Higgs and Vacuum (In)Stability

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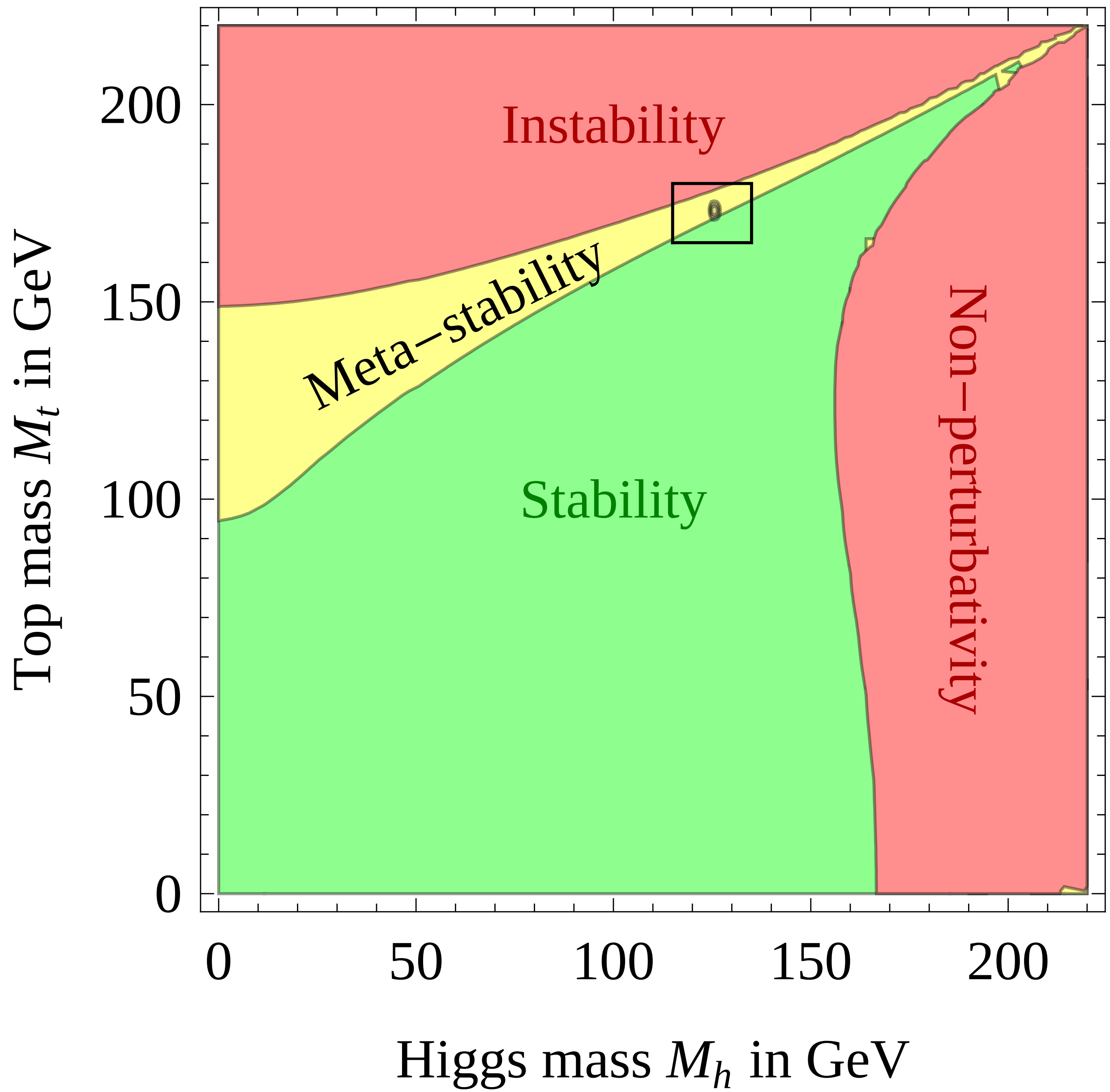
Trusting the SM up to the Planck scale



The Higgs potential

Including quantum corrections, parameters get renormalised around the vev:

$$V(H) \approx \underbrace{-\frac{m^2(\mu\sim h)}{2}|H|^2}_{\text{negligible at } h\gg v} + \lambda(\mu \sim h)|H|^4 \qquad H = \begin{pmatrix} 0 \\ v + h/\sqrt{2} \end{pmatrix}$$

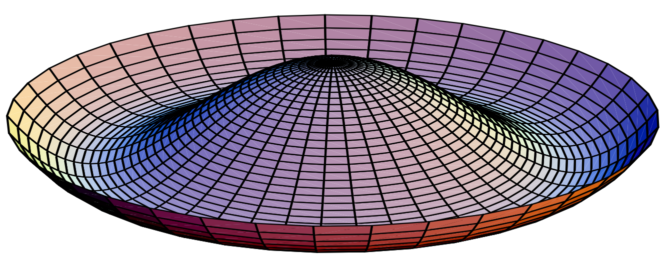
RGE running of λ:

$$(4\pi)^2 \frac{d\lambda}{d\ln\mu} = -6y_t^4 + \frac{9}{8}g_2^4 + \frac{27}{200}g_1^4 + \frac{9}{20}g_2^2 g_1^2 + \lambda(12y_t^2 - 9g_2^2 + \frac{9g_1^2}{5}) + 24\lambda^2 + \text{higher loops}$$

A too big M_h makes $\lambda(\mu)$ non-perturbative at large energy

A too heavy top makes $\lambda(\mu)$ negative at large energy...

...so the SM potential falls down at large h vev



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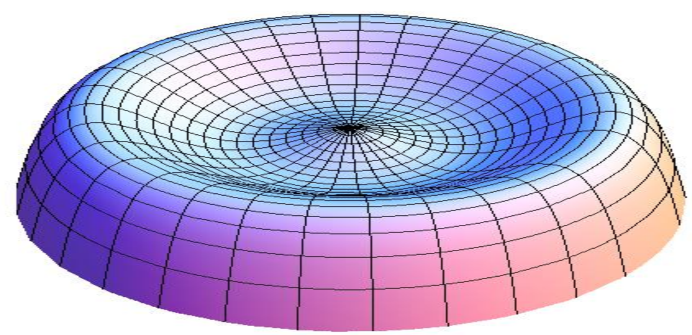
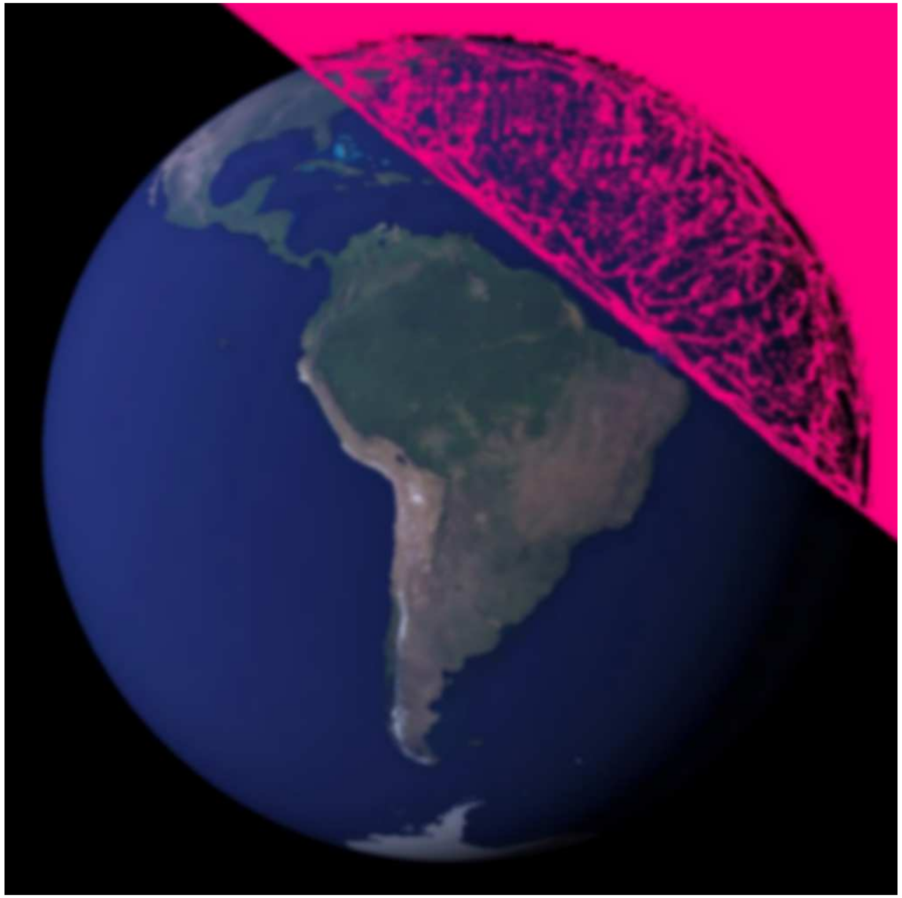


If your mexican hat turns out to be a dog bowl you have a problem...

