The Galileo Galilei Institute for Theoretical Physics

Arcetri, Florence

Electroweak Baryogenesis & Higgs self-coupling measurement

With B. Jain, M. Son; arXiv:1709.03232

With B. Fucks, J. Kim; arXiv:1704.04298

KOREA UNIVERSITY

Collider Physics and the Cosmos

Seung J. Lee

Aug 28, 2017 - Oct 14, 2017

Oct 10, 2017, Florence

@ Colliders

- Naturalness / Fine-tuning problem - how EW symmetry is broken
- Dark Matter (à la missing ET)
- Inflation (low scale case)
- Baryon Asymmetry of the Universe: Electroweak Baryogenesis? <-> Higgs Potential

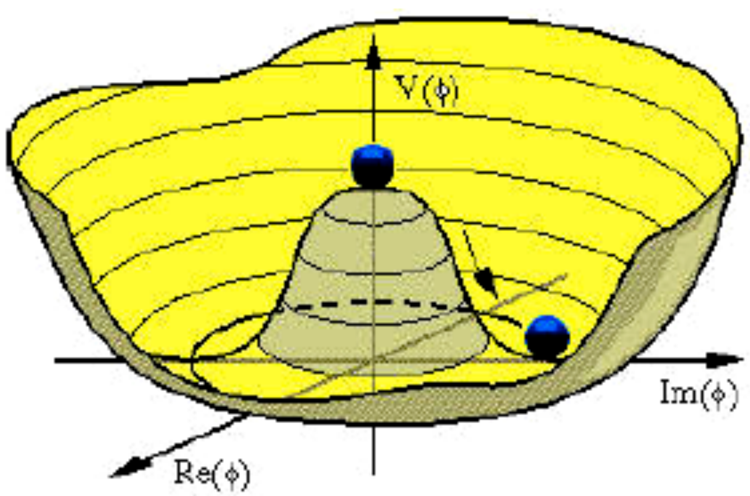
.

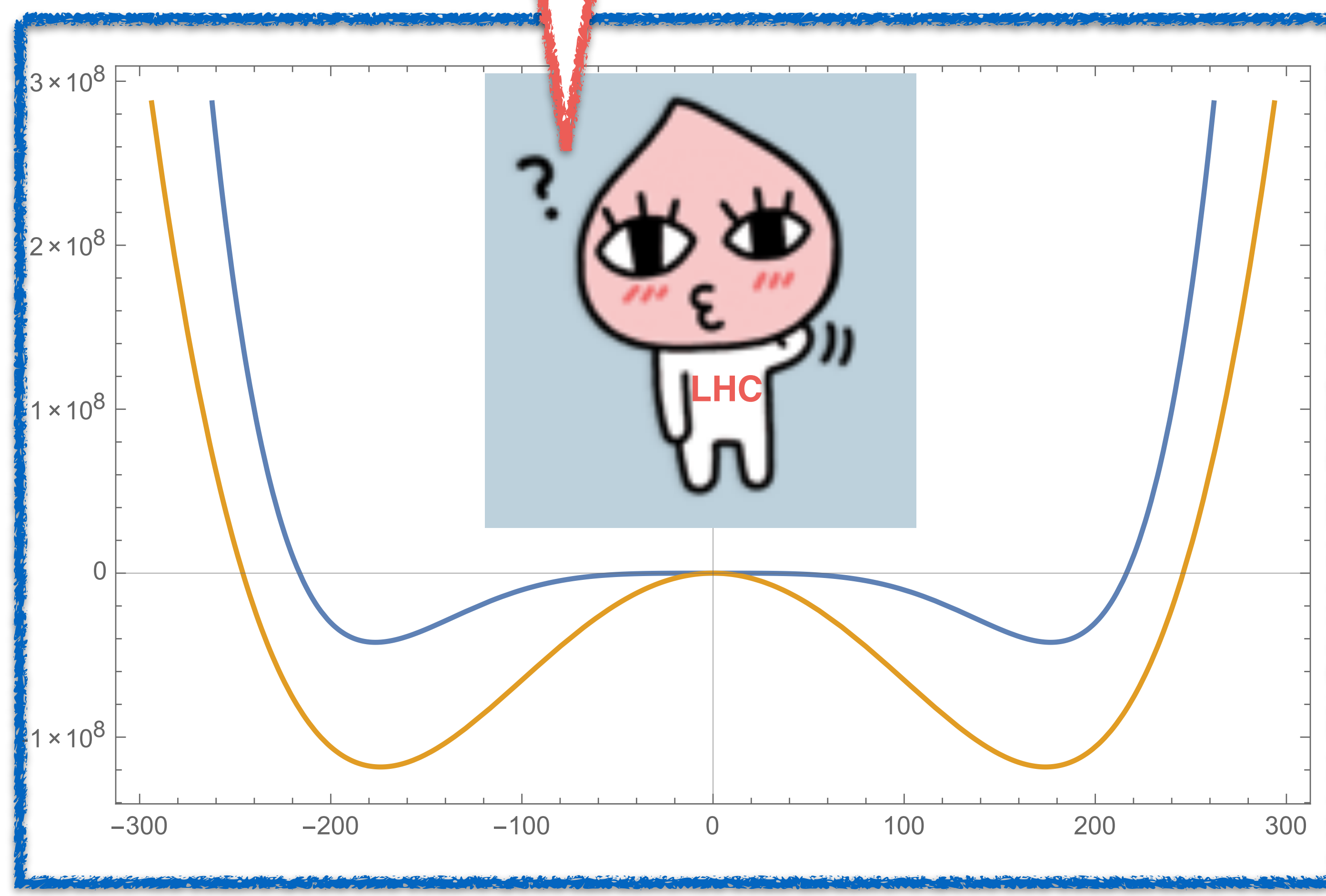
.

.

How well do we understand the EWSB, i.e. Higgs Potential?

$$V(\tilde{h}) \overset{?}{=} \tilde{v}^{2\Delta}\tilde{m}^{4-2\Delta}\sum_{n=0}^{\infty}\frac{c_n}{n!}\left(\frac{\tilde{h}}{\tilde{v}^{\Delta}}\right)^n$$





- The global picture of the Higgs potential is still unknown!

Probing the EWSB mechanism

Kim, Fuks, SL 16', 17'

- ✦ Establishing the SM nature of the electroweak symmetry breaking mechanism
  - ✤ Finding a Higgs is the first step ✅
  - ✤ Deriving the form of the scalar potential is necessary ❌
- ✦ Establishing the SM nature of the Higgs boson ?
  - ✤ Measurements of the Yukawa interaction strengths (*i.e.*, the fermion masses)

- ✦ The scalar potential

$$V_{\rm h} = \frac{m_h^2}{2} h^2 + (1+\kappa_3)\lambda_{hhh}^{\rm SM} v h^3 + \frac{1}{4}(1+\kappa_4)\lambda_{hhhh}^{\rm SM} h^4 \quad \text{with} \quad \lambda_{hhh}^{\rm SM} = \lambda_{hhhh}^{\rm SM} = \frac{m_h^2}{2v^2}$$

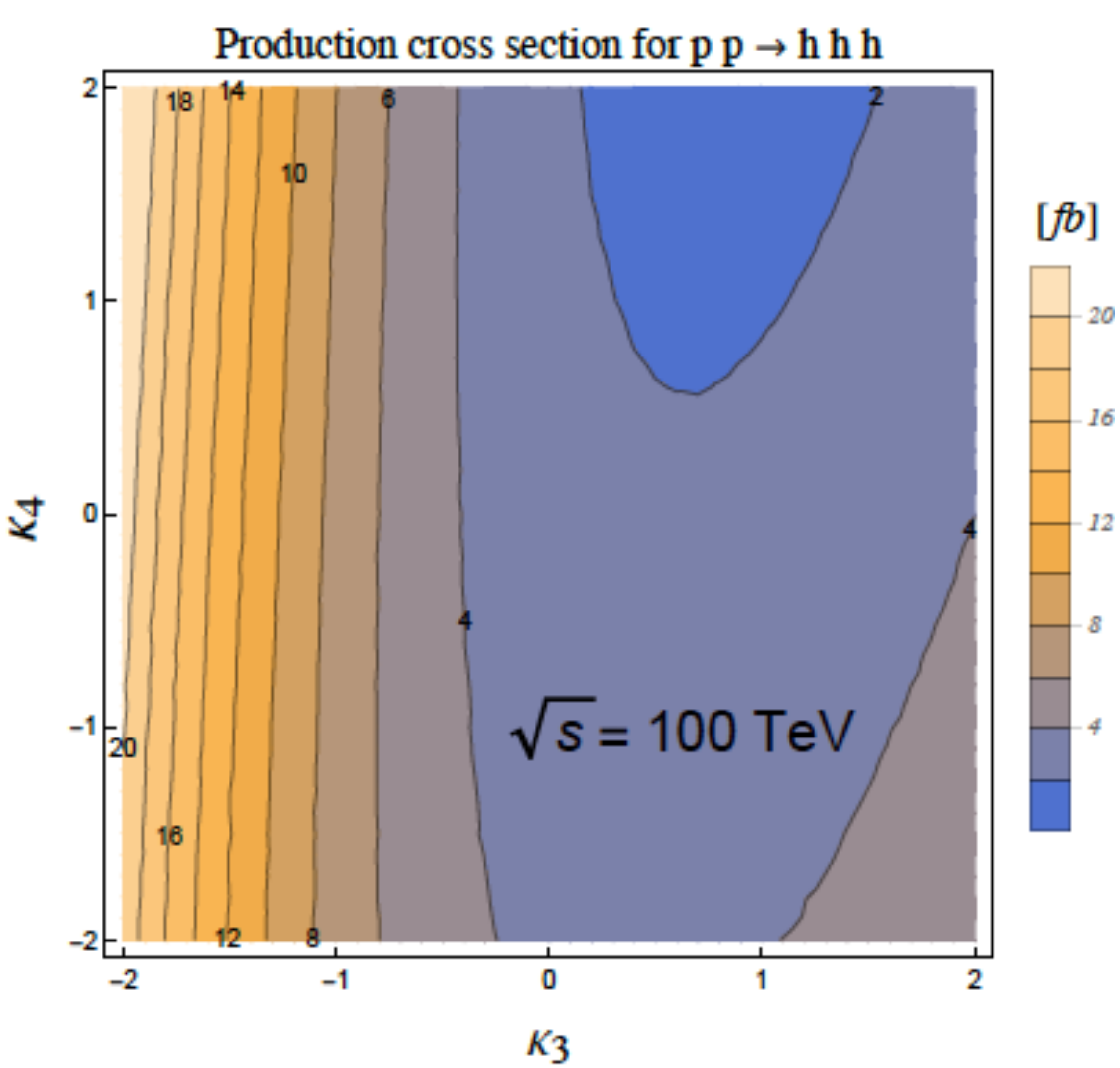
Measured

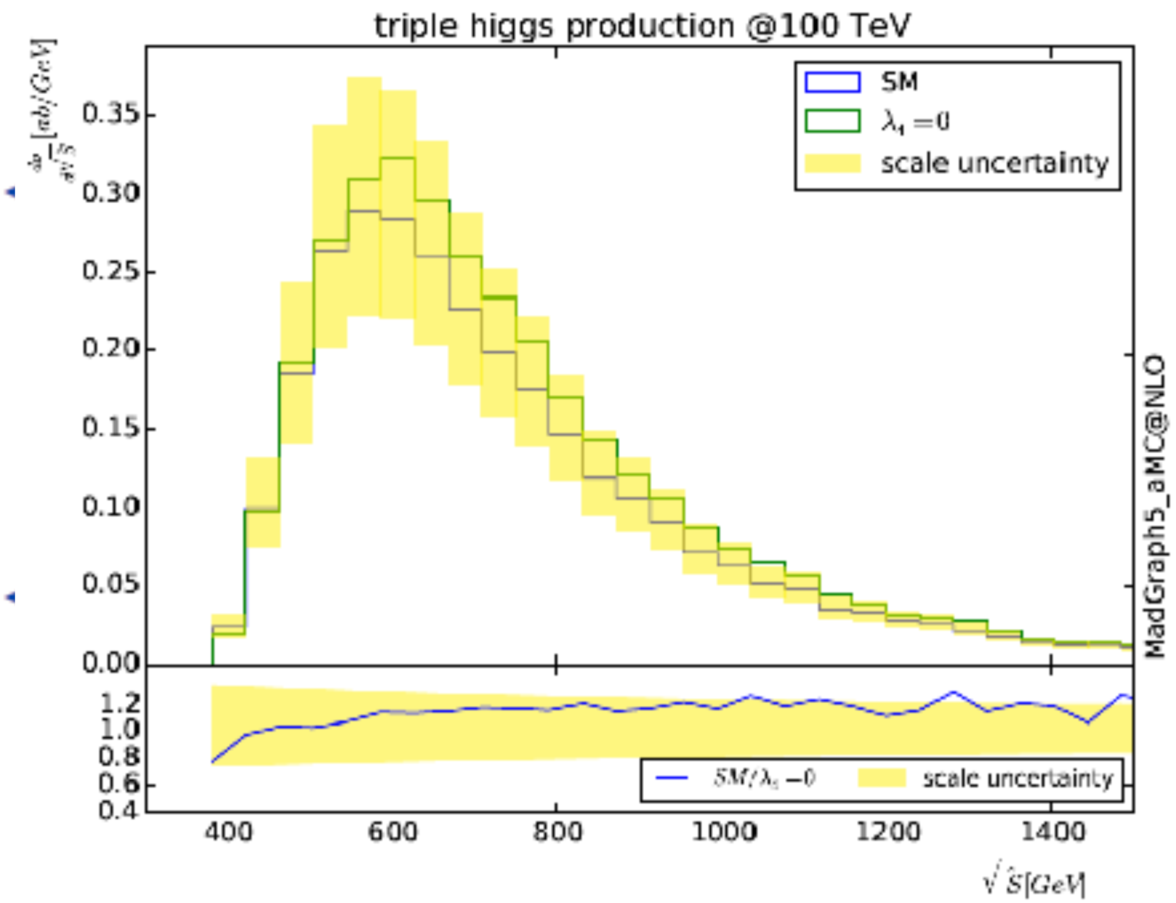
is $\kappa_3 = 0$?
Di-Higgs production
at the LHC can help

is $\kappa_4 = 0$?
No way for the LHC
(hhh cross section of ~0.01 fb)

Idea: probing triple-Higgs production at the FCC → Sensitivity to both κ parameters

Triple Higgs production at the FCC

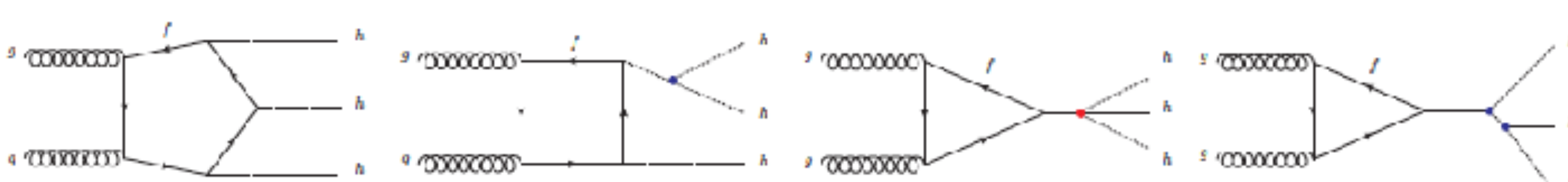




values

Could it be constrained?

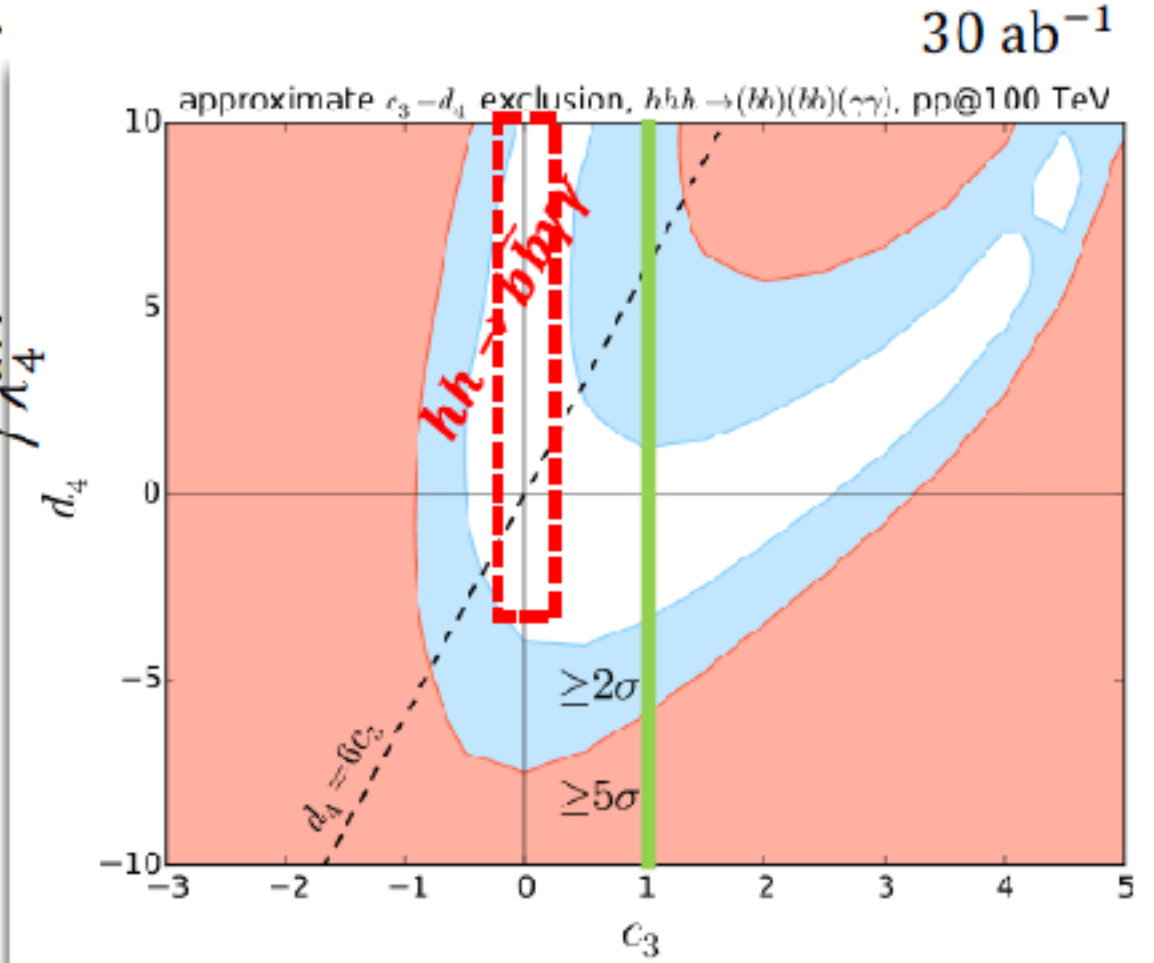
$$(1+\kappa_3)\lambda_{hhh}^{\rm SM} v h^3 + \frac{1}{4}(1+\kappa_4)\lambda_{hhhh}^{\rm SM} h^4$$



Cubic vs Quartic

$hhh \to b\bar{b}b\bar{b}\gamma\gamma$ BR=0.232 %

$\lambda_4 / \lambda_4^{SM} - 1 = d_4$

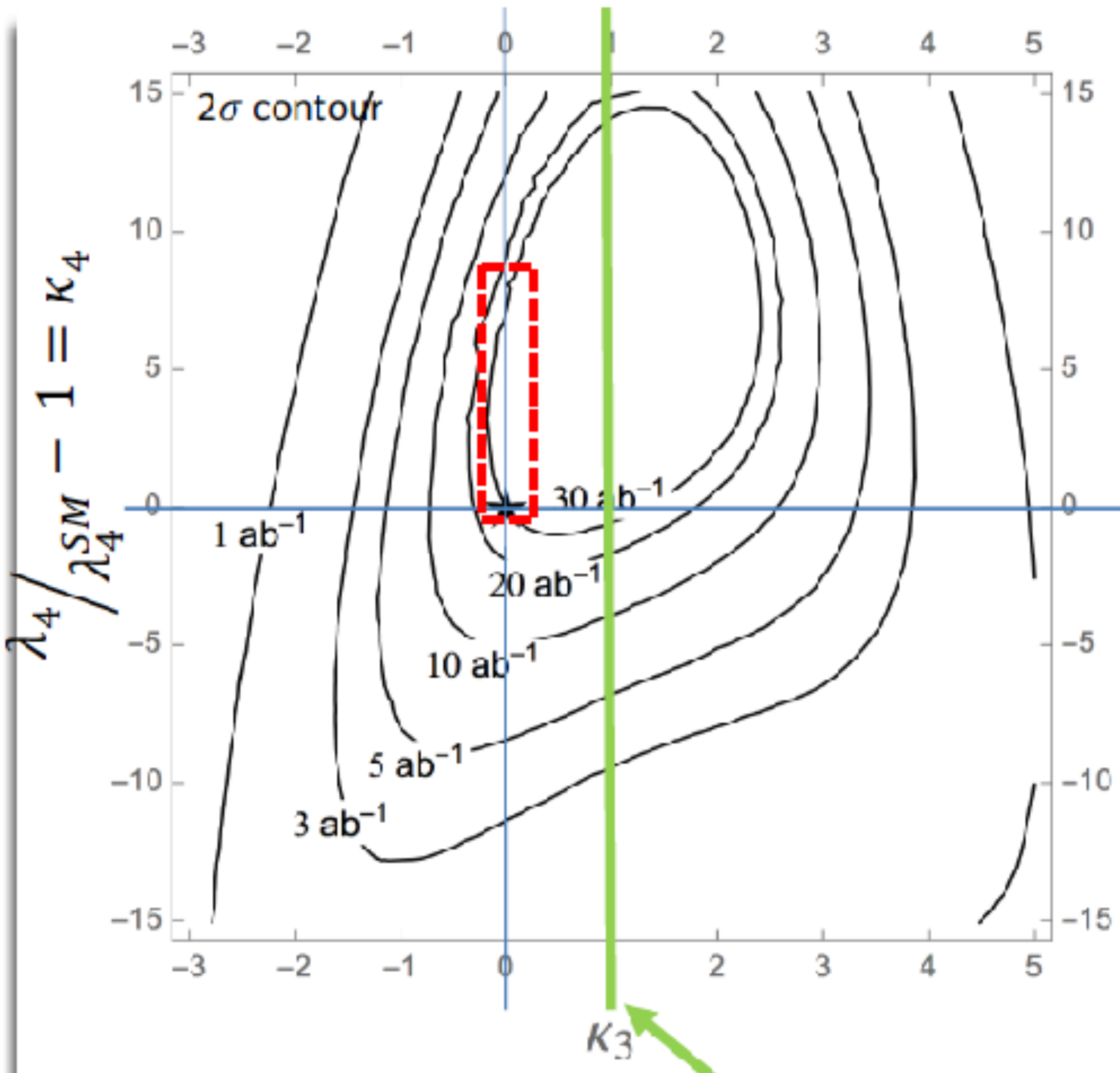


$\lambda_3 / \lambda_3^{SM} - 1 = c_3$

Papaefstathiou, Sakurai 15'
Kim, Fuks, SL 16'

