Freeze-in

Making sense of *very* weakly coupled dark matter candidates

GGI Workshop "Collider Physics and the Cosmos"
Florence, Italy

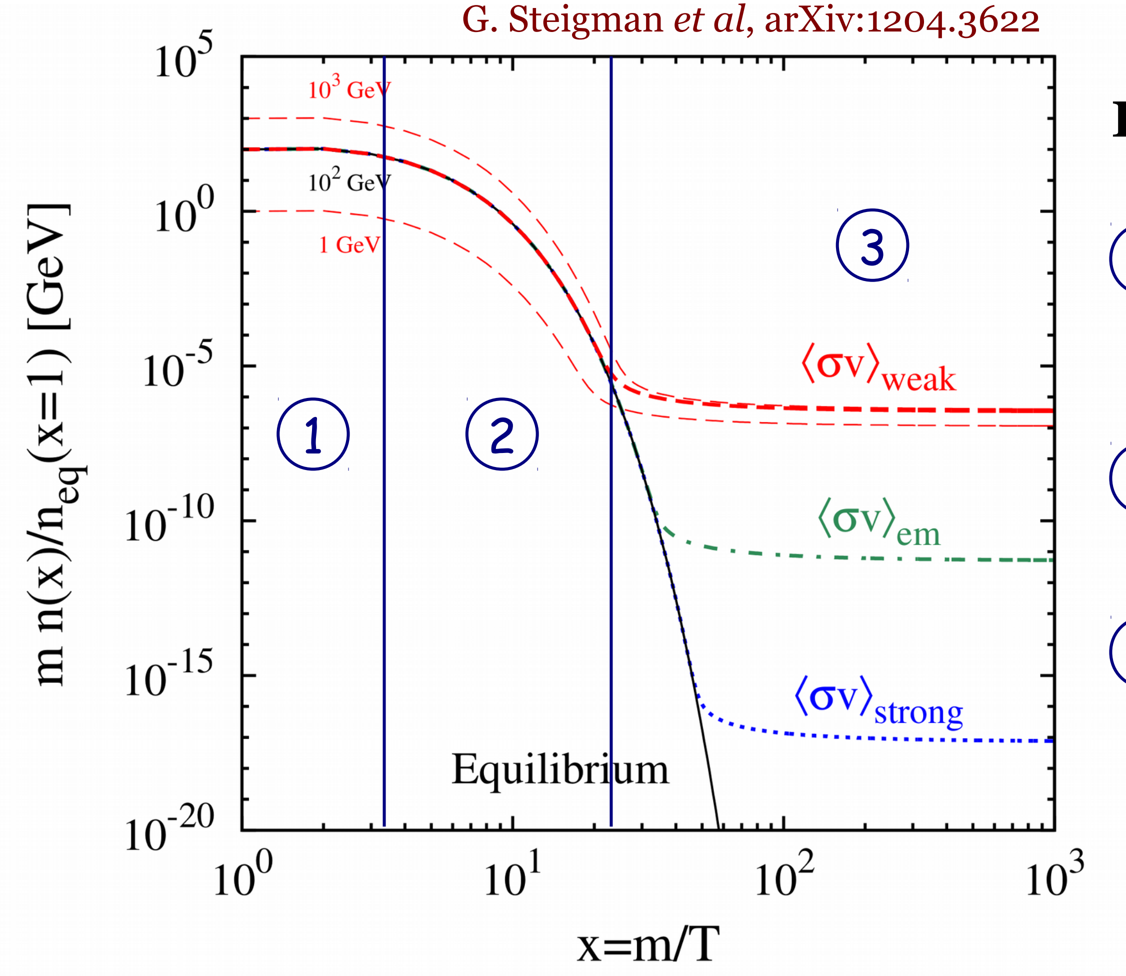
Andreas Goudelis
LPTHE - Jussieu

Outline

- Reminder: standard freeze-out lore
- The general Boltzmann equation and (two of) its limits
- Types of freeze-in
- Experimental signatures of freeze-in models

The standard result: thermal freeze-out

Number density evolution for strong enough DM-SM interactions



Physically:

1. DM + DM ↔ SM + SM efficient in both directions.
2. DM + DM ← SM + SM disfavoured.
3. $n_{DM} <\sigma v> < H$: Equilibrium lost → Freeze-out.

