# Higgs boson and cosmology – can we learn anything about fundamental physics?

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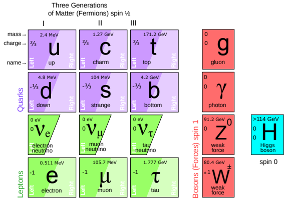
# Outline

1. Standard Model and the reality of the Universe
   - Standard Model is in great shape!
   - Top-quark and Higgs-boson masses and vacuum stability
2. Higgs as an inflaton?
   - Tree level story
   - Adding RG corrections
   - Role of RG corrections in various regimes
   - Surviving the false vacuum in the Hot Universe
3. Questions?

# Lesson from LHC so far – Standard Model is good




- SM works in all laboratory/collider experiments (electroweak, strong)
- LHC 2012 – final piece of the model discovered – Higgs boson
  - Mass measured $\sim 125\,\text{GeV}$ – weak coupling! Perturbative and predictive for high energies
- Add gravity
  - get cosmology
  - get Planck scale $M_P \sim 1.22 \times 10^{19}\,\text{GeV}$ as the highest energy to worry about

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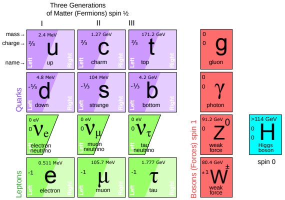


\+ Einstein gravity

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# Many things in cosmology are not explained by SM

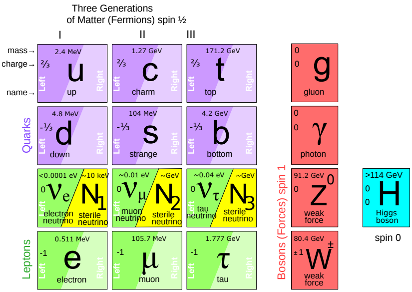
## Experimental observations

- Dark Matter
- Baryon asymmetry of the Universe
- Inflation (nearly scale invariant spectrum of initial density perturbations)

## Laboratory also asks for SM extensions

- Neutrino oscillations

# Possible: New physics only at low scales – $\nu$MSM




## Role of sterile neutrinos

$N_1$ $M_1 \sim 1 - 50$keV: (Warm) Dark Matter

$N_{2,3}$ $M_{2,3} \sim$ several GeV:
Gives masses for active neutrinos, Baryogenesys

Forgetting about three problems!

# What to do with the problems?

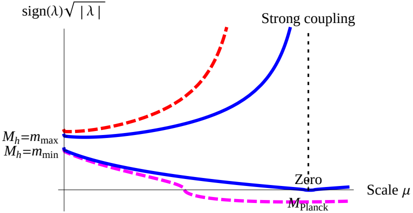
- Inflationary mechanism required
- Higgs is weakly coupled

but not completely trouble free

# Standard Model self-consistency and Radiative Corrections

- Higgs self coupling constant $\lambda$ changes with energy due to radiative corrections.

$$(4\pi)^2\beta_\lambda = 24\lambda^2 - 6y_t^4 + \frac{3}{8}(2g_2^4 + (g_2^2 + g_1^2)^2) + (-9g_2^2 - 3g_1^2 + 12y_t^2)\lambda$$

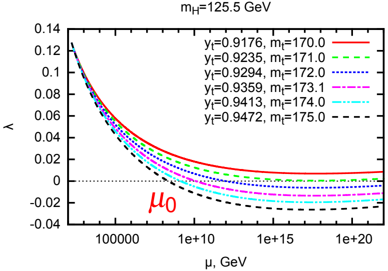


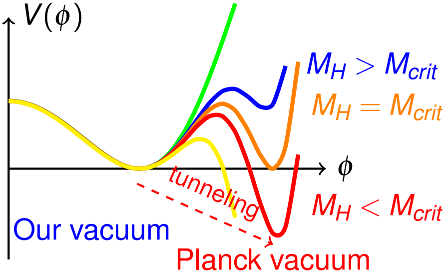

- Behaviour is determined by the masses of the Higgs boson $m_H = \sqrt{2\lambda}v$ and other heavy particles (top quark $m_t = y_t v/\sqrt{2}$)
- If Higgs is heavy $M_H > 170$ GeV – the model enters *strong coupling* at some low energy scale – new physics emerges.

