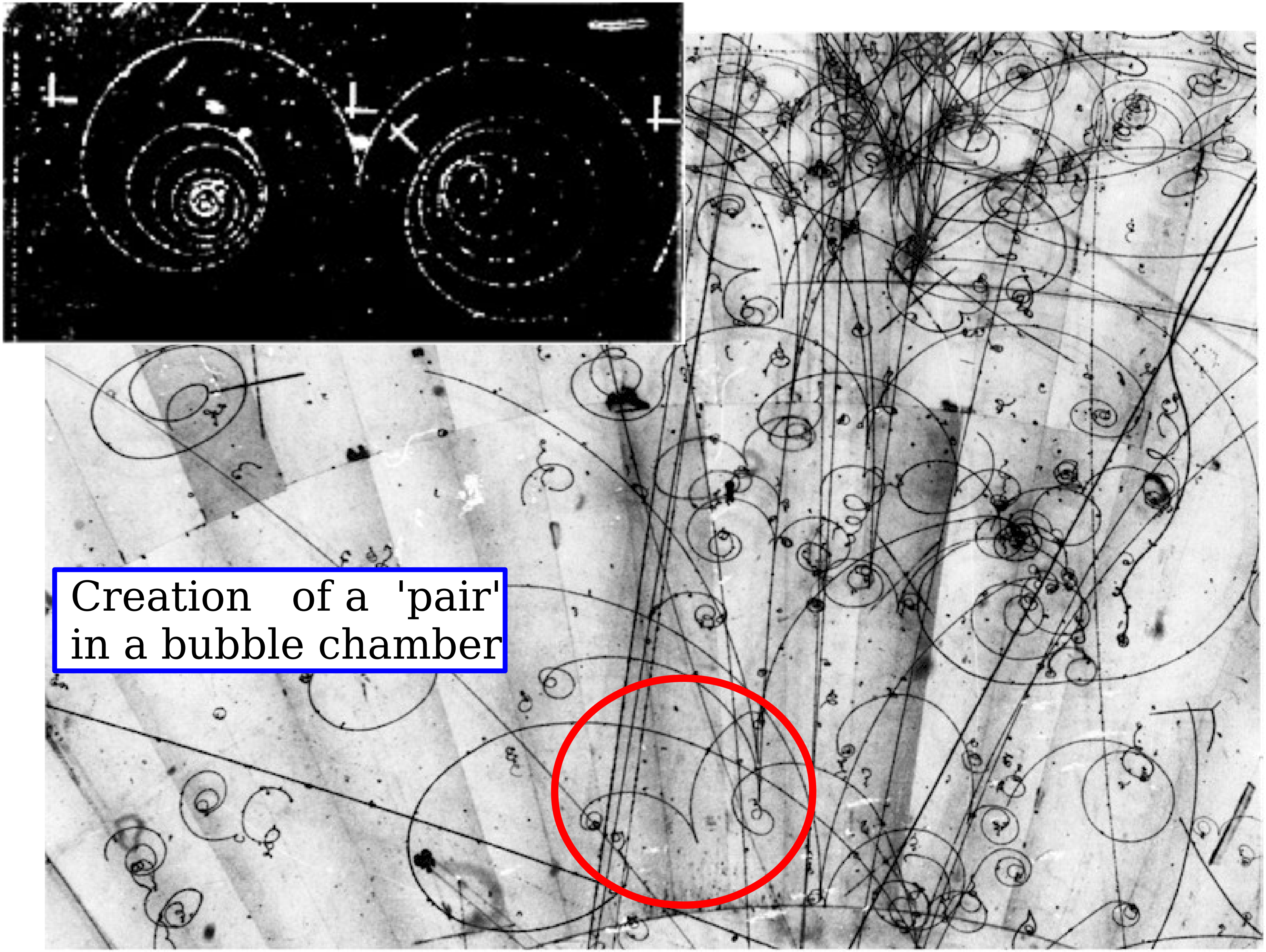
$\varphi(v)$ Bremsstrahlung



$\varphi(v)$

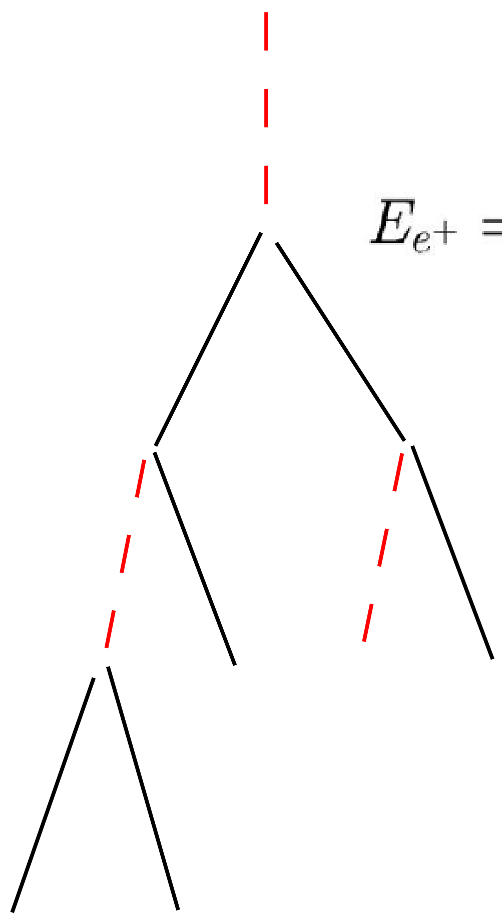
Bremsstrahlung





Creation of a 'pair' in a bubble chamber

ELECTROMAGNETIC SHOWERS