How does this picture arise?

Evolution of a particle species

Consider a particle 1 interacting with three other particles through 1+2 $\leftrightarrow$ 3+4 in a FLRW Universe. The evolution of its distribution is described by a Boltzmann equation:

$$\underbrace{\mathcal{L}\left[f_1\right]}_{\text{Liouville operator}} = \underbrace{\mathcal{C}\left[f_1\right]}_{\text{Collision operator}}$$

Liouville operator

Phase-space density evolution
for particle species 1

Collision operator

Dynamics

Let's integrate and write things a bit more explicitly…

Evolution of a particle species

Consider a particle 1 interacting with three other particles through 1+2 ↔ 3+4 in a FLRW Universe. The evolution of its distribution is described by a Boltzmann equation:

$$\frac{d}{dt}\left[\int f_1(E,t)\frac{g_1 d^3p_1}{(2\pi)^3}\right] + 3H\int f_1(E,t)\frac{g_1 d^3p_1}{(2\pi)^3} =$$

$$-\sum_{\text{spins}}\int\left[f_1 f_2(1\pm f_3)(1\pm f_4)\left|\mathcal{M}_{12\to 34}\right|^2 - f_3 f_4(1\pm f_1)(1\pm f_2)\left|\mathcal{M}_{34\to 12}\right|^2\right]$$

$$\times(2\pi)^4\delta^4(p_1+p_2-p_3-p_4)\times\prod_{i=1}^{4}\left(\frac{d^3p_i}{(2\pi)^3\,2E_i}\right)$$

Evolution of a particle species

Consider a particle 1 interacting with three other particles through 1+2 ↔ 3+4 in a FLRW Universe. The evolution of its distribution is described by a Boltzmann equation:

Hubble parameter

Known

$\equiv n(t)$

Usually the quantity we're most interested in computing, although full distribution carries extremely useful information too.

$$\frac{d}{dt}\left[\int f_1(E,t)\,\frac{g_1 d^3p_1}{(2\pi)^3}\right] + \underbrace{3H}_{\text{Hubble parameter}} \underbrace{\int f_1(E,t)\,\frac{g_1 d^3p_1}{(2\pi)^3}}_{\equiv n(t)} =$$

$$-\sum_{\text{spins}}\int\left[\underbrace{f_1 f_2 (1\pm f_3)(1\pm f_4)}_{\text{1, 2, 3, 4 distribution functions}}\underbrace{\left|\mathcal{M}_{12\to34}\right|^2}_{\text{Matrix elements}} - \underbrace{f_3 f_4 (1\pm f_1)(1\pm f_2)}_{\text{1, 2, 3, 4 distribution functions}}\left|\mathcal{M}_{34\to12}\right|^2\right]$$

$$\times (2\pi)^4\,\delta^4(p_1+p_2-p_3-p_4)\times\prod_{i=1}^{4}\left(\frac{d^3p_i}{(2\pi)^3\,2E_i}\right)$$

Matrix elements

Known (model-building part)

1, 2, 3, 4 distribution functions

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Matrix elements

Known (model-building part)

1, 2, 3, 4 distribution functions

To compute $f_1(E, t_0)$ in full generality, we also need:

· The distributions of 2, 3, 4 at all temperatures.

· The distribution of 1 at some initial temperature.

(or, eventually, solve a coupled set of 4 such equations knowing $f_{1,2,3,4}$ at some temperature)

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Consider a particle 1 interacting with three other particles through 1+2 ↔ 3+4 in a FLRW Universe. The evolution of its distribution is described by a Boltzmann equation:

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$\equiv n\left(t\right)$

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$$\frac{d}{dt}\left[\int f_1\left(E,t\right)\frac{g_1 d^3 p_1}{\left(2\pi\right)^3}\right] + 3H\int f_1\left(E,t\right)\frac{g_1 d^3 p_1}{\left(2\pi\right)^3} =$$

$$-\sum_{\text{spins}}\int\left[f_1 f_2\left(1\pm f_3\right)\left(1\pm f_4\right)\left|\mathcal{M}_{12\to 34}\right|^2 - f_3 f_4\left(1\pm f_1\right)\left(1\pm f_2\right)\left|\mathcal{M}_{34\to 12}\right|^2\right]$$

$$\times\left(2\pi\right)^4\delta^4\left(p_1+p_2-p_3-p_4\right)\times\prod_{i=1}^{4}\left(\frac{d^3 p_i}{\left(2\pi\right)^3 2E_i}\right)$$