Vacuum decay

If our vacuum is only a local minimum of the potential, at some point **quantum tunnelling towards the true minimum** will happen.
Vacuum decay was studied by Coleman and is 'similar' to boiling of water (quantum field theory is formally similar to thermal field theory for matter...). A bubble of negative-energy true vacuum can appear anywhere and anytime and start expanding at the speed of light.



Computing vacuum decay in the SM

The probability density of vacuum decay is $d\wp/dV\,dt = e^{-S}/R^4$, suppressed by the action S of the classical field configuration $h(r)$ that interpolates vacua

$$h(\infty) = \text{unstable vacuum} \qquad h(0) \approx \text{other side of the potential barrier}$$

In the SM at tree level there is a family of bounces

$$h(r) = \sqrt{\frac{2}{|\lambda|}}\frac{2R}{r^2+R^2} \qquad r^2 = \vec{x}^2 - t^2, \qquad S = \frac{8\pi^2}{3}\frac{1}{|\lambda|}\,.$$

Our vacuum is **unstable** (decay rate faster than the age of the universe T_U) if

$$S \lesssim \frac{1}{4}\ln M_{\rm Pl}T_U \quad \Rightarrow \quad \lambda \lesssim -0.05$$

At loop level λ runs. Using λ renormalized at $\sim 1/R$ roughly agrees with the full one-loop computation of $S[h]$.

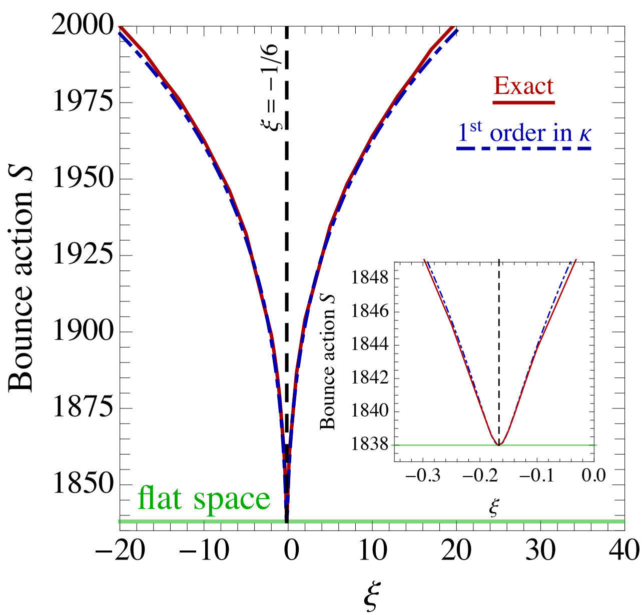
For $M_h \approx 115\,\text{GeV}$ the vacuum decay bound is saturated and $1/R \approx 10^{17}\,\text{GeV}$.
For $M_h \approx 125\,\text{GeV}$, $1/R \sim M_{\rm Pl}$, but vacuum decay is negligibly slow, $S \gg 500$.

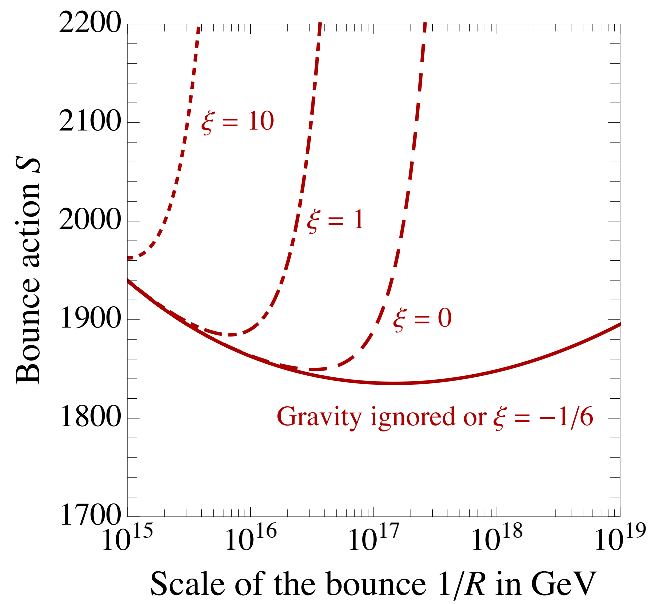
Gravity corrections to vacuum decay

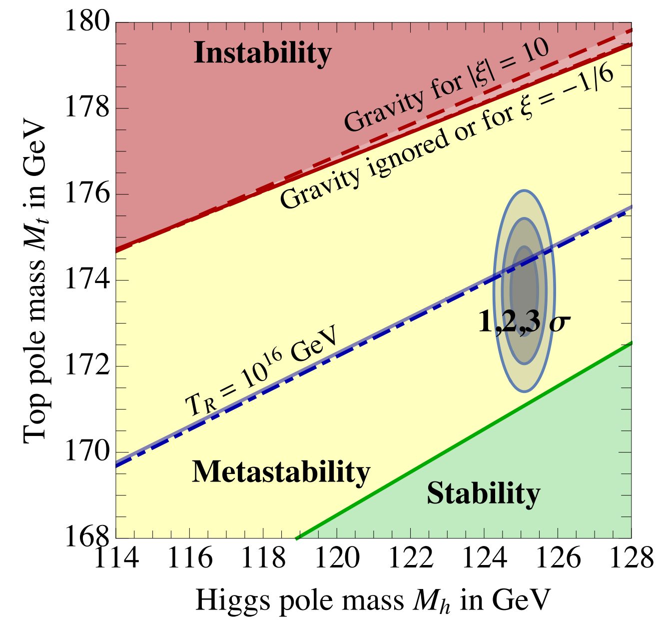
$$S_E = \int d^4x \sqrt{g}\left[\frac{(\partial_\mu h)^2}{2} + V(h) - \frac{\mathcal{R}}{2\kappa} - \frac{\mathcal{R}}{2}\xi h^2\right].$$

Makes sense to compute at leading order in $\kappa = 1/\bar{M}_{\rm Pl}^2$. Simple general result: metrics becomes $ds^2 = dr^2 + (r^2 + \kappa\rho(r))d\Omega^2$; bounce action changes by

$$\Delta S_{\rm gravity} = \frac{6\pi^2}{\bar{M}_{\rm Pl}^2}\int dr\ r\rho'^2 \geq 0 \qquad \rho' = \frac{r^2}{6}\left[\frac{h_0'^2}{2} - V(h_0) - \frac{6}{r}\xi h_0 h_0'\right]$$