$$E_{e^+} = E_\gamma \; u$$

$$E_\gamma = E_e \; v$$

$\psi(u)$ Pair Production

$\varphi(v)$ Brems-strahlung

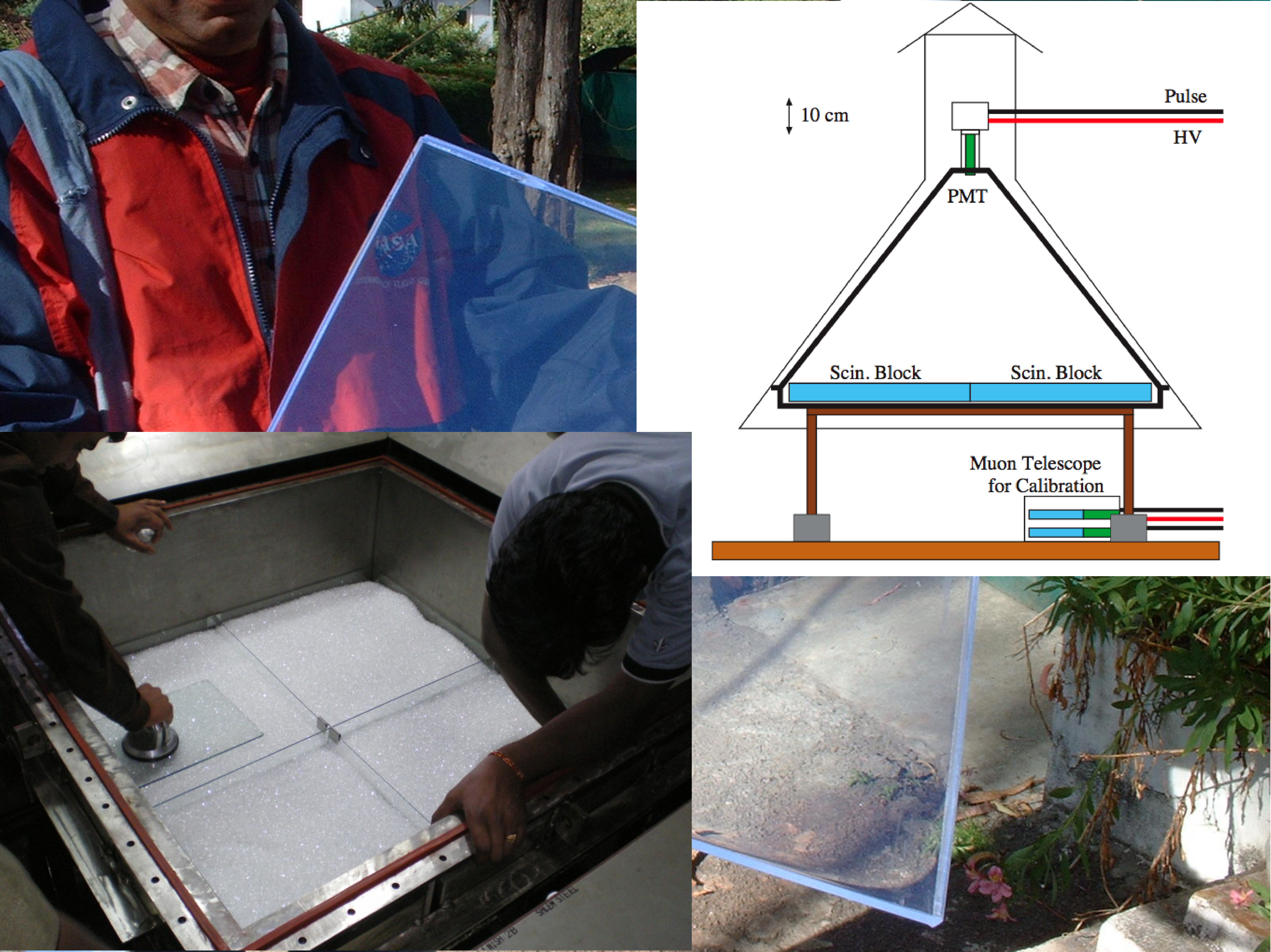
Radiation Length
(Energy independent)