# Lower Higgs masses: RG corrections push Higgs coupling to negative values

- For Higgs masses $M_H < M_{\text{critical}}$ coupling constant is negative above some scale $\mu_0$.
- The Higgs potential may become negative!
  - Our world is not in the lowest energy state!
  - Problems at some scale $\mu_0 > 10^{10}$ GeV?

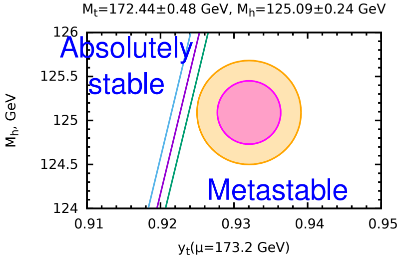
Higgs potential $V(\phi) \simeq \lambda(\phi)\frac{\phi^4}{4}$



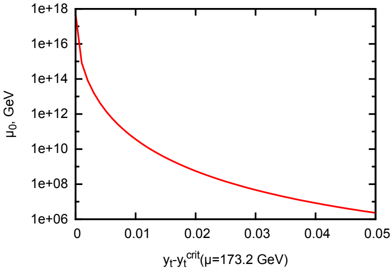



# LHC result: SM is definitely perturbative up to Planck scale, and probably has metastable SM vacuum

Experimental values for $y_t$




Scale $\mu_0$ for $\lambda(\mu_0) = 0$




We live close to the metastability boundary – but on which side?!

Future measurements of top Yukawa and Higgs mass are essential!

# Vacuum stability – what it means?

- Stable Electroweak vacuum – looks safe
- Metastable – is it ok?

# Inflation versus vacuum stability

| Stable SM vacuum | inflaton & Higgs independent | inflaton & Higgs interacting | inflaton = Higgs |
|---|---|---|---|
| Large $r$ | Yes | Yes | Yes (threshold corr.) |
| Small $r$ | Yes | Yes | Yes |
| Planck scale corections | Any | Any | Scale inv. |

| Metastable SM vacuum | inflaton & Higgs independent | inflaton & Higgs interacting | inflaton = Higgs |
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| Large $r$ | No | Yes Model dep. | Hard |
| Small $r$ | Yes $r < 10^{-9}$ | Yes Model dep. | Yes (threshold corr.) |
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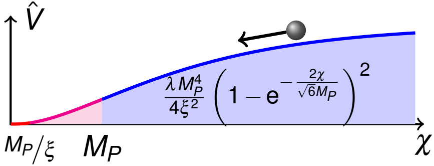
# Higgs inflation at tree level

**Scalar part of the (Jordan frame) action**

$$S_J = \int d^4x\sqrt{-g}\left\{-\frac{M_P^2}{2}R - \xi\frac{h^2}{2}R + g_{\mu\nu}\frac{\partial^\mu h\partial^\nu h}{2} - \frac{\lambda}{4}(h^2-v^2)^2\right\}$$

**To get observed $\delta T/T \sim 10^{-5}$**

$$\frac{\sqrt{\lambda}}{\xi} = \frac{1}{49000}$$

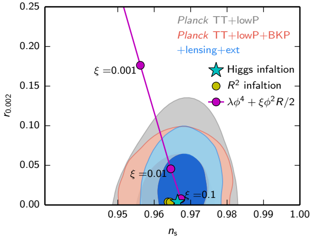



Conformal transformation: $\hat{g}_{\mu\nu} = \sqrt{1+\frac{\xi\phi^2}{M_P^2}}\,g_{\mu\nu}$,