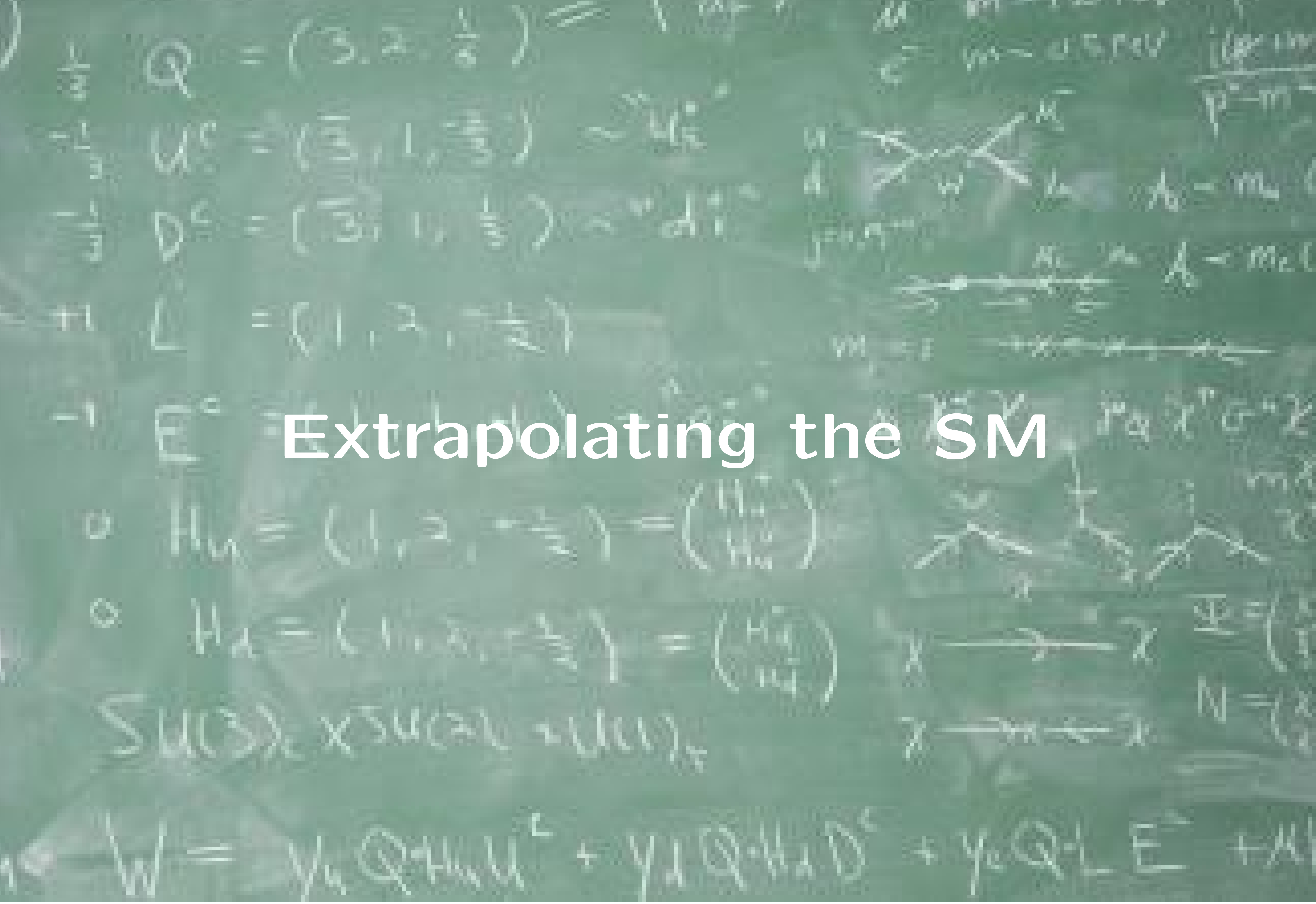






New physics can add new decay channels (extra scalars; string landscape).

Extrapolating the SM



The SM parameters

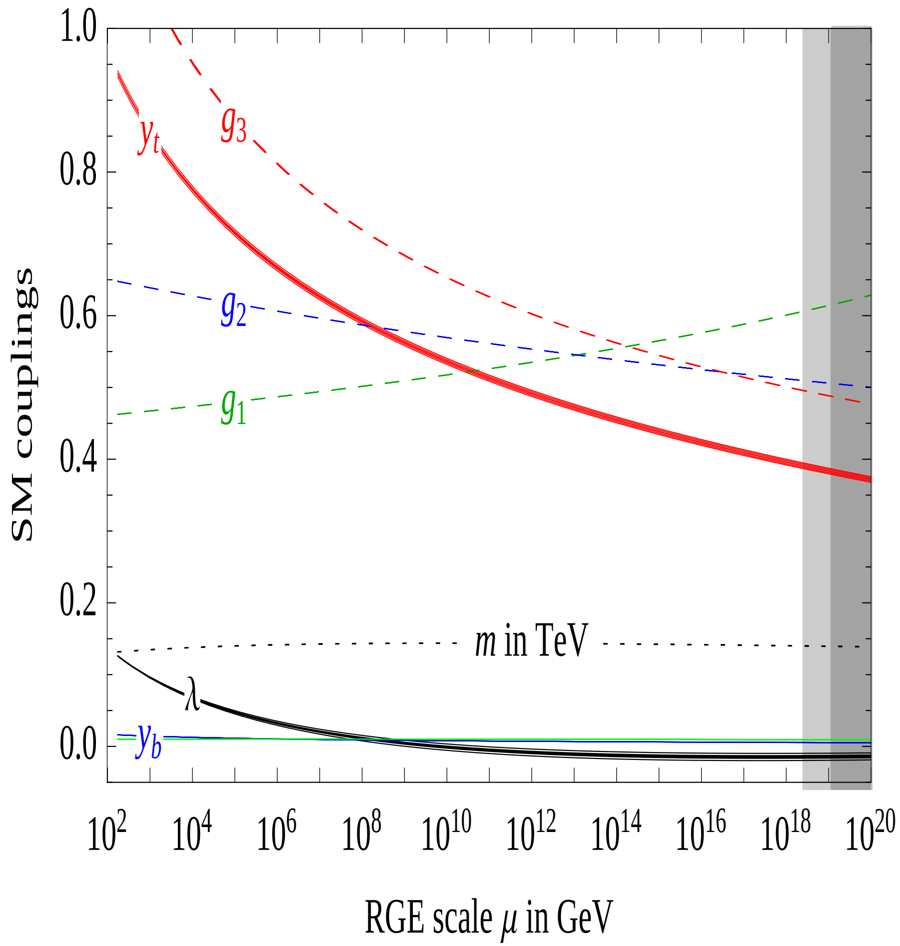
M_h and M_t lie around the meta/stability border so more precision is needed.

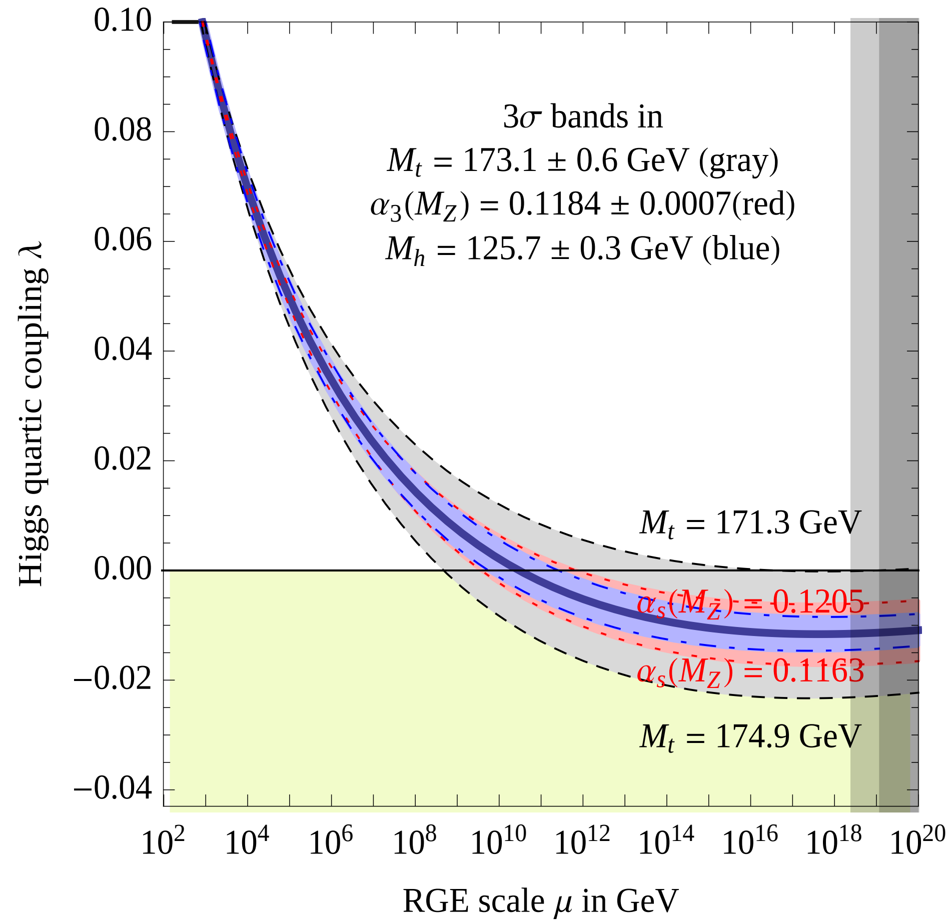
SM parameters extracted from data at 2 loop accuracy: at $\bar{\mu} = M_t$

$$
\begin{aligned}
g_2 &= 0.64822 + 0.00004 \left(\frac{M_t}{\text{GeV}} - 173.34\right) + 0.00011 \frac{M_W - 80.384\,\text{GeV}}{0.014\,\text{GeV}} \\
g_Y &= 0.35761 + 0.00011 \left(\frac{M_t}{\text{GeV}} - 173.34\right) - 0.00021 \frac{M_W - 80.384\,\text{GeV}}{0.014\,\text{GeV}} \\
y_t &= 0.9356 + 0.0055 \left(\frac{M_t}{\text{GeV}} - 173.34\right) - 0.0004 \frac{\alpha_3(M_Z) - 0.1184}{0.0007} \pm 0.0005_{\text{th}} \\
\lambda &= 0.1271 + 0.0021 \left(\frac{M_h}{\text{GeV}} - 125.15\right) - 0.00004 \left(\frac{M_t}{\text{GeV}} - 173.34\right) \pm 0.0003_{\text{th}} \\
\frac{m}{\text{GeV}} &= 132.03 + 0.94 \left(\frac{M_h}{\text{GeV}} - 125.15\right) + 0.17 \left(\frac{M_t}{\text{GeV}} - 173.34\right) \pm 0.15_{\text{th}}.
\end{aligned}
$$

Renormalization to large energies done with 3 loop RGE.