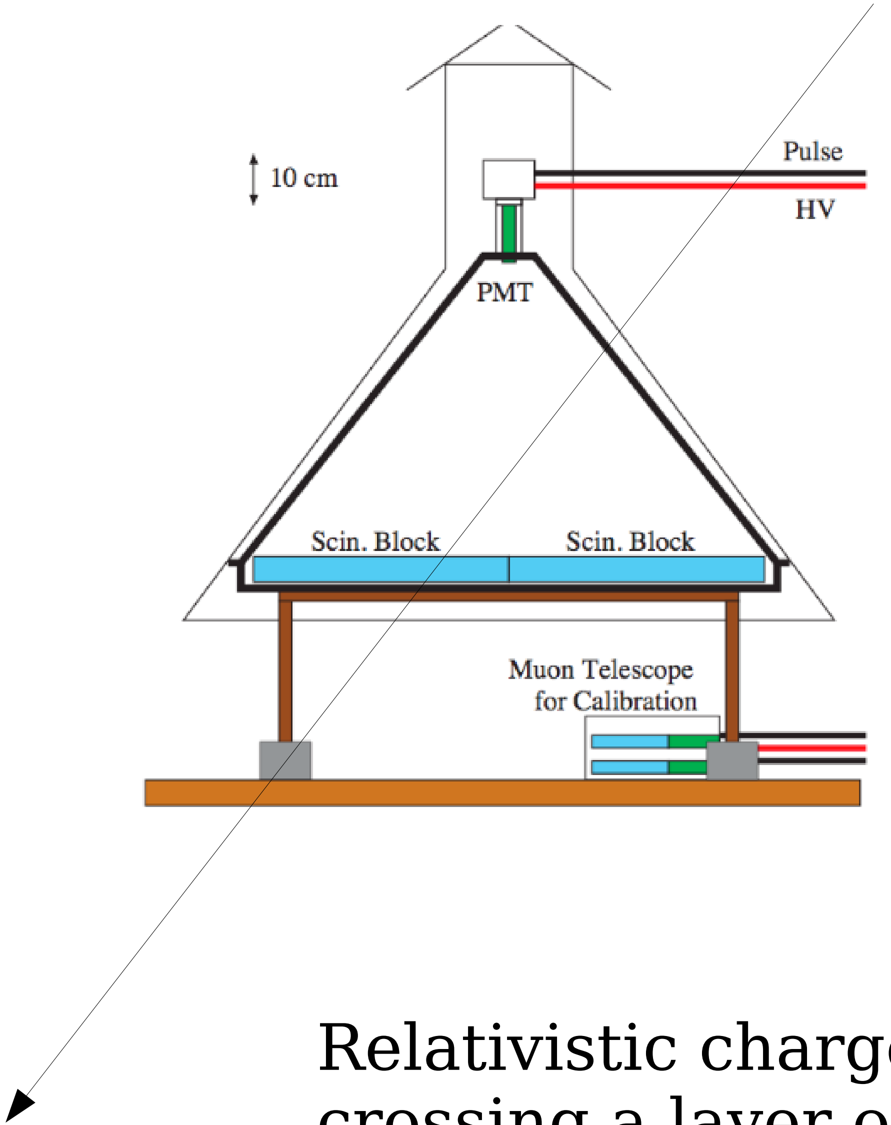
Vertices :
theoretically understood
(and scaling)

Energy Loss of electrons and positrons via collisions with the electron of the medium

