Fundamental (Partial) Compositeness

Francesco Sannino

with Strumia, Tesi e Vigiani, 1607.01659

Cacciapaglia, Gertov, Thomsen 1704.07845

Pica et al. 1612.09336, ….

Cacciapaglia 1402.0233, ….

D·IAS

CP³ Origins
Cosmology & Particle Physics

The Standard Model ado

Fields:

Gauge fields + fermions + scalars

Interactions:

Gauge: SU(3) x SU(2) x U(1) at EW scale

Yukawa: Fermion masses/Flavour

Culprit: Higgs

Scalar self-interaction

Gauge - Yukawa theories

$$L = \boxed{-\frac{1}{2}F^2 + i\overline{Q}\gamma_\mu D^\mu Q} + \boxed{y(\overline{Q}_L H Q_R + \text{h.c.})}$$

Yukawa

Gauge

$$\boxed{\mathrm{Tr}\left[\mathrm{DH}^\dagger\mathrm{DH}\right]} - \boxed{\lambda_\mathrm{u}\mathrm{Tr}\left[(\mathrm{H}^\dagger\mathrm{H})^2\right] - \lambda_\mathrm{v}\mathrm{Tr}\left[(\mathrm{H}^\dagger\mathrm{H})\right]^2}$$

Scalar selfinteractions

4D: standard model, dark matter, …

3D: condensed matter, phase transitions

2D: graphene, …

4plusD: extra dimensions, string theory, …

Universal description of physical phenomena

Partial Composite Goldstone Higgs

Higgs is a composite pseudo goldstone SM doublet

D.B. Kaplan & H. Georgi, 84

SM fermions masses via (effective) Lagrangian operators

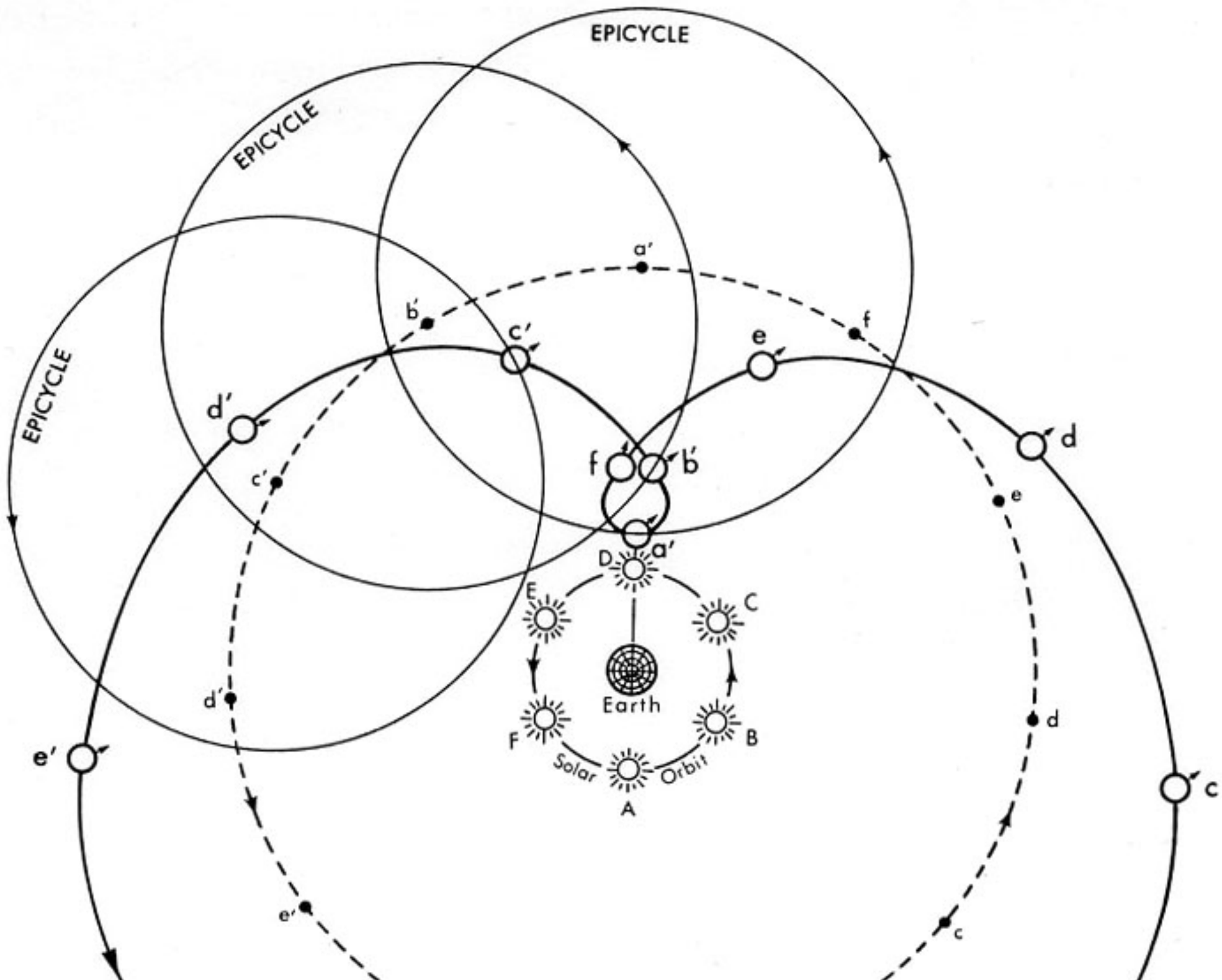
$$f\mathcal{B}$$

D.B. Kaplan, 91

Extra-dim, no-lagrangian approaches, (in)effective theories

It talks like a Higgs, it walks like a Higgs, but it is not the Higgs

Ptolemaic Fermion Mass-Generation



Fermion Masses

$$\bar{q}Hq \longrightarrow \frac{\bar{q}q\ \bar{Q}Q}{\Lambda^2}$$

$$\bar{q}q\ \bar{Q}Q \longrightarrow \bar{q}q\ \bar{q}q$$

FM generation equivalent of Ptolemaic's ubiquitous circle:

Require a single scale for all generations. FCNC requires

$$\Lambda > 10^3\ \mathrm{TeV} \longrightarrow$$ Too small top mass

Epicycle: *Walk* your way out. Need large anomalous dim.

$$D(\bar{Q}Q) = 3 - \gamma_m\ , \qquad \gamma_m \to 2$$

Status of Walking Epicycles

- State-of-the-art: 4 and 5-loops anomalous dim.
- Lattice results support 4 loops
- Small anomalous dim. unless strong QQQQ

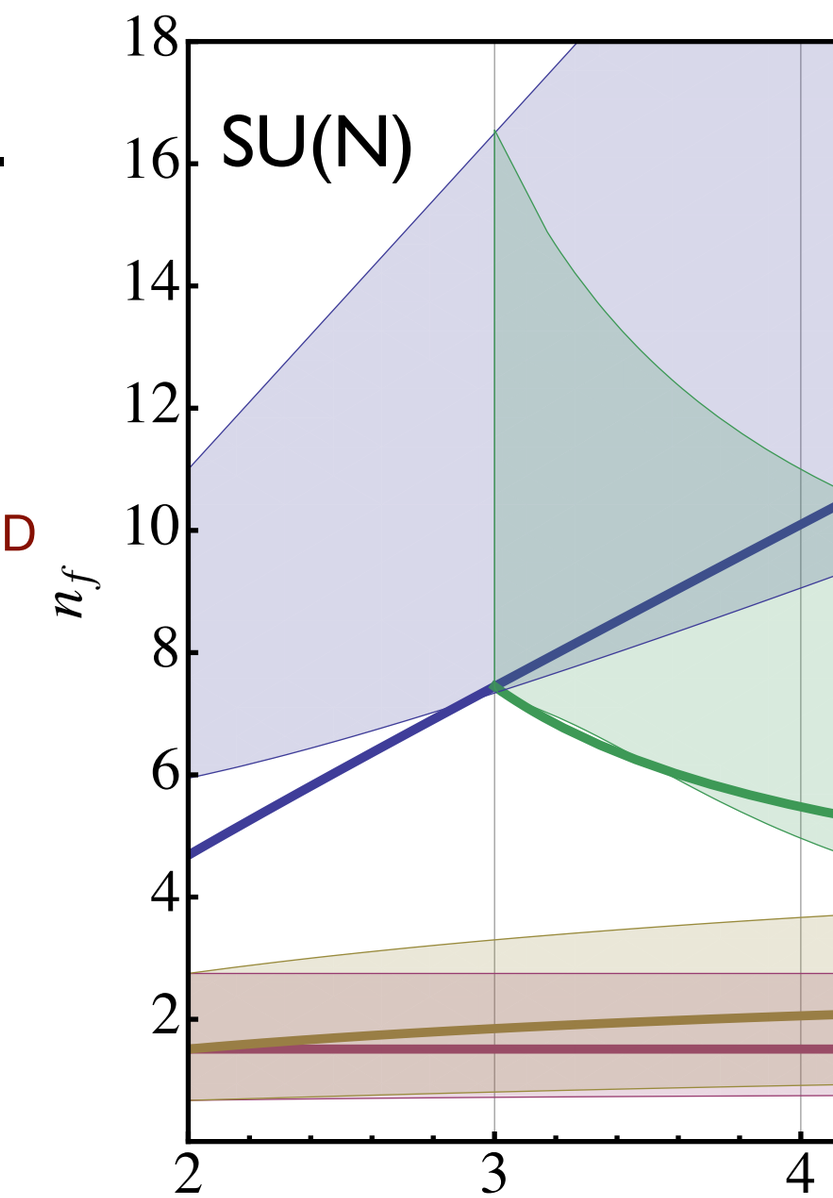
First walking theory observed Pica, Rantaharju, Sannino, 1704.03977 PRD