$hhh \to b\bar{b}b\bar{b}\tau^+\tau^-$ $BR = 6.46$ %

$\lambda_4 / \lambda_4^{SM} - 1 = \kappa_4$



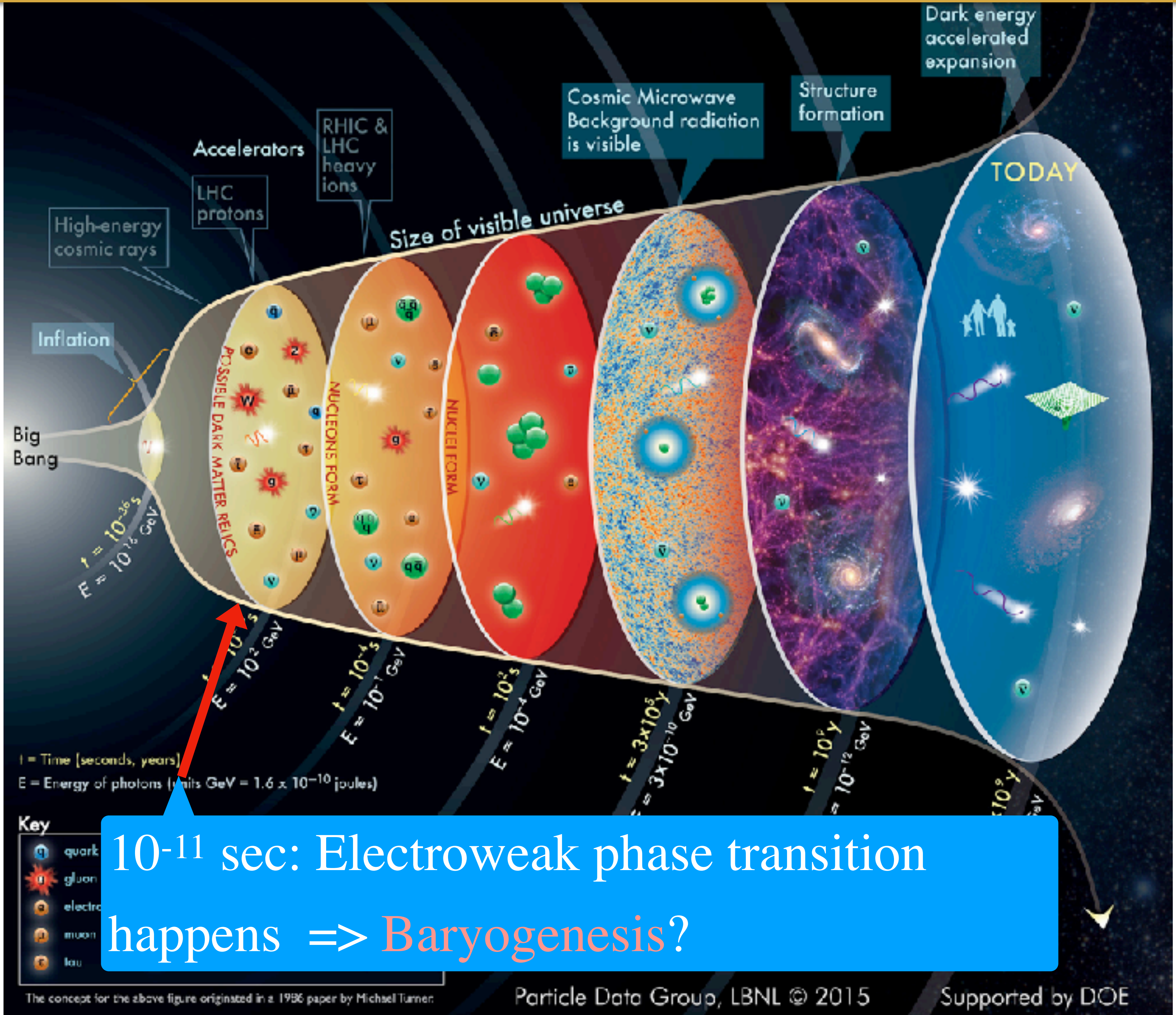
$\lambda_3 / \lambda_3^{SM} - 1 = \kappa_3$

Kim, Fuks, SL 17'

What if we observe a large κ_3 at HL LHC?

From Son's slide

History of the Universe



10^{-11} sec: Electroweak phase transition happens => Baryogenesis?

Baryon Asymmetry of the Universe

✦There is no evidence of antimatter in the universe (only p̄ in cosmic rays): $n_B \gg n_{\overline{B}}$

✦Baryon-to-photon ratio may have not changed since nucleosynthesis:

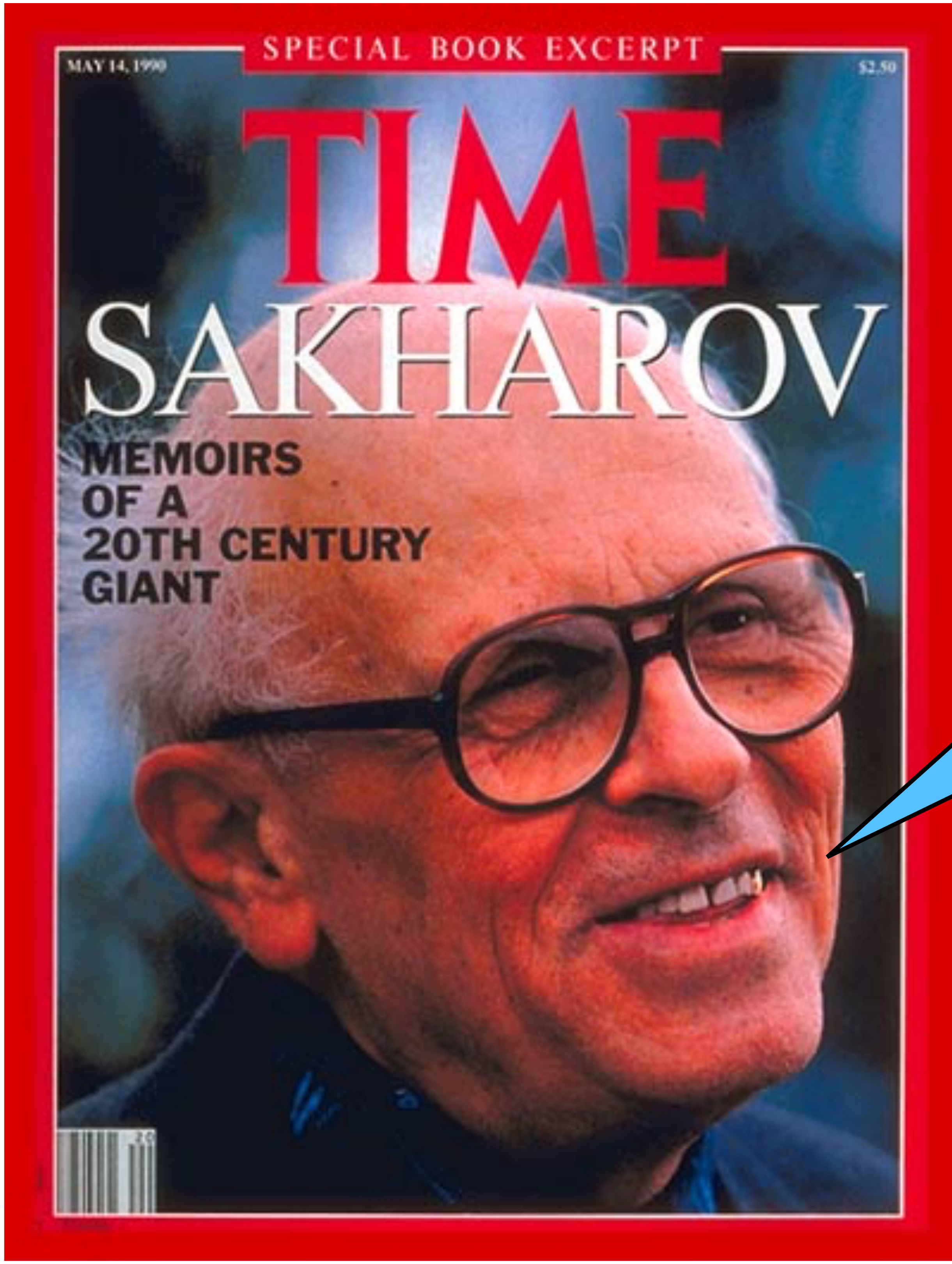
$$\eta \equiv \frac{n_B}{n_\gamma} \sim (0.3 - 1.0) \times 10^{-9} \qquad s = \frac{\pi^4}{45\zeta(3)} 3.91\ n_\gamma = 7.04\ n_\gamma$$

✦But, @ early universe (T > 100 MeV): creation and annihilation of quark-antiquark pairs $n_q,\ n_{\bar{q}} \approx n_\gamma$

$$\frac{n_q - n_{\bar{q}}}{n_q + n_{\bar{q}}} \sim 10^{-9}$$

How was this small number generated in the course of the cosmological evolution?

Baryon Asymmetry of the Universe

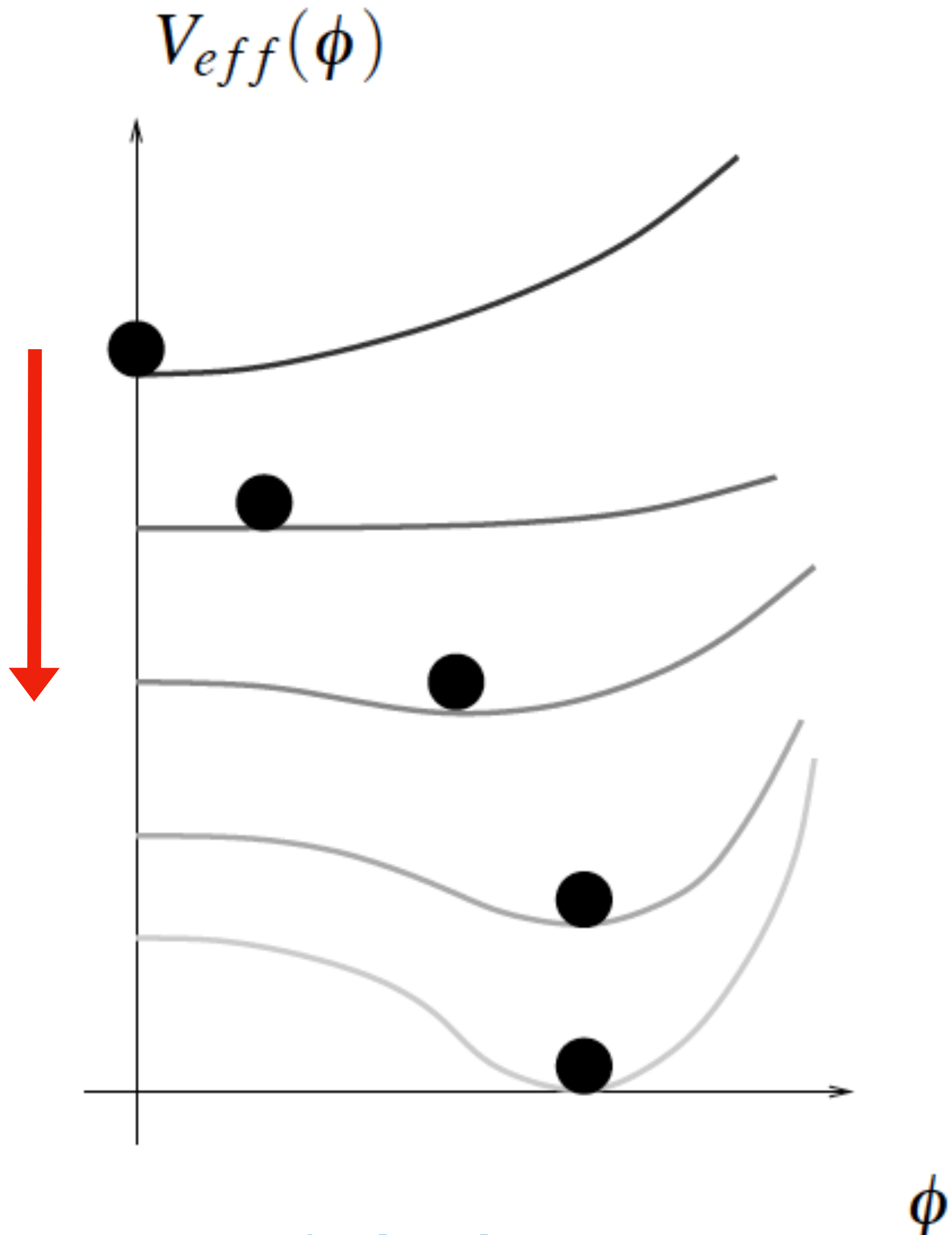


1) B- Violation

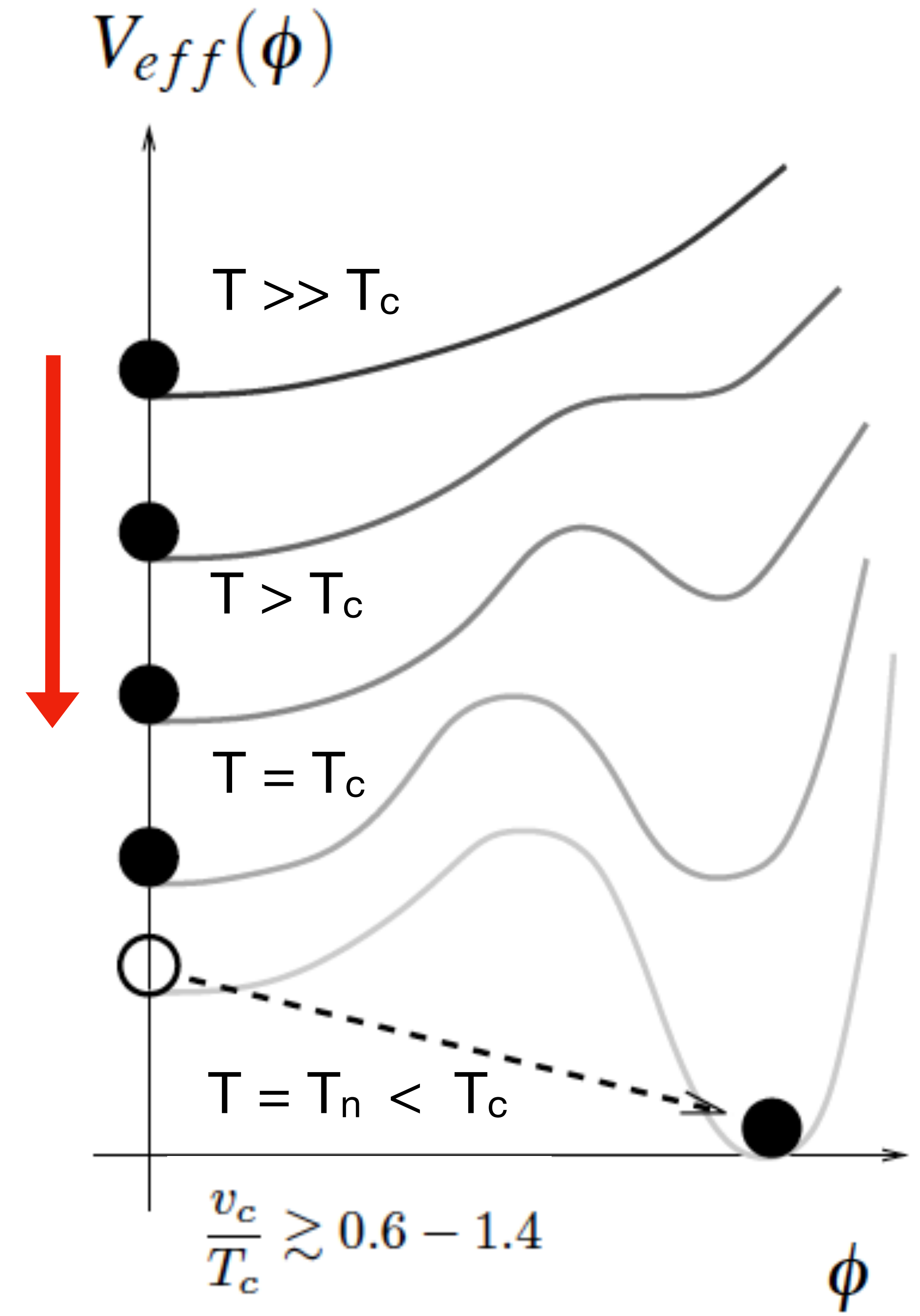
2) C- and CP- Violation

3) Thermal In-equilibrium

History of the Universe: 1st order vs 2nd order



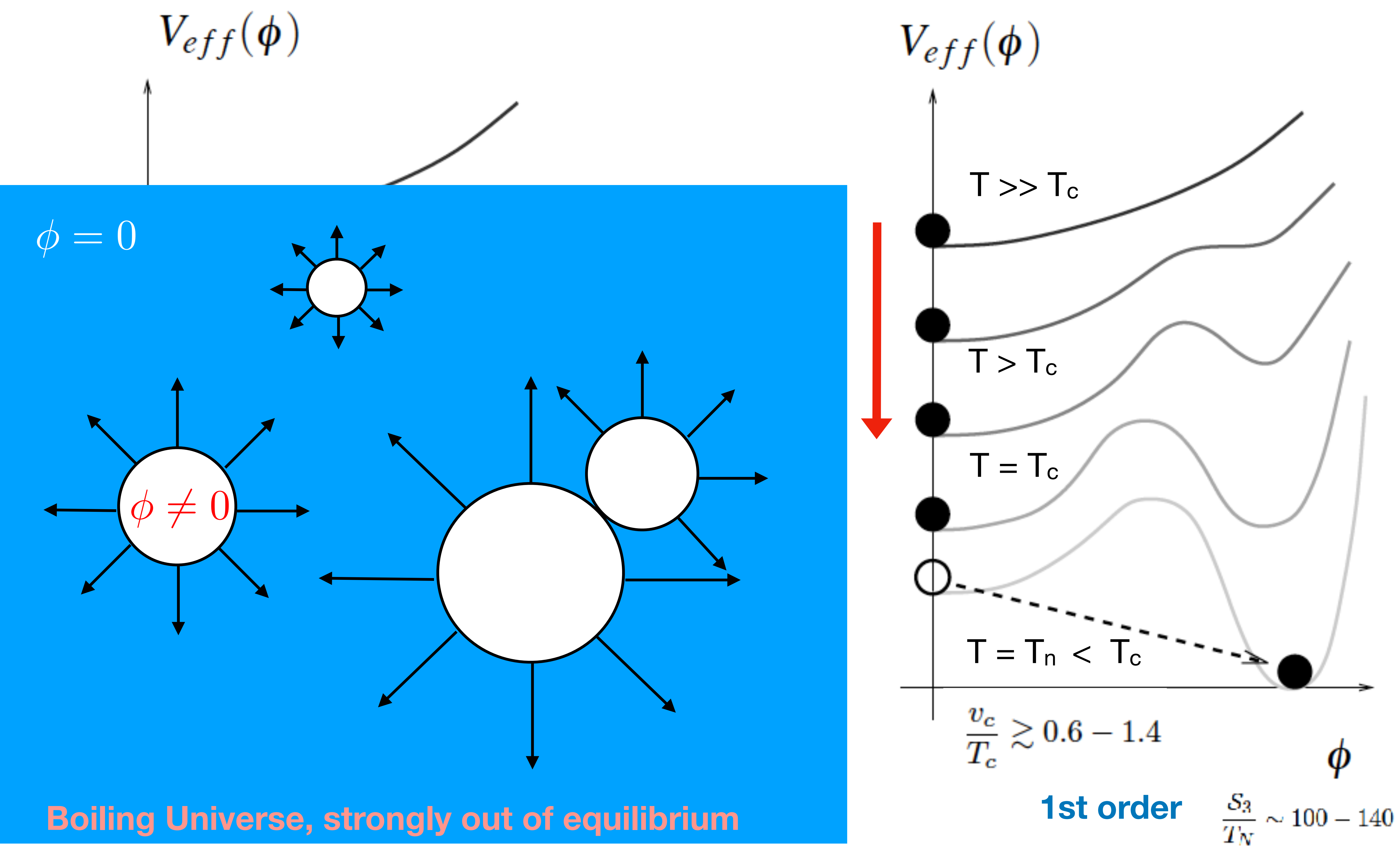
2nd order



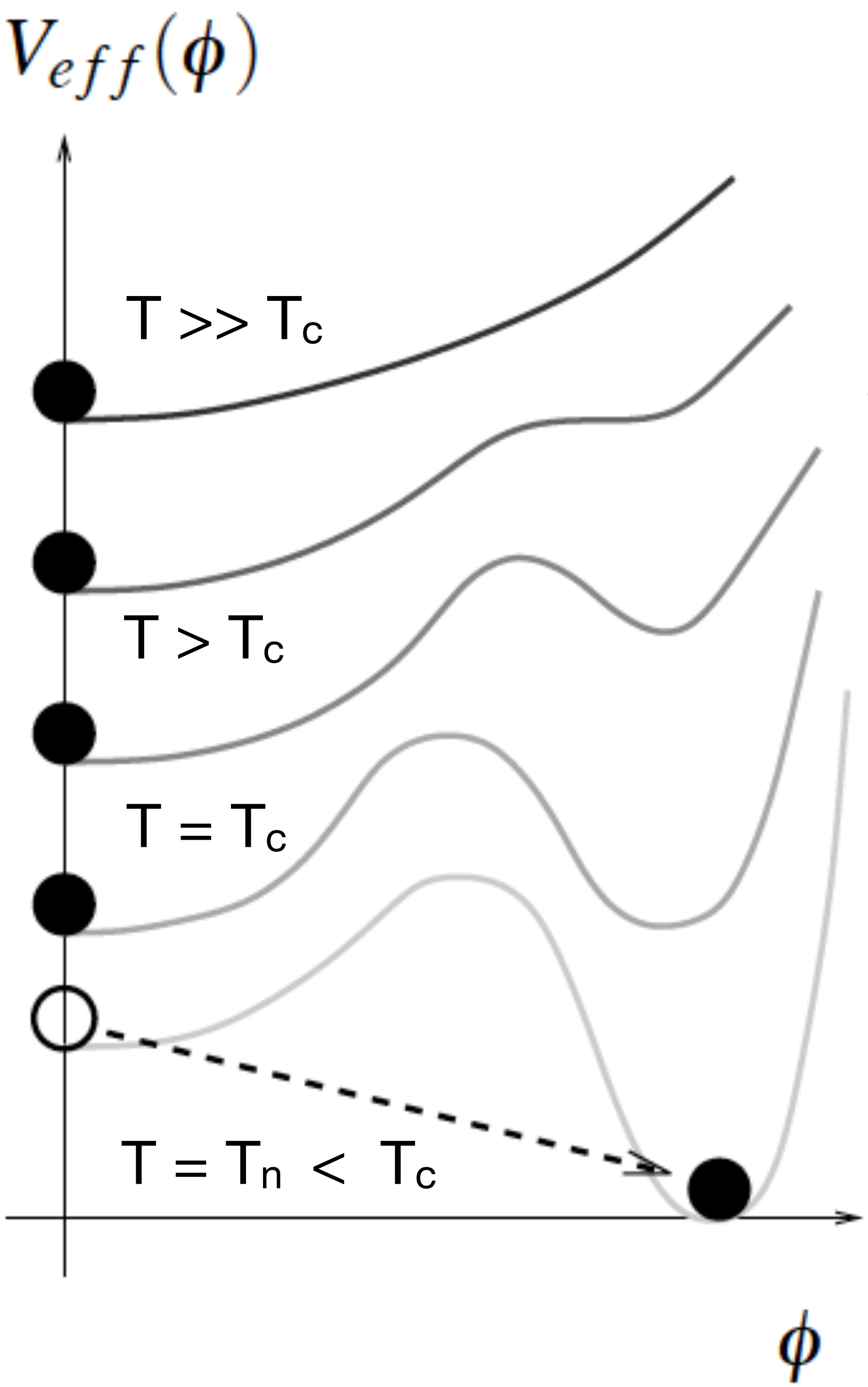
1st order

$\frac{S_3}{T_N} \sim 100 - 140$

History of the Universe: 1st order vs 2nd order



History of the Universe: 1st order vs 2nd order



1st order

$$V_{eff} = (-m^2 + \alpha T^2)|\phi|^2 - \frac{\beta}{3} T|\phi|^3 + \frac{\lambda}{4}|\phi|^4$$