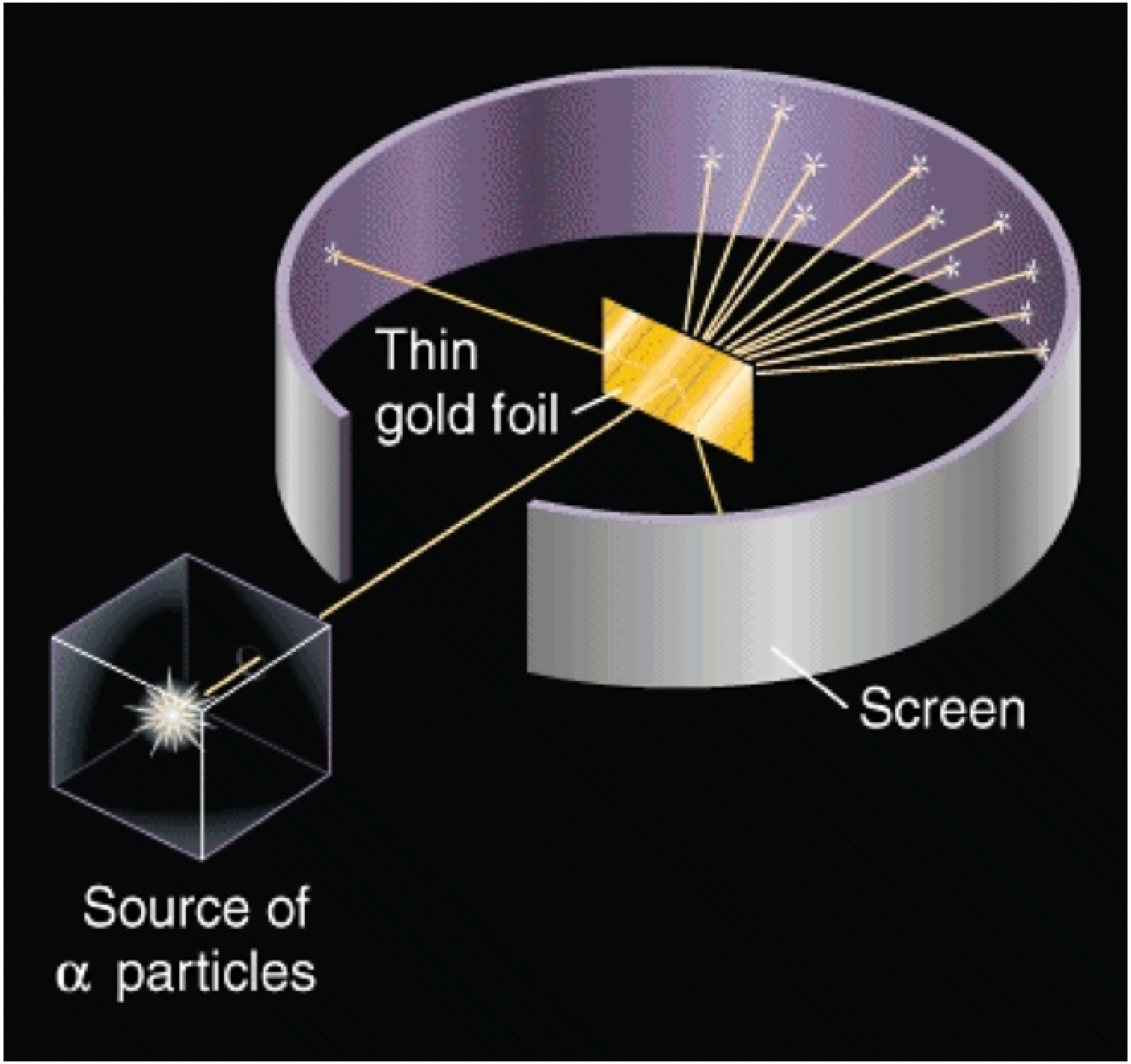

GRAPES Experiment


15


Pulse
10 cm
HV
PMT
Scin. Block
Scin. Block
Muon Telescope
for Calibration

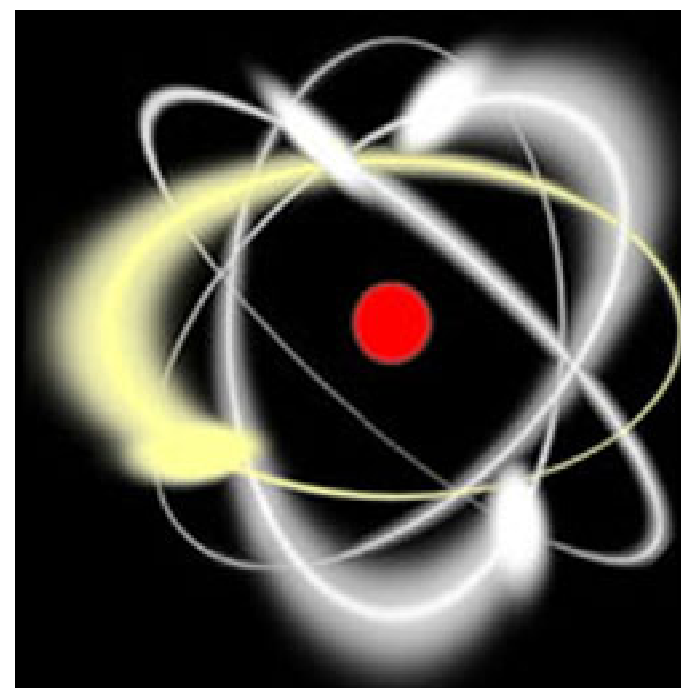


Relativistic charged particle
crossing a layer of material
deposits energy   ionizing the atoms
of the atomos



