Heavy neutrinos meet the triple Higgs coupling

Collider Physics and the Cosmos, **Galileo Galilei Institute, Florence**

Julien Baglio | 10/10/2017

ITP Tübingen (DE) & IPPP Durham (UK)

Outline

Once upon a time...

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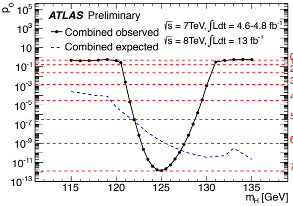
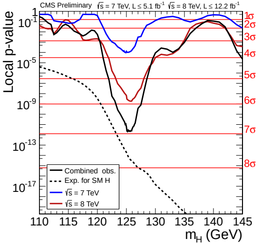
4/7/2012: CERN presents the discovery of a bosonic particle
Its properties are compatible with those of the Higgs boson





$M_H \simeq 125$ GeV
[ATLAS, PLB 716 (2012) 1; CMS *ibid* 30]

2013 analyses have confirmed the discovery of a Higgs boson: spin 0, couplings to fermions and bosons as a function of their masses ⇒ 2013 Nobel Prize awarded to Englert and Higgs

Key question: what is the exact nature of the observed Higgs boson? Standard Model (SM)-like or more exotics?

Once upon a time...

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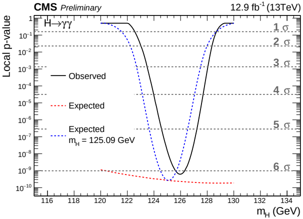
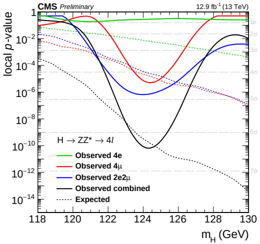
Observed again last year in 13 TeV data [CMS-PAS-HIG-16-020, CMS-PAS-HIG-16-033]





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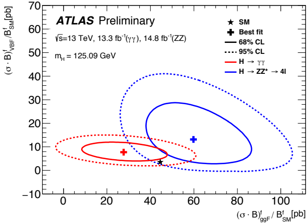
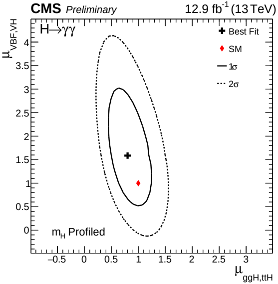
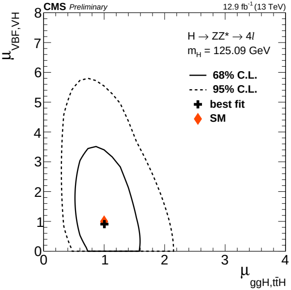
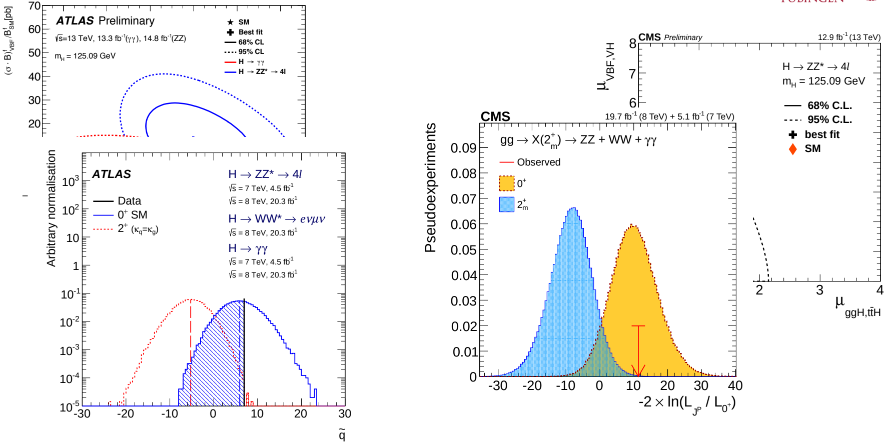
Coining the scalar boson