Dietrich and Sannino, 2006

Pica and Sannino, 2010

Ryttov and Shrock, 2010, 2016

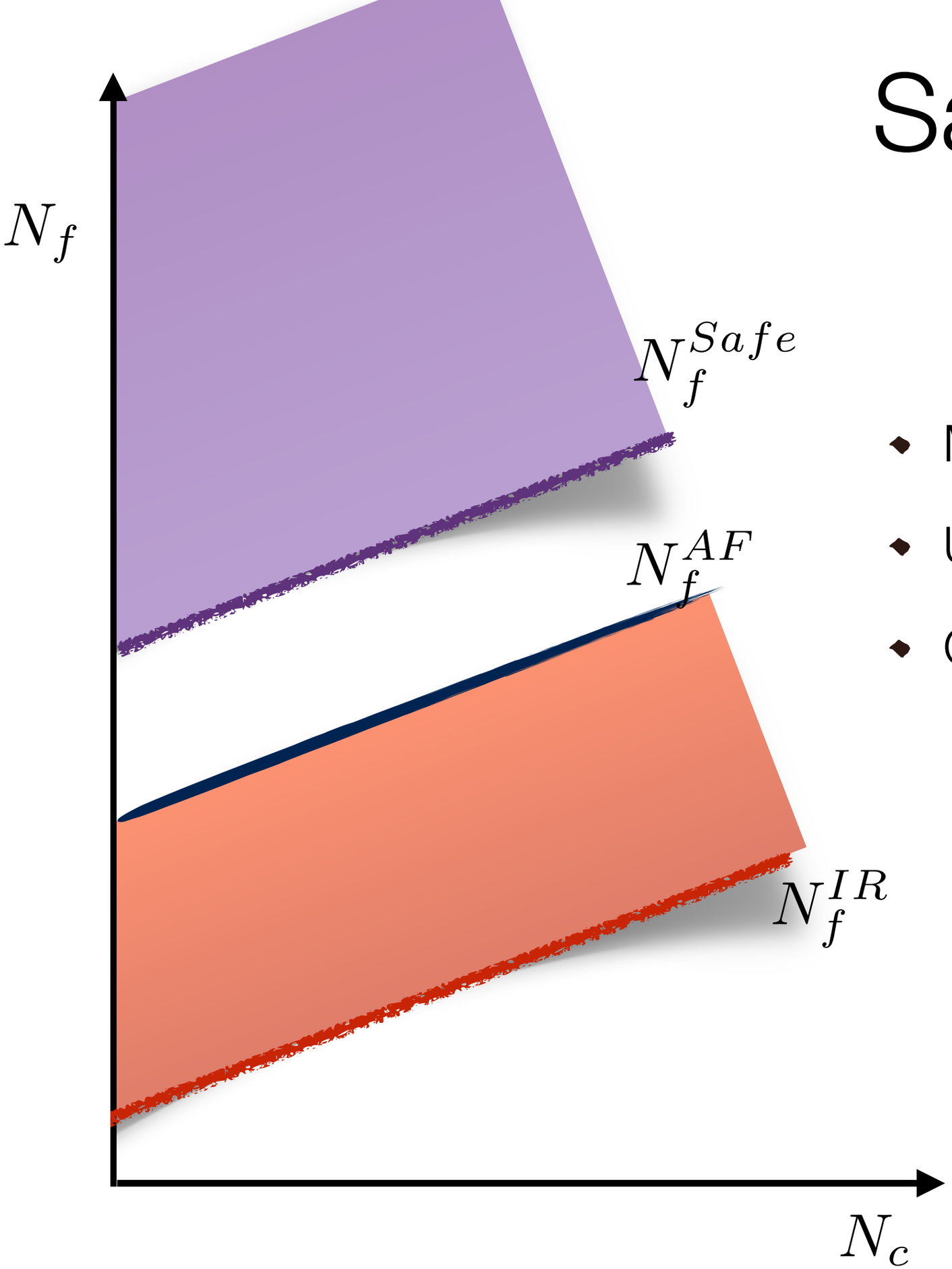
Giedt 2015



Walking epicycle is challenging

Conformal Window 1.0

Safe QCD ado

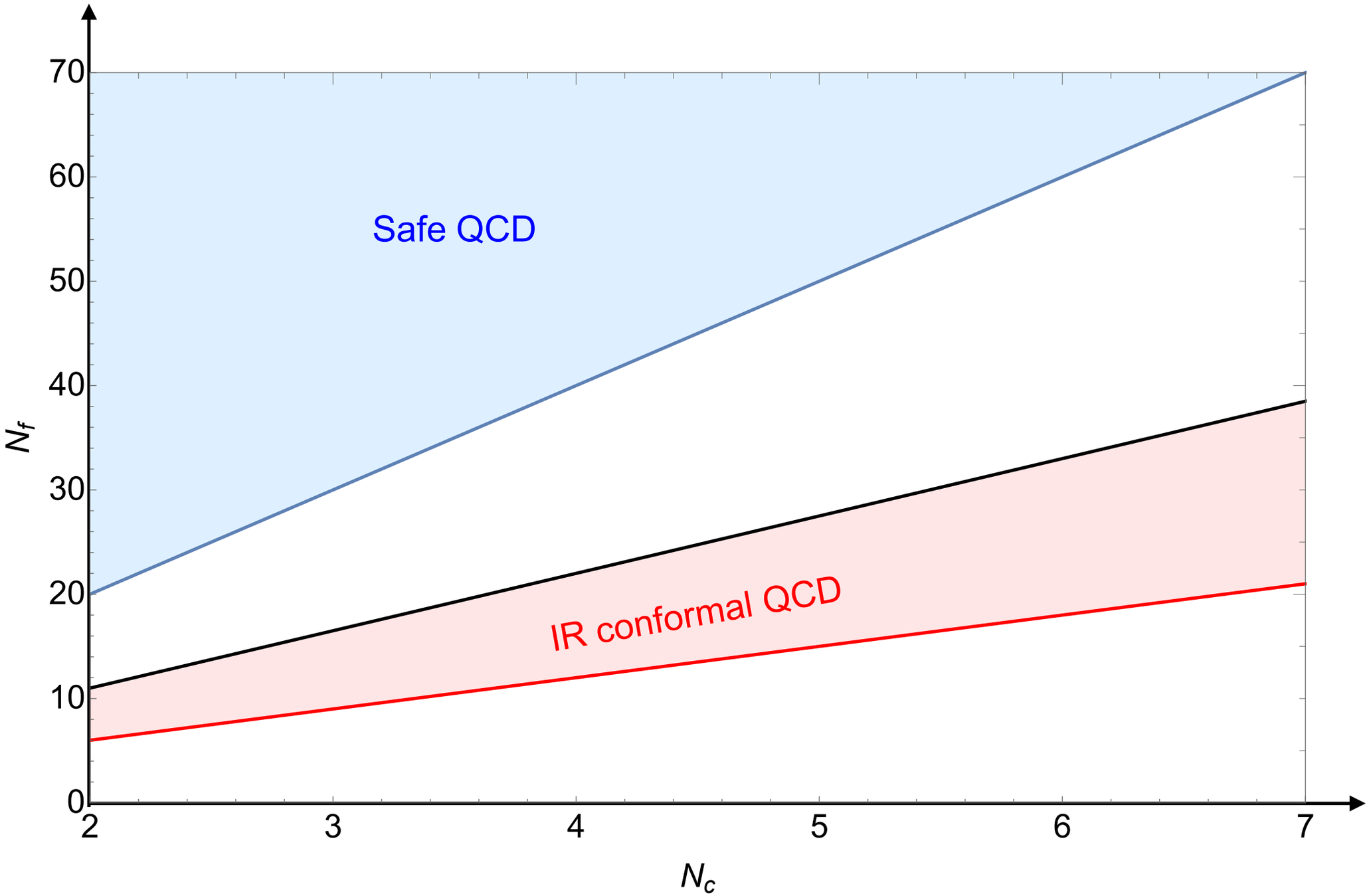


- Must exist a critical Safe Nf
- Unsafe region in Nf-Nc
- Continuous (Walking) transition?

Sannino, ERG 2016, Heidelberg

Antipin and Sannino, to appear
Pica and Sannino 1011.5917, PRD

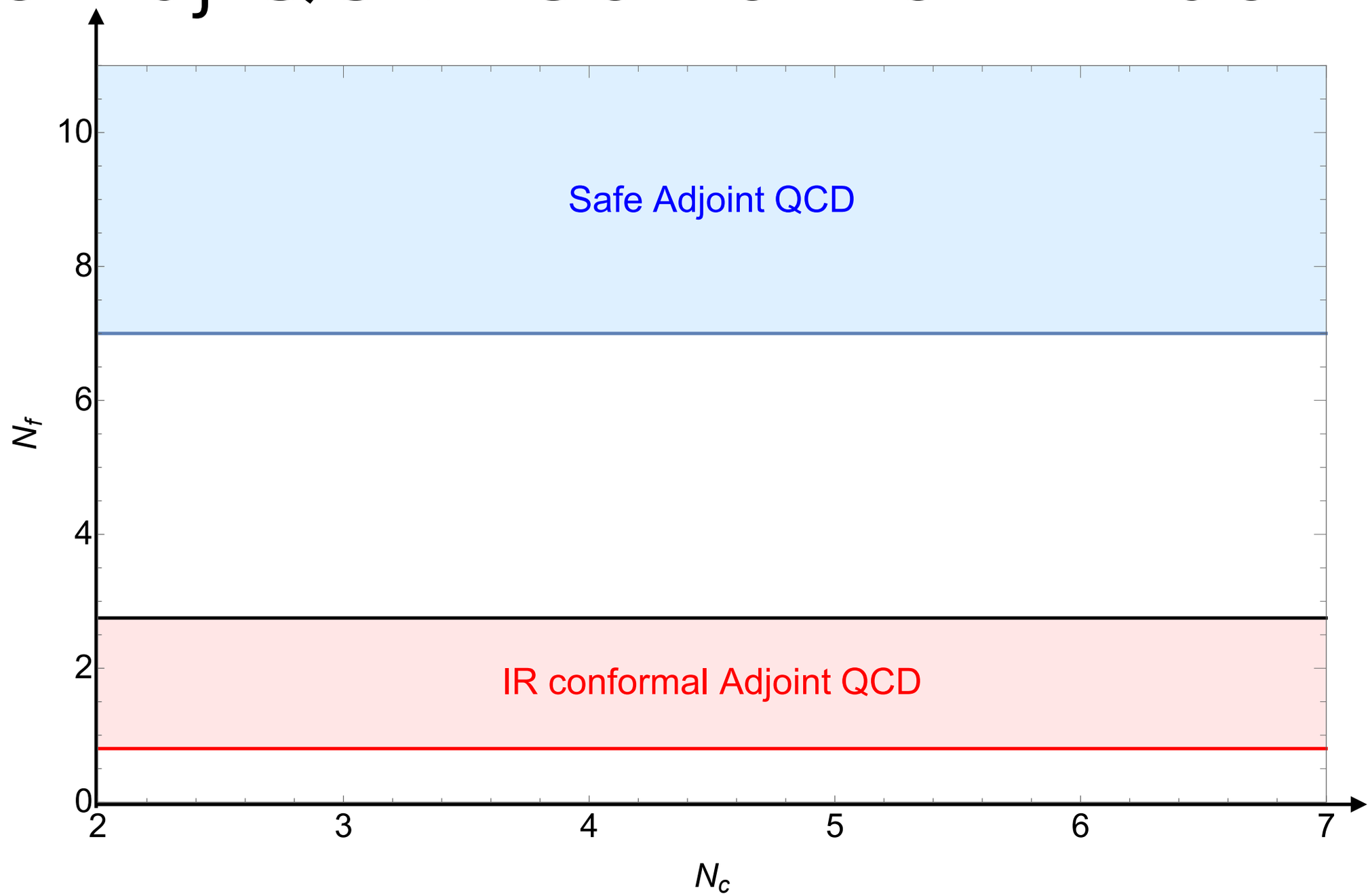
Safe QCD: Conformal Window 2.0



$$\gamma_m^* \overset{N_f \to \infty}{\to} \frac{(N_c^2 - 1)}{2 N_c N_f}$$

Sannino, ERG 2016, Heidelberg
Antipin and Sannino, 1709.02354

Safe Adj QCD: Conformal Window 2.0



Sannino, ERG 2016, Heidelberg
Antipin and Sannino, 1709.02354

Fermion Masses II

Fundamental partial compositeness $\bar{q}\ QQQ$

D.B. Kaplan, 91

Epicycle: Large PC Baryon anomalous dimensions

$$\frac{3}{2} \leq D(QQQ) = \frac{9}{2} - \gamma^B \leq \frac{5}{2} \qquad 2 \leq \gamma^B \leq 3$$

Epicycle of the epicycle: Requires also suppression of

$$\bar{q}q\ \bar{Q}Q \qquad \bar{Q}Q\ \bar{Q}Q \longrightarrow \gamma^B > \gamma_m$$