From the EW scale to the Planck scale

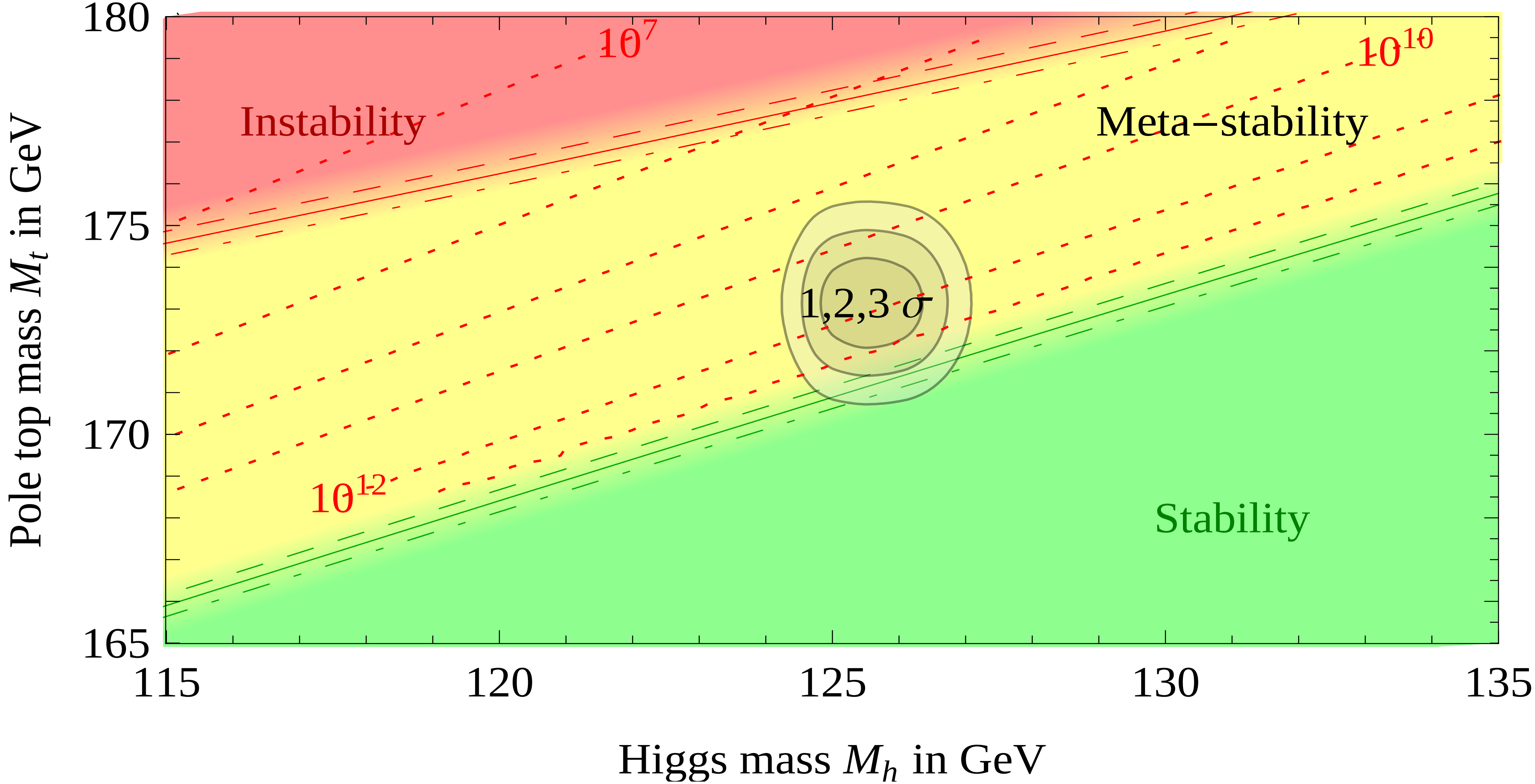




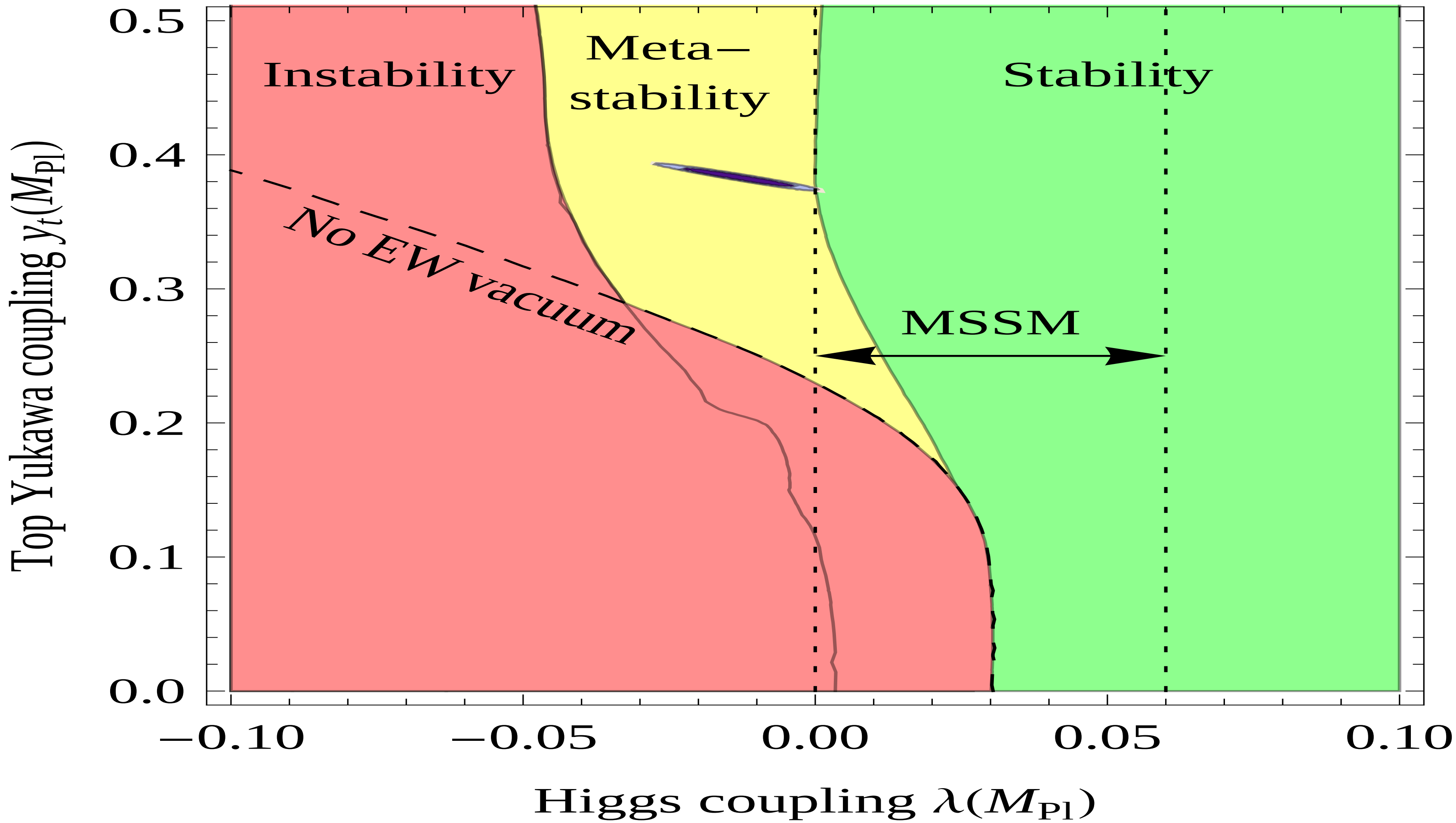
$$\lambda(\mu = M_{\rm Pl}) = -0.0144 + 0.0028\left(\frac{M_h}{\rm GeV} - 125\right) \pm 0.0047_{M_t} \pm 0.0018_{\alpha_s} \pm 0.0028_{\rm th}$$

For the measured masses both λ and its β-function vanish around $M_{\rm Pl}$!!?