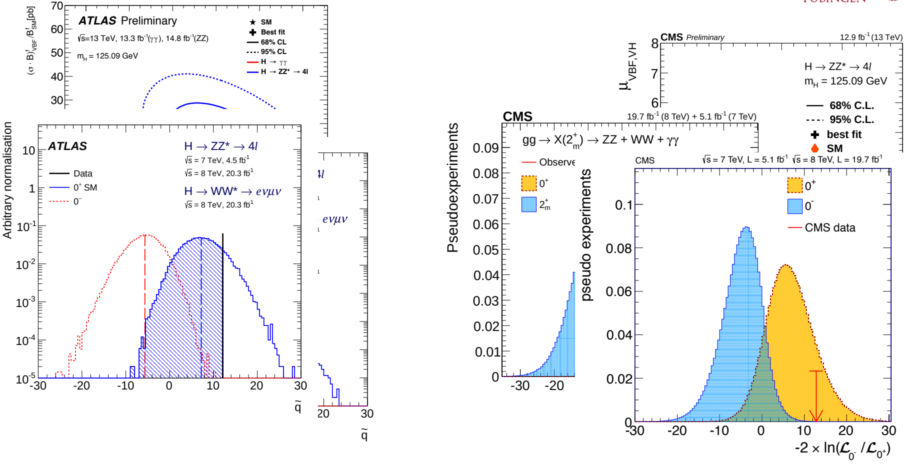




Coining the scalar boson



Coining the scalar boson



Coining the scalar boson

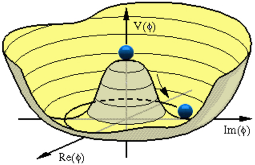
CP-even spin 0 hypothesis strongly preferred, no significant deviations from SM couplings: data up to now points toward an SM Higgs boson...

[ATLAS Collaboration, EPJC 75 (2015) 476, ATLAS-CONF-2016-081; CMS Collaboration, PRD 89 (2014) 092007, PRD 92 (2015) 012004, CMS-PAS-HIG-16-020, CMS-PAS-HIG-16-033]

The SM ultimate test: probing the scalar potential

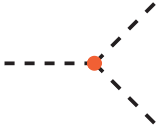
- **From the scalar potential before EWSB (ϕ as the Higgs field):**

$$V(\phi) = -m^2|\phi|^2 + \lambda|\phi|^4$$

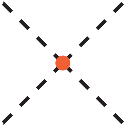


- **To $V(\phi)$ after EWSB, with $M_H^2 = 2m^2$, $v^2 = m^2/\lambda$:**

$$\phi = \begin{pmatrix} 0 \\ \frac{v + H(x)}{\sqrt{2}} \end{pmatrix} \Rightarrow V(H) = \frac{1}{2}M_H^2 H^2 + \frac{1}{2}\frac{M_H^2}{v}H^3 + \frac{1}{8}\frac{M_H^2}{v^2}H^4 + \text{constant}$$



$$\frac{3M_H^2}{v} \times (-\imath)$$



$$\frac{3M_H^2}{v^2} \times (-\imath)$$

Neutrino properties

- **Neutrino oscillations:** observed experimentally in 1998 [Super-Kamiokande, PRL 81 (1998) 1562]
  $\Rightarrow$ neutrinos are massive! $\Rightarrow$ new physics required to account for their mass

- Different mixing pattern from CKM, ν lightness; Majorana or Dirac ν?

- **No information through oscillations about:**
  - **Absolute mass scale:**
    cosmology $\Sigma m_{\nu_i} < 0.23$ eV [Planck, A& A 594 (2016) A13]
    β decays $m_{\nu_e} < 2.05$ eV [Mainz experiment, EPJC 40 (2005) 447; Troitsk, PRD 84 (2011) 112003]
  - **Neutrino nature (Dirac or Majorana):**
    Neutrinoless double β decays $m_{2\beta} < 0.061 - 0.165$ eV [KamLAND-ZEN, PRL 117 (2016) 082503]

Massive neutrinos and New Physics

- **Standard Model:** $L = \binom{\nu_L}{\ell_L}, \tilde{\phi} = \binom{H^{0*}}{H^-}$
  - No right-handed neutrino $\nu_R \Rightarrow$ No Dirac mass term

$$\mathcal{L}_{\rm mass} = -Y_\nu \bar{L}\tilde{\phi}\nu_R + \text{h.c.}$$

  - No Higgs triplet $T \Rightarrow$ No Majorana mass term

$$\mathcal{L}_{\rm mass} = -\frac{1}{2} m\bar{L}TL^c + \text{h.c.}$$

- **Necessary to go beyond the Standard Model for ν mass**
  - Radiative models?
  - R-parity violation in supersymmetry?
  - Seesaw mechanisms? $\rightarrow \nu$ mass at tree-level
    $\rightarrow$ **heavy sterile fermions**
    $\Rightarrow$ **neutrino portal for Dark Matter?**

Dirac neutrinos?