Status of Partial Compositeness Epicycles

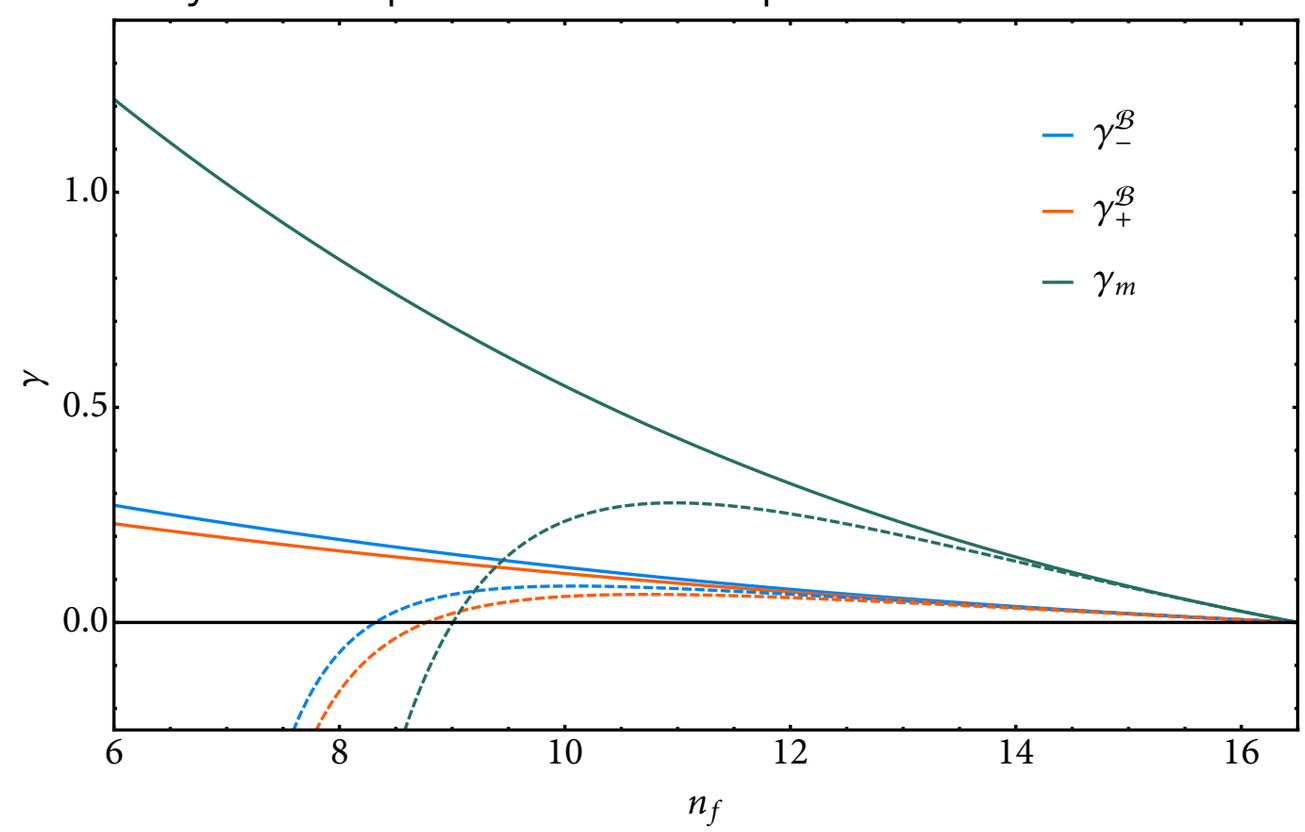
Tiny PC Baryons anomalous dim (solid estimate beyond PT)

$$4\gamma^B \sim \gamma_m$$

unsuppressed operators

$\bar{q}q\ \bar{Q}Q$ $\qquad$ $\bar{Q}Q\ \bar{Q}Q$

Physical expansion & 3-loops results



Pica and Sannino 1604.02572

Ryttov 1604.00687

Partial compositeness epicycle is very challenging

Copernican-like approach

Allow for at least 2 distinct FM generation scales ('beyond the circle')

$$\frac{\bar{q}q\ \bar{Q}Q}{\Lambda_q^2} + \frac{\bar{t}t\ \bar{Q}Q}{\Lambda_{\rm top}^2}$$

$$1 - 3\ \mathrm{TeV} \approx \Lambda_{\rm top} \ll \Lambda_q \sim \Lambda$$

New states at the TeV Scale!

so far..

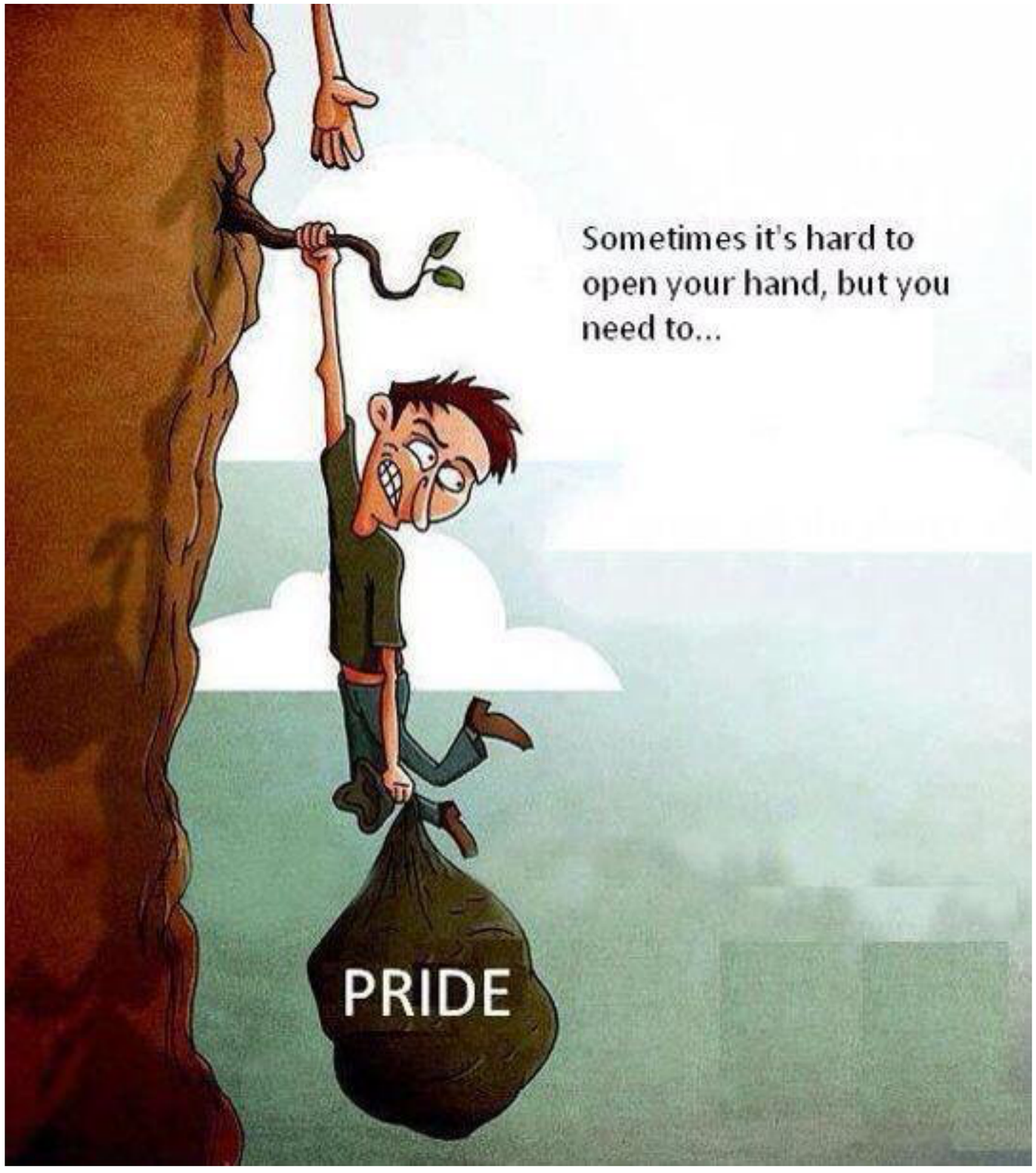
- QQX, QQQ UV attempts by Gerghetta, Ferretti, Vecchi
- Phenomenology by Cacciapaglia, De Andrea, Buttazzo, Tesi, De Curtis, Redi, Isidori, Nardecchia, Pomarol,…
- No theory linking composite TC baryons to SM fermions exists

No UV complete fermion theory exists able to give masses to SM fermions

In the following:

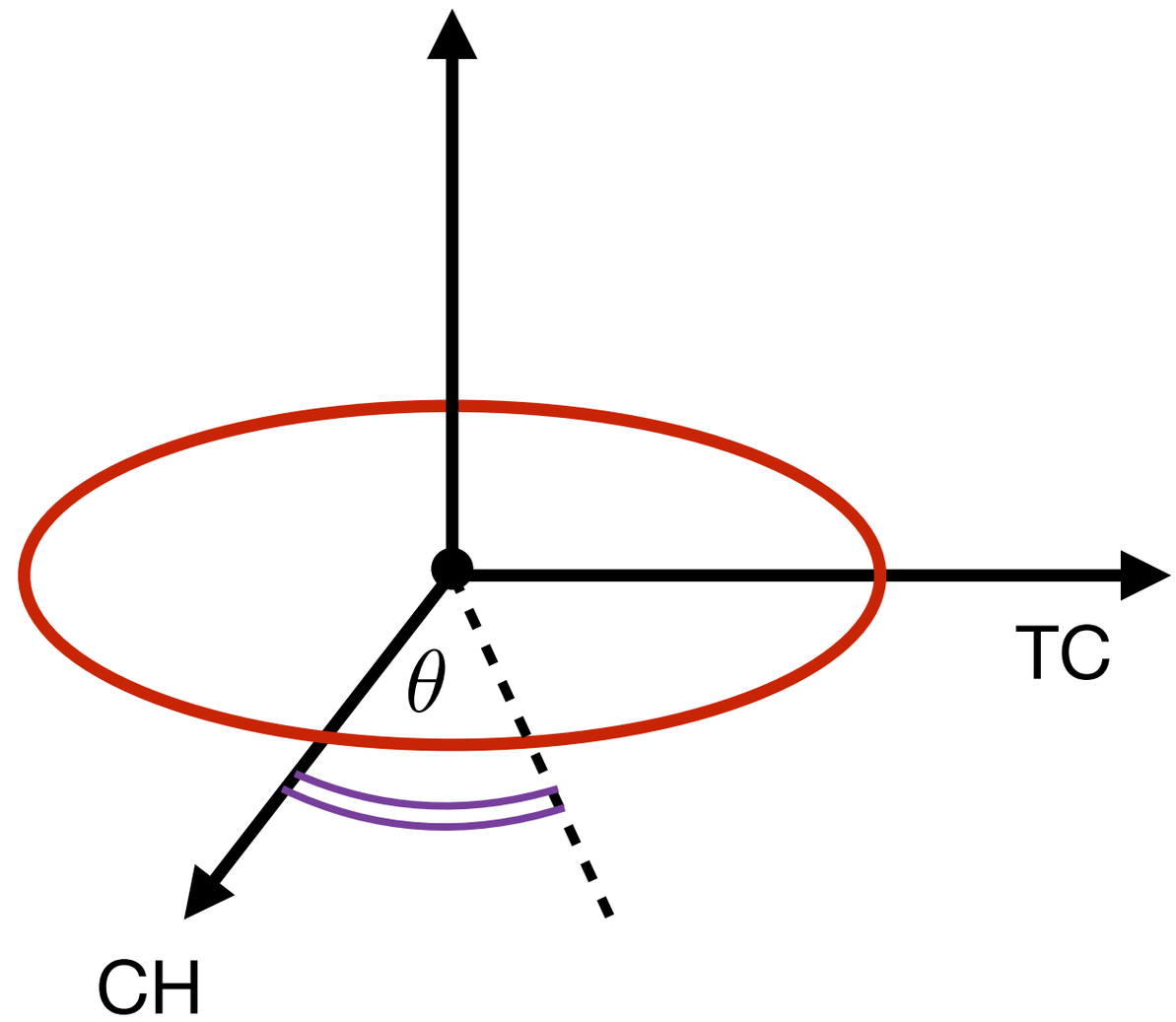
$$Q = \mathcal{F}$$

Fermion mass generation with only TC fermions


Sometimes it's hard to open your hand, but you need to...
PRIDE

Fundamental (Partial) Compositeness

- Unified TC and CH framework
- Fund 4D underlying theory
- Spectrum via lattice



(Mostly) Effective Theories

Appelquist, Sannino, 98, 99

Ryttov, Sannino, 2008

Katz, Nelson Walker, 2005

Gripaios, Pomarol, Riva, Serra, 2009

Galloway, Evans, Luty, Tacchi, 2010

Cacciapaglia and Sannino 1402.0233 JHEP

(Partial) Composite Pandora Box

Sannino, Strumia, Tesi, Vigiani, 1607.01659

Extend fundamental dynamics to include techniscalars

$$\mathcal{F} \quad S$$

- Richer composite spectrum
- Large anomalous dim. not needed
- "All" SM fermion masses generated

$$f\mathcal{B} = f(\mathcal{F}S)$$

Renormalizable operator

Fundamental composite dynamics generating 'all' SM fermion masses

Fundamental theory backbone

$$\mathcal{L} = \mathcal{L}_{\rm SM}^{H=0} + \mathcal{L}_{\rm kin} + \mathcal{L}_Y - V$$

$$\mathcal{L}_{\rm kin} = -\frac{1}{4}\mathcal{G}_{\mu\nu}^2 + \overline{\Psi}_{\mathcal{F}}(i\gamma_\mu D^\mu - m_{\mathcal{F}})\Psi_{\mathcal{F}} + \boxed{(|D_\mu \mathcal{S}|^2 - m_{\mathcal{S}}^2|\mathcal{S}|^2)}$$

$$\mathcal{L}_Y \sim (Q\mathcal{F}\boxed{\mathcal{S}_q^*} + Q_R\mathcal{F}^c\boxed{\mathcal{S}_q}) + (L\mathcal{F}\boxed{\mathcal{S}_\ell^*} + L_R\mathcal{F}^c\boxed{\mathcal{S}_\ell})$$

$$V = \lambda_{\mathcal{S}}{\rm Tr}(\mathcal{S}\mathcal{S}^\dagger)^2 + \lambda'_{\mathcal{S}}{\rm Tr}(\mathcal{S}\mathcal{S}^\dagger\mathcal{S}\mathcal{S}^\dagger)$$

Quarks and leptons acquire mass via mixing with TC-Baryon

How does it work?

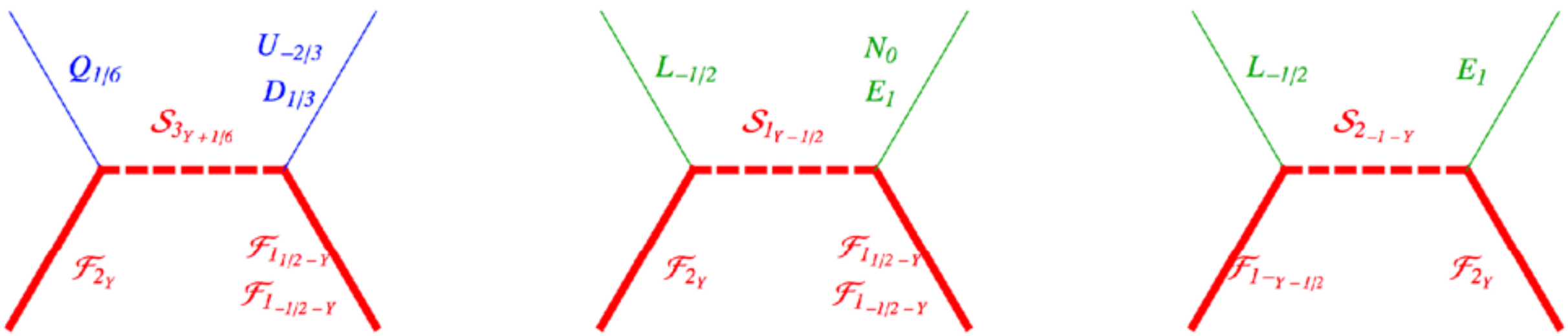
$\mathcal{F}\mathcal{F} \longrightarrow$ Composite Higgs

$\mathcal{F}S \longrightarrow$ Composite Baryon

$$f\mathcal{B} = f(\mathcal{F}S)$$

S N_{TC} x N_s matrix

N_s info on QCD color and generations



Top mass

$$y_t \sim \frac{y_Q y_U}{g_{TC}} \qquad y_Q \sim 1 \qquad y_U \sim g_{TC}$$

$$(4\pi^2)\frac{\partial g_{TC}}{\partial \ln \mu} = b g_{TC}^3 \qquad (4\pi^2)\frac{\partial y_f}{\partial \ln \mu} = f_f y_f^3 - f_g g_{TC}^2 y_f$$

$$f_f = \frac{N + 2n_f + 1}{2} \qquad f_g = 6C_N = 6 \begin{cases} (N^2-1)/2N & \text{for } G_{\mathrm{TC}} = \mathrm{SU}(N) \\ (N-1)/2 & \text{for } G_{\mathrm{TC}} = \mathrm{SO}(N) \\ N(N+1)/4N & \text{for } G_{\mathrm{TC}} = \mathrm{Sp}(N) \end{cases}$$

Quasi fixed point $\quad y_f = g_{\mathrm{TC}}\sqrt{(f_g + b)/f_f} \sim g_{\mathrm{TC}}$