Meta-stability favoured at about 2σ



In terms of Planck-scale parameters

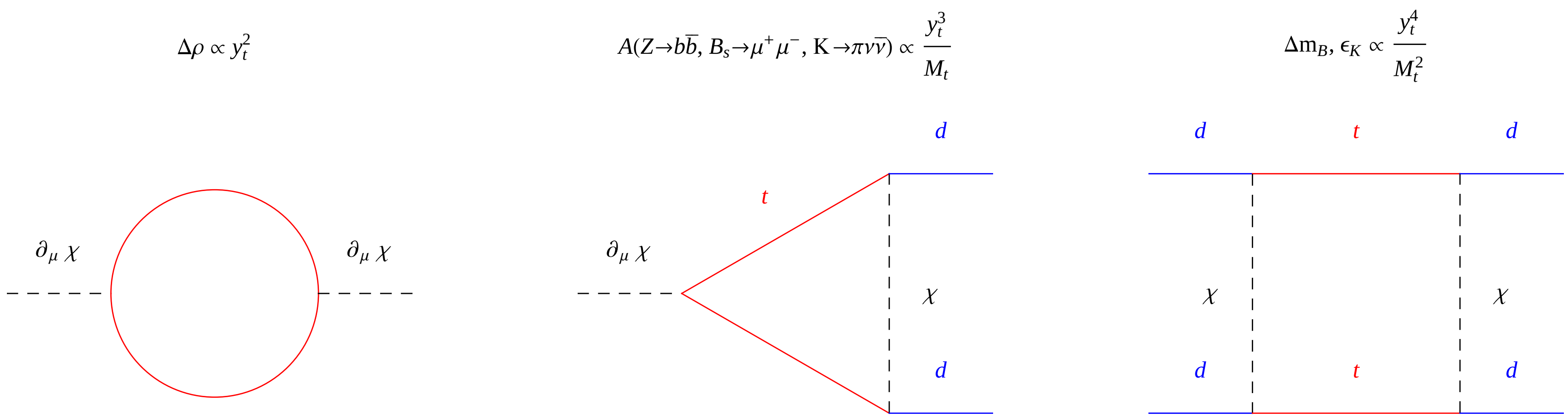


1) There is a new red region, with $\lambda < 0$ at the weak scale.

2) **If $\boldsymbol{\lambda}(M_{\mathsf{Pl}}) = 0$, then M_t has the minimal value needed for stability.**
Indeed smaller M_t leads to negative $\beta_\lambda(M_{\mathsf{Pl}})$ such that λ transists negative

The top mass is the main uncertainty

- $M_t = 173.3 \pm 0.8\,\text{GeV}$ from LHC2015, TeVatron. Doubts:
  i) the pole mass is defined up to $\pm\mathcal{O}(\Lambda_{\text{QCD}}) \sim \pm 0.3\,\text{GeV}$;
  ii) MonteCarlo used to reconstruct the pole t mass from its decay products, that contain j, ν and initial state radiation. Distributions ok, but it's like reconstructing the pig mass from sausages (don't ask how they are made).
- $M_t = 174.6 \pm 1.9\,\text{GeV}$ from top production cross sections.
- $M_t = 177.0 \pm 2.6\,\text{GeV}$ from EW data, dominated by M_W (LHC will measure it 2-3 times better). Observables with a quadratic dependence on M_t:



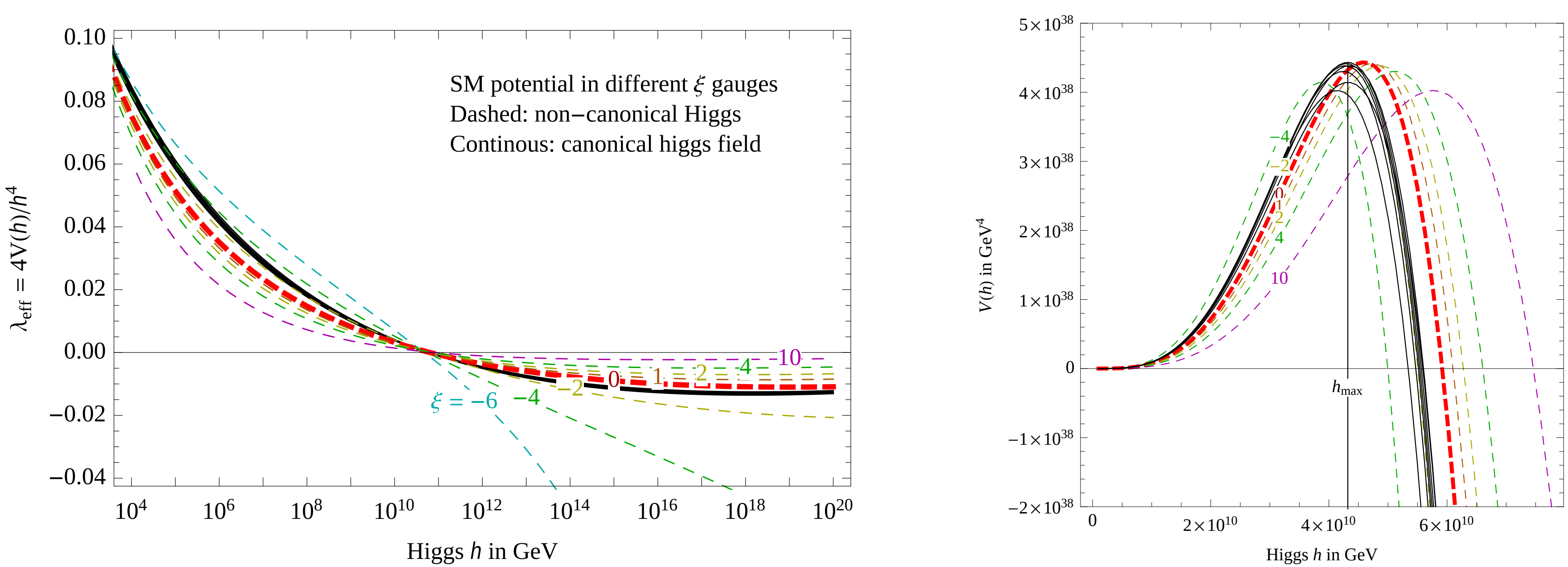
- $M_t = 173.9 \pm 7.8\,\text{GeV}$ from flavor (Δm_{B_s}).
  Uncertainty will decrease down to 1.7 GeV (2025) thanks to $B_s \to \mu^+\mu^-$.
- A linear collider at the top threshold would measure M_t up to $\pm 0.05\,\text{GeV}$.

The SM effective potential

Trouble 1: resummation of **IR-divergences** needed.
Right, but they only appear at 3 loops in Landau gauge $\xi = 0$.

Trouble 2: V_{eff} **is gauge dependent**, the instability scale depends on ξ.
The full action $S[h,\xi]$ is morally gauge-invariant: the dependence on ξ can be reabsorbed into a redefinition of $h_\xi(h)$. Like a change of coordinates in Higgs space. In practice, expanding in derivatives

$$\mathscr{L}_{\text{eff}} = Z_{\text{eff}}(h,\xi)\frac{(\partial_\mu h)^2}{2} - \lambda_{\text{eff}}(h,\xi)\frac{h^4}{4} + \cdots = \frac{(\partial_\mu h_{\text{can}})^2}{2} - \lambda_{\text{can}}(h_{\text{can}})\frac{h_{\text{can}}^4}{4} + \cdots$$



The instability scale

- $V_{\max}$ and $V_{\min}$ and the stability condition $\Delta V = 0$ are gauge-independent.
- The bounce action is gauge-independent, because it is a classical solution.
- The classical e.o.m. is gauge-independent, at least at leading order in ∂.
- The same for Langevin and Fokker-Planck equations (used later).