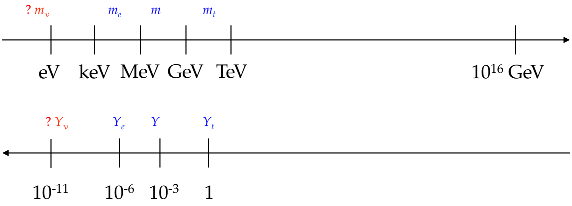
- Add gauge singlet (sterile), right-handed neutrinos $\nu_R \Rightarrow \nu = \nu_L + \nu_R$

$$\mathcal{L}^{\text{leptons}}_{\text{mass}} = -Y_\ell \bar{L}\phi\ell_R - Y_\nu \bar{L}\tilde{\phi}\nu_R + \text{h.c.}$$

$\Rightarrow$ After electroweak symmetry breaking:

$$\mathcal{L}^{\text{leptons}}_{\text{mass}} = -m_\ell \bar{\ell}_L \ell_R - m_D \bar{\nu}_L \nu_R + \text{h.c.}$$

$\Rightarrow$ **3** light active neutrinos: $m_\nu \lesssim 1\text{eV} \Rightarrow Y^\nu \lesssim 10^{-11}$



Majorana neutrinos?

- Add gauge singlet (sterile), right-handed neutrinos ν_R

$$\mathcal{L}_{\text{mass}}^{\text{leptons}} = -Y_\ell \bar{L}\phi\ell_R - Y_\nu \bar{L}\tilde{\phi}\nu_R - \frac{1}{2}M_R\overline{\nu_R}\nu_R^c + \text{h.c.}$$

$\Rightarrow$ After electroweak symmetry breaking:

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$3\,\nu_R \Rightarrow$ **6** mass eigenstates: $\boldsymbol{\nu = \nu^c}$

- ν_R gauge singlets
  $\Rightarrow M_R$ not related to SM dynamics, not protected by symmetries
  $\Rightarrow M_R$ between 0 and M_P
- $M_R\overline{\nu_R}\nu_R^c$ violates lepton number conservation $\Delta L = 2$

Linking the Higgs sector and neutrinos

How to search for heavy neutrino with $m_\nu > \mathcal{O}(1\text{ TeV})$?

Use the Higgs sector to probe neutrino mass models

- TeV-scale neutrinos + Large Yukawa couplings

  $\Rightarrow$ Possibly large deviations from SM properties in the Higgs sector

- ***HH* production:** one of the main motivation for high-luminosity LHC and future colliders
  $\Rightarrow$ need to study the impact of BSM on λ_{HHH} $\Rightarrow$ impact of heavy neutrino(s) on λ_{HHH}?
  - Sizeable SM 1-loop corrections ($\mathcal{O}(10\%)$) $\Rightarrow$ Quantum corrections cannot be neglected
  - Sensitive to diagonal Yukawa couplings Y_ν

Neutrino effects on the triple Higgs coupling The case of the 3+1 model

[J.B., Weiland, PRD 94 (2016) 013002]

Sensitivity to λ_{HHH}

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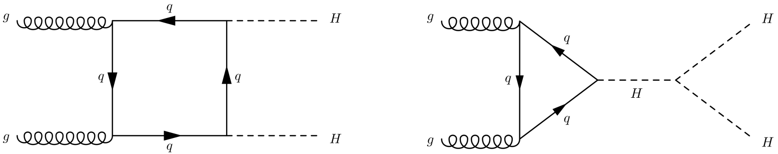
λ_{HHH} extracted from HH production

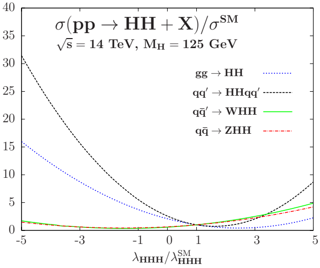


Experimental prospects for the sensitivity to λ_{HHH}:

- HL-LHC: $\sim$ 50 % for ATLAS or CMS [CMS-PAS-FTR-15-002]
  $\sim$ 35 % combined
- ILC: 27 % at 500 GeV with 4 ab^{-1} [Fujii *et al*, arXiv:1506.05992]
  10 % at 1 TeV with 5 ab^{-1}
- FCC-hh: 8 % per experiment with 3 ab^{-1} using only $b\bar{b}\gamma\gamma$ [Je, Ren, Yao, PRD 93 (2016) 015003]
  $\sim$ 5 % combining all channels

Sensitivity to λ_{HHH}

λ_{HHH} extracted from *HH* production



[J.B *et al.* , JHEP 04 (2013) 151]

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SM 1-loop corrections

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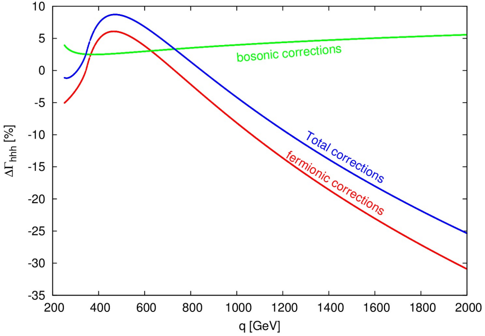


taken from [Arhrib *et al*, JHEP 12 (2015) 007]