Requirement from UV physics – No corrections $\frac{h^n}{M_P^{4-n}}$ allowed

# CMB parameters are predicted

## Exactly like preferred by CMB




### For large $\xi$ Higgs inflation

spectral index $\quad n \simeq 1 - \frac{8(4N+9)}{(4N+3)^2} \simeq 0.97$

tensor/scalar ratio $\quad r \simeq \frac{192}{(4N+3)^2} \simeq 0.0033$

$$\delta T/T \sim 10^{-5} \quad \Longrightarrow \quad \frac{\xi}{\sqrt{\lambda}} \simeq 47000$$

What happens if we try to take into account loop corrections?

# RG improved potential for Higgs inflation

The standard rule would be to write potential and replace constant with constant at the relevant mass scale:

$$U_{\text{RG improved}}(\chi) = \frac{\lambda(\mu)}{4}\frac{M_P^4}{\xi^2}\left(1 - e^{-\frac{2\chi}{\sqrt{6}M_P}}\right)^2$$

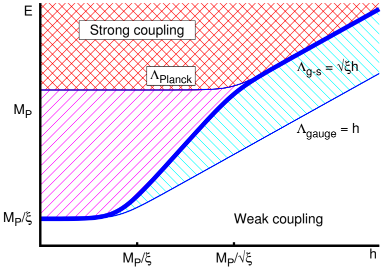
with

$$\mu^2 = \alpha^2 m_t^2(\chi) = \alpha^2 \frac{y_t^2(\mu)}{2}\frac{M_P^2}{\xi}\left(1 - e^{-\frac{2\chi}{\sqrt{6}M_P}}\right)$$

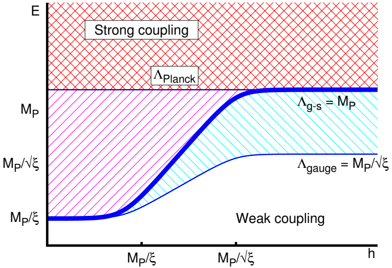
Problem: How to gete $\lambda(\mu)$ at high energy scales?

# Theory has background dependent tree level unitarity violation

## Jordan frame




## Einstein frame




Relation between cut-offs in different frames:

$$\Lambda_{\text{Jordan}} = \Lambda_{\text{Einstein}}\Omega$$

Relevant scales

Hubble scale $H \sim \lambda^{1/2}\frac{M_P}{\xi}$

Energy density at inflation

$$V^{1/4} \sim \lambda^{1/4}\frac{M_P}{\sqrt{\xi}}$$

# Approximate symmetry at inflation

$$\mathscr{L} = \frac{(\partial_\mu \chi)^2}{2} - U(\chi)$$

$$U(\chi) = U_0 \left( 1 + \sum_{n=1}^{\infty} u_n e^{-\frac{n\chi}{M}} \right) = U_0 \left( 1 + \sum_{k=0}^{\infty} \frac{1}{k!} \left[ \frac{\delta\chi}{M} \right]^k \sum_{n=1}^{\infty} n^k u_n e^{-\frac{n\bar{\chi}}{M}} \right)$$

Effective action has the form ($M = \sqrt{6} M_P / 2$)

$$\mathscr{L} = f^{(1)}(\chi) \frac{(\partial_\mu \chi)^2}{2} - U(\chi) + f^{(2)}(\chi) \frac{(\partial^2 \chi)^2}{M^2} + f^{(3)}(\chi) \frac{(\partial \chi)^4}{M^4} + \cdots$$

All the divergences are absorbed in $u_n$ and in $f^{(n)} \sim \sum f_l e^{-n\chi/M}$

## UV complete theory requirement

Shift symmetry is respected

$$\chi \mapsto \chi + \text{const}$$

(or equivalently scale symmetry in the Jordan frame)