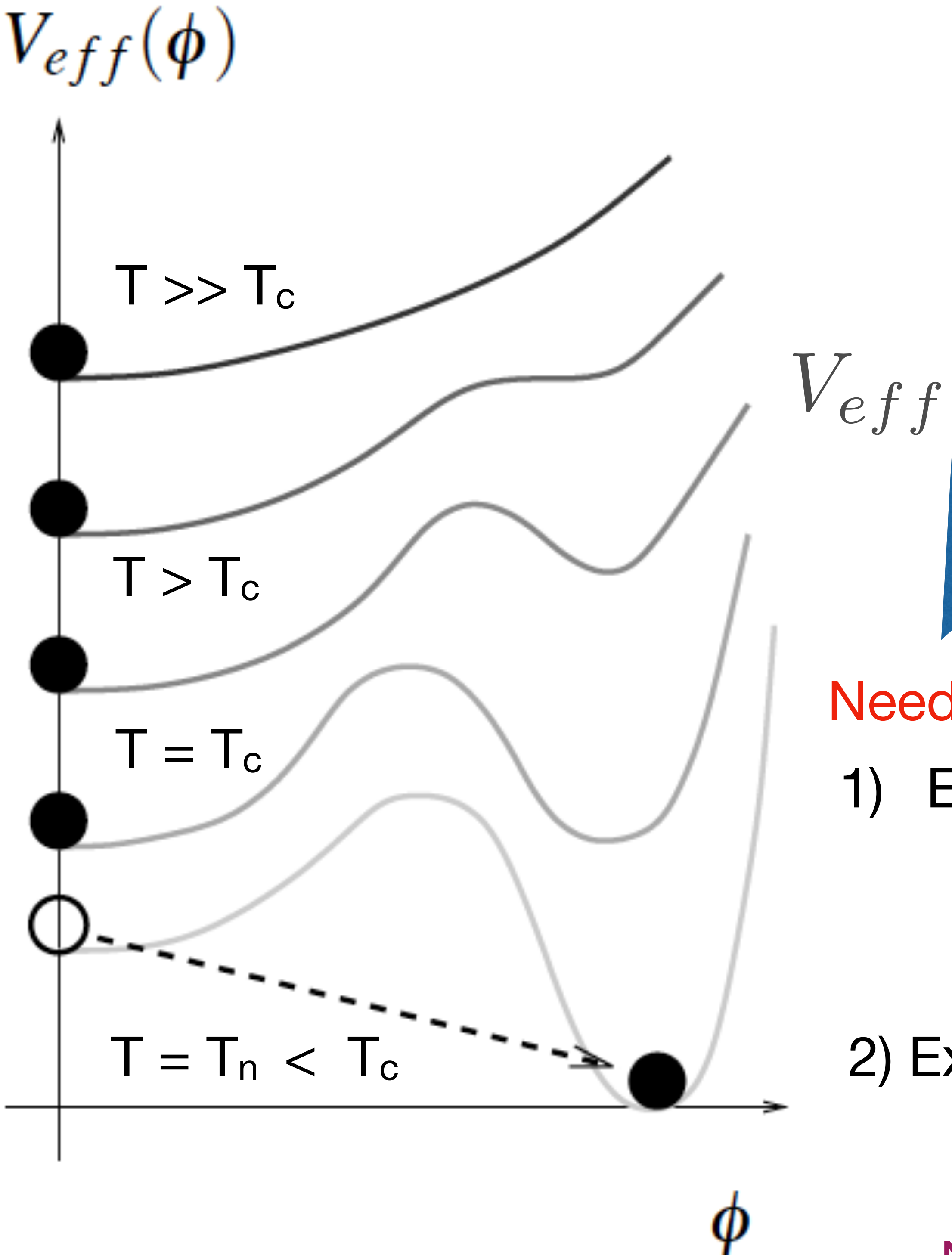
Need Beyond the SM physics enters here:

1) Extra bosons: -should interact strongly with Higgs
   -should be present in plasma at $T \sim 100$ GeV (cannot be too heavy)

2) Extra source of CP violation: model dependent - not in this discussion

N.B.: other possibilities not discussed here include: Leptogenesis, asymmetric DM (Subir Sarkar's talk), etc

History of the Universe: 1st order vs 2nd order



1st order

Here we discuss two possibilities:

1)the Higgs portal with the singlet scalar under the SM gauge group with the Z_2 symmetry

2) the EFT approach with higher-dimensional operators

Need Beyond the SM physics enters here:

1) Extra bosons: -should interact strongly with Higgs
-should be present in plasma at T ~ 100 GeV (cannot be too heavy)

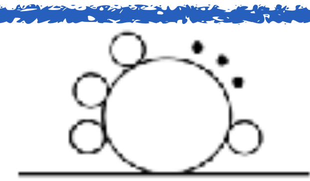
2) Extra source of CP violation: model dependent
- not in this discussion

N.B.: other possibilities not discussed here include: Leptogenesis, asymmetric DM (Subir Sarkar's talk), etc

Effective Potential @ finite T

♦ Truncated Full Dressing (TFD):

$$V_{eff}(\phi_i, T) = V_{tree}(\phi_i) + V_{CW}(m_i^2(\phi) + \Pi_i) + V_T(m_i^2(\phi) + \Pi_i, T)$$



Resum leading type of loops

thermal mass (can be computed by solving finite-T gap equations)

obtained in the high-T approximation

Coleman Weinberg Potential

$$V_{CW}(m_i^2(\phi) + \Pi_i) = \sum_i (-1)^{F_i} \frac{g_i}{64\pi^2} \left[m_i^4(\phi) \left(\log \frac{m_i^2(\phi) + \Pi_i}{m_i^2(v) + \Pi_i} - \frac{3}{2} \right) + 2 \left(m_i^2(\phi) + \Pi_i \right) \left(m_i^2(v) + \Pi_i \right) \right]$$

Thermal potential @ finite T

$$V_T(m_i^2(\phi) + \Pi_i, T) = \sum_i (-1)^{F_i} \frac{g_i T^4}{2\pi^2} \int_0^\infty dx\, x^2 \log \left[1 \mp \exp\left(- \sqrt{x^2 + \left(m_i^2(\phi) + \Pi_i \right) / T^2} \right) \right]$$

$$m_i^2(h) = m^2 + \text{coupling} \times h^2$$

For $v_c \gtrsim T_c$ and $\gtrsim \mathcal{O}(1)$ coupling, integral needs to be exactly evaluated

Prescription A

♦ Validity of High-T approx. / Validity of perturbation

⇒ no so rigorously treated in most literature for BSM physics

N.B.: Curtin, Meade, Ramani 16'

We just point out a few important issues we observed.

Effective Potential @ finite T

- A self-consistent High-T approximation

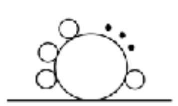
$$V_{eff}(\phi,T) \equiv V_{tree}(\phi) + V_{CW}(m_i^2(\phi)) + V_T(m_i^2(\phi),T) + V_{ring}(m_i^2(\phi),T)$$

Coleman Weinberg Potential

$$V_{CW}(m_i^2(\phi)) = \sum_i (-1)^{F_i} \frac{g_i}{64\pi^2}\left[m_i^4(\phi)\left(\log\frac{m_i^2(\phi)}{m_i^2(v)} - \frac{3}{2}\right) + 2m_i^2(\phi)m_i^2(v)\right]$$

Thermal potential @ finite T

$$V_T(m_i^2(\phi),T) = \sum_i (-1)^{F_i} \frac{g_i T^4}{2\pi^2} J_{B/F}\left(\frac{m_i^2(\phi)}{T^2}\right)$$



Ring term

$$V_{ring}(m_i^2(\phi),T) = -\sum_i \frac{T}{12\pi} Tr\left[m_i^3(\phi_i) - (m_i^2(\phi) + \Pi_i(0))^{3/2}\right]$$

(at high-T expansion)

$$\alpha = m/T \ll 1$$

can induce the 1st order PT via thermal effects

$$\begin{cases} J_B(\alpha^2) = \int_0^\infty dx\, x^2 \ln\left(1 - e^{-\sqrt{x^2+\alpha^2}}\right) \sim \frac{\pi^2}{12}\alpha^2 - \frac{\pi}{6}\alpha^3 - \frac{\pi^4}{45} - \frac{1}{32}\alpha^4 \ln\left(\frac{\alpha^2}{a_b}\right) \\ J_F(\alpha^2) = \int_0^\infty dx\, x^2 \ln\left(1 + e^{-\sqrt{x^2+\alpha^2}}\right) \sim -\frac{\pi^2}{24}\alpha^2 + \frac{7\pi^4}{360} - \frac{1}{32}\alpha^4 \ln\left(\frac{\alpha^2}{a_f}\right) \end{cases}$$

Prescription B

Benchmark Scenarios

- Higgs Portal (SM + singlet scalar S with Z_2 symmetry)

$$V_{tree} = -\frac{\mu^2}{2}h^2 + \frac{\lambda}{4}h^4 + \frac{1}{2}\lambda_{HS}h^2S^2 + \frac{1}{2}\mu_S^2S^2 + \frac{1}{4}\lambda_S S^4$$

  - $\langle S \rangle = 0$ vs $\langle S \rangle \neq 0$ (no mixing vs mixing with Higgs)
  - N_s (# of scalar multiplicity) for weaker coupling

“no-lose" theorem for testing EWBG in future colliders

- EFT with higher dimensional operators

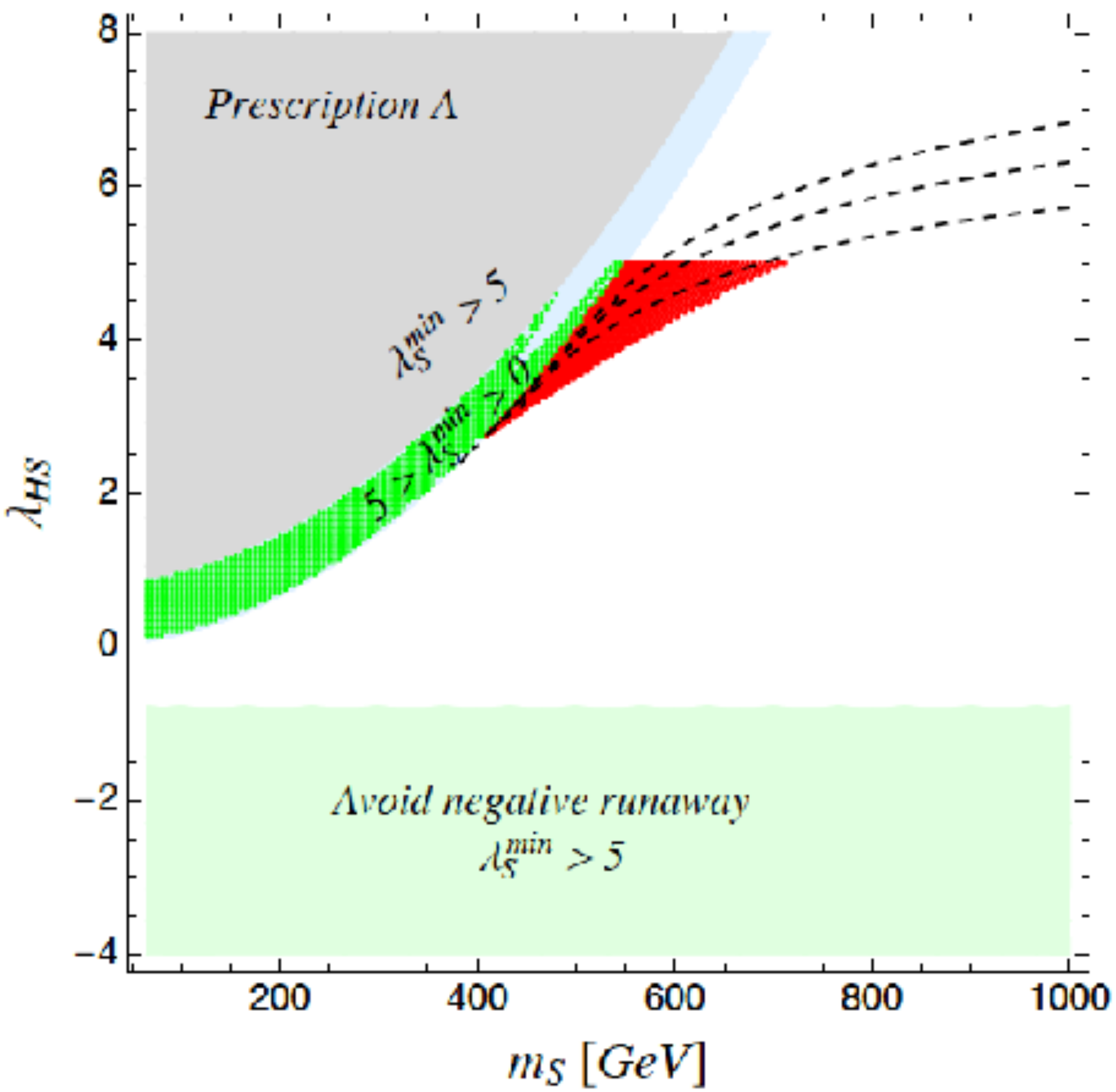
Benchmark Scenarios

"no-lose" theorem for testing EWBG in future colliders

- **Higgs Portal**
  (SM + singlet scalar S with Z_2 symmetry)

$$V_{tree} \approx \frac{\lambda}{4}h^4 + \frac{1}{2}\lambda_{HS}h^2S^2 + \frac{1}{4}\lambda_S S^4 \,,$$
$$= \frac{1}{4}\left[\left(\sqrt{\lambda}h^2 - \sqrt{\lambda_S}S^2\right)^2 + 2h^2S^2\left(\lambda_{HS} + \sqrt{\lambda\,\lambda_S}\right)\right]$$

Exact V_T



1. One-step strong 1st phase transition (**RED dots**)

$$V(0,0) \rightarrow V(v,0) \quad , \langle S \rangle = 0$$

2. Two-step strong 1st phase transition (**GREEN dots**)

$$V(0,0) \rightarrow V(0,v_s) \rightarrow V(v,0)$$

$$V(0,v_s) > V(v,0) \rightarrow$$

$$\lambda_S > \lambda_S^{\min} \equiv \lambda \frac{m_{0\,s}^4}{m^4} = \frac{2(m_s^2 - v^2\lambda_{HS})^2}{m_h^2 v^2}$$

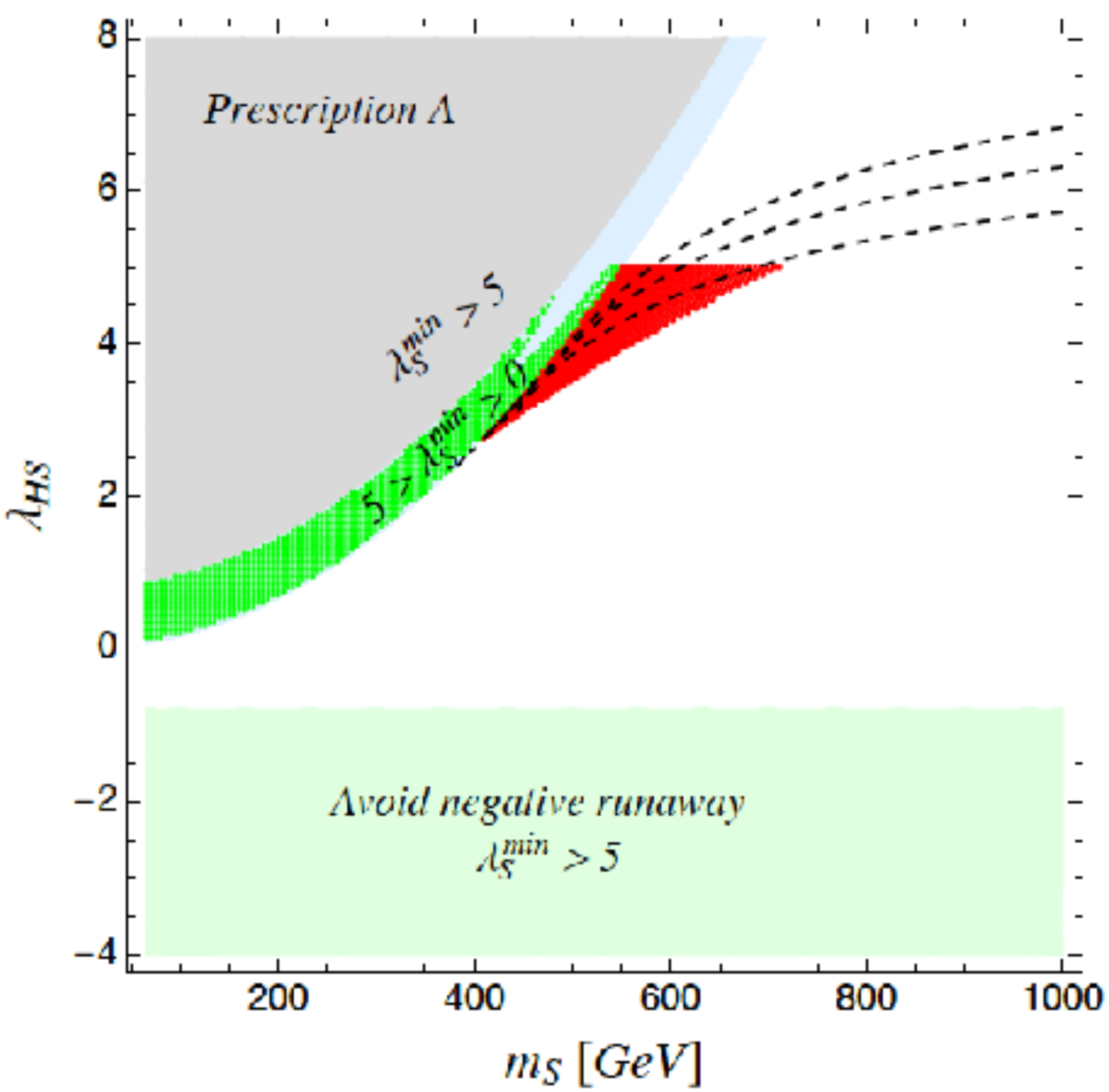
Benchmark Scenarios

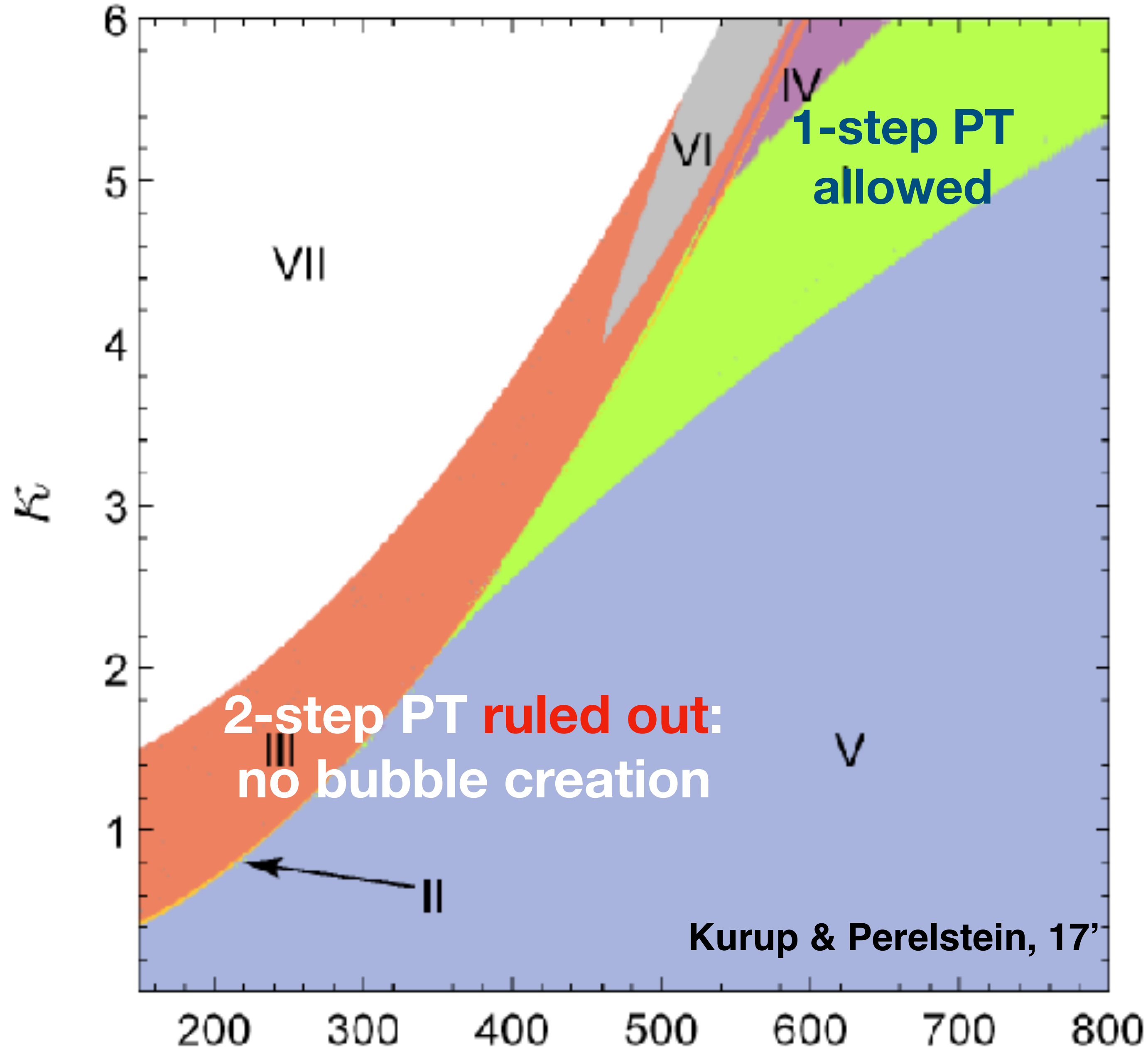
"no-lose" theorem for testing EWBG in future colliders