1, 2, 3, 4 distribution functions

Matrix elements

Known (model-building part)

Imagine now actually having a realistic model with lots of particles involved in such reactions. Long story short:

$\boxed{\text{It's a pretty complicated problem to tackle in full generality!}}$

Back to freeze-out

Given the complexity of the problem, we try to find interesting limits. Assume that particle 1 possesses substantial interactions with 2, 3 and 4. Then:

e.g. Gondolo, Gelmini, Nucl. Phys. B (1991)

$$\frac{d}{dt}\left[\overbrace{\int f_1(E,t)\frac{g_1 d^3p_1}{(2\pi)^3}}^{\equiv n(t)}\right] + 3H\int f_1(E,t)\frac{g_1 d^3p_1}{(2\pi)^3} =$$

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Replace with $\left|\mathcal{M}_{12\to 34}\right|^2$

Unitarity

Ignore

Decoupling during radiation domination

$\sim f_3^{eq} \text{ x } f_4^{eq} = f_1^{eq} \text{ x } f_2^{eq}$

Typically the SM particles, thermalize quickly + Detailed balancing

After quite a bit of algebra, we get the usual expression:

$$\dot{n}_1 + 3Hn_1 = -\langle\sigma v\rangle\left(n_1 n_2 - n_1^{\rm eq} n_2^{\rm eq}\right)$$

where $\qquad \langle\sigma v\rangle = \dfrac{\int \sigma v\; dn_1^{\rm eq}\; dn_2^{\rm eq}}{\int dn_1^{\rm eq}\; dn_2^{\rm eq}}$

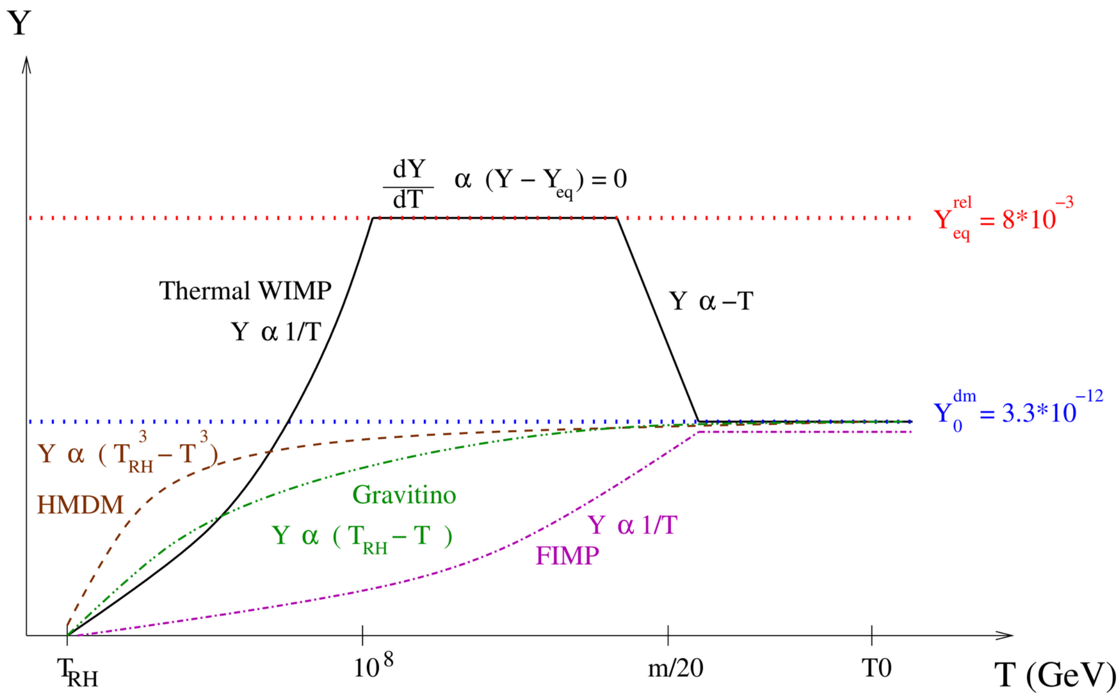
Back to freeze-out: in practice

Solve the – much simplified – equation

$$\dot{n}_1 + 3Hn_1 = -\langle\sigma v\rangle\left(n_1 n_2 - n_1^{\rm eq} n_2^{\rm eq}\right)$$

(or small variations of it)

starting from and early enough temperature so that equilibrium is guaranteed. Actually, even if we set the initial density to zero, we'd get a similar end result.