# Adding required counterterms to the action

- In principle – HI is not renormalizable, all counterterms appear at some loop order
- Let us try to add only the *required* counterterms at each order in loop expansion

$$\mathscr{L} = \frac{(\partial\chi)^2}{2} - \frac{\lambda}{4}F^4(\chi) + i\bar{\psi}_t\partial\!\!\!/\psi_t + \frac{y_t}{\sqrt{2}}F(\chi)\bar{\psi}_t\psi_t$$

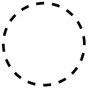
$$F(\chi) \equiv \frac{h(\chi)}{\Omega(\chi)} \approx \left\{ \begin{array}{ll} \chi & , \chi < \frac{M_P}{\xi} \\ \frac{M_P}{\sqrt{\xi}}\left(1 - e^{-\sqrt{2/3}\chi/M_P}\right)^{1/2} & , \chi > \frac{M_P}{\xi} \end{array} \right\}$$

Doing quantum calculations we should add

$$\mathscr{L} + \mathscr{L}_{\text{1-loop}} + \delta\mathscr{L}_{\text{1-loop c.t.}} + \cdots$$

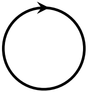
# Counterterms: $\lambda$ modification

Calculating vacuum energy



$$= \frac{1}{2}\mathrm{Tr}\ln\left[\Box - \left(\frac{\lambda}{4}(F^4)''\right)^2\right]$$

$$= \frac{9\lambda^2}{64\pi^2}\left(\frac{2}{\bar{\varepsilon}} - \ln\frac{\lambda(F^4)''}{4\mu^2} + \frac{3}{2}\right)\left(F'^2 + \frac{1}{3}F''F\right)^2 F^4,$$



$$= -\mathrm{Tr}\ln\left[i\partial\!\!\!/ + y_t F\right]$$

$$= -\frac{y_t^4}{64\pi^2}\left(\frac{2}{\bar{\varepsilon}} - \ln\frac{y_t^2 F^2}{2\mu^2} + \frac{3}{2}\right)F^4$$

# Counterterms: $\lambda$ modification

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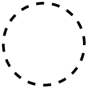
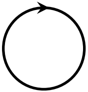


$$= \frac{1}{2}\mathrm{Tr}\ln\left[\Box - \left(\frac{\lambda}{4}(F^4)''\right)^2\right]$$

$$\delta\mathscr{L}_{ct} = \frac{9\lambda^2}{64\pi^2}\left(\frac{2}{\bar{\varepsilon}} + \delta\lambda_{1a}\right)\left(F'^2 + \frac{1}{3}F''F\right)^2 F^4,$$



$$= -\mathrm{Tr}\ln\left[i\partial\!\!\!/ + y_t F\right]$$

$$\delta\mathscr{L}_{ct} = -\frac{y_t^4}{64\pi^2}\left(\frac{2}{\bar{\varepsilon}} + \delta\lambda_{1b}\right)F^4$$

Small $\chi$ : $F'^4 F^4 \sim \chi \sim F^4$

Large $\chi$ : $F'^4 F^4 \sim e^{-4\chi/\sqrt{6}M_P}$, and $F^4 \sim M_P^4/\xi^2$

$\delta\lambda_{1b}$ – just $\lambda$ redefinition, while $\delta\lambda_{1a}$ is not!

# Modified "evolution" of $\lambda(\mu)$

For RG we should in principle write infinite series

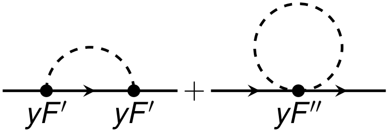
$$\frac{d\lambda}{d\ln\mu} = \beta_\lambda(\lambda, \lambda_1, a\dots)$$
$$\frac{d\lambda_1}{d\ln\mu} = \beta_{\lambda_1}(\lambda, \lambda_1, \dots)$$
$$\dots$$