- **Higgs Portal**
  (SM + singlet scalar S with Z_2 symmetry)

$$V_{tree} \approx \frac{\lambda}{4}h^4 + \frac{1}{2}\lambda_{HS}h^2S^2 + \frac{1}{4}\lambda_S S^4 ,$$
$$= \frac{1}{4}\left[\left(\sqrt{\lambda}h^2 - \sqrt{\lambda_S}S^2\right)^2 + 2h^2S^2\left(\lambda_{HS} + \sqrt{\lambda\,\lambda_S}\right)\right]$$

Exact V_T





Benchmark Scenarios

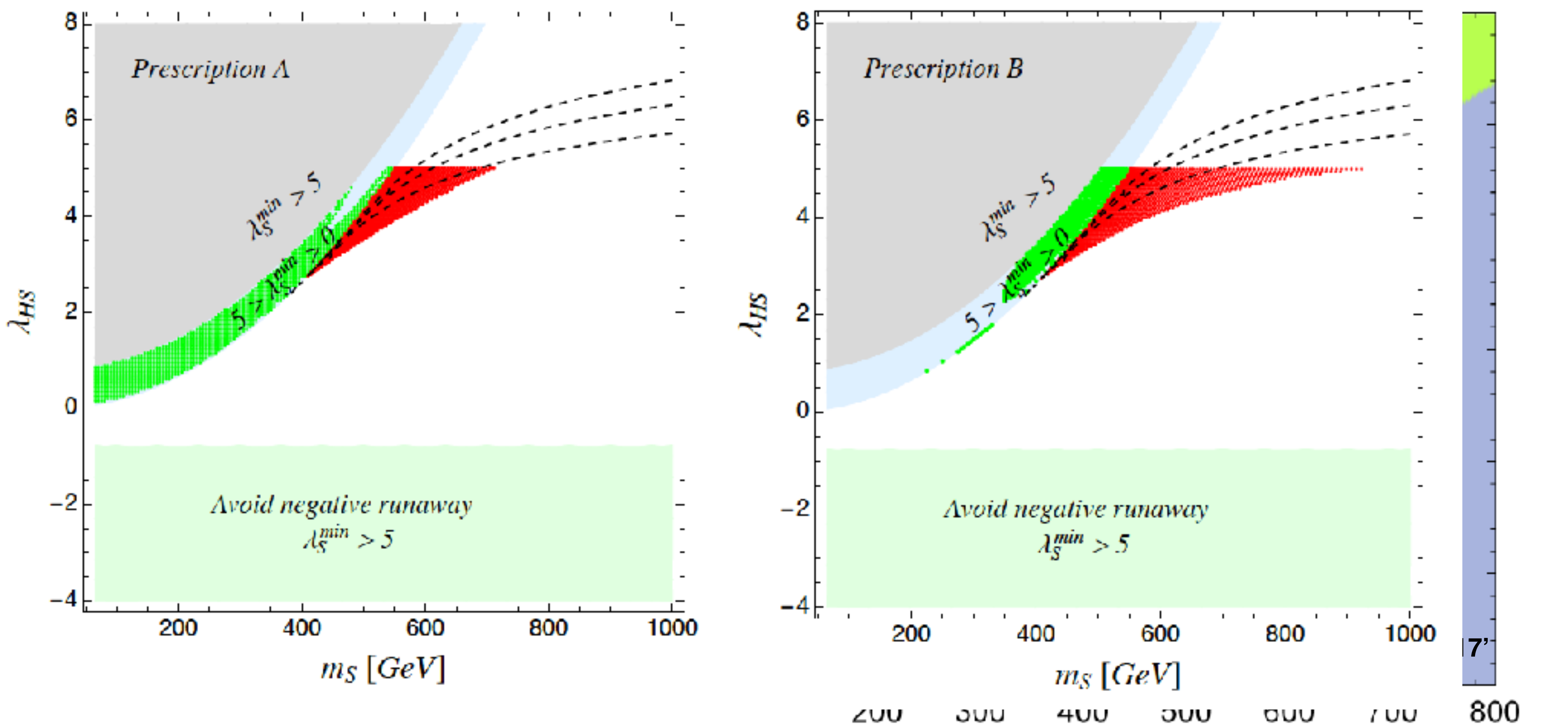
"no-lose" theorem for testing EWBG in future colliders

- **Higgs Portal**

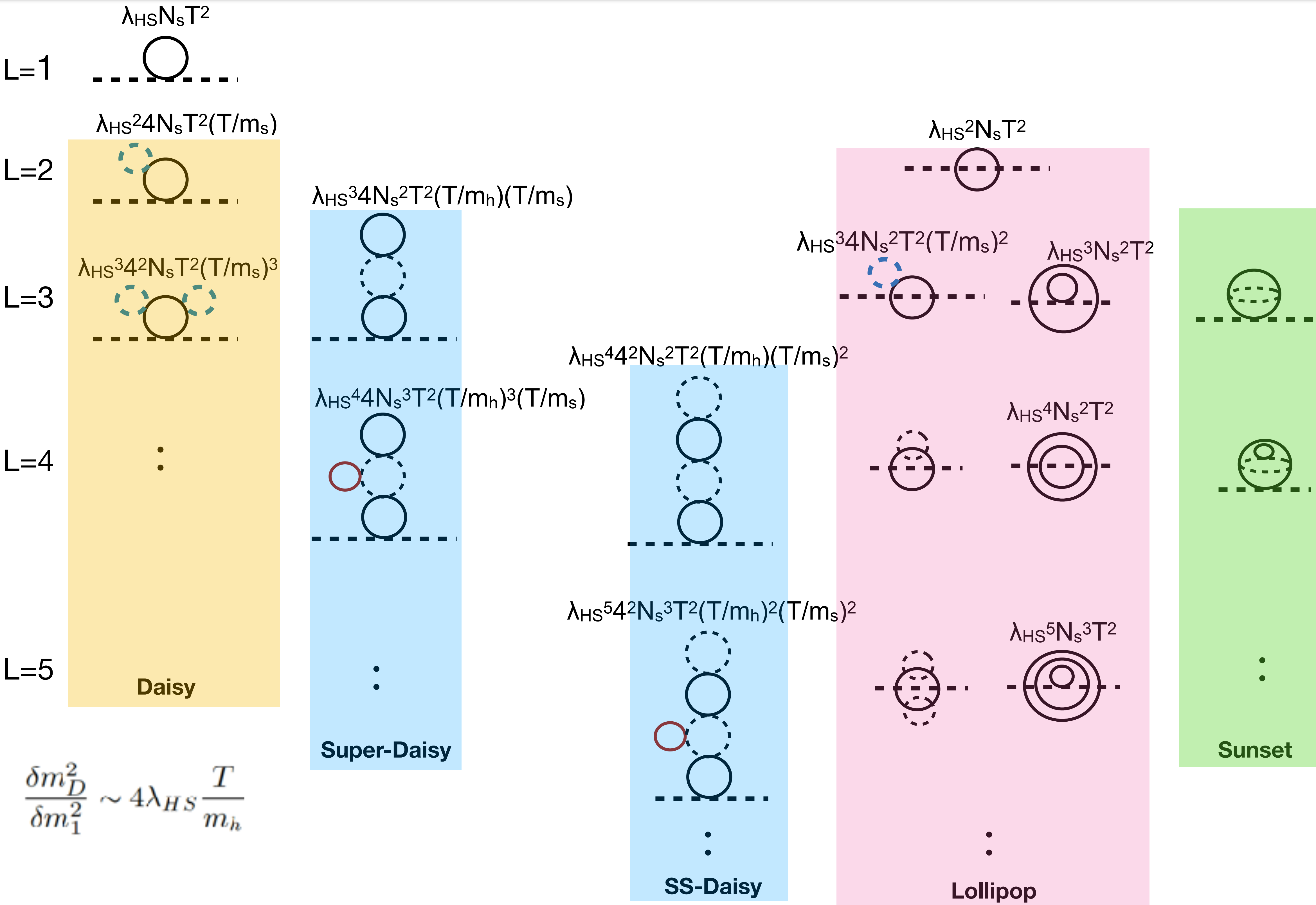
  (SM + singlet scalar S with Z_2 symmetry)

$$V_{tree} \approx \frac{\lambda}{4}h^4 + \frac{1}{2}\lambda_{HS}h^2S^2 + \frac{1}{4}\lambda_S S^4 \,,$$
$$= \frac{1}{4}\left[\left(\sqrt{\lambda}h^2 - \sqrt{\lambda_S}S^2\right)^2 + 2h^2S^2\left(\lambda_{HS} + \sqrt{\lambda\,\lambda_S}\right)\right]$$

Exact V_T | High-T Approximated V_T



Naive Power Counting: break down of PT?



$$\frac{\delta m_D^2}{\delta m_1^2} \sim 4\lambda_{HS}\frac{T}{m_h}$$

$$\frac{\delta m_{LL}^2}{\delta m_D^2} \sim \frac{1}{4}\frac{m_s}{T} \sim \mathcal{O}(1)$$

c.f.: ϕ^4 therory: $\frac{\delta m_{LL}^2}{\delta m_D^2} \sim \frac{1}{4+N_s}\frac{m}{T}$

Benchmark Scenarios

“no-lose" theorem for testing EWBG in future colliders

- Higgs Portal

(SM + singlet scalar S with Z_2 symmetry)

$$V_{tree} \approx \frac{\lambda}{4}h^4 + \frac{1}{2}\lambda_{HS}h^2S^2 + \frac{1}{4}\lambda_S S^4 \,,$$
$$= \frac{1}{4}\left[\left(\sqrt{\lambda}h^2 - \sqrt{\lambda_S}S^2\right)^2 + 2h^2S^2\left(\lambda_{HS} + \sqrt{\lambda\,\lambda_S}\right)\right]$$

for the SFOEPT is satisfied, $v_c > T_c$, with O(1) coupling, $\frac{m^2(v_c)}{T_c^2} \sim \mathcal{O}(1) \times \frac{v_c^2}{T_c^2} \gtrsim 1$

Benchmark Scenarios

"no-lose" theorem for testing EWBG in future colliders

- **Higgs Portal**

  (SM + singlet scalar S with Z_2 symmetry)

$$V_{tree} \approx \frac{\lambda}{4} h^4 + \frac{1}{2}\lambda_{HS} h^2 S^2 + \frac{1}{4}\lambda_S S^4 \ ,$$
$$= \frac{1}{4}\left[\left(\sqrt{\lambda} h^2 - \sqrt{\lambda_S} S^2\right)^2 + 2h^2 S^2 \left(\lambda_{HS} + \sqrt{\lambda\,\lambda_S}\right) \right]$$

for the SFOEPT is satisfied, $v_c > T_c$, with O(1) coupling,

$$\frac{m^2(v_c)}{T_c^2} \sim \mathcal{O}(1) \times \frac{v_c^2}{T_c^2} \gtrsim 1$$

High-T approximation is violated!

Benchmark Scenarios

"no-lose" theorem for testing EWBG in future colliders

- **Higgs Portal**
  (SM + singlet scalar S with Z_2 symmetry)

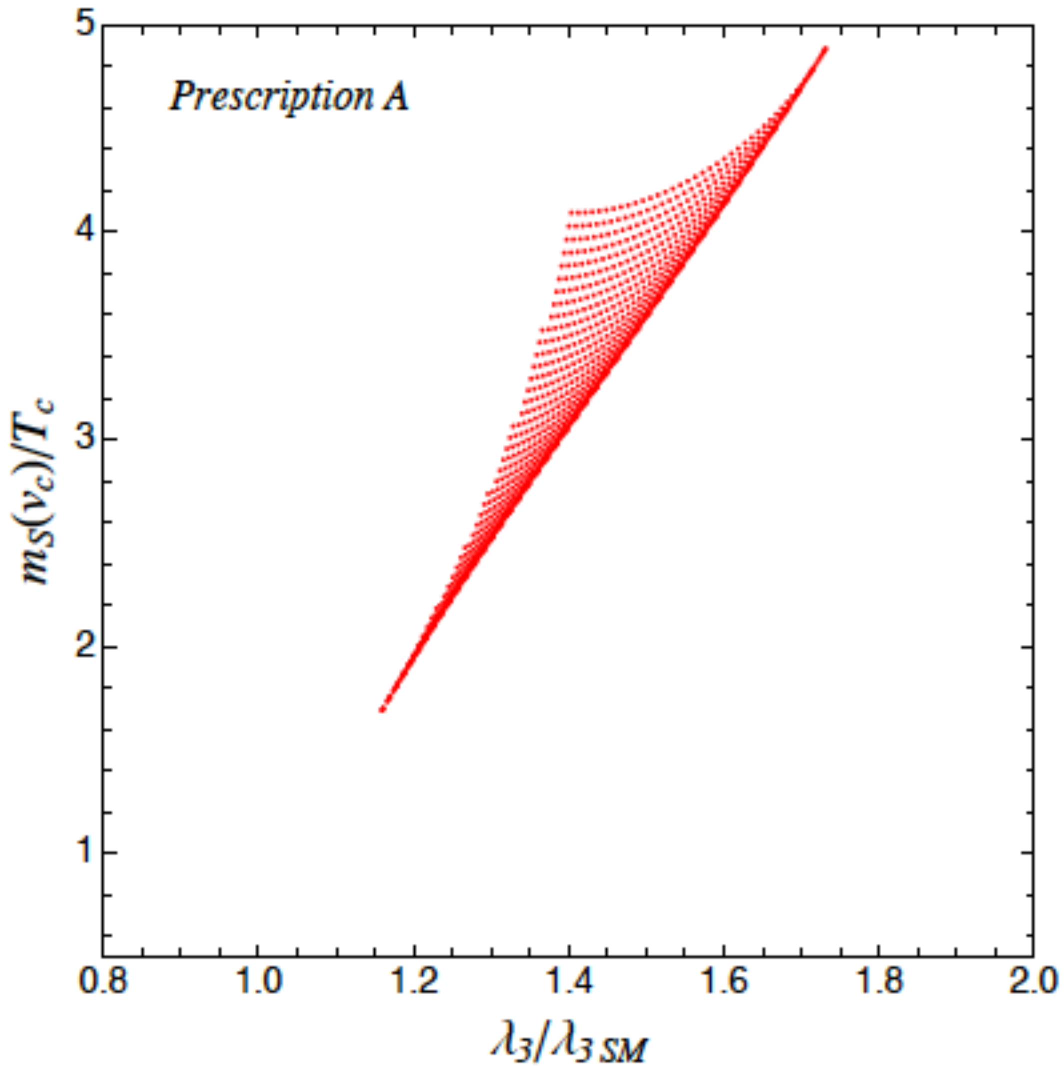
$$V_{tree} \approx \frac{\lambda}{4}h^4 + \frac{1}{2}\lambda_{HS}h^2S^2 + \frac{1}{4}\lambda_S S^4 ,$$
$$= \frac{1}{4}\left[\left(\sqrt{\lambda}h^2 - \sqrt{\lambda_S}S^2\right)^2 + 2h^2S^2\left(\lambda_{HS} + \sqrt{\lambda\,\lambda_S}\right)\right]$$

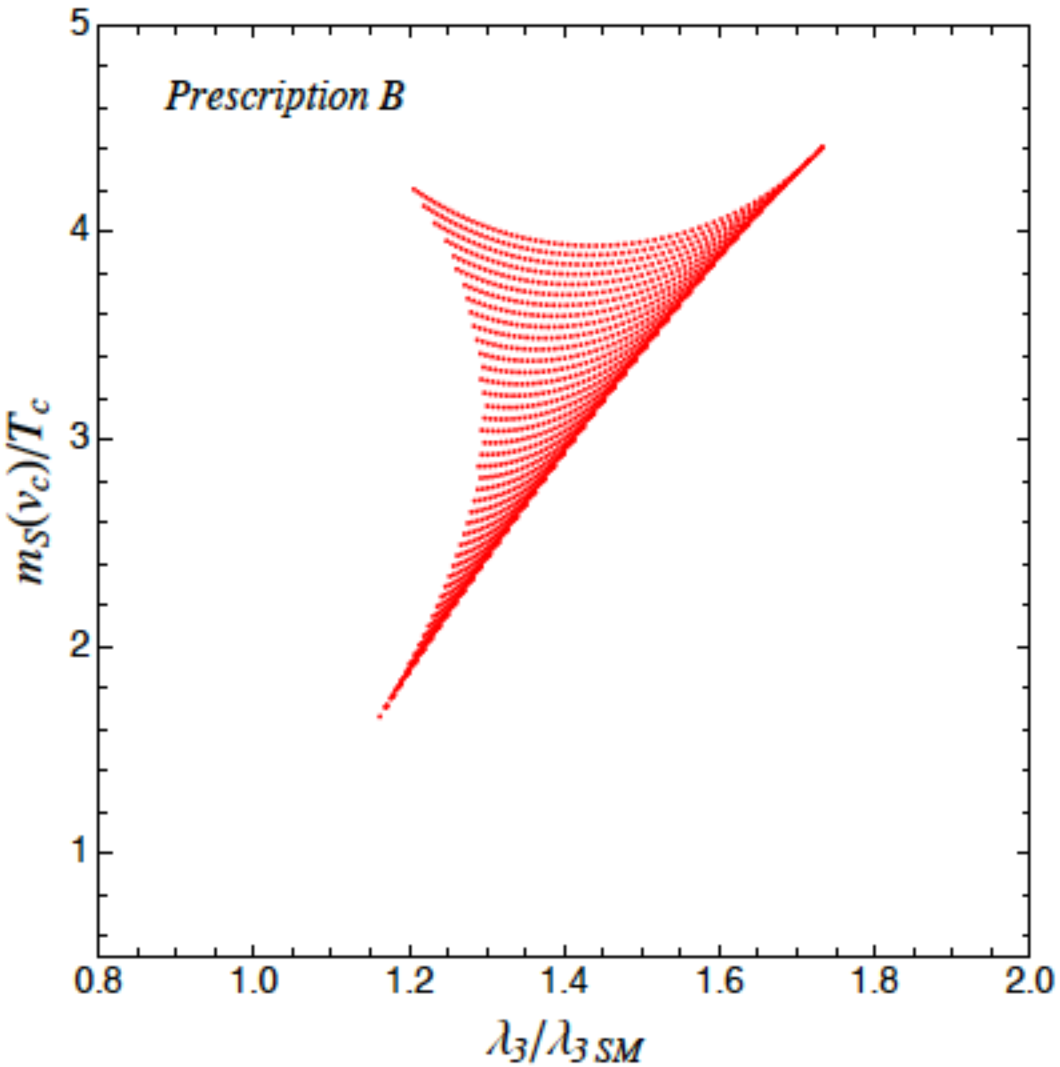
for the SFOEPT is satisfied, $v_c > T_c$, with O(1) coupling,

$$\frac{m^2(v_c)}{T_c^2} \sim \mathcal{O}(1) \times \frac{v_c^2}{T_c^2} \gtrsim 1$$

High-T approximation is violated!