- tree-level: $\lambda^0_{HHH} = -\frac{3M_H^2}{v}$

- Dominant contribution from top-quark loops [Kanemura *et al*, PRD 70 (2004) 115002]

$$\lambda_{HHH}(q^2, m_H^2, m_H^2) = -\frac{3m_H^2}{v}\left[1 - \frac{1}{16\pi^2}\frac{16m_t^4}{v^2 m_H^2} \times \left\{1 + \mathcal{O}\left(\frac{m_H^2}{m_t^2}, \frac{q^2}{m_t^2}\right)\right\}\right]$$

- Opposite sign for the threshold ($\sqrt{q^2} = 2m_t$) and m_t^4 contributions

Simplified 3+1 Dirac model

- A well-known class of renormalizable models for neutrino (ν) mass: seesaw models

  $\rightarrow$ To simplify the study of low-scale seesaw effects from a heavy ν: Simplified models

  $\rightarrow$ Illustrate effects of new fermionic coupling through neutrino portal

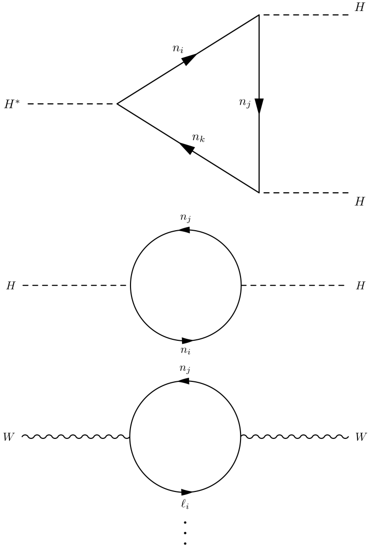
- Simplified model with 3 light ν ($m_n = 1$ eV) and 1 heavy sterile ν (m_4) parametrized by ν masses and active-sterile mixing B_{ij}

$$
\begin{aligned}
\mathcal{L} \ni & -\frac{g_2}{\sqrt{2}} \bar{\ell}_i \gamma^\mu W_\mu^- B_{ij} P_L n_j + \text{h.c.} \\
& -\frac{g_2}{2M_W} \bar{n}_i \left(B^\dagger B\right)_{ij} H (m_{n_i} P_L + m_{n_j} P_R) n_j \\
& -\frac{g_2}{2\cos\theta_W} \bar{n}_i \gamma^\mu Z_\mu \left(B^\dagger B\right)_{ij} P_L n_j
\end{aligned}
$$

$$
B_{3\times 4} = \begin{pmatrix} B_{e1} & B_{e2} & B_{e3} & \boldsymbol{B_{e4}} \\ B_{\mu 1} & B_{\mu 2} & B_{\mu 3} & \boldsymbol{B_{\mu 4}} \\ B_{\tau 1} & B_{\tau 2} & B_{\tau 3} & \boldsymbol{B_{\tau 4}} \end{pmatrix}
$$

Active-sterile mixing matrix B constructed from the PMNS matrix

New contributions to the triple Higgs coupling



- Heavy ν generates new 1-loop diagrams and new counterterms
- Counterterm to the triple Higgs coupling:

$$\frac{\delta\lambda_{HHH}}{\lambda^0_{HHH}} = \frac{3}{2}\delta Z_H + \delta t_H \frac{e}{2M_W \sin\theta_W M_H^2} + \delta Z_e + \frac{\delta M_H^2}{M_H^2} - \frac{1}{2}\frac{\delta M_W^2}{M_W^2} + \frac{1}{2}\cot^2\theta_W\left(\frac{\delta M_W^2}{M_W^2} - \frac{\delta M_Z^2}{M_Z^2}\right)$$

- Tools for the calculation:
  → `FeynArts/FormCalc` and `LoopTools 2.12` for the 1-loop integrals
  → implementation of a Model File for the neutrino interactions

Constraints on the model

Theoretical constraints

- Loose (tight) perturbativity bound:

$$\left(\frac{\max|C_{i4}|\, g_2\, m_{n_4}}{2M_W}\right)^3 < 16\pi\ (2\pi)$$

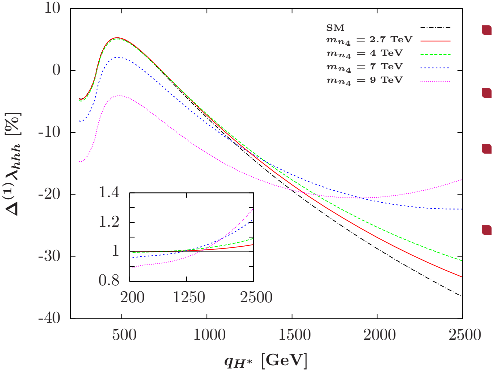
- Width limit: $\Gamma_{n_4} \leq 0.6\, m_{n_4}$