Y. Mambrini, habilitation thesis

Remarks:

· Freeze-out has no memory: as long as equilibrium is reached at some point, what happens before doesn't matter.

NB1: As long as T_{RH} is high enough.

NB2: Good or bad? Depends on taste.

· As $<\sigma v>$ increases, n_∞ decreases (dark matter annihilates for a longer period).

The other side of the spectrum: freeze-in

Assume, now that particle 1 instead possesses feeble (*very* weak) couplings with 2, 3 and 4.
Reasonable assumption (perhaps): its initial density is negligible.

arXiv:hep-ph/0106249,
arXiv:0911.1120, arXiv:1706.07442
...and many more

$$\frac{d}{dt}\left[\int f_1\left(E,t\right)\frac{g_1 d^3p_1}{\left(2\pi\right)^3}\right] + 3H\overbrace{\int f_1\left(E,t\right)\frac{g_1 d^3p_1}{\left(2\pi\right)^3}}^{\equiv n(t)} =$$

$$-\sum_{\text{spins}}\int\left[\underbrace{f_1 f_2\left(1\pm f_3\right)\left(1\pm f_4\right)\left|\mathcal{M}_{12\to34}\right|^2}_{\text{Ignore}} - \underbrace{f_3 f_4}_{\sim f_3^{eq}\times f_4^{eq}}\underbrace{\left(1\pm f_1\right)\left(1\pm f_2\right)}_{\text{Ignore}}\left|\mathcal{M}_{34\to12}\right|^2\right]$$

$$\times\left(2\pi\right)^4\delta^4\left(p_1+p_2-p_3-p_4\right)\times\prod_{i=1}^{4}\left(\frac{d^3p_i}{\left(2\pi\right)^3 2E_i}\right)$$

Ignore
Feeble coupling + small density

Ignore
Small density

$\sim f_3^{eq} \times f_4^{eq}$
Typically the SM particles, thermalize quickly

· No DM annihilation term.

· No equilibrium between 1 and the other particles.

Basic types of freeze-in : scattering

The basic premise of the freeze-in mechanism is that DM interacts extremely weakly with the Standard Model particles. One can then envisage two basic scenarios:

I) Dark matter could be produced directly from 2 → 2 processes, annihilations of Standard Model (or other bath) particles: a + b → χ + χ

$$\dot{n}_\chi + 3Hn_\chi = \int \prod_{i=a,b,\chi,\bar{\chi}} \frac{d^3 \left|\vec{p}_i\right|}{(2\pi)^3 2E_i} (2\pi)^4 \delta^4 \left(p_a + p_b - p_\chi - p_{\bar{\chi}}\right) \left|\mathcal{M}\right|^2 f_a f_b$$

Or, eventually, by defining $\tilde{K}_1 (s, T) \equiv \frac{1}{4p_{a,b}^{CM} T e^{\sum_{i=\text{in}} \mu_i / T}} \int dE_+ dE_- f_a f_b$

Reduces to Bessel function of the 1st kind for Maxwell-Boltzmann statistics.

$$\boxed{\dot{n}_\chi + 3Hn_\chi = \frac{T}{128\pi^6} \int \frac{ds}{\sqrt{s}} p_{\chi\bar{\chi}}^{CM} p_{ab}^{CM} \tilde{K}_1 (s, T) \left|\mathcal{M}\right|^2 d\Omega}$$

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· No good argument to assume Maxwell-Boltzmann distribution.

Can induce ~factor 2 difference in the DM abundance computation for B-E statistics, example coming up…

· Not necessarily IR dominated mechanism, details depend on model.