The instability scale can be characterised trough the maximal height $V_{\max}$ of the potential barrier

$$\log_{10}\frac{V_{\max}^{1/4}}{\text{GeV}} = 9.5+0.7\left(\frac{M_h}{\text{GeV}}-125.15\right)-1.0\left(\frac{M_t}{\text{GeV}}-173.34\right)+0.3\,\frac{\alpha_3(M_Z)-0.1184}{0.0007}$$

Around its maximum V has a universal form:

$$V(h) \approx -b\ln\left(\frac{h^2}{h^2_{\max}\sqrt{e}}\right)\frac{h^4}{4} \qquad V_{\max} = \frac{b}{8}h^4_{\max} \qquad b \approx \frac{0.16}{(4\pi)^4}$$

So: $h_{\max} \approx 3\ 10^{10}\,\text{GeV}$ for central values of SM parameters.

Can be compared with the inflationary Hubble rate $H \sim 2.5\ 10^{14}\,\text{GeV}\sqrt{r}$.

Cosmological Higgstory

Cosmology of the meta-stable Higgs

If the SM Higgs potential is unstable for $h > h_{\text{max}}$, can cosmology end up in the observed meta-stable small $h = v$? Or it's impossible? Unnatural?

1) During **inflation** the Higgs h fluctuates: $\delta h \sim H$. Space splits in 3 regions:

- regions where the Higgs remains below its instability scale, $|h| < h_{\text{max}}$;
- regions where the Higgs fluctuates above the instability scale, $|h| > h_{\text{max}}$, and remains fluctuating there without falling;
- regions where the Higgs falls down to its Planck-scale minimum.

2) Little happens during **pre-heating** and **re-heating** to T_{RH};

3) During **thermal big-bang** orange bubbles get burnt. The big thermal mass for the Higgs, $m^2 \approx 0.2T^2$ brings $h \to 0$, which is special because corresponds to unbroken SU(2). Extra thermal decay is slow.

4) **Later** Gyr. Red bubbles expand eating the universe. Vacuum decay is slow.

Higgs mass during inflation

During inflation with Hubble H, the Higgs potential becomes $V \approx V_{\rm SM} + \frac{1}{2}m^2h^2$.

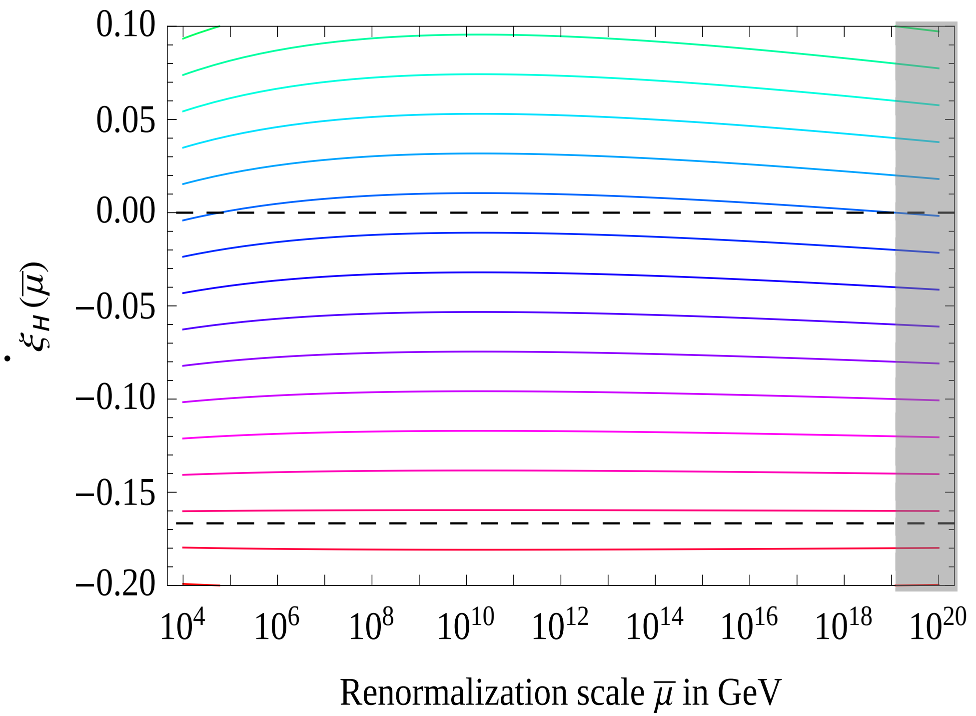
Various sources can contribute to extra inflationary Higgs mass m:

1. the inflaton vev ϕ in presence of a quartic $\lambda h^2\phi^2$: $m^2 = 2\lambda\phi^2$;
2. thermal mass from continuous inflaton decay, $m \lesssim H$;
3. the gravitational coupling $-\xi h^2 R/2$ gives $m^2 = \xi R \overset{\rm infl}{=} -12\xi H$.

SM RGE running induces $|\xi| \gtrsim 0.01$

$$\frac{d\xi}{d\ln\bar\mu} = \frac{\xi + \frac{1}{6}}{(4\pi)^2}\left(6y_t^2 - \frac{9}{2}(g_2^2 + \frac{g_1^2}{5}) + 12\lambda\right) + \cdots$$

So we focus on 3.



Higgs inflationary fluctuations

If $m < \frac{3}{2}H$, the higgs fluctuates as approximated by the Langevin equation:

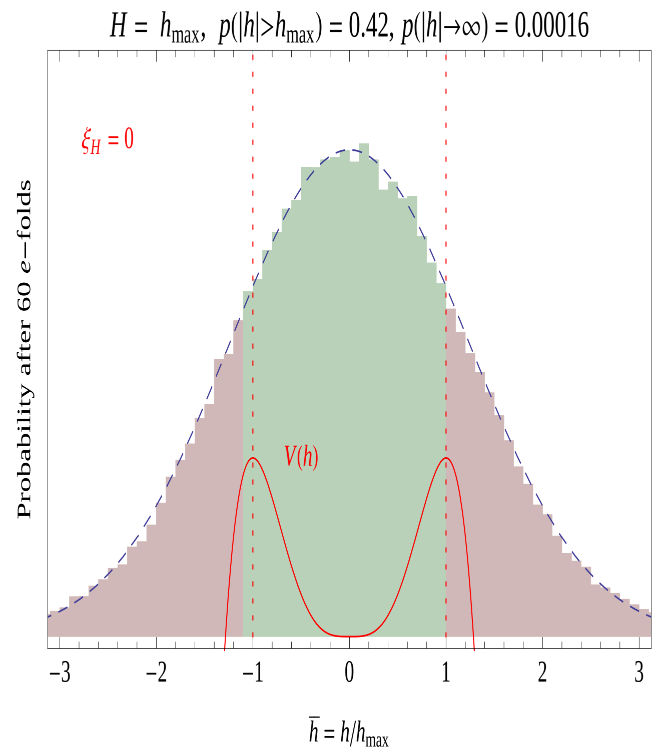
$$\frac{dh}{dt} + \frac{1}{3H}\frac{dV(h)}{dh} = \eta(t), \qquad \langle \eta(t)\eta(t')\rangle = \frac{H^3}{4\pi^2}\delta(t-t').$$

E.g. neglecting V, ξ, after N e-folds one gets a Gaussian with variance

$$\sqrt{\langle h^2 \rangle} = \frac{H}{2\pi}\sqrt{N}.$$

Space-time splits into **three** regions:

- $h < h_{\rm max}$ up to the end of inflation.
- h is fluctuating above $h_{\rm max}$ at the end of inflation.
- h falls to the true deep (Planckian?) minimum.



The falling rate can also be computed as de-Sitter Hawking-Moss tunneling.

Red bubbles are bad

Do 'red' bubbles where the higgs falls to $h \sim M_{\rm Pl}$ expand? Eating all the space?

1) YES, because things fall to lower energy.
2) NO, because space with negative $V \sim -M_{\rm Pl}^4$ shrinks.
3) MAYBE such regions are behind black-hole horizons. Could be DM?

General relativistic computation in thin-wall approximation clarifies:

3) The volume energy is $\Delta \equiv V_{\rm out} - V_{\rm in} - (4\pi G\sigma)^2$: its last term is the Newton-like gravitational energy of the surface only. Red bubbles expand safely behind a black hole horizon if $\Delta < 0$ (they are bad DM because of iso-curvatures). However bubbles form with $\Delta > 0$.
2) If seen from inside, AdS bubbles contract and collapse, hitting a singularity in a finite short time $t = \sqrt{3/|V_{\rm in}|}$.
1) If seen from outside, AdS bubbles expand forever. During inflation bubbles make swiss cheese. After inflation bubbles eat all Minkowski.