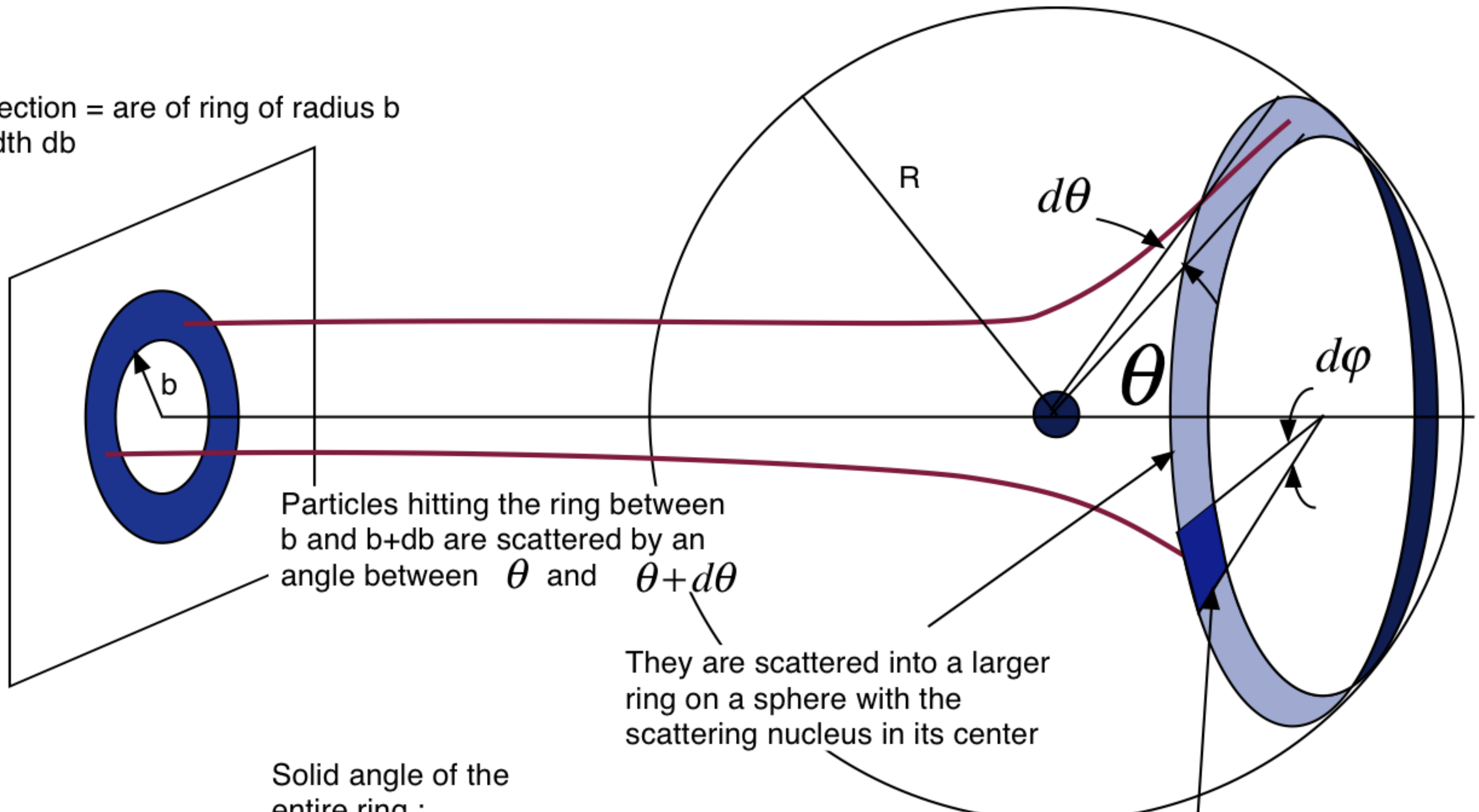
Rutherford
Experiment

[Identification of
the atomic nucleus]



Rutherford and Geiger

Scattering between two point particles With electric charge



Solid angle of the entire ring :

$$d\Omega=\frac{2\pi R\sin(\theta)Rd\theta}{R^2}=2\pi\sin(\theta)d\theta$$

solid angle of small area:

$$d\Omega=\frac{d\varphi R\sin(\theta)Rd\theta}{R^2}=\sin(\theta)d\theta d\varphi$$