- "Ultraviolet freeze-in": 1410.6157 (NR operators)
- Freeze-in from heavy bath particles (in progress)

Basic types of freeze-in : decays - 1

The basic premise of the freeze-in mechanism is that DM interacts extremely weakly with the Standard Model particles. One can then envisage two basic scenarios:

IIa) Dark matter could be produced from the decay of a heavier particle in equilibrium with the thermal bath: Y → χ + χ (but χ is a FIMP)

$$\dot{n}_\chi + 3Hn_\chi = \int \prod_{i=Y,\chi,\bar{\chi}} \frac{d^3 |\vec{p}_i|}{(2\pi)^3 \, 2E_i} (2\pi)^4 \delta^4 (p_Y - p_\chi - p_{\bar{\chi}}) |\mathcal{M}|^2 f_Y^{\rm eq}$$

Or, eventually, by defining $\hat{K}_1 (m_Y, T) \equiv \frac{1}{m_Y T} \int dE_Y \sqrt{E_Y^2 - m_Y^2} f_Y^{\rm eq}$

$$\boxed{\dot{n}_\chi + 3Hn_\chi = \Gamma_{Y \to \chi\bar{\chi}} n_Y^{\rm eq} \frac{\hat{K}_1 (m_Y, T)}{K_1 (m_Y, T)}}$$

· The heavy particle can be Z2-even or Z2-odd.

"Mediator" - decays into DM pairs. "NLOP" - decays into single DM particles.

· Since the heavy particles are in equilibrium, no need to write down any dedicated Boltzmann equation.

Unless freeze-in after Y freeze-out, *cf* next slide

Basic types of freeze-in : decays - 2

The basic premise of the freeze-in mechanism is that DM interacts extremely weakly with the Standard Model particles. One can then envisage two basic scenarios:

IIb) Dark matter could be produced from the decay of a heavier particle which is *not* in equilibrium with the thermal bath: Y → χ + χ (both χ and Y are FIMPs)

$$\dot{n}_\chi + 3Hn_\chi = \int \prod_{i=Y,\chi,\bar{\chi}} \frac{d^3 |\vec{p}_i|}{(2\pi)^3 \, 2E_i} (2\pi)^4 \, \delta^4 \left(p_Y - p_\chi - p_{\bar{\chi}}\right) |\mathcal{M}|^2 \, f_Y$$

> Slightly trickier case, since one needs to write down a Boltzmann equation for the decaying particle as well.

A couple of limiting cases:

- If the decaying particle freezes out, solve its Boltzmann equation separately and rescale the final abundance.

- If the decaying particle freezes in, compute the corresponding yield separately and rescale the final abundance.