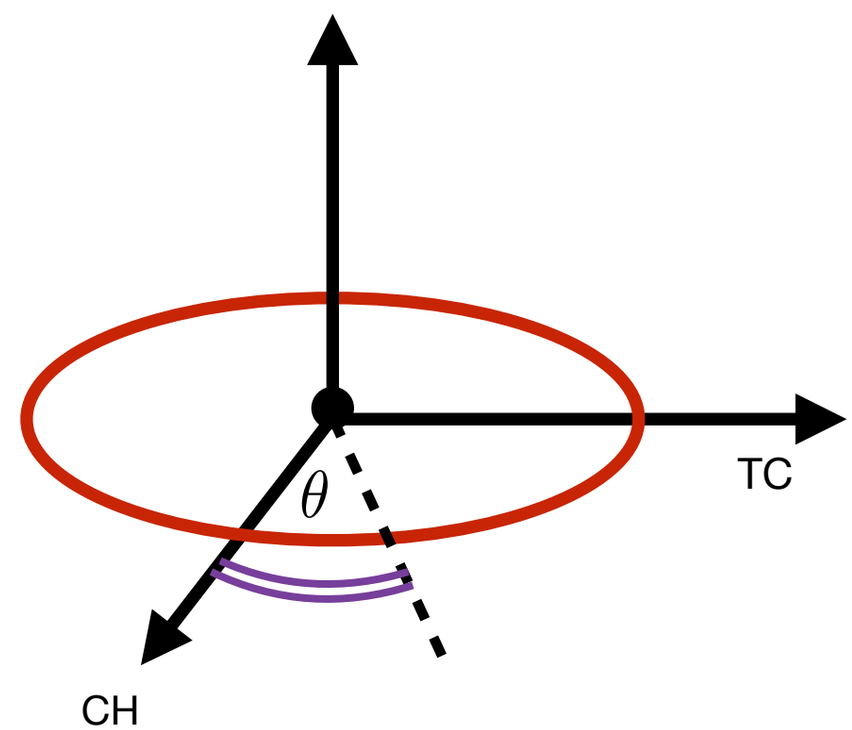
Top Yukawa naturally of the expected size

Higgs sector

$$\Lambda_{TC} \sim g_{TC} f_{TC} \sim 4\pi f_{TC}$$

Use chiral Lagrangians

$$\mathcal{F}\mathcal{F} = f_{TC}^2 \Lambda_{TC} \mathcal{U} \qquad \mathcal{U} = \exp \frac{2i\Pi}{f_{TC}}$$



- Gauge bosons quantum corrections
- Top corrections
- Explicit breaking of global symmetry

$$-M_h^2 \sim c_m \left(\sum m_{\mathcal{F}_i}\right) \Lambda_{TC} + \left(c_g \frac{3(3g_2^2 + g_Y^2)}{64\pi^2} - c_y 3 \frac{y_t^2}{16\pi^2}\right) \Lambda_{TC}^2$$

$$\lambda_H \sim \frac{c_y y_Q^2 y_U^2}{4(4\pi)^2} - \frac{c_g g_{TC}^2 (3g_2^2 + g_Y^2)}{16(4\pi)^2} \sim \frac{y_t^2}{N}$$

Flavour

$$y_{ff'} \approx y_f \cdot \ell\left(\frac{f_{\mathrm{TC}}\Lambda_{\mathrm{TC}}}{m_{\mathcal{S}}^2}\right) \cdot y_{f'}^T \qquad \ell = \text{TC-loop function}$$

SM spurion structure

Coupling	Flavor symmetry of SM fermions					Flavor of TC-scalars	
	$U(3)_L$	$U(3)_E$	$U(3)_Q$	$U(3)_U$	$U(3)_D$	$U(3)_{\mathcal{S}_N}$	$U(3)_{\mathcal{S}_{U^c}}$
y_L	3	1	1	1	1	3	1
y_E	1	3	1	1	1	$\bar{3}$	1
y_Q	1	1	3	1	1	1	3
y_U	1	1	1	3	1	1	$\bar{3}$
y_D	1	1	1	1	3	1	$\bar{3}$
$m^2_{\mathcal{S}_N}$	1	1	1	1	1	$3 \otimes \bar{3}$	1
$m^2_{\mathcal{S}_{U^c}}$	1	1	1	1	1	1	$3 \otimes \bar{3}$

3 mix matrices in y_f + 2 in m^2_S

- Very rich structure
- Hundreds of composite states to discover
- Effective theories used so far are ineffective

Underlying theories

Fields	Gauge	Global symmetry of fermions			Global, scalars	
	$\mathrm{SU}(N)_{\mathrm{TC}}$	$\mathrm{SU}(N_F)_L$	$\mathrm{SU}(N_F)_R$	$\mathrm{U}(1)_V$	$\mathrm{SU}(N_S)$	$\mathrm{U}(1)_S$
$\mathcal{F}$	N	N_F	1	$+1$	1	0
$\mathcal{F}^c$	$\bar{N}$	1	$\bar{N}_F$	-1	1	0
$\mathcal{S}$	N	1	1	0	N_S	1

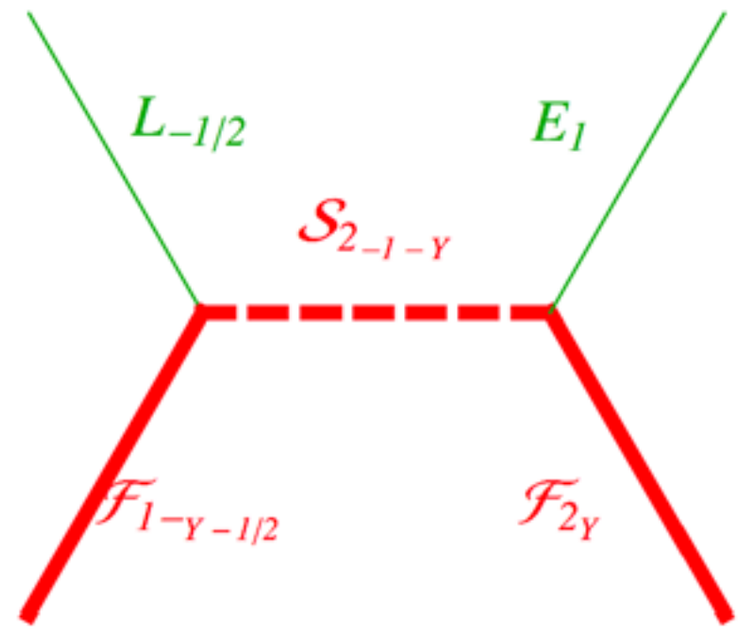
	$\mathrm{SO}(N)_{\mathrm{TC}}$	$\mathrm{SU}(N_F)$	$\mathrm{O}(N_S)$
$\mathcal{F}$	N	N_F	1
$\mathcal{S}$	N	1	N_S

	$\mathrm{Sp}(N)_{\mathrm{TC}}$	$\mathrm{SU}(N_F)$	$\mathrm{Sp}(2N_S)$
$\mathcal{F}$	N	N_F	1
$\mathcal{S}$	N	1	$2N_S$