Conclusion: zero red bubbles in our past light-cone.

Orange bubbles are (likely) good

Regions with $h > h_{\text{max}}$ at the end of inflation can fall to $h = 0$ or fall to $h \sim M_{\text{Pl}}$.

Minimal scenario:

- the inflaton releases its energy during a matter-dominated epoch until $t \sim 1/\Gamma_\phi$. In this phase $m^2 = -3\xi H/a^{3/2}$.

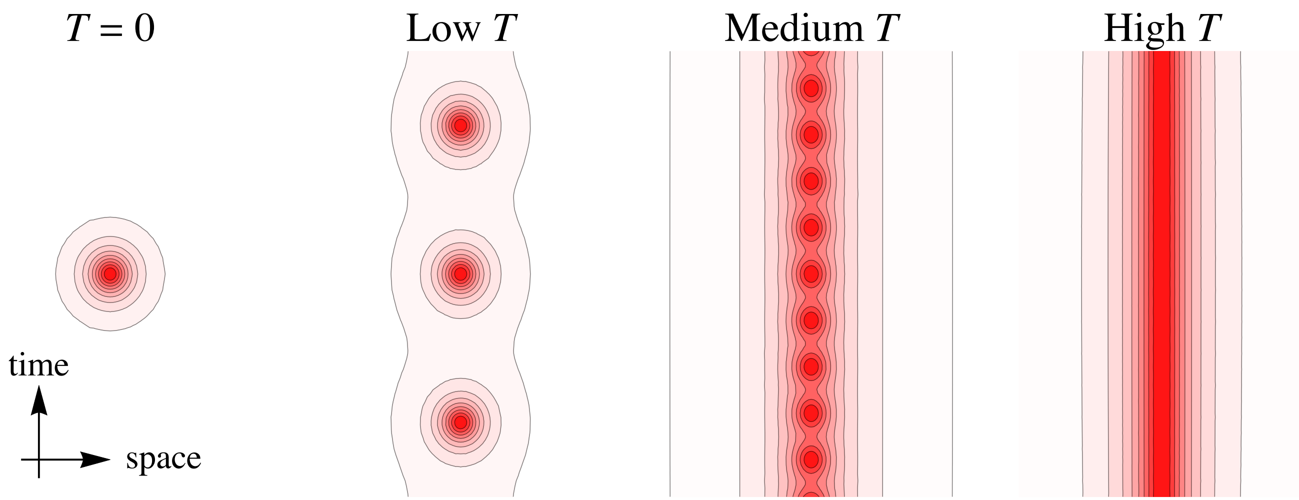
- later the radiation-dominated epoch starts with $T \sim T_{\text{RH}} \sim \sqrt{M_{\text{Pl}}\Gamma_\phi}$. **The Higgs acquires a thermal mass** $m^2 \sim +T^2$, which is typically large, even if the inflaton decays gravitationally, $\Gamma_\phi \sim M_\phi^3/M_{\text{Pl}}^2$.

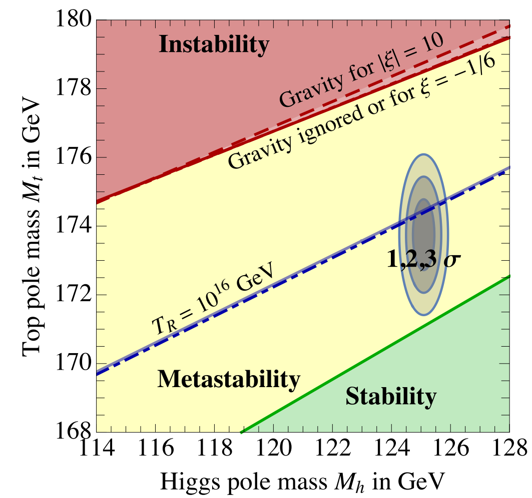
Rough conclusion: orange bubbles tend to fall towards $h = 0$.

Thermal corrections to vacuum decay

Thermal fluctuations do not induce thermal vacuum decay for $M_h \gtrsim 120\,\text{GeV}$.

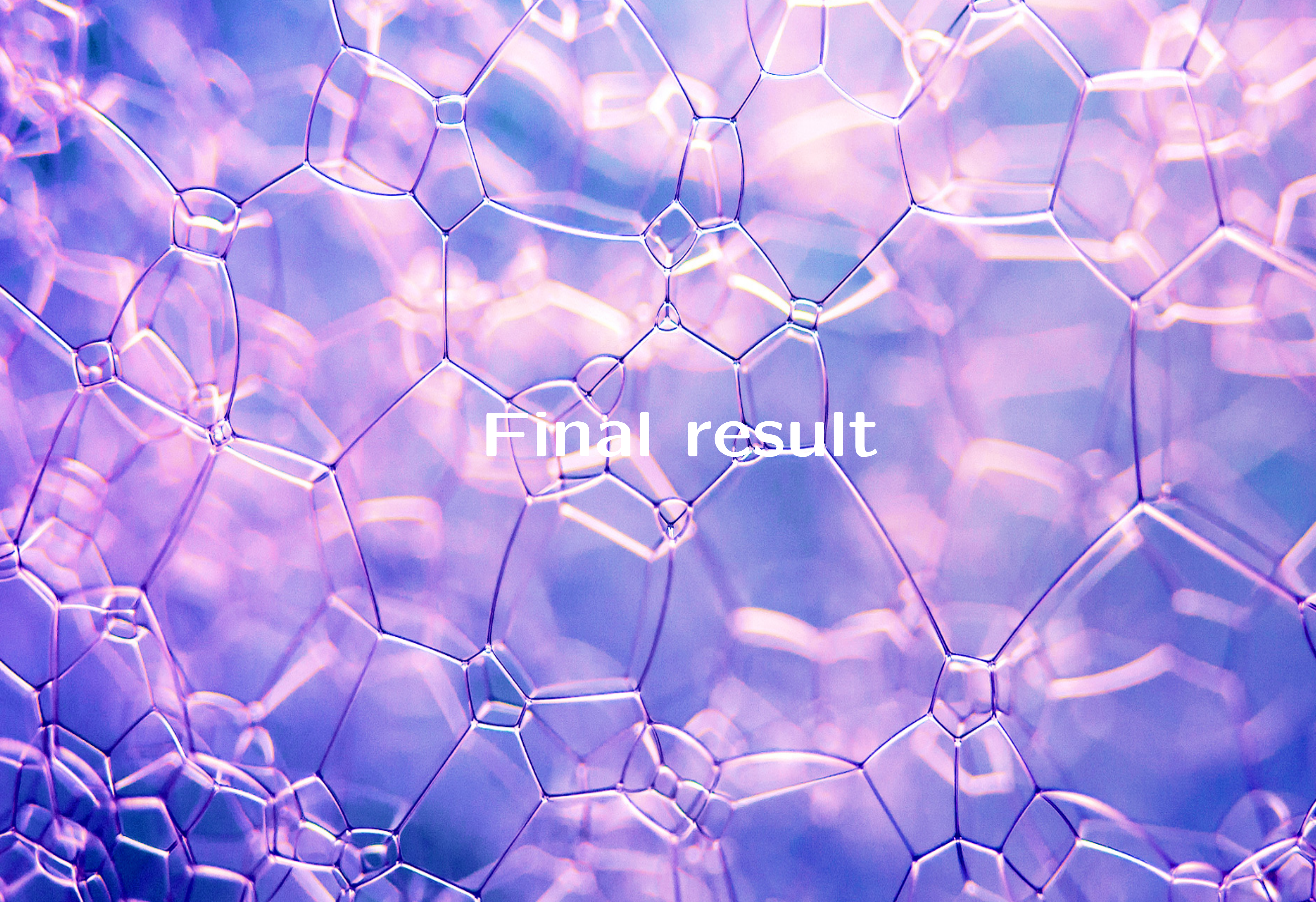
Standard approximation: the bounce is O(4)-symmetric at $T \ll M$ and O(3)-symmetric at $T \gg M$, where M is the mass scale in the potential.





The SM potential is ≈ scale-invariant. Nevertheless, the hi-T approx is ok.