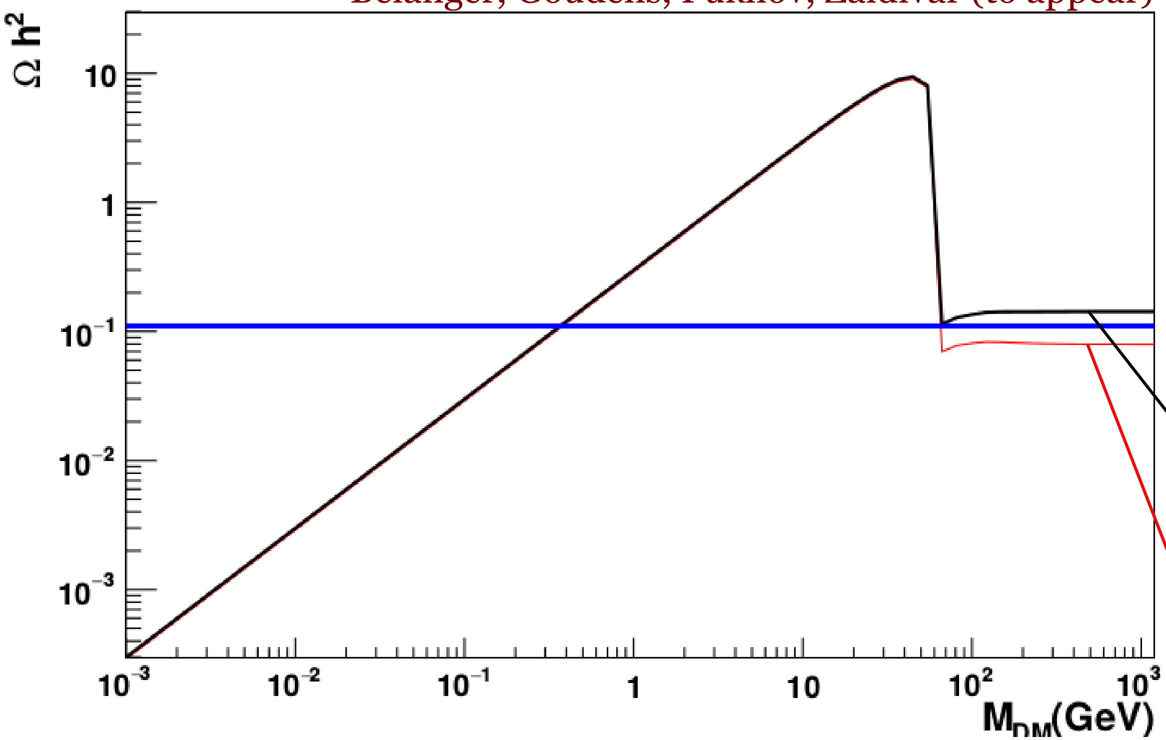
A very simple example

Consider the singlet scalar DM model

$$\mathcal{L} = \mathcal{L}_{\rm SM} - \frac{\mu_s^2}{2} s^2 + \frac{\lambda_s}{4} s^4 + \lambda_{Hs} H^\dagger H s^2$$

Model first studied in the context of freeze-in by C. Yaguna in arXiv:1105.1654

Bélanger, Goudelis, Pukhov, Zaldivar (to appear)



Example for parameter choices as:

· $\lambda_{Hs} = 10^{-11}$

· $m_h = 125$ GeV

Properly accounting for statistics of SM particles.

Assuming Maxwell-Boltzmann statistics.

Statistics can be important!
Price to pay: integrations more complicated.

Freeze-out vs freeze-in: some comments

· Naively, the freeze-in equation is simpler than the freeze-out one: quite similar but only including the DM production term.

· When working in full generality (i.e. for a general particle physics model), though, things are much more involved:

- Equilibrium erases all memory: no dependence on the initial conditions.

- In freeze-out, no need to keep track of the decays of heavier particles (unless they happen late, i.e. after freeze-out).

Equilibrium is restored extremely fast

- On the model-building side: more than one particles in the spectrum can be feebly coupled, need to write down Boltzmann equations for them too.

Need to keep track of the evolution of all states and the way they contribute to the DM abundance.

· Again on the model-building side: feeble couplings need to be radiatively stable!

· The most complicated case: the intermediate regime.

What about signatures ?

So we have another mechanism to explain dark matter. Can we test it? Certainly not in full generality, but

There are actually numerous handles!

Arguably, both remarks also apply to freeze-out

If there are heavier particles in the spectrum

Primordial nucleosynthesis

Probed lifetimes depend on nature of decay products.

Mono-X searches @ LHC

If parent particle neutral and detector-stable.

Charged track searches @ LHC

If parent particle charged and detector-stable.

Long-lived particle searches

May require new experiments (at least partly).

Otherwise

Direct/Indirect detection

In very special limits?

Structure formation

If DM warm/ self-interacting.

Non-collider opportunities

Primordial nucleosynthesis

· If a heavy particle decays into DM + SM, the visible decay products could alter the abundance of light elements in the Universe.

Reminder: T_D~100 keV (~100 sec)

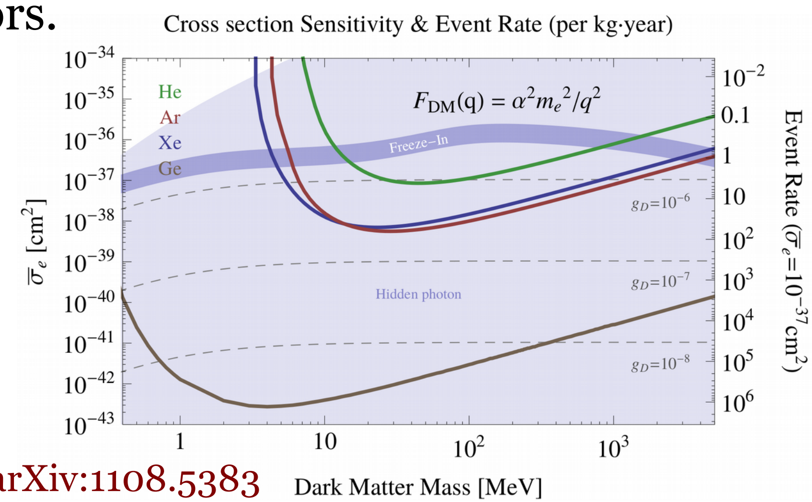
· The exact constraint on the heavy particle lifetime depends strongly on the decay products (hadrons/photons).

e.g. S. Banerjee, G. Bélanger, B. Mukhopahyaya, P. D. Serpico, arXiv:1603.08834

· Important point: BBN constrains *long* lifetimes.