Experimental constraints

- PMNS matrix: best fit of normal hierarchy with no CP-violation [Gonzalez-Garcia, Maltoni, Schwetz, JHEP 11 (2014) 052]
- Lepton flavor violating Higgs decays $\mathrm{BR}(\mu \to e\gamma)$, $\mathrm{BR}(\tau \to e\gamma)$, $\mathrm{BR}(\tau \to \mu\gamma)$ [MEG, EPJC 76 (2016) 434]
- Neutrinoless beta decay: escaped as neutrino are Dirac
- Strongest experimental constraints on n_4: Electroweak precision observables [del Aguila, de Blas, Pérez-Victoria, PRD 78 (2008) 013010]

$$|B_{e4}| \leq 0.041$$
$$|B_{\mu4}| \leq 0.030$$
$$|B_{\tau4}| \leq 0.087$$

Momentum dependence



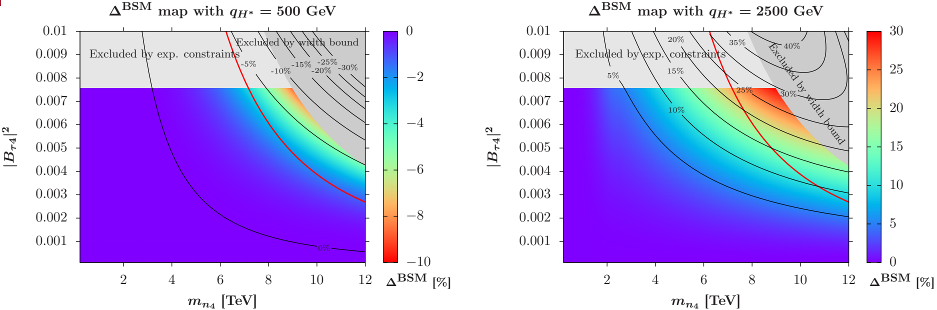
- $\Delta^{(1)}\lambda_{HHH} = \frac{1}{\lambda^0}\left(\lambda^{1r}_{HHH} - \lambda^0\right)$
- $B_{\tau 4} = 0.087, B_{e4} = B_{\mu 4} = 0$
- Deviation of the BSM correction with respect to the SM correction in the insert
- $C_{44} m_{n_4} = m_t \Rightarrow m_{n_4} = 2.7\,\text{TeV}$

  tight perturbativity bound: $m_{n_4} = 7\,\text{TeV}$

  width bound: $m_{n_4} = 9\,\text{TeV}$

- Largest positive correction at $q^*_H \simeq 500\,\text{GeV}$, heavy ν decreases it
- Large negative correction at large q^*_H, heavy ν increases it

Contour maps in a 3+1 simplified model



- $\Delta^{\rm BSM} = \frac{1}{\lambda^{1r,\rm SM}_{HHH}} \left(\lambda^{1r,\rm full}_{HHH} - \lambda^{1r,\rm SM}_{HHH} \right)$
- Red line: tight perturbativity bound
- Heavy ν effects at the limit of HL-LHC sensitivity (35 %)
- Heavy ν effects clearly visible at the ILC (10 %) and FCC-hh (5 %)
- Similar plots for B_{e4} and $B_{\mu 4}$

Neutrino effects on the triple Higgs coupling
The inverse seesaw

[J.B., Weiland, JHEP 1704 (2017) 038]

From the 3+1 Dirac model to the inverse seesaw

- TeV-scale neutrino induces sizeable corrections to λ_{HHH}
  - Decrease at $q_H^* \simeq 500\,\mathrm{GeV}$
  - Increase at large q_H^*
- Effects could be used to constrain the active-sterile mixing at the ILC and FCC-hh
- What are the effects in an appealing low-scale seesaw model ?
  - Inverse seesaw $\rightarrow$ Additional constraints need to be included

The inverse seesaw (ISS) mechanism

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- Lower seesaw scale from approximately conserved lepton number
- Add fermionic gauge singlets ν_R ($L = +1$) and X ($L = -1$)