Exact V_T | High-T Approximated V_T





Benchmark Scenarios

"no-lose" theorem for testing EWBG in future colliders

- **Higgs Portal**
  (SM + singlet scalar S with Z_2 symmetry)

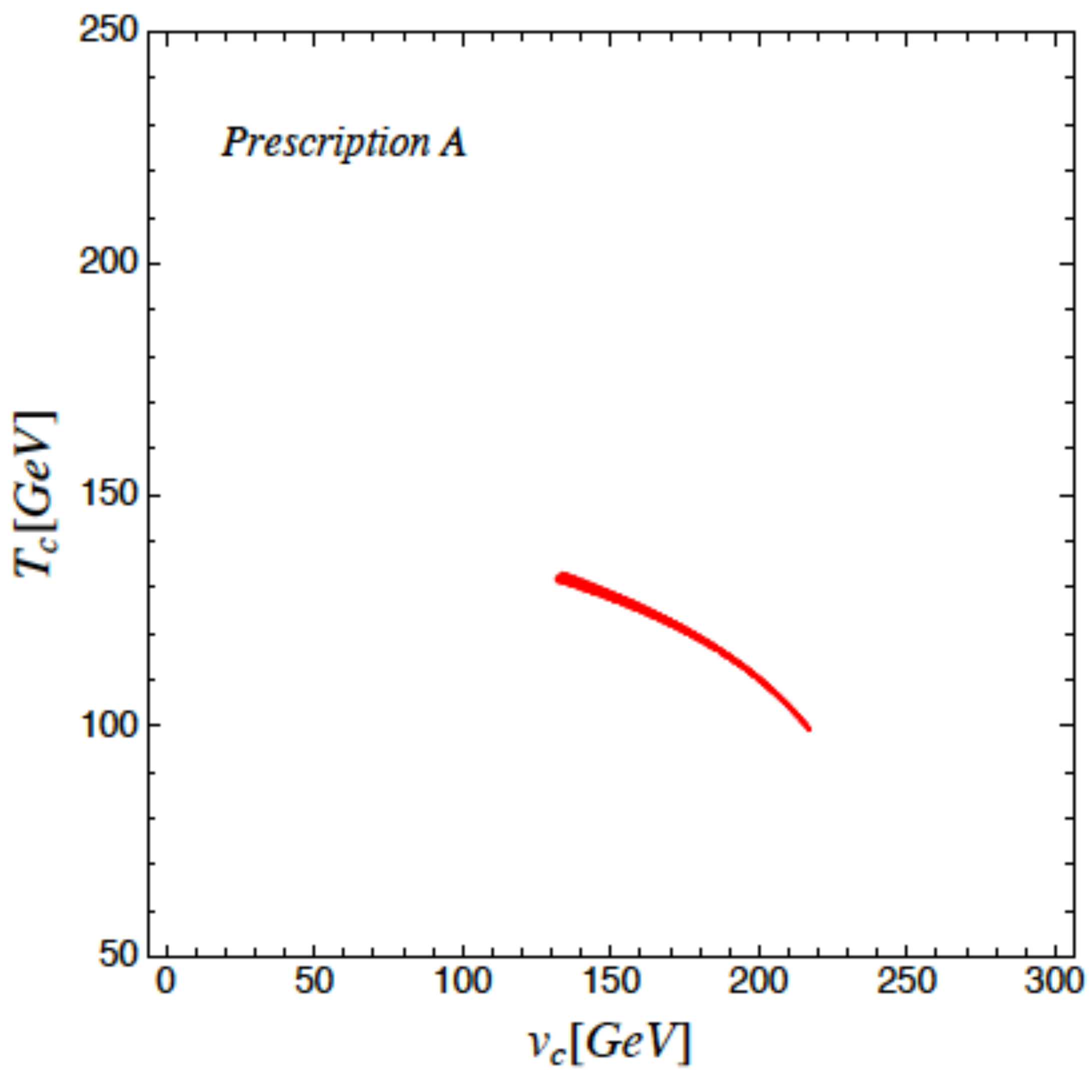
$$V_{tree} \approx \frac{\lambda}{4} h^4 + \frac{1}{2}\lambda_{HS} h^2 S^2 + \frac{1}{4}\lambda_S S^4 \,,$$
$$= \frac{1}{4}\left[\left(\sqrt{\lambda} h^2 - \sqrt{\lambda_S} S^2\right)^2 + 2h^2 S^2 \left(\lambda_{HS} + \sqrt{\lambda \lambda_S}\right) \right]$$

for the SFOEPT is satisfied, $v_c > T_c$, with O(1) coupling,

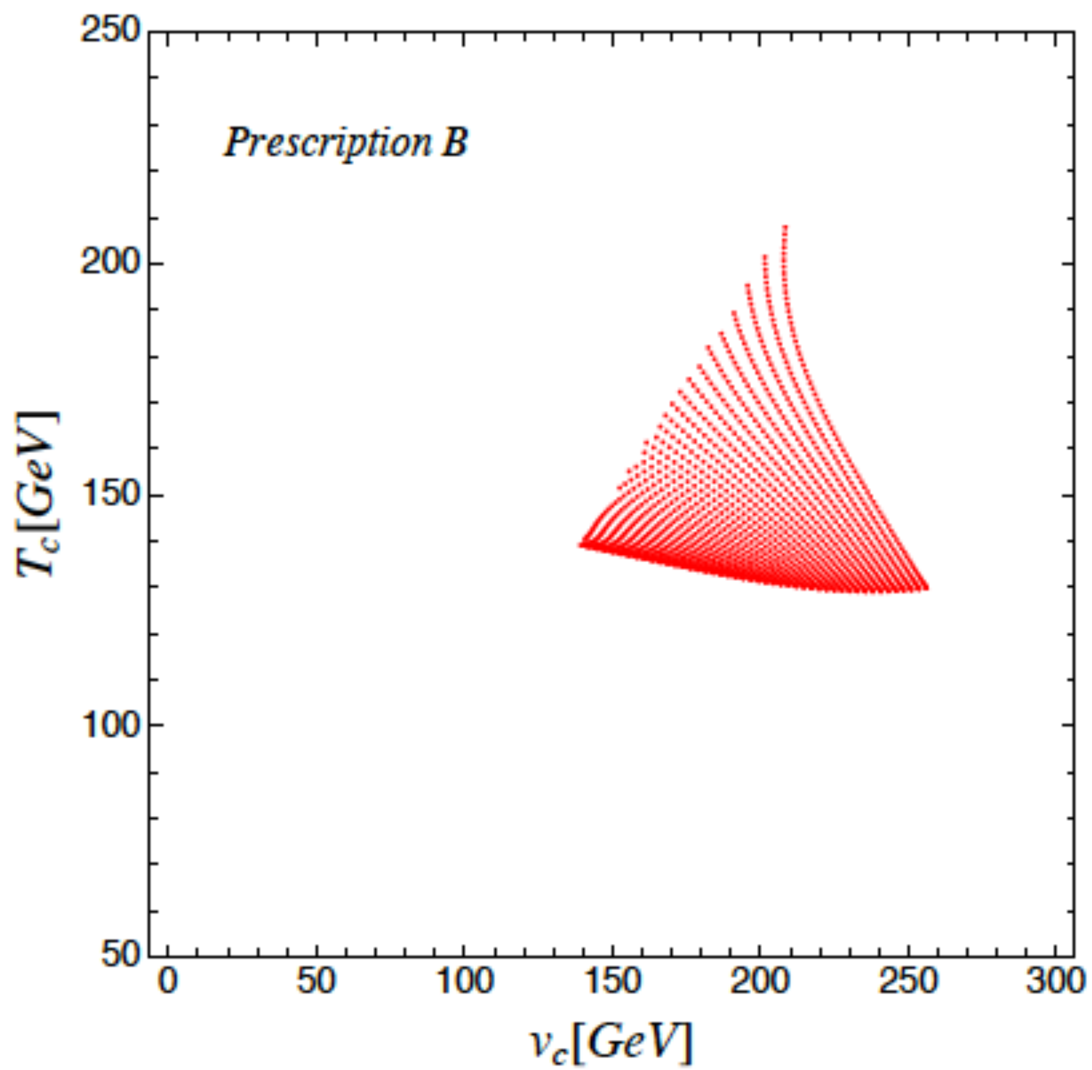
$$\frac{m^2(v_c)}{T_c^2} \sim \mathcal{O}(1) \times \frac{v_c^2}{T_c^2} \gtrsim 1$$

High-T approximation is violated!

Exact V_T



High-T Approximated V_T



Benchmark Scenarios

"no-lose" theorem for testing EWBG in future colliders

- **Higgs Portal**

  (SM + singlet scalar S with Z_2 symmetry)

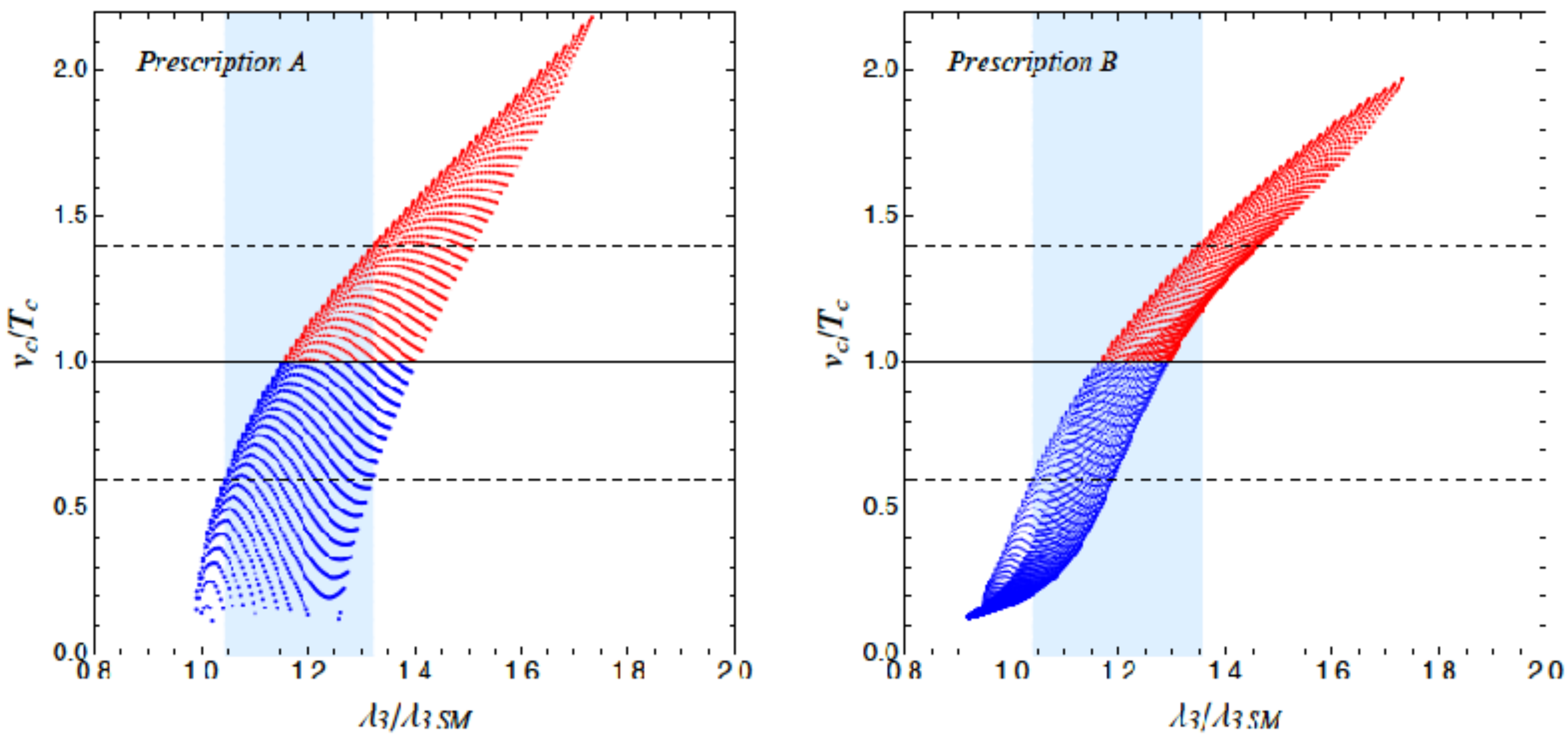
$$V_{tree} \approx \frac{\lambda}{4}h^4 + \frac{1}{2}\lambda_{HS}h^2S^2 + \frac{1}{4}\lambda_S S^4\,,$$
$$= \frac{1}{4}\left[\left(\sqrt{\lambda}h^2 - \sqrt{\lambda_S}S^2\right)^2 + 2h^2S^2\left(\lambda_{HS} + \sqrt{\lambda\,\lambda_S}\right)\right]$$

for the SFOEPT is satisfied, $v_c > T_c$, with O(1) coupling,

$$\frac{m^2(v_c)}{T_c^2} \sim \mathcal{O}(1) \times \frac{v_c^2}{T_c^2} \gtrsim 1$$

High-T approximation is violated!

Exact V_T | High-T Approximated V_T



$$\mu_S = [10, 900]\ \text{GeV}$$

Benchmark Scenarios

"no-lose" theorem for testing EWBG in future colliders

- **Higgs Portal**

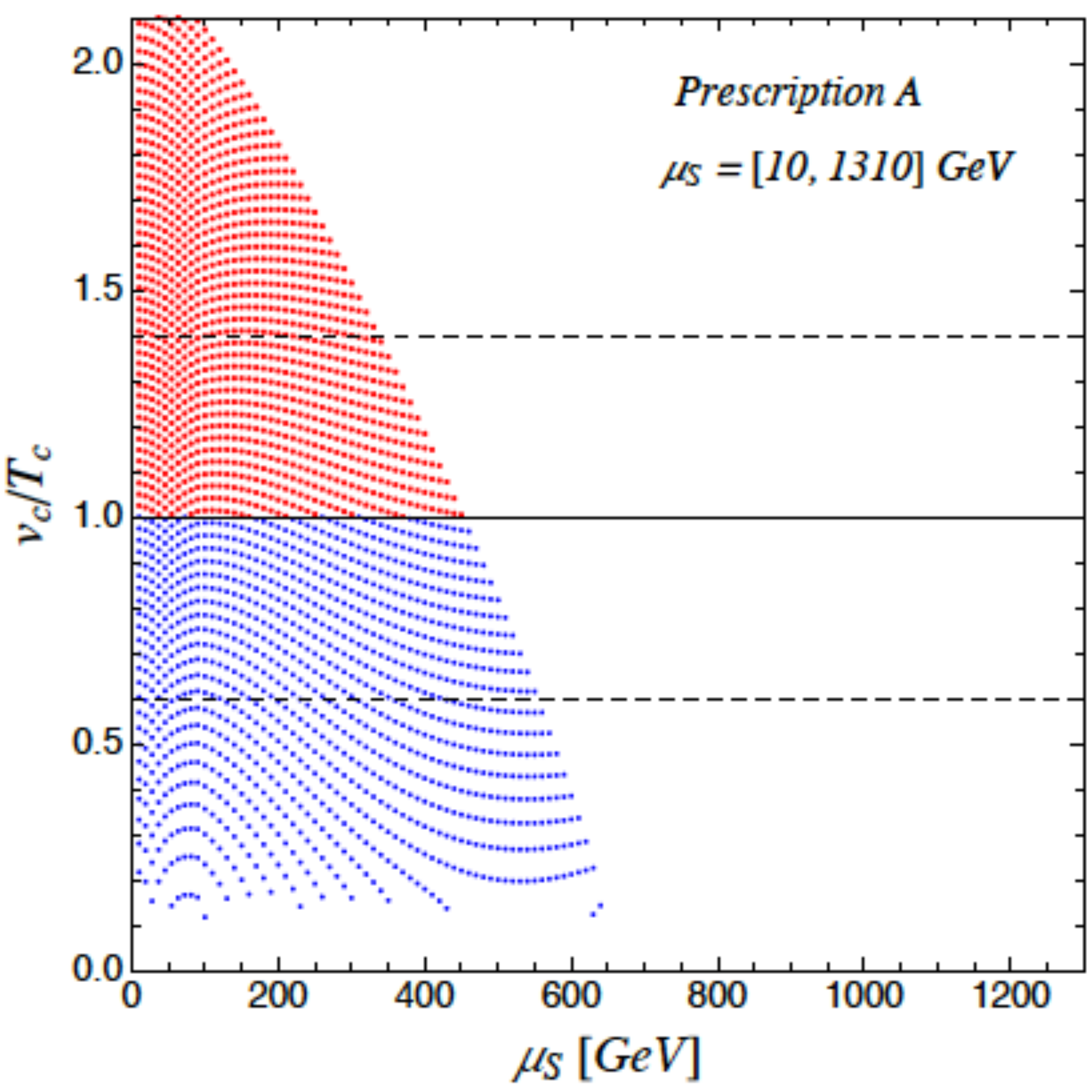
  (SM + singlet scalar S with Z_2 symmetry)

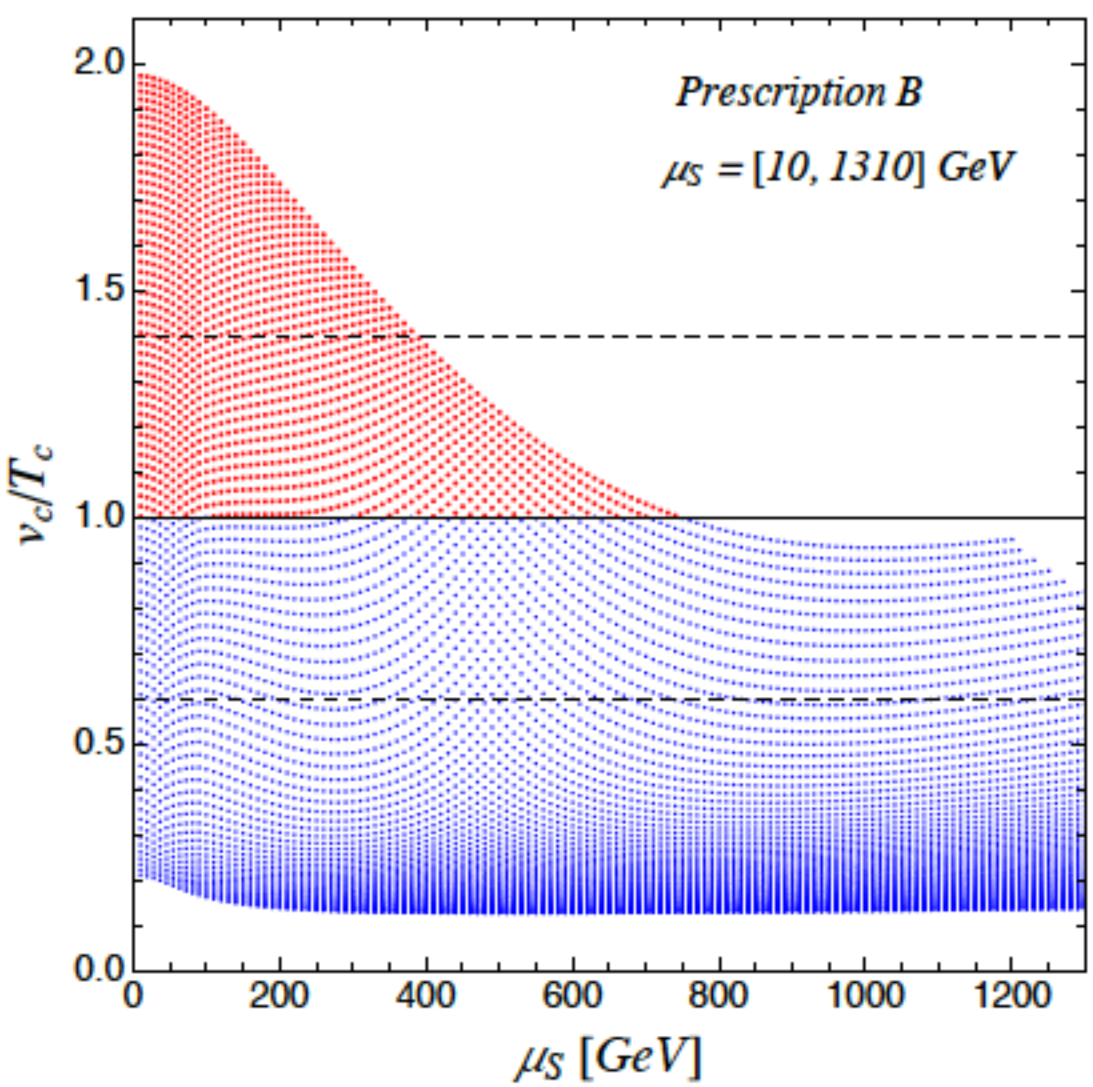
$$V_{tree} \approx \frac{\lambda}{4}h^4 + \frac{1}{2}\lambda_{HS}h^2S^2 + \frac{1}{4}\lambda_S S^4 \,,$$
$$= \frac{1}{4}\left[\left(\sqrt{\lambda}h^2 - \sqrt{\lambda_S}S^2\right)^2 + 2h^2S^2\left(\lambda_{HS} + \sqrt{\lambda\,\lambda_S}\right)\right]$$

for the SFOEPT is satisfied, $v_c > T_c$, with O(1) coupling, $\frac{m^2(v_c)}{T_c^2} \sim \mathcal{O}(1) \times \frac{v_c^2}{T_c^2} \gtrsim 1$

High-T approximation is violated!