Minimal (Partial) Compositeness

Minimal = 4D minimal matter and simplest gauge-groups

- SP(2) TC Gauge Group
- 2 TC Dirac Flavours
- 12 TC complex scalars

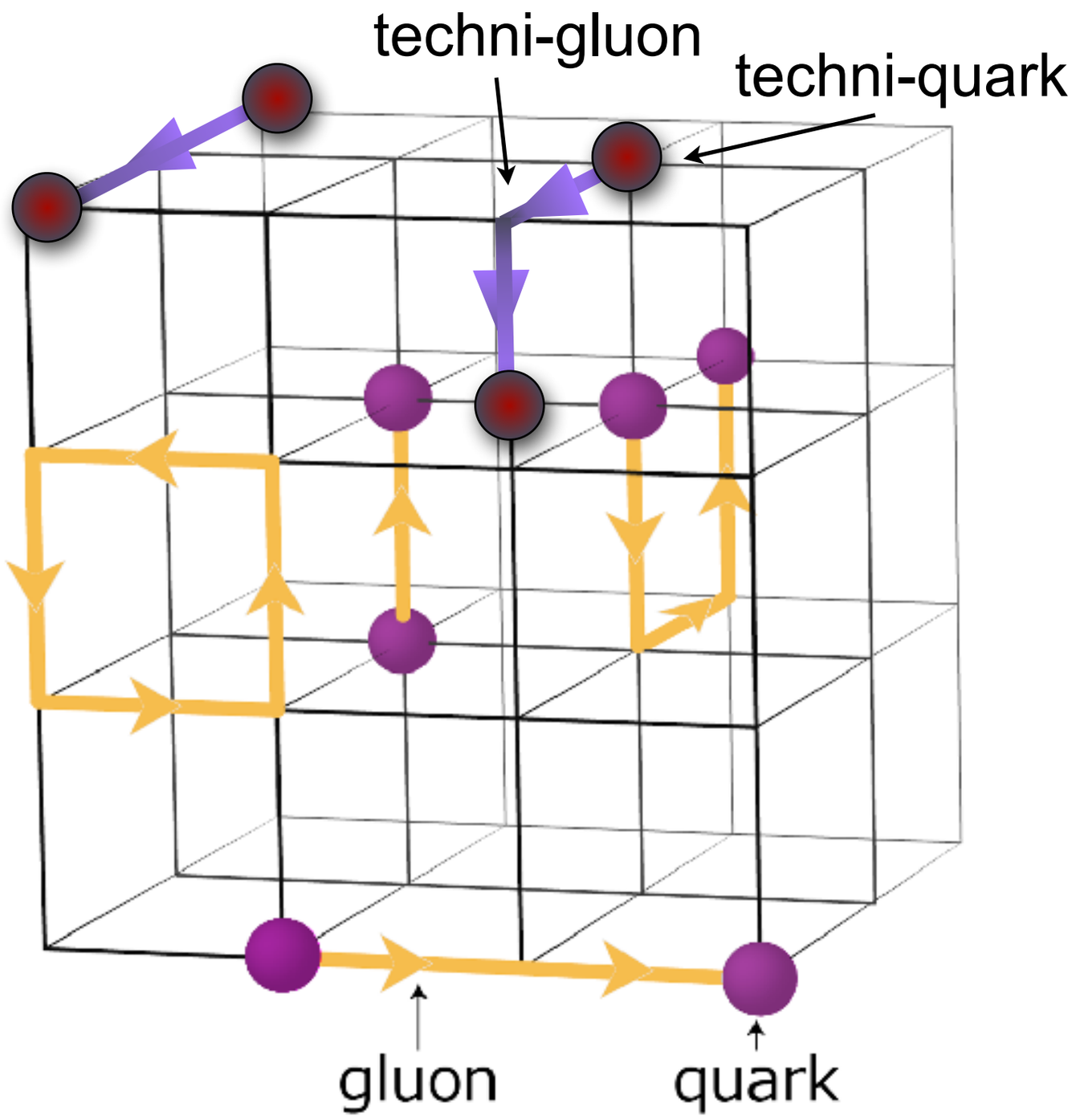


$$(\mathcal{F}_L \oplus \mathcal{F}_{E^c} \oplus \mathcal{F}_N) \oplus 3 \times (\mathcal{S}_{L^c} \oplus \mathcal{S}_{D^c})$$

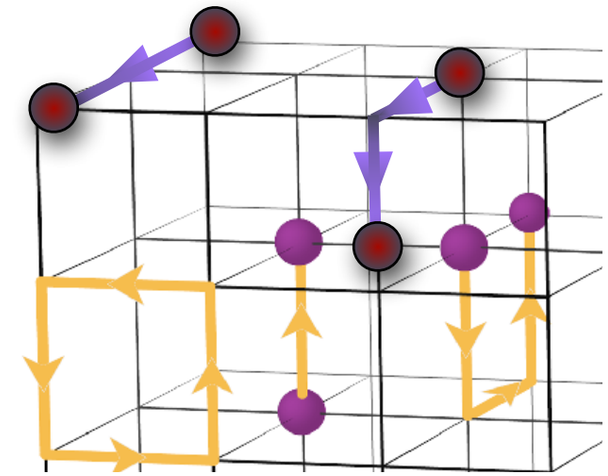
$$\begin{aligned} \mathcal{L}_Y &= y_L\, L\mathcal{F}_N^c \mathcal{S}_{L^c} + y_E\, E\mathcal{F}_L \mathcal{S}_{L^c}^* + \\ &+ (y_U\, U\mathcal{F}_{E^c}^c + y_D\, D\mathcal{F}_N^c)\mathcal{S}_{D^c} + y_Q\, Q\mathcal{F}_L \mathcal{S}_{D^c}^* + \text{h.c.} \end{aligned}$$

Complete effective theory at the EW Cacciapaglia, Gertov, F.S. Thomsen 1704.07845

Composite Dynamics on the Lattice



SU(**2**) + 2 Dirac Flavors



- SU(4) global symmetry breaks to Sp(4)
- 5 Goldstone bosons
- Goldstone form factors

Lewis, Pica, Sannino, 1109.3513
Hietanen, Lewis, Pica, Sannino, 1308.4130
Hietanen, Lewis, Pica, Sannino, 1404.2794

Lattice predictions for TC & CH Spin-One states

$$m_V = \frac{3.2(5)}{\sin\theta}\ \mathrm{TeV} \qquad m_A = \frac{3.6(9)}{\sin\theta}\ \mathrm{TeV}$$

Arthur, Drach, Hansen, Hietanen, Pica, Sannino, 1602.06559

In progress:

- Scalar spectrum
- Scattering amplitudes

What next?

- Fund partial compositeness flavour framework [with Stangl, Straub,Thomsen]
- New (minimal) composite states (276 SS bound states)
- Add TC-scalars on the lattice and determine the spectrum
- Baryogenesis
- Electroweak phase transition
- Dark matter
- Neutrino masses
- Safe composite dynamics

We pursue fundamental theories of Nature

Backup slides

Fundamental lagrangian

The fundamental Lagrangian for the underlying dynamics:

$$\mathcal{L} = \mathcal{L}_{\mathrm{SM-h}} + \mathcal{L}_{\mathrm{TC}} + \mathcal{L}_y.$$

The TC sector features new strong dynamics, from a gauge group $G_{\mathrm{TC}} = \mathrm{Sp}(N)_{\mathrm{TC}}$;

$$\begin{aligned}\mathcal{L}_{\mathrm{TC}} = &- \tfrac{1}{4}\mathcal{G}_{\mu\nu}\mathcal{G}^{\mu\nu} + i\overline{\mathcal{F}}\bar{\sigma}^{\mu}D_{\mu}\mathcal{F} - \left(\tfrac{1}{2}\mathcal{F}m_{\mathcal{F}}\epsilon_{\mathrm{TC}}\mathcal{F} \;+\; \mathrm{h.c.}\right) \\ &+ \left(D_{\mu}\mathcal{S}\right)^{\dagger}\left(D^{\mu}\mathcal{S}\right) - \mathcal{S}^{\dagger}m_{\mathcal{S}}^{2}\mathcal{S} - V(\mathcal{S}),\end{aligned}$$

In the minimal FPC model, the symmetries of the strong dynamics ar

$$\mathrm{SU}(4)_{\mathcal{F}} \quad \text{and} \quad \mathrm{Sp}(24)_{\mathcal{S}}$$

with breaking parameters $m_{\mathcal{F}}$ and $m_{\mathcal{S}}^2$. [1704.07845]

Fundamental Yukawas

A partial compositeness-like mass generation is achieved by identifying composite Baryons by $\mathcal{B} \sim \mathcal{F}\mathcal{S}$.
The coupling to SM fermions in the underlying Lagrangian is done by fundamental Yukawa couplings

$$\mathcal{L}_y = \sum_{f \in \{Q,u,d,L,e,\nu\}} y_f f \mathcal{F}\mathcal{S}^{(\dagger)} = \psi^i{}_a \mathcal{F}^a \epsilon_{ij} \mathcal{S}^j .$$

Giving masses to all generations of SM fermions requires 3 generations of $\mathcal{S}$.