Rutherford Cross section for the scattering between charged particles, one moving with velocity v the other at rest

$$\left.\frac{d\sigma}{d\Omega}\right|^{\text{Rutherford}} = \frac{(q_1\, q_2)^2}{m_e^2\, v^4\, (1-\cos\theta)^2}$$

$$e^- \; p \to e^- \; p$$

$$\left.\frac{d\sigma}{d\Omega}\right|_{ep\to ep}^{\text{Rutherford}} = \frac{(q_1\, q_2)^2}{m_e^2\, v^4\, (1-\cos\theta)^2}$$

$$\boxed{\begin{array}{l} \left.\dfrac{d\sigma}{d\Omega}\right|_{ep\to ep}^{\text{Rutherford}} = \left(\dfrac{q_1\, q_2}{Q^2}\right)^2 4m_e^2 \\[2em] Q^2 = |\vec{p}_i - \vec{p}_f|^2 \qquad\qquad 0 \le Q^2 \lesssim (2m\, v)^2 \end{array}}$$

$$\left.\frac{d\sigma}{d\Omega}\right|_{\mathrm{Rutherford}} = \frac{(q_1\, q_2)^2}{m^2\, v^4\, (1-\cos\theta)^2}$$

$$m^2\, v^4 = (mv \times v)^2 = (pv)^2$$

$$\left.\frac{d\sigma}{d\Omega}\right|_{\mathrm{Rutherford}} = \frac{(q_1\, q_2)^2}{p^2\, v^2\, (1-\cos\theta)^2}$$

$$Q^2 = -(p - p')^2$$

$$= -(E - E')^2 + (\vec{p} - \vec{p'})^2$$

$$Q^2 \simeq 2p^2(1 - \cos\theta) \simeq 2\, m_e\, T$$

$$d\Omega = 2\pi\, d\cos\theta$$

$$\left.\frac{d\sigma}{dQ^2}\right|_{\text{collisions}} = 4\pi\,\frac{e^4}{Q^4\,\beta^2}$$

$$\left.\frac{d\sigma}{dT}\right|_{\text{collisions}} = 2\pi\,\frac{e^4}{m_e\,c^2\,\beta^2\,T^2}$$

$$\left.\frac{d\sigma}{dQ^2}\right|_{\text{collisions}} = 4\pi\,\frac{e^4}{Q^4\,\beta^2}$$

$$\left.\frac{d\sigma}{dT}\right|_{\text{collisions}} = 2\pi\,\frac{e^4}{m_e\,c^2\,\beta^2\,T^2}$$

$$\left.\frac{d\sigma}{dT}\right|_{\text{collision}} = 2\pi\,\frac{e^4}{m_e\,c^2\,\beta^2\,T^2}\,\left(1-\beta^2\frac{T}{T_{\max}}\right)$$

$$\left.\frac{d\sigma}{d\Omega}\right|_{\text{Rutherford}} \propto \frac{1}{(1-\cos\theta)^2} = \frac{4}{\sin^4(\theta/2)} \propto \frac{1}{\theta^4}$$

$$\left.\frac{d\sigma}{dT}\right|_{\text{Rutherford}} \propto \frac{1}{T^2}$$

$$\left.\frac{dE}{d\ell}\right|_{\text{collisions}} = n\ Z\ \int dT\ T\ \frac{d\sigma}{dT}$$

$$dX = d\ell\ \rho = d\ell\ n\ m_{\text{nucleon}}\,A = d\ell\ n\ \frac{A}{N_{\text{Avogadro}}}$$

$$\left.\frac{dE}{dX}\right|_{\text{collisions}} = N_{\text{Avogadro}}\,\frac{Z}{A}\ \int dT\ T\ \frac{d\sigma}{dT}$$

$$\left.\frac{dE}{dX}\right|_{\text{collisions}} = N_{\text{Avogadro}}\,\frac{Z}{A}\,2\pi\frac{e^4}{m_e\,c^2}\,\frac{1}{\beta^2}\,\int_{T_{\text{min}}}^{T_{\text{max}}} dT\;T\;\frac{1}{T^2}\left[1-\beta^2\,\frac{T}{T_{\text{max}}}\right]$$

$$\left.\frac{dE}{dX}\right|_{\text{collisions}} = N_{\text{Avogadro}}\,\frac{Z}{A}\,2\pi\frac{e^4}{m_e\,c^2}\frac{1}{\beta^2}\,\left[\ln\left(\frac{T_{\text{max}}}{T_{\text{min}}}\right)-\beta^2\right]$$