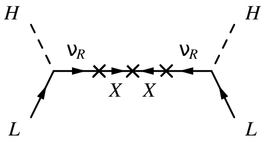
  [Mohapatra, PRL 56 (1986) 561; Mohapatra, Valle, PRD 34 (1986) 1642; Bernabéu *et al.* , PLB 187 (1987) 303]

$$\mathcal{L}_{\rm ISS} = -Y_\nu \overline{L}\tilde{\phi}\nu_R - M_R \overline{\nu_R^c} X - \frac{1}{2}\mu_X \overline{X^c} X + \text{h.c.}$$

with $m_D = Y_\nu v/\sqrt{2}$, $M^\nu = \begin{pmatrix} 0 & m_D & 0 \\ m_D^T & 0 & M_R \\ 0 & M_R^T & \mu_X \end{pmatrix}$

$$m_\nu \approx \frac{m_D^2}{M_R^2}\mu_X$$

$$m_{N_1,N_2} \approx \mp M_R + \frac{\mu_X}{2}$$



2 scales: μ_X and M_R

- Decouple neutrino mass generation from active-sterile mixing
- Inverse seesaw: $Y_\nu \sim \mathcal{O}(1)$ and $M_R \sim 1\,\text{TeV}$
  $\Rightarrow$ within reach of the LHC and low energy experiments

Most relevant constraints for the ISS

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- Accommodate low-energy neutrino data using parametrization

$$v Y_\nu^T = V^\dagger \mathrm{diag}(\sqrt{M_1}\,, \sqrt{M_2}\,, \sqrt{M_3})\, R\, \mathrm{diag}(\sqrt{m_1}\,, \sqrt{m_2}\,, \sqrt{m_3}) U_{PMNS}^\dagger$$

$$M = M_R \mu_X^{-1} M_R^T \text{ (Casas-Ibarra parametrization)}$$

or

$$\mu_X = M_R^T\, Y_\nu^{-1}\, U^*_{\mathrm{PMNS}} m_\nu U^\dagger_{\mathrm{PMNS}}\, Y_\nu^{T\,-1} M_R v^2 \qquad \text{and beyond}$$

- Charged lepton flavor violation
  → For example: $\mathrm{Br}(\mu \to e\gamma) < 4.2 \times 10^{-13}$ [MEG, EPJC 76 (2016) 434]

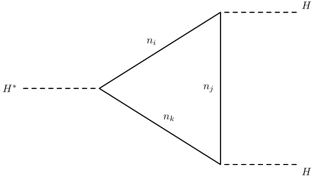
- Global fit to EWPO and lepton universality tests [Fernandez-Martinez *et al.* , JHEP 1608 (2016) 033]

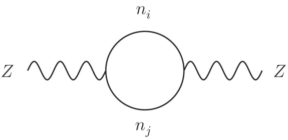
- Electric dipole moment: 0 with real PMNS and mass matrices

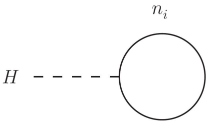
- Invisible Higgs decays: $M_R > m_H$, does not apply

- Yukawa perturbativity: $|\frac{Y_\nu^2}{4\pi}| < 1.5$

Calculation in the ISS







- Similar diagrams to the 3+1 Dirac scenario but with Majorana neutrinos
- Evaluated with `FeynArts`, `FormCalc` and `LoopTools`
- More heavy neutrinos
  $\Rightarrow$ effects generically larger than 3+1 model

Analytical formulae for both Dirac and Majorana fermions

Results using the Casas-Ibarra parametrization

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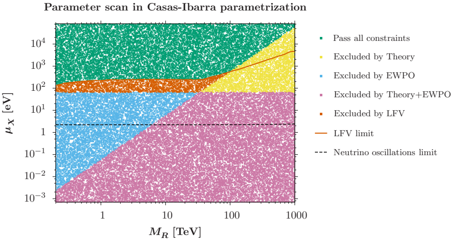
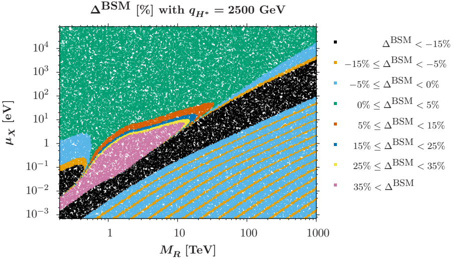




- Random scan: 180000 points with degenerate (diagonal) M_R and μ_X, θ_i angles of the matrix R,

$$
\begin{aligned}
0 &\leq \theta_i \leq 2\pi, \ (i = 1, 2, 3) \\
0.2\ \text{TeV} &\leq M_R \leq 1000\ \text{TeV} \\
7 \times 10^{-4}\ \text{eV} &\leq \mu_X \leq 8.26 \times 10^4\ \text{eV}
\end{aligned}
$$

- $\Delta^{\mathrm{BSM}} = \frac{1}{\lambda_{HHH}^{1r,\mathrm{SM}}} \left(\lambda_{HHH}^{1r,\mathrm{full}} - \lambda_{HHH}^{1r,\mathrm{SM}} \right)$

- Strongest constraints:
  - Lepton flavor violation, mainly $\mu \to e\gamma$
  - Yukawa perturbativity (and neutrino width)

- Large effects necessarily excluded by LFV constraints ?