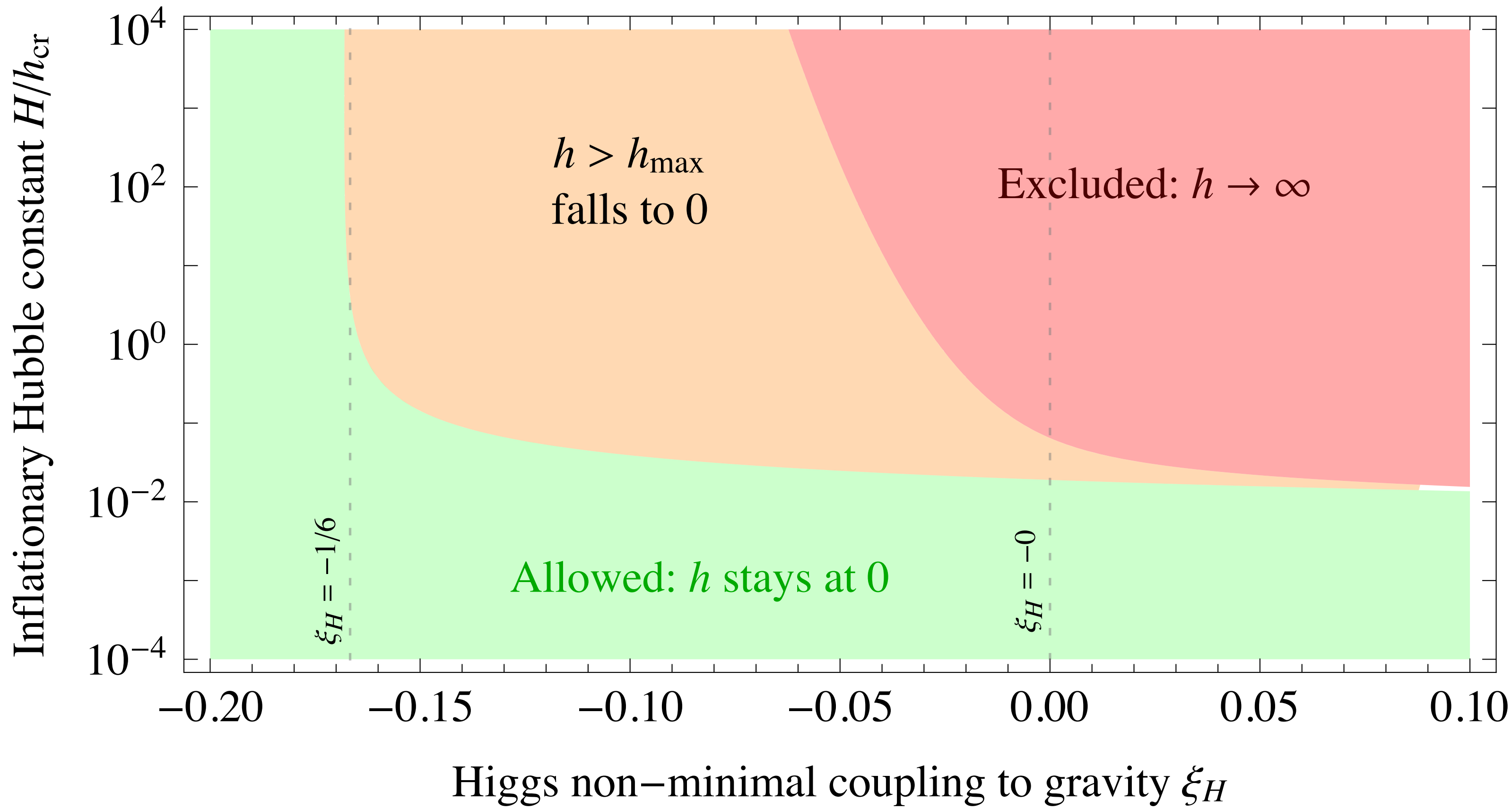
Final result



Final result

An upper bound on (inflationary Hubble scale H)/(Higgs instability scale $h_{\max}$) that depends on the non-minimal Higgs coupling to gravity ξ. For $\xi < -0.05$

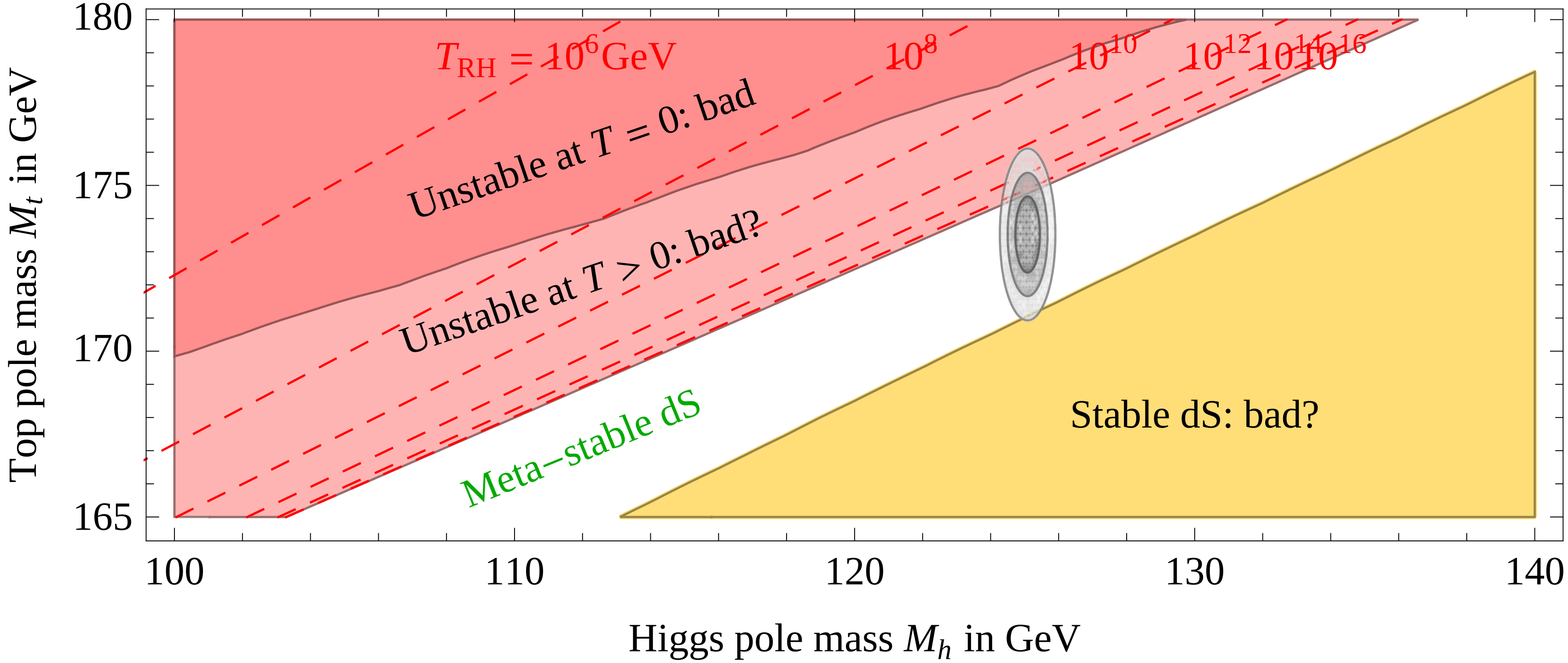
H can be orders of magnitude above the instability scale $h_{\max}$



Even if $h_{\max}$ is small, tensor modes can be observable, $r = 0.1(H/8\ 10^{13}\,\text{GeV})^2$.
A model-dependent Higgs/inflaton quartic can make constraints even weaker.

Meta-stability needed to tame Λ?

Today $\Lambda > 0$: some theorists argue that de Sitter is inconsistent and must decay. The SM Higgs instability offers this way out, in a restricted range:



$$171\,\text{GeV} < M_t < 175\,\text{GeV}$$

Conclusions

Main result of LHC so far: $M_h \approx 125\,\mathrm{GeV}$:

Around the stability/meta-stability border.
Precision computations favour meta-stability at $\approx 2\sigma$.
Important to compute higher orders and to be sure that $M_t > 171.5\,\mathrm{GeV}$.

Meta-stability allows a sensible cosmological history.

From

One loop SM vacuum decay: hep-ph/0104016; 1707.08124.

Precision computations of SM parameters: 1205.6497; 1307.3536...

Gravity vacuum decay: 0712.0242; 1608.02555...

Higgstory: 1505.04825, 1706.00792...

Etc