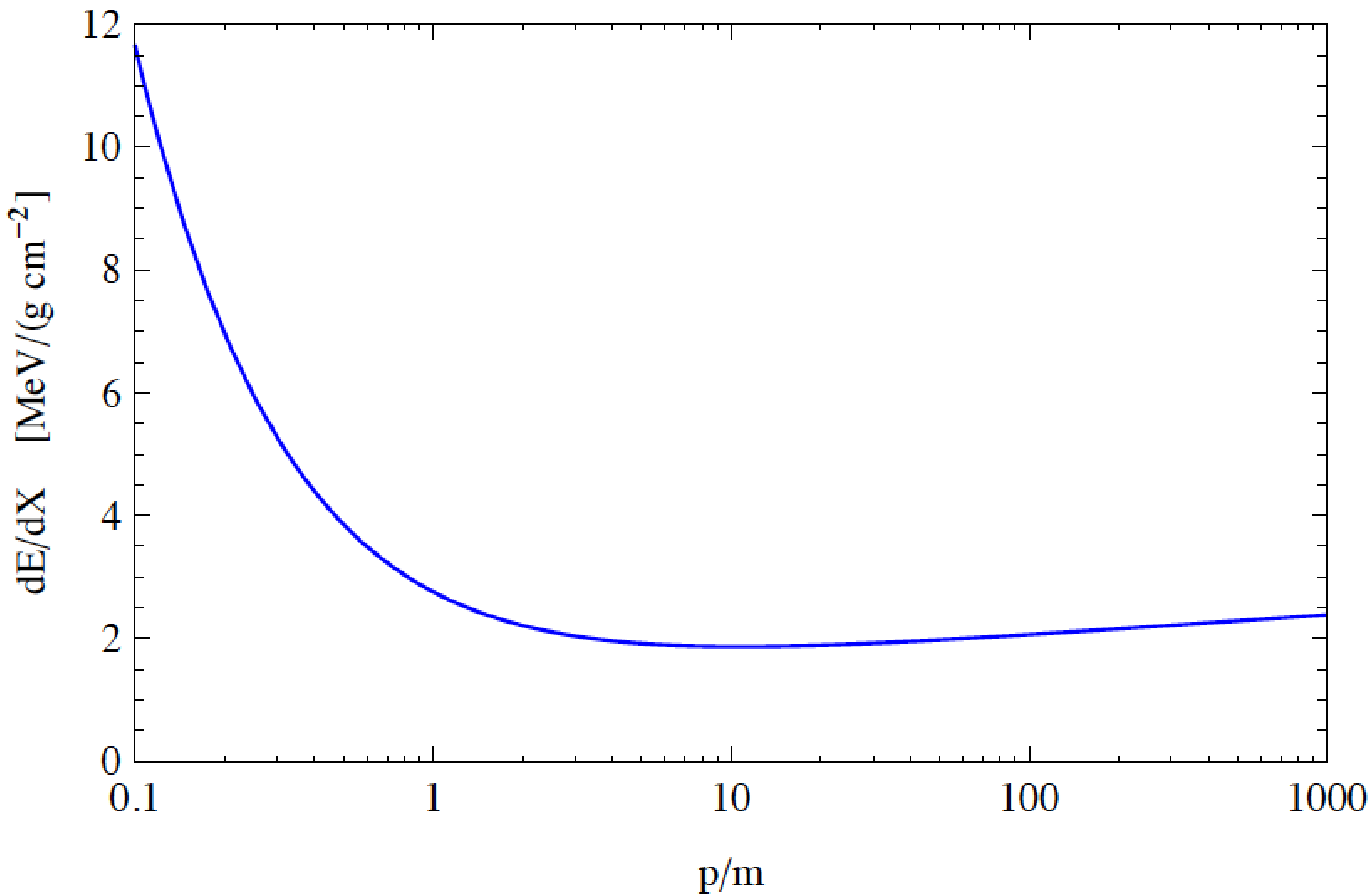
$$\left.\frac{dE}{dX}\right|_{\text{collisions}} = N_{\text{Avogadro}}\,\frac{Z}{A}\,2\pi\frac{e^4}{m_e\,c^2}\,\frac{1}{\beta^2}\,\int_{T_{\min}}^{T_{\max}} dT\;T\;\frac{1}{T^2}\left[1-\beta^2\,\frac{T}{T_{\max}}\right]$$

$$\left.\frac{dE}{dX}\right|_{\text{collisions}} = N_{\text{Avogadro}}\,\frac{Z}{A}\,2\pi\frac{e^4}{m_e\,c^2}\frac{1}{\beta^2}\,\left[\ln\left(\frac{T_{\max}}{T_{\min}}\right)-\beta^2\right]$$