- Assuming $\delta_i$ are small and have the same hierarchy, as the loop expansion, we truncate this to just first equation.
- Neglect change of $\delta\lambda_1$ between $\mu \sim M_P/\xi$ and $M_P/\sqrt{\xi}$

$$\lambda(\mu) \to \lambda(\mu) + \delta\lambda \left[ \left( F'^2 + \frac{1}{3} F'' F \right)^2 - 1 \right],$$

# Counterterms: Top Yukawa coupling

Calculating propagation of the top quark in the background $\chi$




$$\begin{aligned} \delta\mathscr{L}_{\mathrm{ct}} &\sim \left(\#\frac{y_t^3}{\bar{\varepsilon}} + \delta y_{t1}\right) F'^2 F \bar{\psi}\psi \\ &+ \left(\#\frac{y_t\lambda}{\bar{\varepsilon}} + \delta y_{t2}\right) F''(F^4)''\bar{\psi}\psi \end{aligned}$$

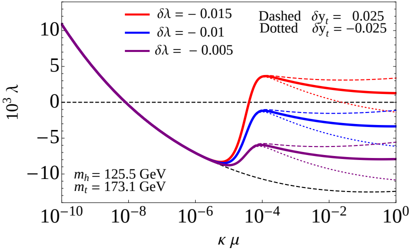
$$y_t(\mu) \to y_t(\mu) + \delta y_t \left[F'^2 - 1\right]$$

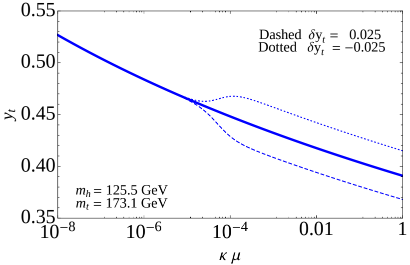
# Threshold effects at $M_P/\xi$ summarized by two new arbitrary constants $\delta\lambda$, $\delta y_t$

$$\lambda(\mu) \to \lambda(\mu) + \delta\lambda\left[\left(F'^2 + \tfrac{1}{3}F''F\right)^2 - 1\right]$$

$$y_t(\mu) \to y_t(\mu) + \delta y_t\left[F'^2 - 1\right]$$

Conservatively – can think of these as parametrization of our lack of knowledge of physics at $M_P/\xi$ threshold.



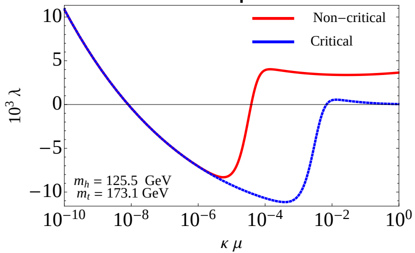



# Modified $\lambda$ evolution modifies the potential

$$\lambda(\mu) \to \lambda(\mu) + \delta\lambda \left[ \left( F'^2 + \frac{1}{3} F'' F \right)^2 - 1 \right]$$

$$y_t(\mu) \to y_t(\mu) + \delta y_t \left[ F'^2 - 1 \right]$$

Effect on the potential




(Red curve: $\xi = 1500$, $\delta y_t = 0.025$, $\delta\lambda = -0.015$)

# Consequences of these “threshold effects”

- Inflation with large $\xi$
- Inflation with small $\xi$
- What if the vacuum is metastable with $\mu_0$ below $M_P/\xi$?