Exact V_T | High-T Approximated V_T





Benchmark Scenarios

"no-lose" theorem for testing EWBG in future colliders

- **Higgs Portal**
  (SM + singlet scalar S with Z_2 symmetry)

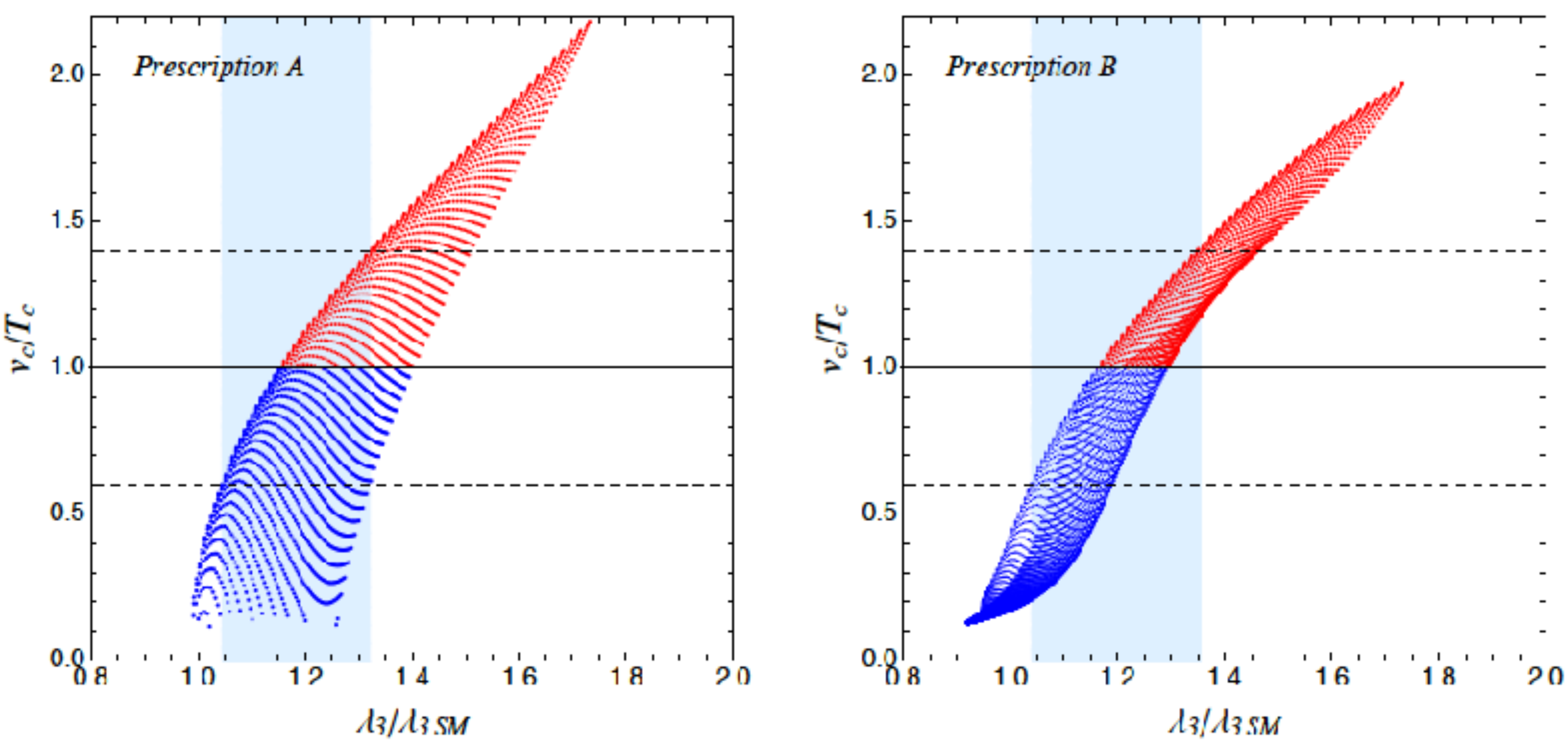
$$V_{tree} \approx \frac{\lambda}{4}h^4 + \frac{1}{2}\lambda_{HS}h^2S^2 + \frac{1}{4}\lambda_S S^4 \,,$$
$$= \frac{1}{4}\left[\left(\sqrt{\lambda}h^2 - \sqrt{\lambda_S}S^2\right)^2 + 2h^2S^2\left(\lambda_{HS} + \sqrt{\lambda\,\lambda_S}\right)\right]$$

for the SFOEPT is satisfied, $v_c > T_c$, with O(1) coupling,

$$\frac{m^2(v_c)}{T_c^2} \sim \mathcal{O}(1) \times \frac{v_c^2}{T_c^2} \gtrsim 1$$

High-T approximation is violated!

Exact V_T | High-T Approximated V_T



$$\mu_S = [10, 900]\ \text{GeV}$$

Benchmark Scenarios

"no-lose" theorem for testing EWBG in future colliders

- **Higgs Portal**
  (SM + singlet scalar S with Z_2 symmetry)

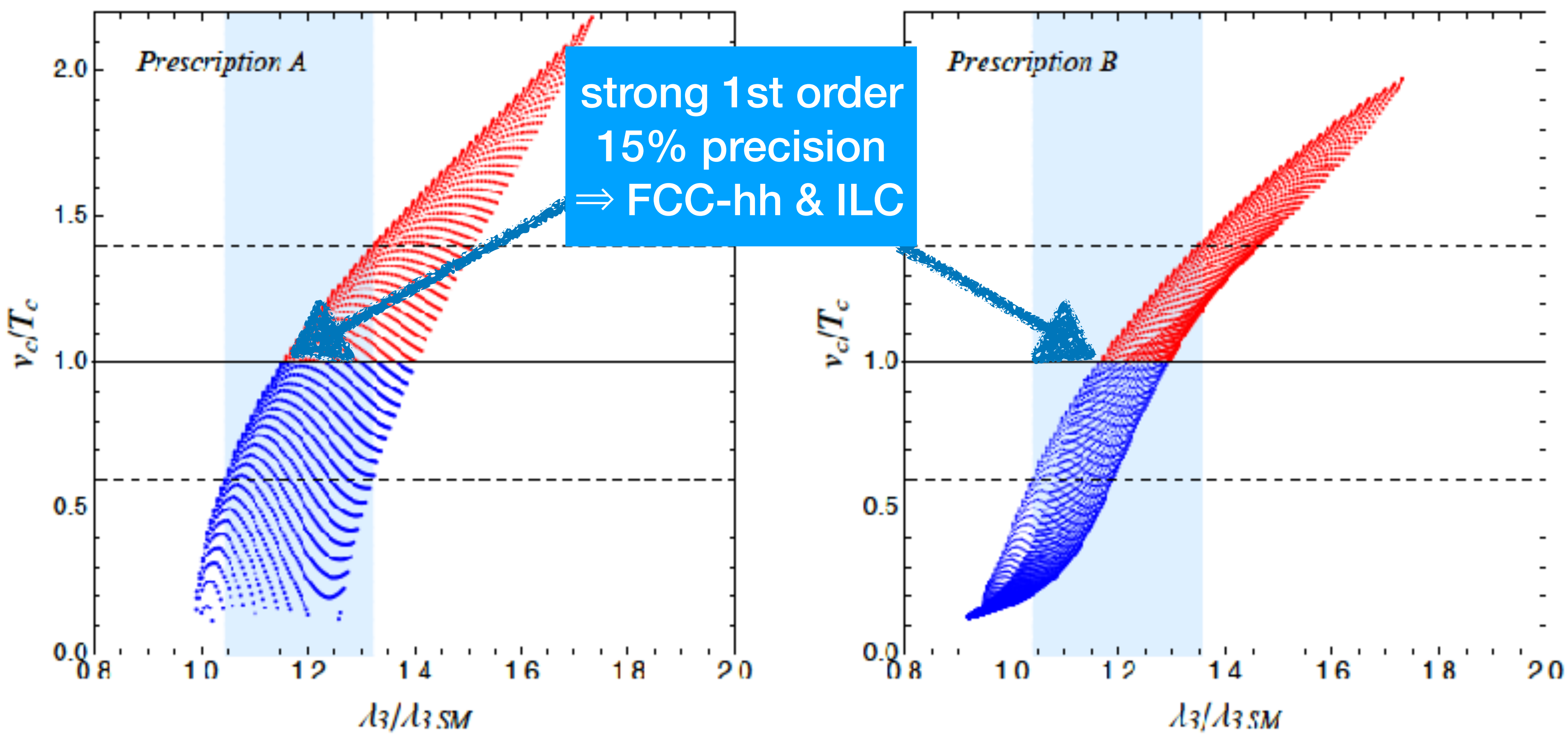
$$V_{tree} \approx \frac{\lambda}{4}h^4 + \frac{1}{2}\lambda_{HS}h^2S^2 + \frac{1}{4}\lambda_S S^4 \;,$$
$$= \frac{1}{4}\left[\left(\sqrt{\lambda}h^2 - \sqrt{\lambda_S}S^2\right)^2 + 2h^2S^2\left(\lambda_{HS} + \sqrt{\lambda\,\lambda_S}\right)\right]$$

for the SFOEPT is satisfied, $v_c > T_c$, with O(1) coupling,

$$\frac{m^2(v_c)}{T_c^2} \sim \mathcal{O}(1) \times \frac{v_c^2}{T_c^2} \gtrsim 1$$

High-T approximation is violated!

Exact V_T High-T Approximated V_T



$$\mu_S = [10, 900]\;\;\text{GeV}$$

Benchmark Scenarios

"no-lose" theorem for testing EWBG in future colliders

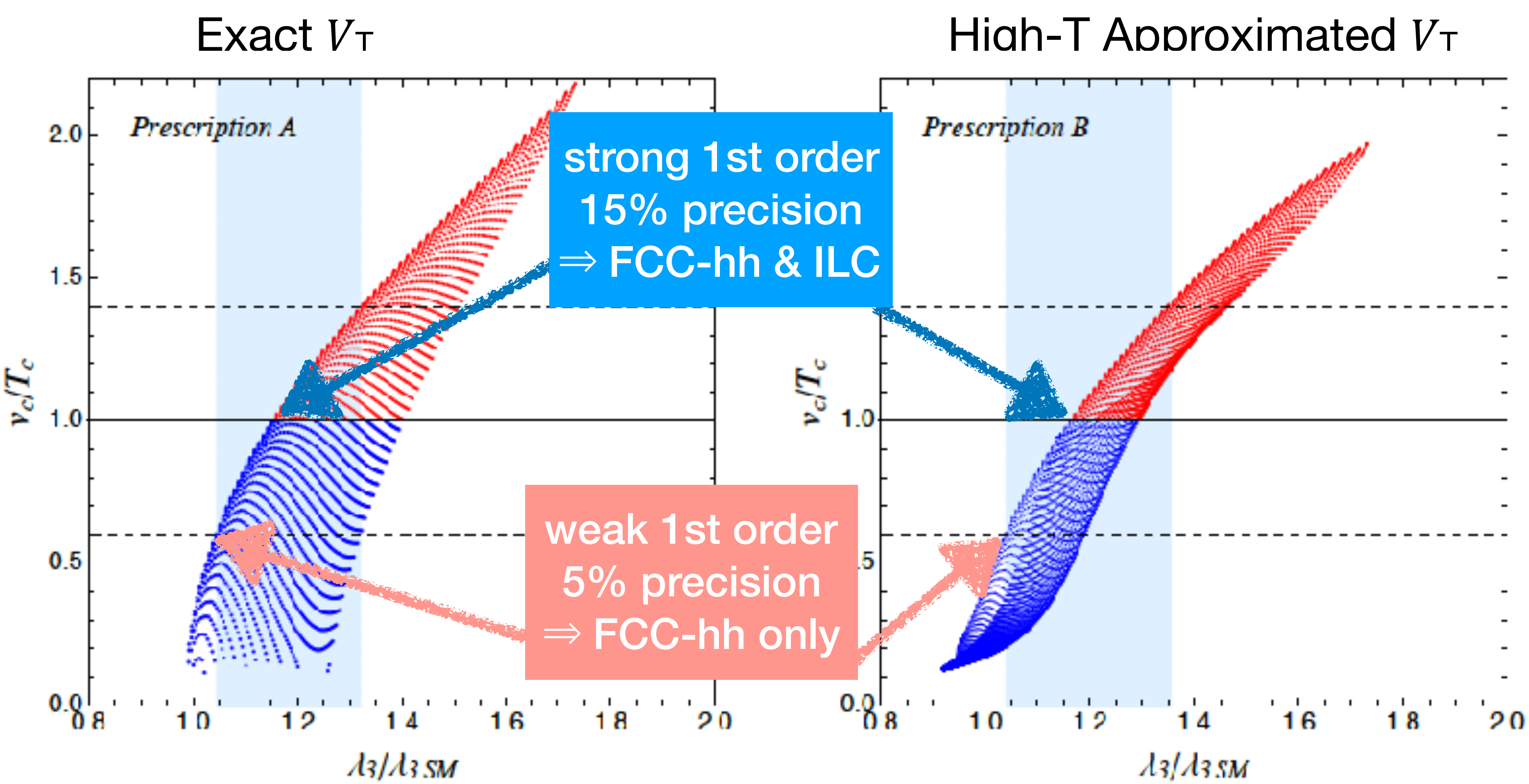
- **Higgs Portal**

  (SM + singlet scalar S with Z_2 symmetry)

$$V_{tree} \approx \frac{\lambda}{4}h^4 + \frac{1}{2}\lambda_{HS}h^2S^2 + \frac{1}{4}\lambda_S S^4 ,$$
$$= \frac{1}{4}\left[\left(\sqrt{\lambda}h^2 - \sqrt{\lambda_S}S^2\right)^2 + 2h^2S^2\left(\lambda_{HS} + \sqrt{\lambda\,\lambda_S}\right)\right]$$

for the SFOEPT is satisfied, $v_c > T_c$, with O(1) coupling, $\frac{m^2(v_c)}{T_c^2} \sim \mathcal{O}(1) \times \frac{v_c^2}{T_c^2} \gtrsim 1$

High-T approximation is violated!



$$\mu_S = [10, 900]\ \text{GeV}$$

Benchmark Scenarios

- EFT with higher dimensional operators
  - Simplest case: only inclusion of O_6

$$\mathcal{O}_6 \sim |H|^6 \quad \text{vs} \quad \mathcal{O}_H \sim (\partial_\mu |H|^2)^2 \quad : \quad V_{tree} = -\frac{\mu^2}{2}h^2 + \frac{\lambda}{4}h^4 + \frac{1}{8}\frac{c_6}{v^2}\frac{m_h^2}{2v^2}h^6$$

$$-\mu^2 + \lambda h^2 + \frac{3c_6 m_h^2}{8v^4}h^4\Big|_{h=v} = 0$$

$$d^2V_{tree}(h)/dh^2|_{h=v} = m_h^2 = -\mu^2 + 3\lambda v^2 + \frac{15}{8}c_6\, m_h^2 = 2\lambda v^2 + \frac{3}{2}c_6\, m_h^2$$

$$\lambda = \frac{m_h^2}{2v^2}\left(1 - \frac{3}{2}c_6\right)$$

$$-\mu^2 = -\frac{m_h^2}{2}\left(1 - \frac{3}{4}c_6\right)$$

$$\lambda_3 = \frac{d^3V_{tree}(h)}{dh^3}\Big|_{h=v} = 6\lambda v + \frac{15}{2}\frac{c_6 m_h^2}{v} = \frac{3m_h^2}{v}(1 + c_6)$$

$$\lambda_4 = \frac{d^4V_{tree}(h)}{dh^4}\Big|_{h=v} = 6\lambda + \frac{45}{2}\frac{c_6 m_h^2}{v^2} = \frac{3m_h^2}{v^2}(1 + 6\,c_6)$$

$$\hookrightarrow \quad \frac{\lambda_3}{\lambda_{3\,SM}} - 1 = c_6\ , \qquad \frac{\lambda_4}{\lambda_{4\,SM}} - 1 = 6\,c_6$$

Benchmark Scenarios

- EFT with higher dimensional operators
  - Simplest case: only inclusion of O_6

$$V_{tree} = -\frac{\mu^2}{2}h^2 + \frac{\lambda}{4}h^4 + \frac{1}{8}\frac{c_6}{v^2}\frac{m_h^2}{2v^2}h^6$$

(m^2, λ, c_6) 3 diff. local curvatures
: 1st order PT becomes in principle possible

When keeping only T^2-term as thermal effect, $m^2(T) = m^2 + aT^2$, the analytic solution is possible

$$\left.\frac{dV_{eff}}{dh}\right|_{h=v_c,\, T=T_c} = 0$$ Should be extreme point

$$V(v_c, T_c) = V(0, T_c)$$ Degeneracy of the vaccua

$$v_c^2 = -\frac{4m^2(T_c)}{\lambda} = -\frac{2\lambda v^4}{c_6 m_h^2}$$

$$c_6 = \frac{2}{3}\frac{1}{1 - \frac{2}{3}\frac{v_c^2}{v^2}}$$

$v_c^2 < v^2$ & $\lambda < 0$ ⇒ $\frac{2}{3} < c_6 < 2$

$$\lambda = \frac{m_h^2}{2v^2}\left(1 - \frac{3}{2}c_6\right)$$

- EFT with higher dimensional operators
  - Special case: universal Wilson coefficient (resumming all operators):

$$V_{tree} = -\mu^2|H|^2 + \lambda|H|^4 + \sum_{n=1}^{\infty} \frac{c_{4+2n}}{v^{2n}} \frac{m_h^2}{2v^2}|H|^{4+2n} \qquad c_{4+2n} = c\,(v/f)^{2n} \qquad f \equiv \Lambda/g_*$$

$$= -\frac{\mu^2}{2}h^2 + \frac{\lambda}{4}h^4 + \frac{1}{8}\frac{c}{f^2}\frac{m_h^2}{2v^2}h^6 \frac{1}{1-\frac{h^2}{2f^2}} \qquad \xi \equiv (v/f)^2$$

$$-\mu^2 = -\frac{m_h^2}{2}\left(1 - \frac{3}{4}c\frac{\xi}{1-\xi/2} - \frac{5}{8}c\frac{\xi^2}{(1-\xi/2)^2} - \frac{1}{8}c\frac{\xi^3}{(1-\xi/2)^3}\right)$$

$$\lambda = \frac{m_h^2}{2v^2}\left[1 + c\left(1 - \frac{1}{(1-\xi/2)^3}\right)\right]$$

$$m_h^2(h) = -\mu^2 + 3\lambda h^2 + \frac{m_h^2}{\xi}\sum_{n=1}^{\infty}\frac{c}{2^{n+2}}(n+2)(2n+3)\left(\frac{h}{f}\right)^{2n+2}$$

$$= -\frac{m_h^2}{2}\left(1 - \frac{3}{4}c\frac{\xi}{1-\xi/2} - \frac{5}{8}c\frac{\xi^2}{(1-\xi/2)^2} - \frac{1}{8}c\frac{\xi^3}{(1-\xi/2)^3}\right)$$

$$+ \frac{3}{2}m_h^2\left[1 + c\left(1 - \frac{1}{(1-\xi/2)^3}\right)\right]\frac{h^2}{v^2} + c\,m_h^2\,\xi\,\frac{15}{8}\,\frac{h^4}{v^4}\,\frac{1 - \frac{17}{30}\xi\frac{h^2}{v^2} + \frac{1}{10}\xi^2\frac{h^4}{v^4}}{\left(1 - \xi\frac{h^2}{2v^2}\right)^3}$$

$$\lambda_3 = \left.\frac{d^3V_{tree}(h)}{dh^3}\right|_{h=v} = \frac{3m_h^2}{v}\left[1 + 16\,c\,\frac{\xi}{(2-\xi)^4}\right]$$

$$\lambda_4 = \left.\frac{d^4V_{tree}(h)}{dh^4}\right|_{h=v} = \frac{3m_h^2}{v^2}\left[1 + 32\,c\,\frac{(6+\xi)\xi}{(2-\xi)^5}\right]$$

Benchmark Scenarios

- EFT with higher dimensional operators
  - Special case: universal Wilson coefficient (resumming all operators):

$$V_{tree} = -\mu^2|H|^2 + \lambda|H|^4 + \sum_{n=1}^{\infty} \frac{c_{4+2n}}{v^{2n}} \frac{m_h^2}{2v^2}|H|^{4+2n} \qquad c_{4+2n} = c(v/f)^{2n} \qquad f \equiv \Lambda/g_*$$

$$= -\frac{\mu^2}{2}h^2 + \frac{\lambda}{4}h^4 + \frac{1}{8}\frac{c}{f^2}\frac{m_h^2}{2v^2}h^6 \frac{1}{1-\frac{h^2}{2f^2}} \qquad \xi \equiv (v/f)^2$$

$$-\mu^2 = -\frac{m_h^2}{2}\left(1 - \frac{3}{4}c\frac{\xi}{1-\xi/2} - \frac{5}{8}c\frac{\xi^2}{(1-\xi/2)^2} - \frac{1}{8}c\frac{\xi^3}{(1-\xi/2)^3}\right)$$

$$\lambda = \frac{m_h^2}{2v^2}\left[1 + c\left(1 - \frac{1}{(1-\xi/2)^3}\right)\right]$$

$$m_h^2(h) = -\mu^2 + 3\lambda h^2 + \frac{m_h^2}{\xi}\sum_{n=1}^{\infty}\frac{c}{2^{n+2}}(n+2)(2n+3)\left(\frac{h}{f}\right)^{2n+2}$$

$$= -\frac{m_h^2}{2}\left(1 - \frac{3}{4}c\frac{\xi}{1-\xi/2} - \frac{5}{8}c\frac{\xi^2}{(1-\xi/2)^2} - \frac{1}{8}c\frac{\xi^3}{(1-\xi/2)^3}\right)$$

$$+\frac{3}{2}m_h^2\left[1 + c\left(1 - \frac{1}{(1-\xi/2)^3}\right)\right]\frac{h^2}{v^2} + c\, m_h^2\, \xi\, \frac{15}{8}\frac{h^4}{v^4}\frac{1 - \frac{17}{30}\xi\frac{h^2}{v^2} + \frac{1}{10}\xi^2\frac{h^4}{v^4}}{\left(1 - \xi\frac{h^2}{2v^2}\right)^3}$$

$$\lambda_3 = \frac{d^3V_{tree}(h)}{dh^3}\bigg|_{h=v} = \frac{3m_h^2}{v}\left[1 + 16\,c\frac{\xi}{(2-\xi)^4}\right]$$

$$\lambda_4 = \frac{d^4V_{tree}(h)}{dh^4}\bigg|_{h=v} = \frac{3m_h^2}{v^2}\left[1 + 32\,c\frac{(6+\xi)\xi}{(2-\xi)^5}\right]$$

$$\frac{\lambda_3}{\lambda_{3\,SM}} - 1 = 16\,c\frac{\xi}{(2-\xi)^4}, \qquad \frac{\lambda_4}{\lambda_{4\,SM}} - 1 = 32\,c\frac{(6+\xi)\xi}{(2-\xi)^5} = 2\,\frac{6+\xi}{2-\xi} \times 16\,c\frac{\xi}{(2-\xi)^4}$$

Benchmark Scenarios

- EFT with higher dimensional operators
  - Special case: universal Wilson coefficient (resumming all operators):

$$V_{tree} = -\mu^2|H|^2 + \lambda|H|^4 + \sum_{n=1}^{\infty} \frac{c_{4+2n}}{v^{2n}} \frac{m_h^2}{2v^2}|H|^{4+2n} \qquad c_{4+2n} = c\,(v/f)^{2n} \qquad f \equiv \Lambda/g_*$$

$$= -\frac{\mu^2}{2}h^2 + \frac{\lambda}{4}h^4 + \frac{1}{8}\frac{c}{f^2}\frac{m_h^2}{2v^2}h^6 \frac{1}{1-\frac{h^2}{2f^2}} \qquad \xi \equiv (v/f)^2$$

$$-\mu^2 = -\frac{m_h^2}{2}\left(1 - \frac{3}{4}c\frac{\xi}{1-\xi/2} - \frac{5}{8}c\frac{\xi^2}{(1-\xi/2)^2} - \frac{1}{8}c\frac{\xi^3}{(1-\xi/2)^3}\right)$$

$$\lambda = \frac{m_h^2}{2v^2}\left[1 + c\left(1 - \frac{1}{(1-\xi/2)^3}\right)\right]$$

$$m_h^2(h) = \mu^2 + 3\lambda h^2 + \frac{m_h^2}{\xi}\sum_{n=1}^{\infty}\frac{c}{2^{n+2}}(n+2)(2n+3)\left(\frac{h}{f}\right)^{2n+2}$$

$$= -\frac{m_h^2}{2}\left(1 - \frac{3}{4}c\frac{\xi}{1-\xi/2} - \frac{5}{8}c\frac{\xi^2}{(1-\xi/2)^2} - \frac{1}{8}c\frac{\xi^3}{(1-\xi/2)^3}\right)$$

$$\lambda_3 = \frac{d^3V_{tree}(h)}{dh^3}\Big|_{h=v} = \frac{3m_h^2}{v}\left[1 + 16\,c\frac{\xi}{(2-\xi)^4}\right]$$

$$\lambda_4 = \frac{d^4V_{tree}(h)}{dh^4}\Big|_{h=v} = \frac{3m_h^2}{v^2}\left[1 + 32\,c\frac{(6+\xi)\xi}{(2-\xi)^5}\right]$$