Suppressing LFV constraints

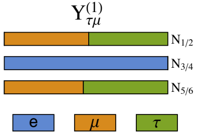
- How to evade the LFV constraints ?

- Approximate formulas for large Y_ν [Arganda *et al.* , PRD 91 (2015) 015001]:

$$\mathrm{Br}^{\mathrm{approx}}_{\mu\to e\gamma} = 8\times 10^{-17}\mathrm{GeV}^{-4}\frac{m_\mu^5}{\Gamma_\mu}\left|\frac{\mathrm{v}^2}{2M_R^2}(Y_\nu Y_\nu^\dagger)_{12}\right|^2$$

- Solution: Textures with $(Y_\nu Y_\nu^\dagger)_{12} = 0$

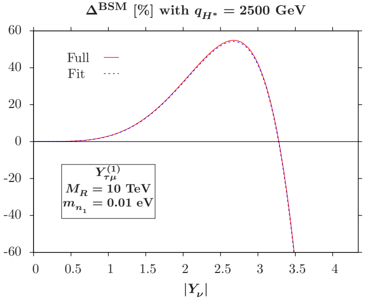
$$Y^{(1)}_{\tau\mu} = |Y_\nu| \begin{pmatrix} 0 & 1 & -1 \\ 0.9 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

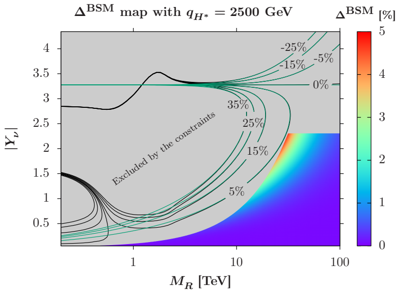


[taken from Arganda *et al.* , PLB 752 (2016) 46]

- Or even take Y_ν diagonal

Results for $Y^{(1)}_{\tau\mu}$



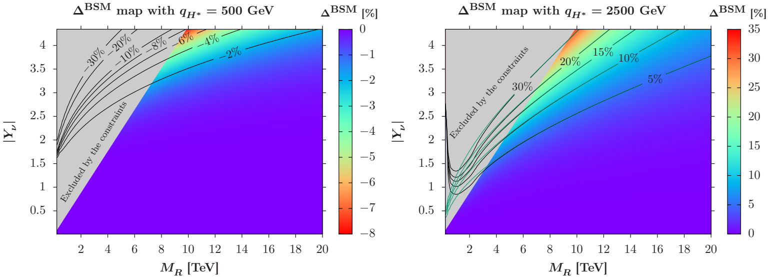


- $\Delta^{\rm BSM} = \frac{1}{\lambda^{1r,\rm SM}_{HHH}}\left(\lambda^{1r,\rm full}_{HHH} - \lambda^{1r,\rm SM}_{HHH}\right)$
- Right: Full calculation in black, approximate formula in green
- Well described at $M_R > 3$ TeV by approximate formula

$$\Delta^{\rm BSM}_{\rm approx} = \frac{(1\ \mathrm{TeV})^2}{M_R^2}\left(8.45\ \mathrm{Tr}(Y_\nu Y_\nu^\dagger Y_\nu Y_\nu^\dagger) - 0.145\ \mathrm{Tr}(Y_\nu Y_\nu^\dagger Y_\nu Y_\nu^\dagger Y_\nu Y_\nu^\dagger)\right)$$

- Can maximize $\Delta^{\rm BSM}$ by taking $Y_\nu \propto \mathrm{I}_3$

Results in the ISS



- $$\Delta^{\rm BSM} = \frac{1}{\lambda^{1r,{\rm SM}}_{HHH}}\left(\lambda^{1r,{\rm full}}_{HHH} - \lambda^{1r,{\rm SM}}_{HHH}\right)$$
- Diagonal Y_ν and hierarchical heavy neutrinos: full calculation in black, approximate formula in green; Maximize the correction
- Results agree with 3+1 Dirac analysis despite stronger constraints
- Heavy ν effects at the limit of HL-LHC (35%) and ILC (10%) sensitivities
  clearly visible at the FCC-hh (5%)
- Complementary to existing observables
  → **Provide a new probe of the $\mathcal{O}(10)$ TeV region of neutrino mass models**