# Large $\xi$ – return to tree level predictions

$$U_{\text{RG improved}}(\chi) = \frac{\lambda(\mu)}{4}\frac{M_P^4}{\xi^2}\left(1 - e^{-\frac{2\chi}{\sqrt{6}M_P}}\right)^2$$

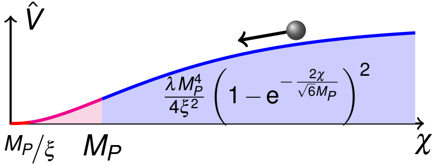
- If $\xi \sim 5\times 10^4\sqrt{\lambda} \gg 1$
  - logarithmic RG running of $\lambda$ is neglidgible
  - threshold "jumps" at $\mu \sim M_P/\xi$ are below inflationary scale – irrelevant for inflationary observables.
- All this story is not needed – we are in general attractor class of inflationary models

spectral index

$$n \simeq 1 - \frac{8(4N+9)}{(4N+3)^2} \simeq 0.97$$

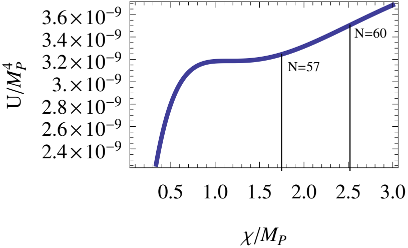
tensor/scalar ratio

$$r \simeq \frac{192}{(4N+3)^2} \simeq 0.0033$$

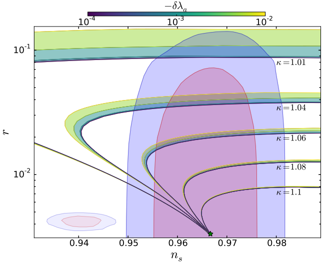


# Small $\xi$ – critical HI

$$U_{\text{RG improved}}(\chi) = \frac{\lambda(\mu)}{4}\frac{M_P^4}{\xi^2}\left(1 - e^{-\frac{2\chi}{\sqrt{6}M_P}}\right)^2$$

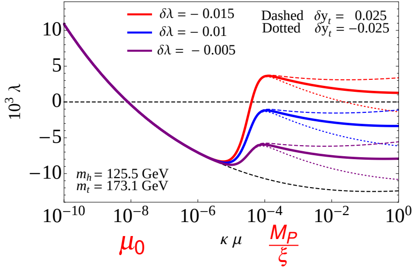



- Small $\xi \lesssim 10$ – $\lambda$ vs. $\delta\lambda$ significant, may give interesting "features" in the potential ("critical inflation", large $r$)
- However – tend to get both inflation *and* $\delta\lambda$ "jumps" in the same scale around $M_P/\xi$
- Loop corrections change result – harder to control



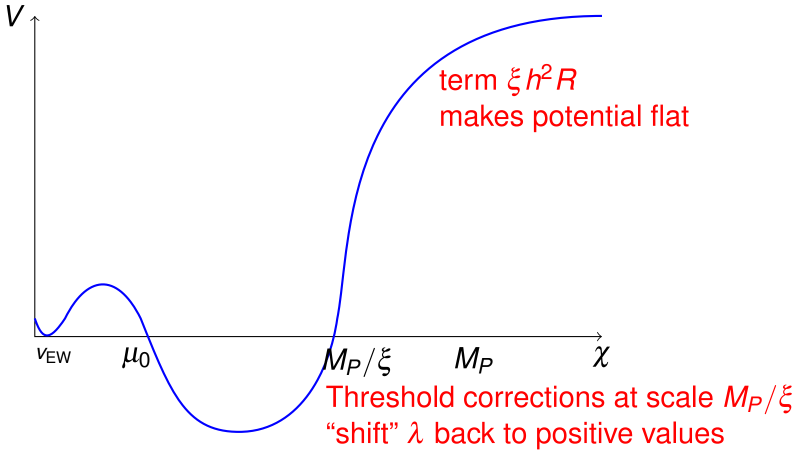
# Interestingly – allows to “cure” metastable vacuum

- Let us we have just metastable SM, with small metastability scale $\mu_0 < M_P/\xi$
- Naively – either no inflation at all, or we end up in the wrong vacuum