$$T_{\max} \simeq m_e\,c^2\;\beta\gamma$$

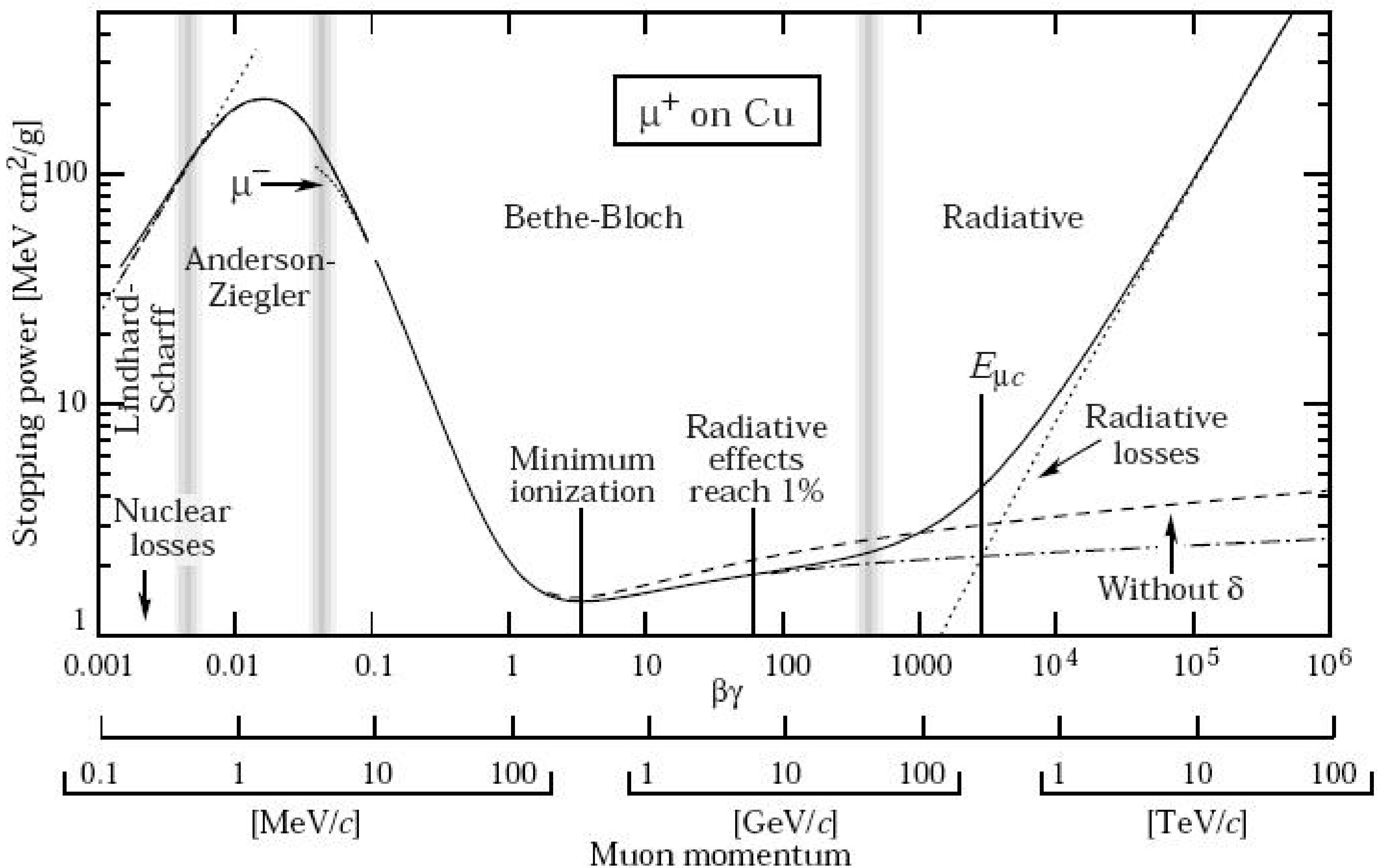
$$T_{\min} \simeq \langle I_{\text{ionization}}\rangle$$

$$\left.\frac{dE}{dX}\right|_{\text{collisions}} \simeq N_{\text{Avogadro}}\,\frac{Z}{A}\,2\pi\frac{e^4}{m_e\,c^2}\frac{1}{\beta^2}\,\left[\ln\left(\frac{2m_e\,c^2\,\beta\gamma}{\langle I\rangle}\right)-\beta^2\right]$$



12
10
8
6
4
2
0
dE/dX [MeV/(g cm^{-2}]
0.1
1
10
100
1000
p/m

INTRODUCTION of
ENERGY LOSS of ELECTRONS for COLLISIONS



Bethe-Bloch formula

$$-\frac{dE}{dx} = Kz^2\frac{Z}{A}\frac{1}{\beta^2}\left[\frac{1}{2}\ln\frac{2m_ec^2\beta^2\gamma^2T_{\max}}{I^2} - \beta^2 - \frac{\delta(\beta\gamma)}{2}\right]$$

Very Simple Approximation:
ENERGY INDEPENDENT LOSS

$$\left.\frac{dE}{dt}\right|_{\text{collisions}} = -\varepsilon$$

Critical energy