Conclusion and outlook

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HH production at hadron colliders: The Higgs frontier at the HL-LHC and at the future colliders

- **Major news since 2012:** a Higgs boson has been observed, now it is time to solve the next big question: **Is it standard or a first window on BSM physics?**

  $\Rightarrow$ **Study of BSM effects on the triple Higgs coupling**

- **Neutrino oscillations: New physics needed to generate ν masses**
  $\Rightarrow$ low scale seesaw mechanism appealing model to generate tree-level ν masses.
  Inverse seesaw: $Y_\nu \sim \mathcal{O}(1)$ and $M_R \sim \mathcal{O}(0.1 - 10)$ TeV

- **Neutrino effects on the triple Higgs coupling:** Corrections as large as 30 %
  $\Rightarrow$ measurable at future colliders

  $\rightarrow$ Probe a new part of the parameter space of the mass generation models

  $\rightarrow$ Larger effects expected with additional heavy neutrinos; generic effect, expected in all models with TeV fermions and large Higgs couplings

  $\rightarrow$ give new constraints on active-sterile neutrino mixing: impact on astrophysics, cosmology, neutrino physics

- **Future work:** corrections to HH production cross-section, UV completion to low-scale seesaw models

Backup slides

Renormalization procedure for the HHH coupling I

- No tadpole: $t_H^{(1)} + \delta t_H = 0 \Rightarrow \delta t_H = -t_H^{(1)}$
- Counterterms:

$$\begin{aligned} M_H^2 &\to M_H^2 + \delta M_H^2 \\ M_W^2 &\to M_W^2 + \delta M_W^2 \\ M_Z^2 &\to M_Z^2 + \delta M_Z^2 \\ e &\to (1 + \delta Z_e) e \\ H &\to \sqrt{Z_H} = (1 + \frac{1}{2}\delta Z_H) H \end{aligned} \tag{1}$$

- Full renormalized 1–loop triple Higgs coupling: $\lambda_{HHH}^{1r} = \lambda^0 + \lambda_{HHH}^{(1)} + \delta\lambda_{HHH}$

$$\begin{aligned} \frac{\delta\lambda_{HHH}}{\lambda^0} = \frac{3}{2}\delta Z_H + \delta t_H \frac{e}{2M_W \sin\theta_W M_H^2} + \delta Z_e + \frac{\delta M_H^2}{M_H^2} \\ - \frac{\delta M_W^2}{2M_W^2} + \frac{1}{2}\frac{\cos^2\theta_W}{\sin^2\theta_W}\left(\frac{\delta M_W^2}{M_W^2} - \frac{\delta M_Z^2}{M_Z^2}\right) \end{aligned}$$

Renormalization procedure for the HHH coupling II

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- OS scheme

$$\delta M_W^2 = Re\Sigma_{WW}^T(M_W^2)$$
$$\delta M_Z^2 = Re\Sigma_{ZZ}^T(M_Z^2)$$
$$\delta M_H^2 = Re\Sigma_{HH}(M_H^2) \tag{2}$$

- Electric charge:

$$\delta Z_e = \frac{\sin\theta_W}{\cos\theta_W}\frac{\mathrm{Re}\boldsymbol{\Sigma}_{\gamma Z}^T(0)}{M_Z^2} - \frac{\mathrm{Re}\boldsymbol{\Sigma}_{\gamma\gamma}^T(M_Z^2)}{M_Z^2}$$

- Higgs field renormalization

$$\delta Z_H = -\mathrm{Re}\frac{\partial\Sigma_{HH}(k^2)}{\partial k^2}\bigg|_{k^2=M_H^2}$$

Next-order terms in the μ_X-parametrization

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- Weaker constraints on diagonal couplings
  $\rightarrow$ Large active-sterile mixing $m_D M_R^{-1}$ for diagonal terms

- Previous parametrizations built on the 1st term in the $m_D M_R^{-1}$ expansion $\rightarrow$ Parametrizations breaks down

- Solution: Build a parametrization including the next order terms

- The next-order μ_X-parametrization is then

$$\mu_X \simeq \left(\mathbf{1} - \frac{1}{2} M_R^{*\,-1} m_D^\dagger m_D M_R^{T\,-1}\right)^{-1} M_R^T m_D^{-1} U_{\mathrm{PMNS}}^* m_\nu U_{\mathrm{PMNS}}^\dagger m_D^{T\,-1} M_R \times \left(\mathbf{1} - \frac{1}{2} M_R^{-1} m_D^T m_D^* M_R^{\dagger\,-1}\right)^{-1}$$