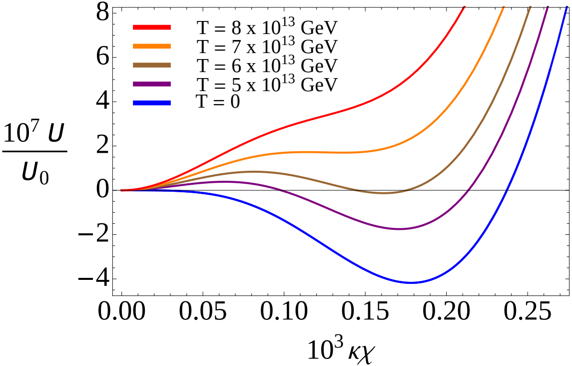
# Higgs inflation and radiative corrections

$$S_J = \int d^4x \sqrt{-g} \left\{ -\frac{M_P^2}{2} R - \xi \frac{h^2}{2} R + g_{\mu\nu} \frac{\partial^\mu h \partial^\nu h}{2} - \frac{\lambda}{4} (h^2 - v^2)^2 \right\}$$




(Not really to scale)

# In the hot enough Universe only one vacuum remains




Thermal potential $\Delta V_T = -\frac{1}{6\pi^2} \sum_{\text{particles}} \int_0^\infty \frac{k^4 dk}{\varepsilon_k(m)} \frac{1}{e^{\varepsilon_k(m)/T} \mp 1}$

- Universe has to be reheated to $T_R \gtrsim 10^{14}$ GeV

# How to get control of what is happening at $M_P/\xi$?

- Usual logic – Perturbative UV-completions
  - Tree level unitarity is violated at $M_P/\xi$
  - Leads to additional degrees of freedom at around $M_P/\xi$
    - Can construct models with additional scalar field perturbative up to $M_P$
- Is it a “no-go” statement?
  Are there possibilities without new particles at $M_p/\xi$?

# Loop corrections vs. frame choice

- $\mu$ is the scale appearing in (dimensional) regularization
- No questions asked in the "usual" case of renormalizable theories – only space/field independent choice gives regularization that is not-breaking renormalizability.
- HI is not renormalizable – multiple choices possible

In Jordan frame: $\mu^2 \propto M_P^2 + \xi h^2$

In Einstein frame: $\mu^2 \propto \text{const}$

Roughly means that effective potential

$U(\phi) \sim \phi^4 \log\left(\frac{\phi^2}{\mu^2}\right)$ or $U(\phi) \sim \phi^4 \log\left(\frac{\phi^2/\Omega^2(\phi)}{\mu^2}\right)$

- How to quantize (with loops) theories with complicated kinetic terms and do this beyond S-matrix calculations?

# What is the field theory for gravity?

- How do we understand the gravity action:
  - Metric – $g_{\mu\nu}(x)$ is an independent field, Connection – $\Gamma^{\lambda}_{\mu\nu} \equiv \frac{g^{\lambda\rho}}{2}(g_{\rho\mu,\nu} + g_{\rho\nu,\mu} - g_{\mu\nu,\rho})$
  - Palatiny – $g_{\mu\nu}(x)$, $\Gamma^{\lambda}_{\mu\nu}(x)$ are independent fields
- Different *classical* dynamics if $\xi \neq 0$
  Can be seen as different transformation under
  $g_{\mu\nu} \to \Omega(x) g_{\mu\nu}$

## Rather different inflationary predictions!

| Metric | Palatini |
|---|---|
| $R \to \Omega^2 R + 6 g^{\mu\nu} \partial_\mu \ln\Omega \partial_\nu \ln\Omega$ | $R \to \Omega^2 R$ |
| $\xi \sim 5 \times 10^4 \sqrt{\lambda}$<br>$r \sim 3.2 \times 10^{-3}$ | $\xi \sim 1.5 \times 10^{10} \lambda$<br>$r \sim 3.5 \times 10^{-14} \lambda^{-1}$ |

# Conclusions: Higgs and inflation

## what is good and what is bad?

**Bad**

Predictions depend on high scale physics

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## what is good and what is bad?

**Bad**

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**Good**

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