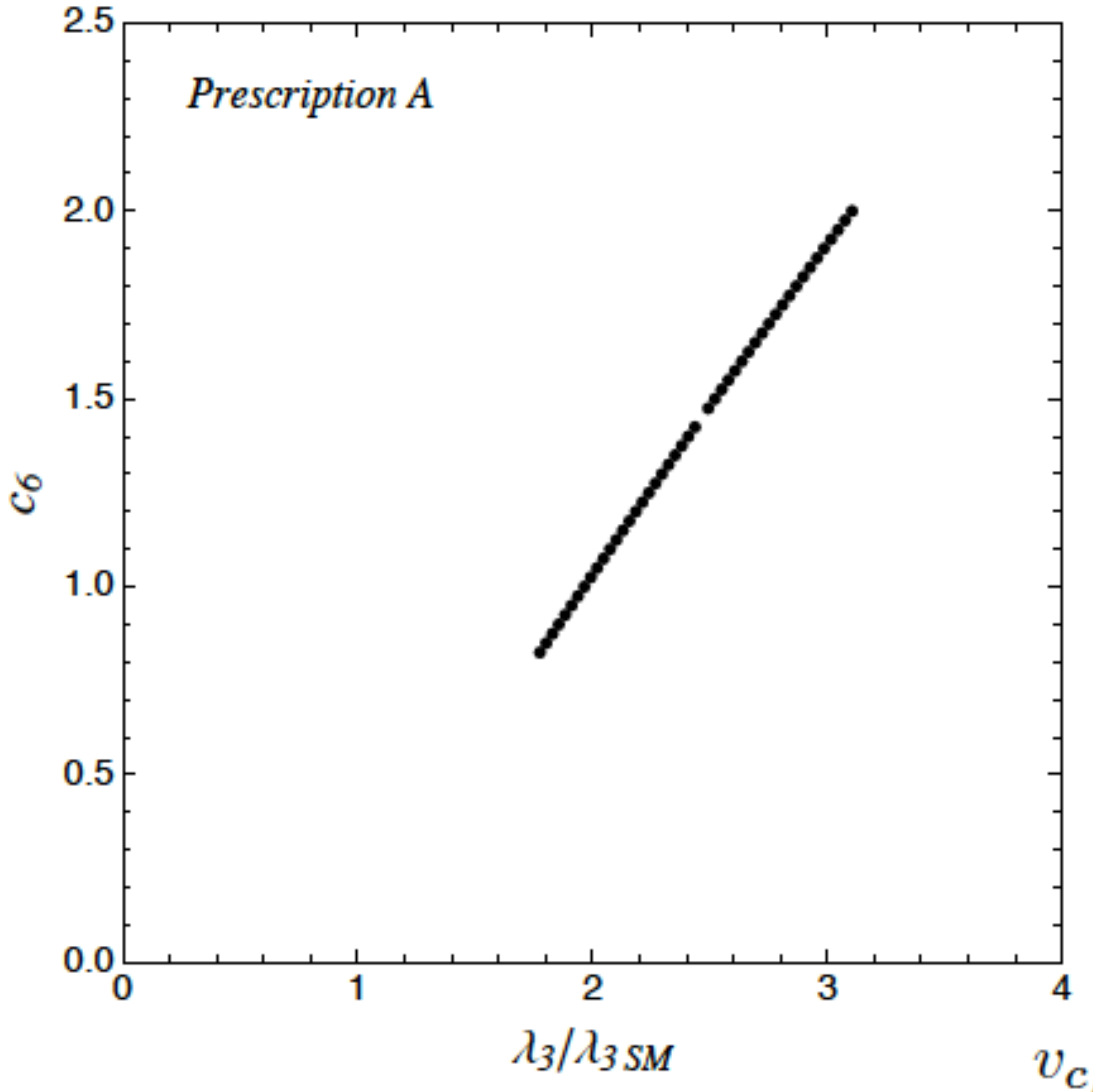
In the limit $f \to v$ (or $\xi \to 1$), the ratio $2(6+\xi)/(2-\xi)$ reaches a maximum value,

$$\frac{\lambda_3}{\lambda_{3\,SM}} - 1 = 16\,c, \qquad \frac{\lambda_4}{\lambda_{4\,SM}} - 1 = 14 \times 16\,c,$$

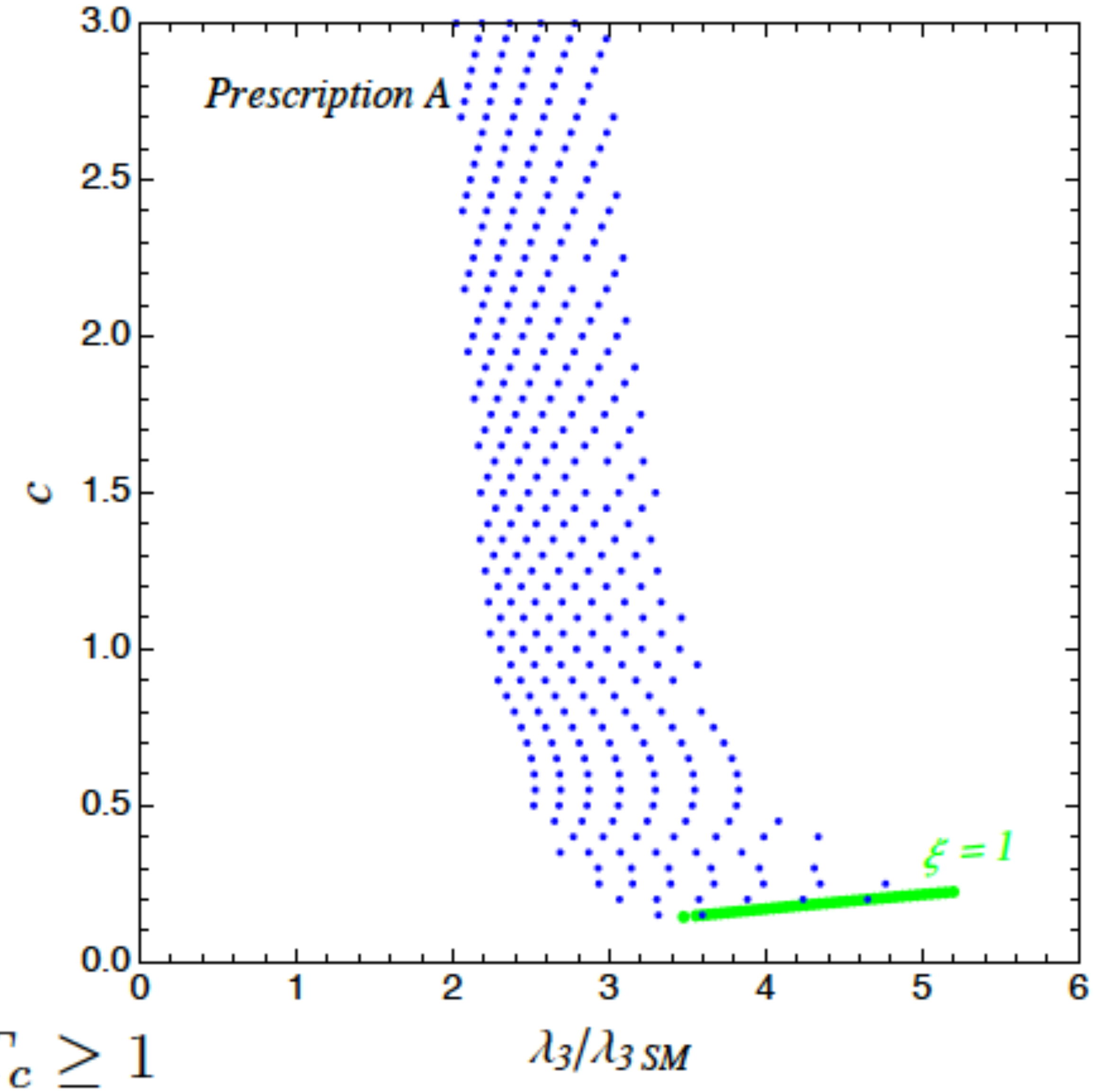
Benchmark Scenarios

- EFT with higher dimensional operator (single and resummed)

$\mathcal{O}_6 \sim |H|^6$



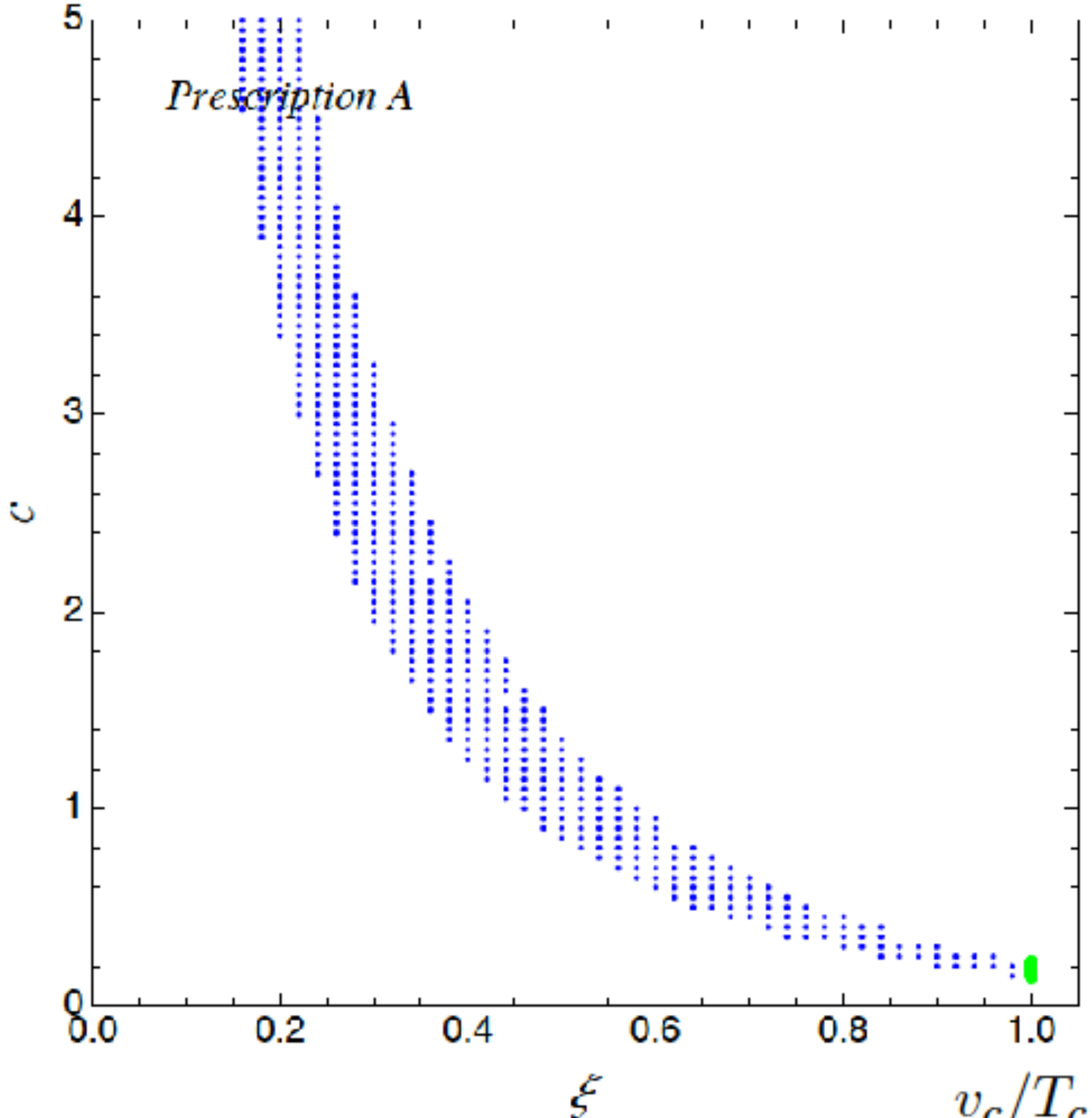
Universal Wilson Coeff. $c_{4+2n} = c(v/f)^{2n}$



$v_c/T_c \geq 1$

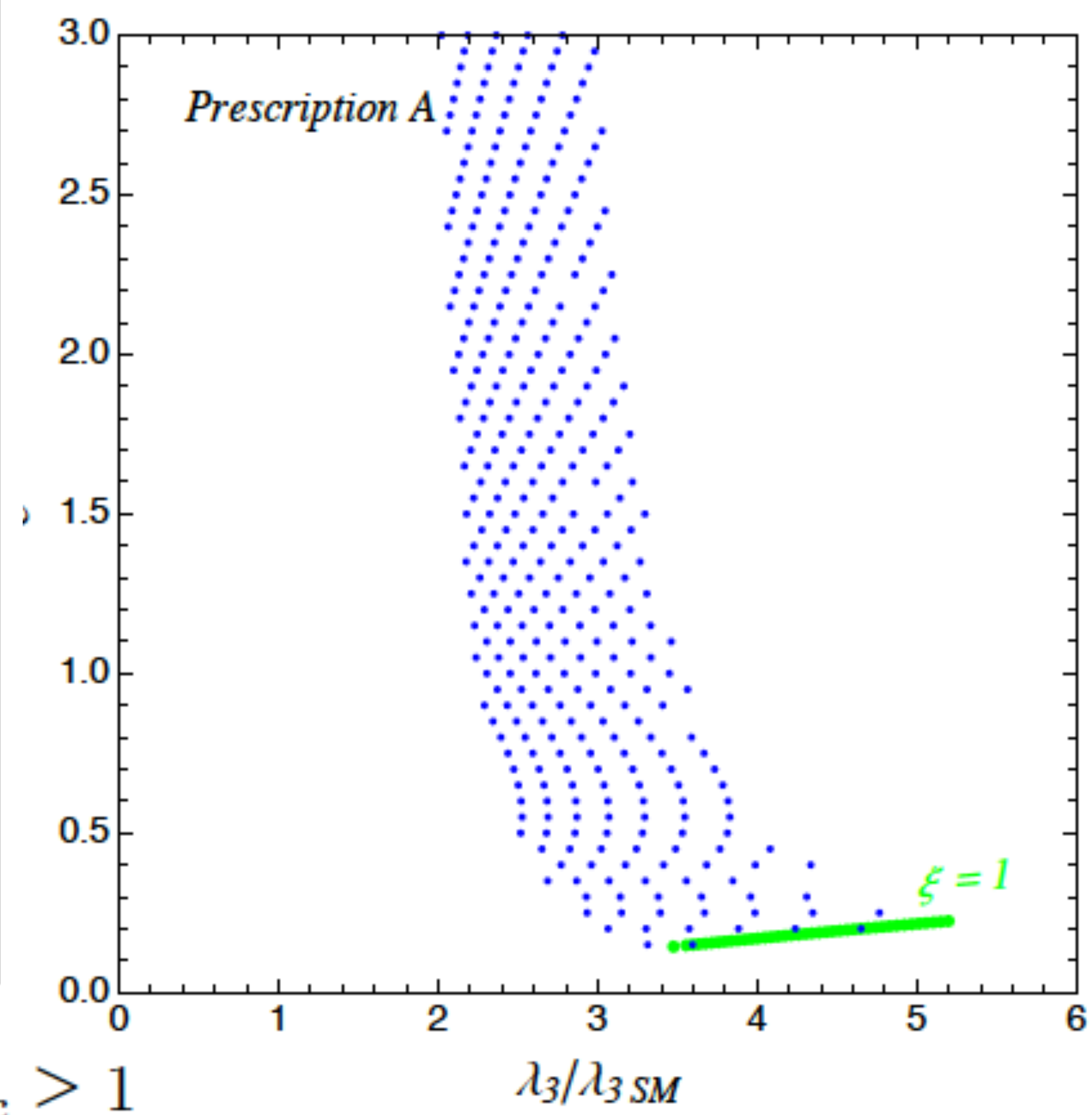
Benchmark Scenarios

- EFT with higher dimensional operator (single and resummed)