Direct/Indirect detection

· Direct detection: based on electron recoils, can probe sub-GeV frozen-in DM with very light mediators.



R. Essig, J. Mardon, T. Volansky, arXiv:1108.5383

· Indirect detection: quite tricky, although some ideas could exist for some limiting cases.

Discussions ongoing w/ B. Zaldivar *et al*

Structure formation

· Self-interactions: *cf* talk by B. Zaldivar this afternoon.

· Warm DM (mostly Lyman-α): does DM have enough time to cool down?

Beware of the DM distribution, production out of equilibrium!

Freeze-in at colliders

All collider searches for FIMPs (that I'm aware of!) rely on the production of some not-so-feebly coupled heavier state that can decay into the FIMP.

Very weak couplings imply long lifetimes for the parent particle

Not so long-lived → **Displaced vertices**

"Very" long-lived → **Detector-stable particles**

Displaced vertices

· Already performed for compressed spectra. Difference: decay products not soft.

· Analysis is non-trivial, because sensitivity depends both on the decay products and on the decay localisation → Many possibilities.

· Not so easy to make a connection to DM physics.

Will comment a bit more on this in a second.

Detector-stable particles

· Charged or neutral?

· Using current detectors? → Searches for charged tracks; Searches for MET

· Or new ones (*e.g.* MATHUSLA proposal)?

J. P. Chou, D. Curtin, H. J. Lubatti, arXiv:1606.06298

Searches for displaced vertices

It turns out that it's not so straightforward to find dark matter models that give rise to displaced vertices within the LHC detectors. I'm aware of two examples:

· Freeze-in during a matter-dominated era: requires much larger couplings than standard freeze-in.

R. T. Co, F. D'Eramo, L. J. Hall, D. Pappadopulo, arXiv:1506.07532
J. A. Evans, J. Shelton, arXiv:1601.01326

· Cases of very light FIMPs, e.g. in the "scotogenic" model.

A. Hessler, A. Ibarra, E. Molinaro, S. Vogl, arXiv:1611.09540

$$c\tau(H^\pm) \simeq 8.3\mathrm{m}\left(\frac{M_1}{10\mathrm{keV}}\right)\left(\frac{100\mathrm{GeV}}{m_{H^\pm}}\right)^2$$

Any other examples?

Searches for detector-stable particles

For standard freeze-in scenarios, the parent particles tend to escape the detector. Using the existing machine, we can distinguish two cases:

Charged parent particle

· Something like a "heavy muon".

· Searches for such states have already been conducted at the LHC. Some scattered studies connecting them to dark matter.

e.g. CMS, arXiv:1305.0491 (LHC@8 TeV, 18.8 fb^{-1})

Is there a corresponding ATLAS search?

Neutral parent particle

· Standard MET searches.

· I'm not aware of any study that reinterprets these searches in terms of non-thermal dark matter models.

High chance that's just my ignorance!

MATHUSLA proposal for the construction of a detector on the surface above ATLAS/CMS

· Original idea: surface ~40000 m^2, height ~25 m, made of inexpensive materials.

J. P. Chou, D. Curtin, H. J. Lubatti, arXiv:1606.06298

· I'm not aware of any dedicated study relating to non-thermal DM in a quantitative way.

Clearly a hot topic!

Outlook

· Freeze-in is a well-established alternative mechanism to explain the dark matter abundance in the Universe.

· It comes in several variations, but the main idea is that DM particles interact very weakly with the SM ones and never attain thermal equilibrium.

· It can be implemented in many (simple or sophisticated) extensions of the Standard Model.

· Despite the fact that it involves small couplings, it may have numerous different experimental signatures (cosmology, astrophysics, intensity frontier, colliders).

· In many cases, the relevant studies are still at an embryonic stage.

Let's discuss in the afternoon!

Advertisement: micrOMEGAs 5 will be able to deal with freeze-in scenarios, to appear soon :)