Techniparticles	$\mathcal{F}_Q$	$\mathcal{F}_u$	$\mathcal{F}_d$	$3 \times S_q$	$3 \times \mathcal{S}_\ell$
TC symmetries		$N_{\mathrm{TC}} \times 4_{\mathcal{F}}$		$N_{\mathrm{TC}} \times 24_{\mathcal{S}}$	
G_{SM}	$(1,2,0)$	$\left(1,1,-\frac{1}{2}\right)$	$\left(1,1,\frac{1}{2}\right)$	$\left(\overline{3},1,-\frac{1}{6}\right)$	$\left(1,1,\frac{1}{2}\right)$

Symmetry breaking

The EW symmetry breaking is realized through the $\mathrm{SU}(4)_{\mathcal{F}}/\mathrm{Sp}(4)$ breaking pattern.

$$\langle \mathcal{F}\mathcal{F} \rangle \sim f_{\mathrm{TC}}^2 \Lambda_{\mathrm{TC}} \Sigma_0.$$

The vacuum alignment, Σ_0, is determined by radiative effects and fundamental fermion masses.
The symmetry is realized nonlinearly in the EFT, with the NGB modes:

$$\mathcal{L}_{\mathrm{EFT}} \supset \tfrac{1}{8}\mathrm{tr}\left[D_\mu \Sigma^\dagger D^\mu \Sigma\right],$$

where

$$\Sigma = \exp\left[\frac{2\sqrt{2}i}{f_{\mathrm{TC}}}\Pi_i(x)X^i\right]\Sigma_0.$$

We identify $v_{\mathrm{EW}} = s_\theta\, f_{\mathrm{TC}}$.

Low-energy EFT

$$\mathcal{L}_{\rm EFT} = \sum_A c_A \mathcal{O}_A + \left(\sum_A c'_A \mathcal{O}'_A \; + \; \text{h.c.} \right)$$

The Building blocks are:

- Symmetries: $\mathrm{SU}(4)_{\mathcal{F}}$ and $\mathrm{Sp}(24)_{\mathcal{S}}$.
- Fields: Σ, A_μ, and $\psi^i{}_a$ (f).
- Spurions/Breaking parameters: y_f, $m_{\mathcal{F}}$, $m^2_{\mathcal{S}}$, $T^I_{\mathcal{F}}$, and $T^I_{\mathcal{S}}$.

$$\psi^i{}_a = \begin{pmatrix} 0 & 0 & y_d d & -y_u u \\ 0 & 0 & y_e e & -y_\nu \nu \\ y_Q q^{(d)} & -y_Q q^{(u)} & 0 & 0 \\ y_\ell l^{(e)} & -y_\ell l^{(\nu)} & \tilde{y}_\nu \nu & 0 \end{pmatrix}$$

Fermion masses

SM fermion masses are generated through the effective Yukawa operator:

$$\mathcal{O}_{\rm Yuk} = -\frac{f_{\rm TC}^2}{2\Lambda_{\rm TC}}\,\left(\psi^{i_1}{}_{a_1}\psi^{i_2}{}_{a_2}\right)\Sigma^{a_1 a_2}\epsilon_{i_1 i_2}\ .$$

The resulting quark masses matrices are

$$(m_u)_{ij} = C_{\rm Yuk}\frac{f_{\rm TC}}{\Lambda_{\rm TC}}\,(y_Q y_u^{\rm T})_{ij} v_{\rm EW} \quad \text{and}$$
$$(m_d)_{ij} = C_{\rm Yuk}\frac{f_{\rm TC}}{\Lambda_{\rm TC}}(y_Q y_d^{\rm T})_{ij} v_{\rm EW}\ ,$$

where $v_{\rm EW} = s_\theta\, f_{\rm TC}$.

Top-bottom masses

Specializing to the third generation quarks, the masses are

$$m_t = |C_{\rm Yuk}\ y_{Q_3} y_t|\,\frac{f_{\rm TC}}{\Lambda_{\rm TC}}\,v_{\rm EW} \quad \text{and} \quad m_b = |C_{\rm Yuk}\ y_{Q_3} y_b|\,\frac{f_{\rm TC}}{\Lambda_{\rm TC}}\,v_{\rm EW}$$

With an $\mathcal{O}(1)$-coefficient we may expect $|y_Q y_t| \sim 4\pi$.

Zbb

Modification of the couplings to the massive gauge bosons:

$$\mathcal{O}_{\Pi f} = \frac{i f_{\rm TC}^2}{2\Lambda_{\rm TC}^2} \left(\bar{\psi}^{i_1 a_1} \bar{\sigma}_\mu \psi^{i_2}{}_{a_2}\right) \left(\Sigma^\dagger \overleftrightarrow{D}^\mu \Sigma\right)_{a_1}{}^{a_2} \epsilon_{i_1 i_2} \,,$$

$$\supset -\frac{g}{2c_w} \frac{f_{\rm TC}^2}{\Lambda_{\rm TC}^2} s_\theta^2 Z_\mu \left|y_{Q_3}^2\right| \overline{b}_{\rm L} \gamma^\mu b_{\rm L}.$$

Comparing to the LEP bounds we find at 95% CL that

$$\left|C_{\Pi f} y_{Q_3}^2\right| s_\theta^2 \frac{f_{\rm TC}^2}{\Lambda_{\rm TC}^2} \leq 3.4 \cdot 10^{-3},$$

$$\Rightarrow \frac{C_{\rm Yuk}}{\sqrt{\left|C_{\Pi f}\right|}} \left|y_t\right| \gtrsim \frac{35\,\text{TeV}}{\Lambda_{\rm TC}} \sim \sqrt{4\pi}\, \frac{10\,\text{TeV}}{\Lambda_{\rm TC}}.$$

4-fermion operators, flavour at last

The EFT contains 5 self-hermitian and 3 complex four fermion operators. The self-hermitians are:

$$\mathcal{O}^1_{4f} = \frac{1}{4\Lambda^2_{\rm TC}}(\psi^{i_1}{}_{a_1}\psi^{i_2}{}_{a_2})(\bar\psi^{i_3 a_3}\bar\psi^{i_4 a_4})\Sigma^{a_1 a_2}\Sigma^\dagger_{a_3 a_4}\epsilon_{i_1 i_2}\epsilon_{i_3 i_4} \ ,$$

$$\mathcal{O}^2_{4f} = \frac{1}{4\Lambda^2_{\rm TC}}(\psi^{i_1}{}_{a_1}\psi^{i_2}{}_{a_2})(\bar\psi^{i_3 a_3}\bar\psi^{i_4 a_4})\left(\delta^{a_1}{}_{a_3}\delta^{a_2}{}_{a_4} - \delta^{a_1}{}_{a_4}\delta^{a_2}{}_{a_3}\right)\epsilon_{i_1 i_2}\epsilon_{i_3 i_4} \ ,$$

$$\mathcal{O}^3_{4f} = \frac{1}{4\Lambda^2_{\rm TC}}(\psi^{i_1}{}_{a_1}\psi^{i_2}{}_{a_2})(\bar\psi^{i_3 a_3}\bar\psi^{i_4 a_4})\Sigma^{a_1 a_2}\Sigma^\dagger_{a_3 a_4}\left(\epsilon_{i_1 i_4}\epsilon_{i_2 i_3} - \epsilon_{i_1 i_3}\epsilon_{i_2 i_4}\right) \ ,$$

$$\mathcal{O}^4_{4f} = \frac{1}{4\Lambda^2_{\rm TC}}(\psi^{i_1}{}_{a_1}\psi^{i_2}{}_{a_2})(\bar\psi^{i_3 a_3}\bar\psi^{i_4 a_4})\left(\delta^{a_1}{}_{a_3}\delta^{a_2}{}_{a_4}\epsilon_{i_1 i_3}\epsilon_{i_2 i_4} + \delta^{a_1}{}_{a_4}\delta^{a_2}{}_{a_3}\epsilon_{i_1 i_4}\epsilon_{i_2 i_3}\right) \ ,$$

$$\mathcal{O}^5_{4f} = \frac{1}{4\Lambda^2_{\rm TC}}(\psi^{i_1}{}_{a_1}\psi^{i_2}{}_{a_2})(\bar\psi^{i_3 a_3}\bar\psi^{i_4 a_4})\left(\delta^{a_1}{}_{a_3}\delta^{a_2}{}_{a_4}\epsilon_{i_1 i_4}\epsilon_{i_2 i_3} + \delta^{a_1}{}_{a_4}\delta^{a_2}{}_{a_3}\epsilon_{i_1 i_3}\epsilon_{i_2 i_4}\right) \ .$$