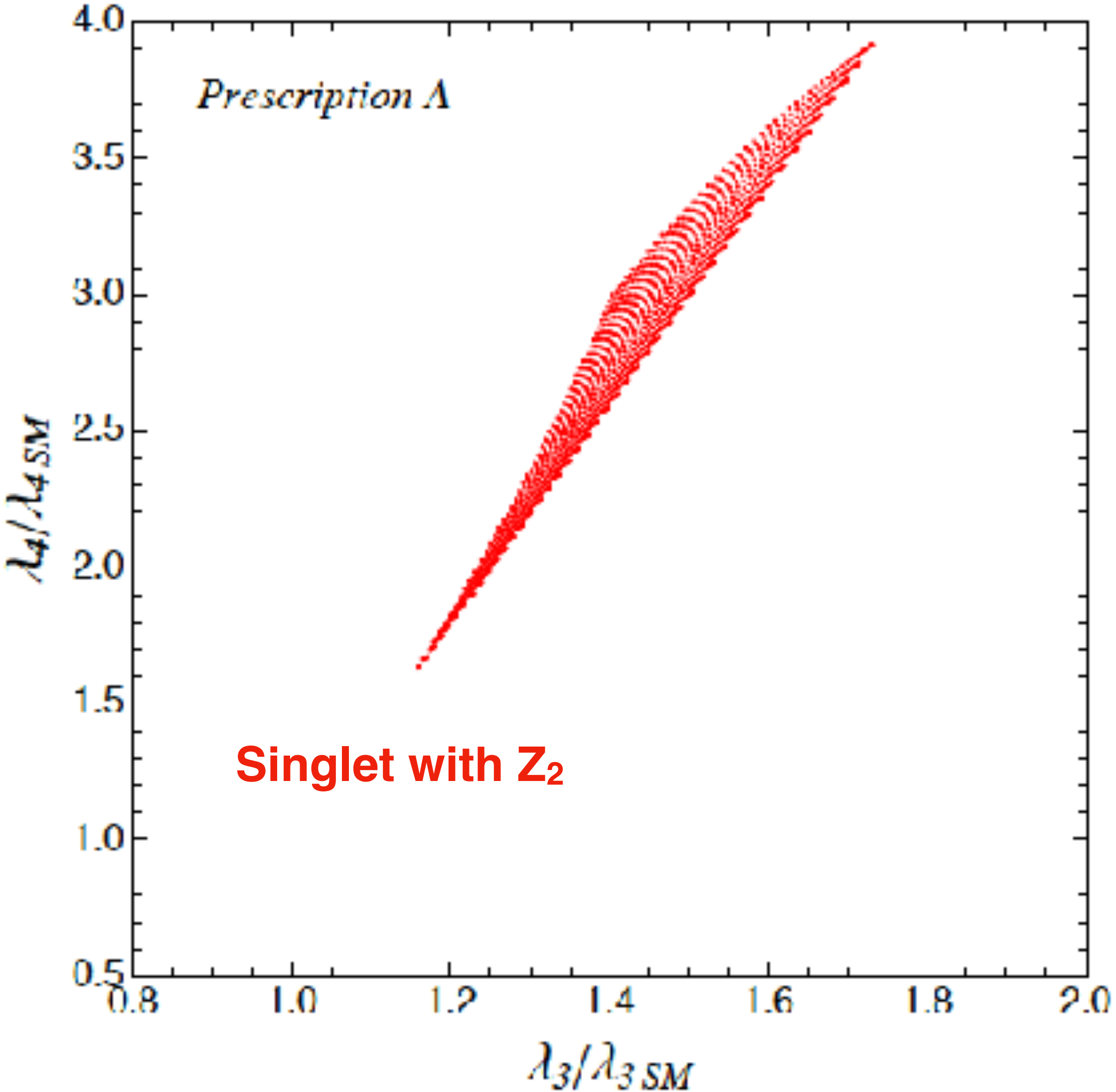
$v_c/T_c \geq 1$

Universal Wilson Coeff. $c_{4+2n} = c\,(v/f)^{2n}$



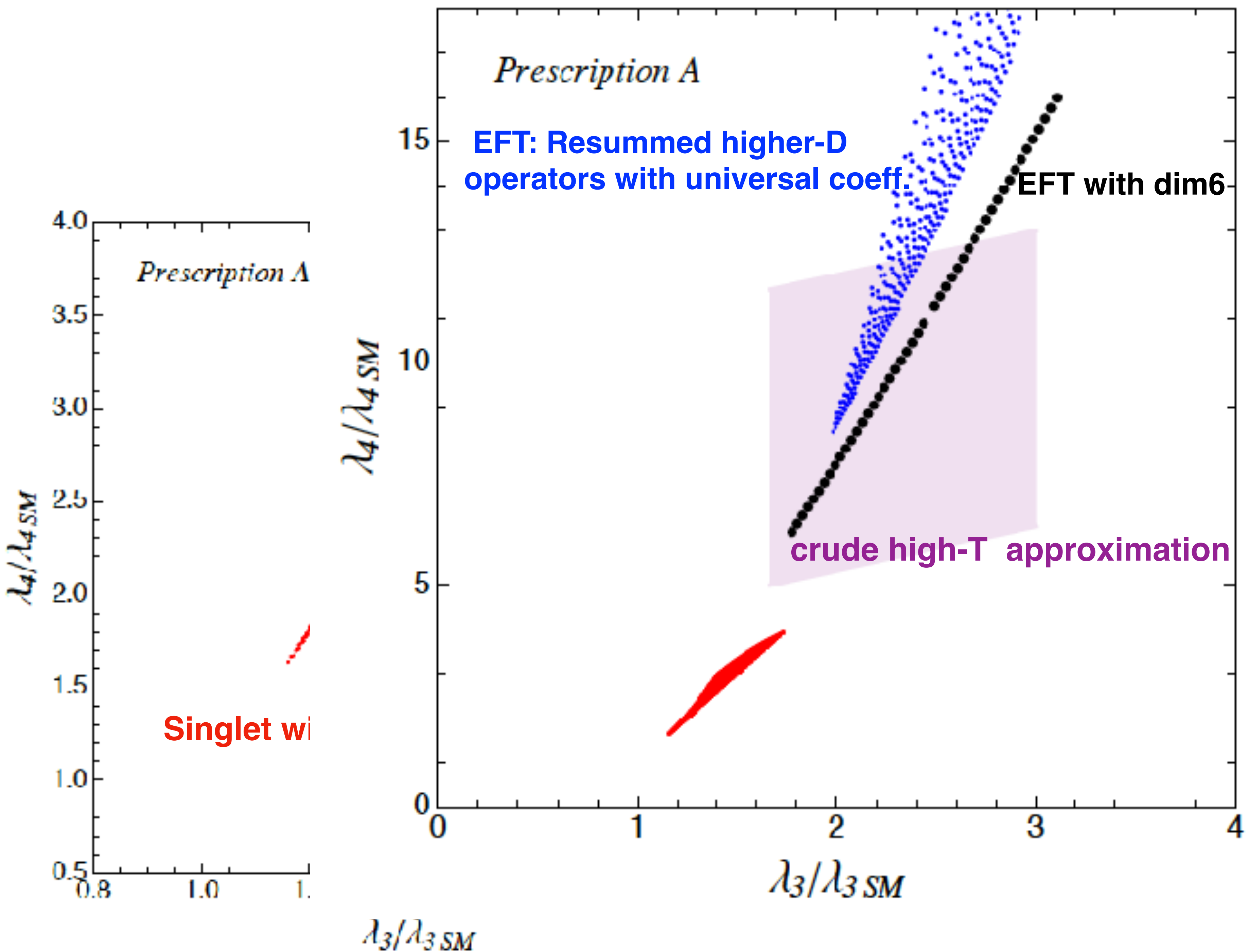
Cubic vs Quartic

$v_c/T_c \geq 1$



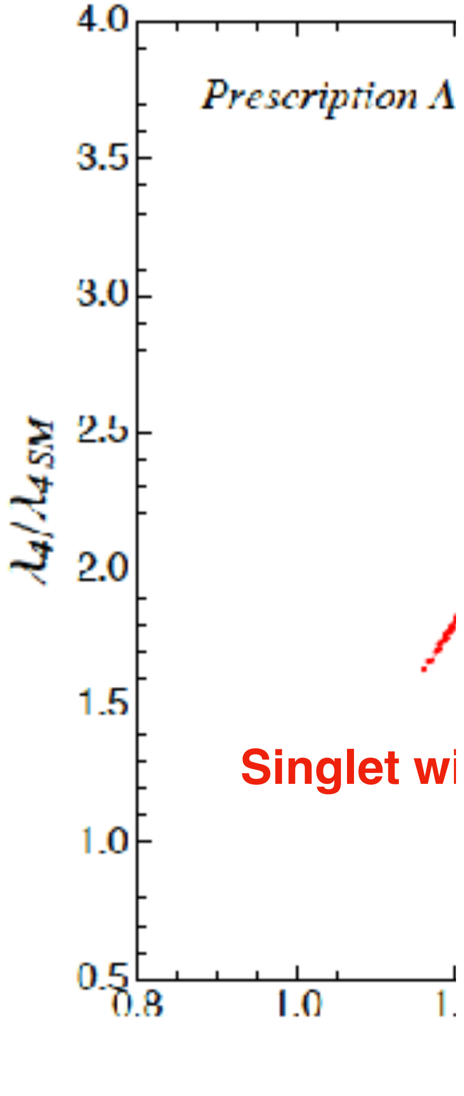
Cubic vs Quartic

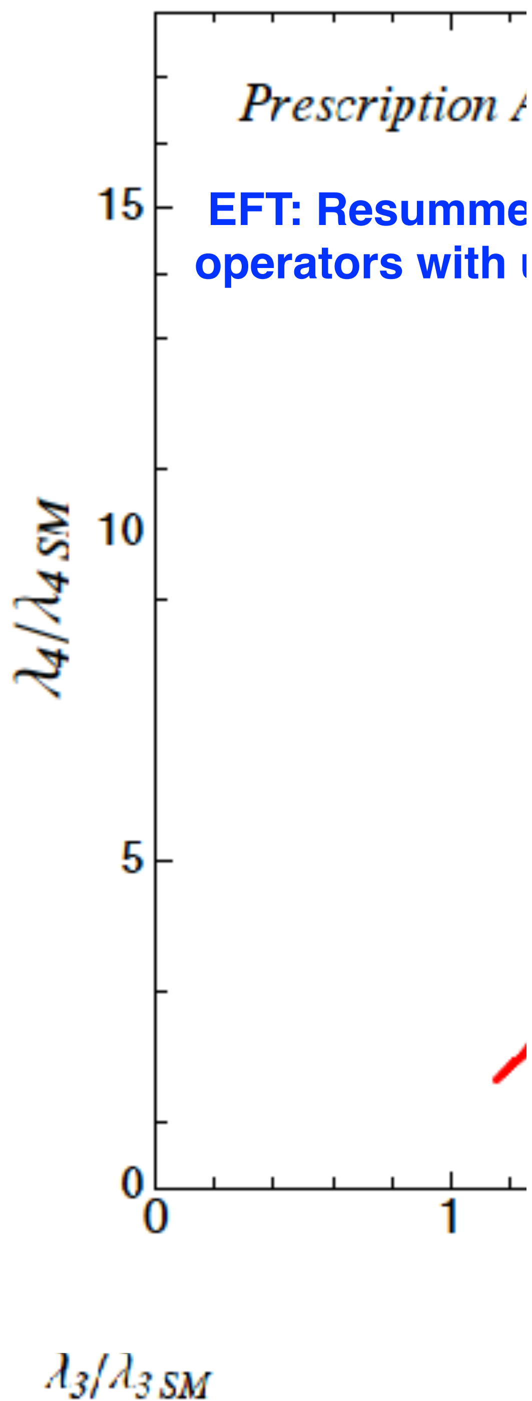
$$v_c/T_c \geq 1$$

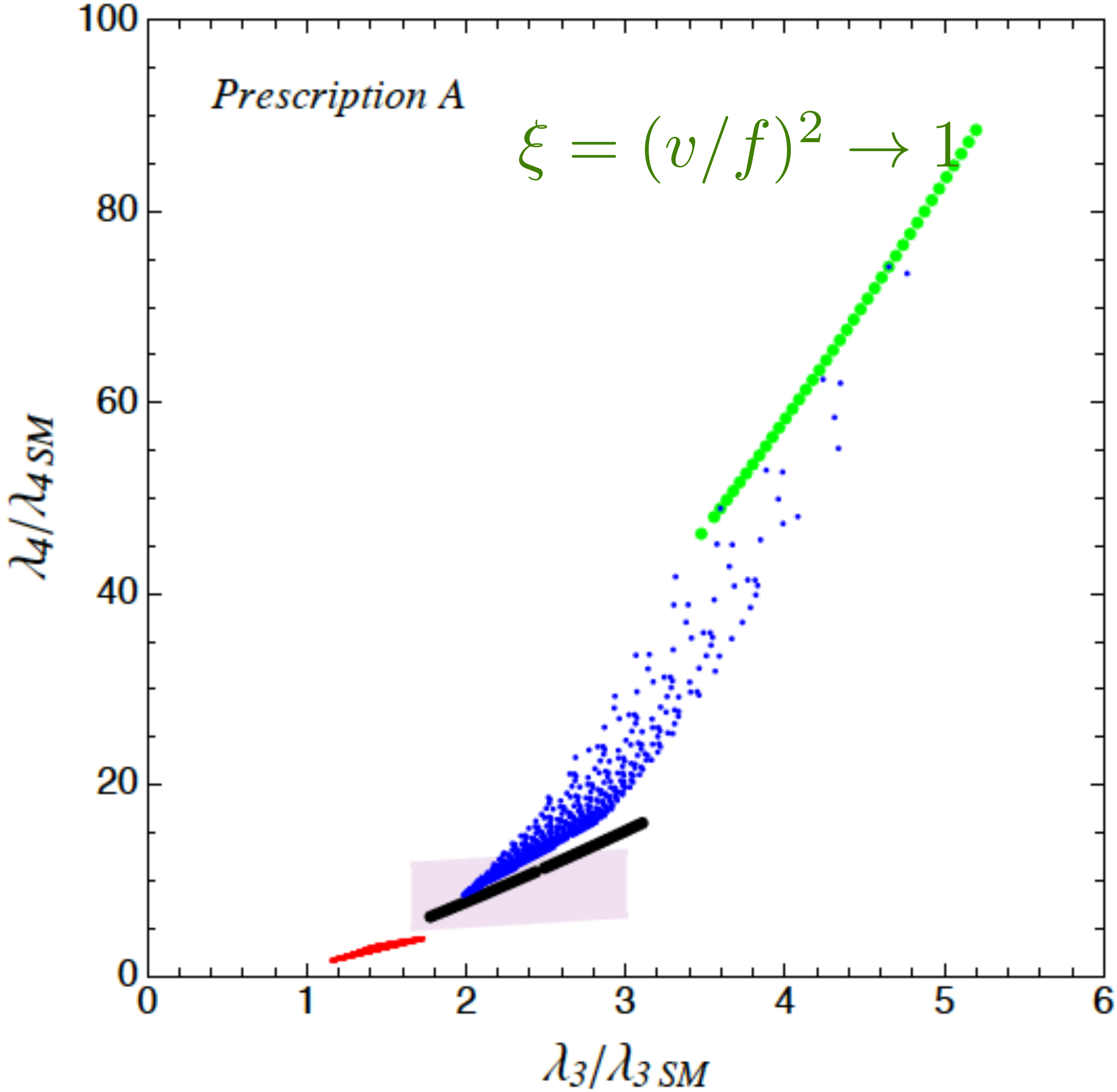


Cubic vs Quartic

$v_c/T_c \geq 1$



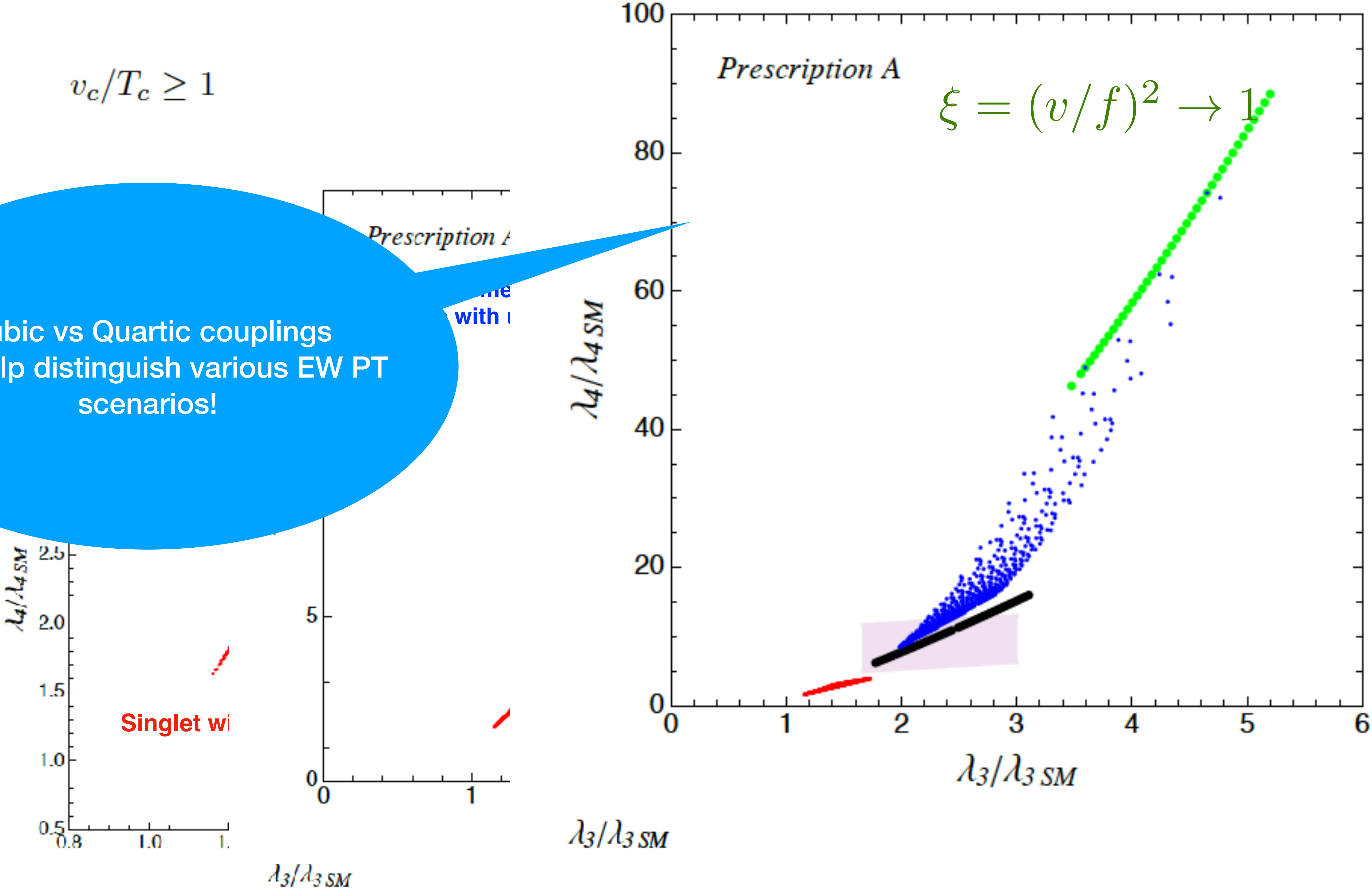




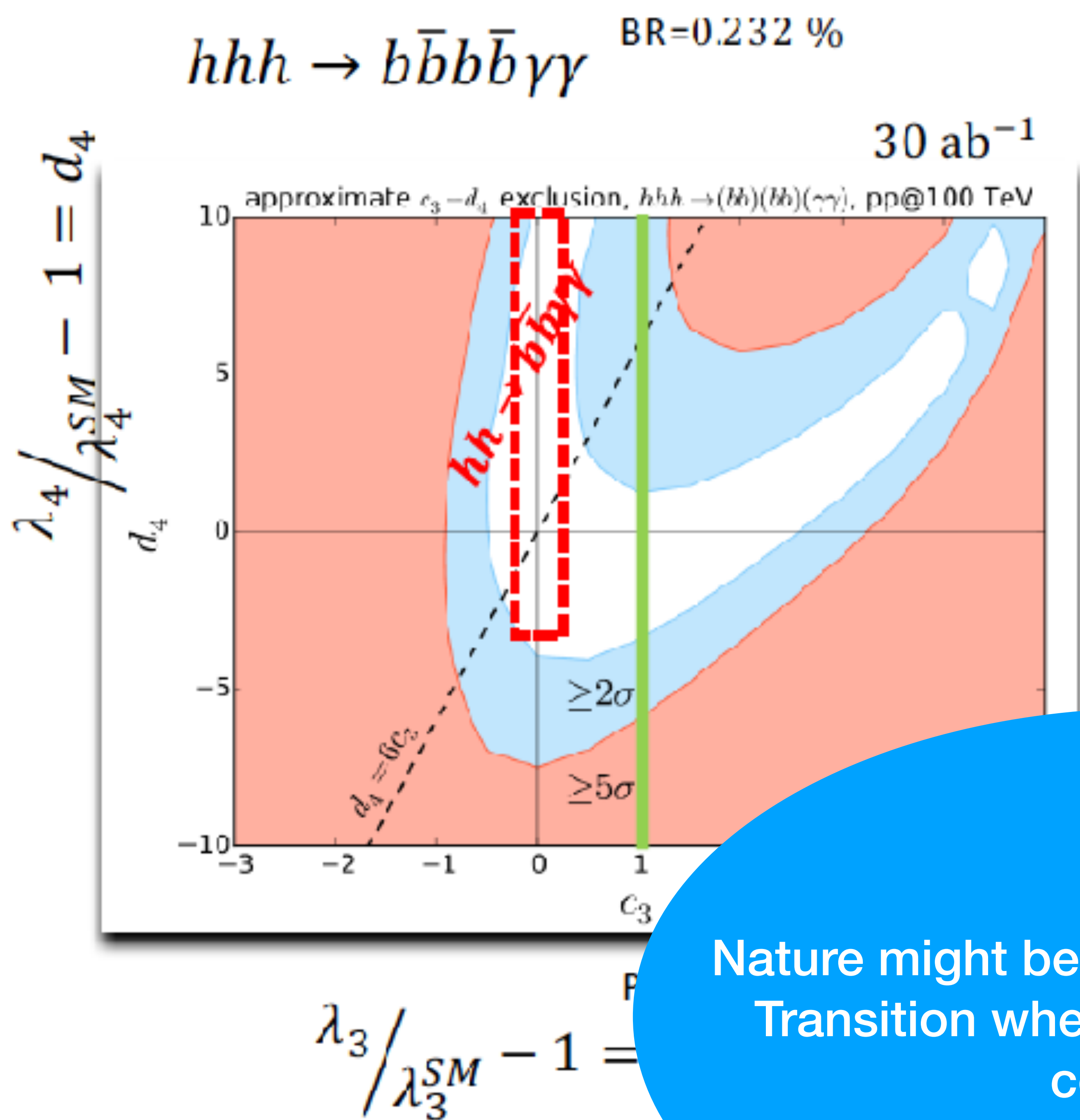
$\lambda_3/\lambda_{3\,SM}$

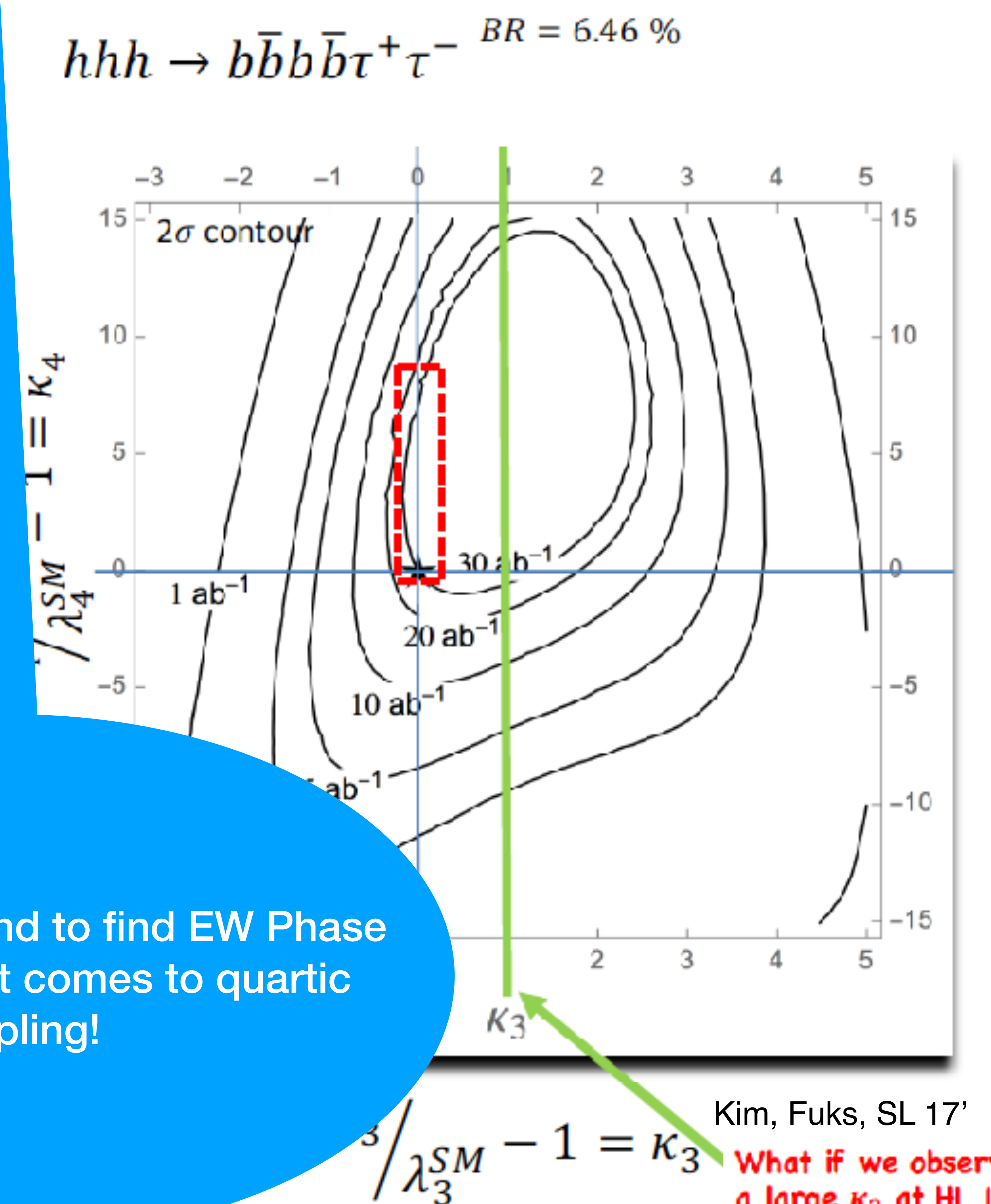
Cubic vs Quartic

$v_c/T_c \geq 1$



Cubic vs Quartic





Nature might be kind to find EW Phase Transition when it comes to quartic coupling!

Kim, Fuks, SL 17’

What if we obser
a large κ_3 at HL

Summary

♦ 1st order Strong EW Phase Transition typically requires a O(1) deviation of Higgs cubic self-coupling:

-Higgs quartic self-coupling can play a role of discriminator between various NP scenarios, and it can be probed at the future collider.

♦ What if we observe any hint of Strong 1st order Phase Transition through the cubic coupling first ?

-Likely strongly coupled dynamics not far away from EW scale ?

♦ Strong 1st order PT based on extra singlet has issues of Validity of high-T approx., and validity of perturbation due to a big coupling

-Precision boundary of λ, might $\sim \mathcal{O}(1)$ fluctuates depending on the prescription.

$\Rightarrow$ More dedicate study is required.


DIALOGO
di
GALILEO GALILEI LINCEO
AL SER.mo FERD. II. GRAN. DVCA DI
TOSCANA
la fine