Novel Application of Non-parametric Density Estimation in MICE

Tanaz Angelina Mohayai

CM49, RAL

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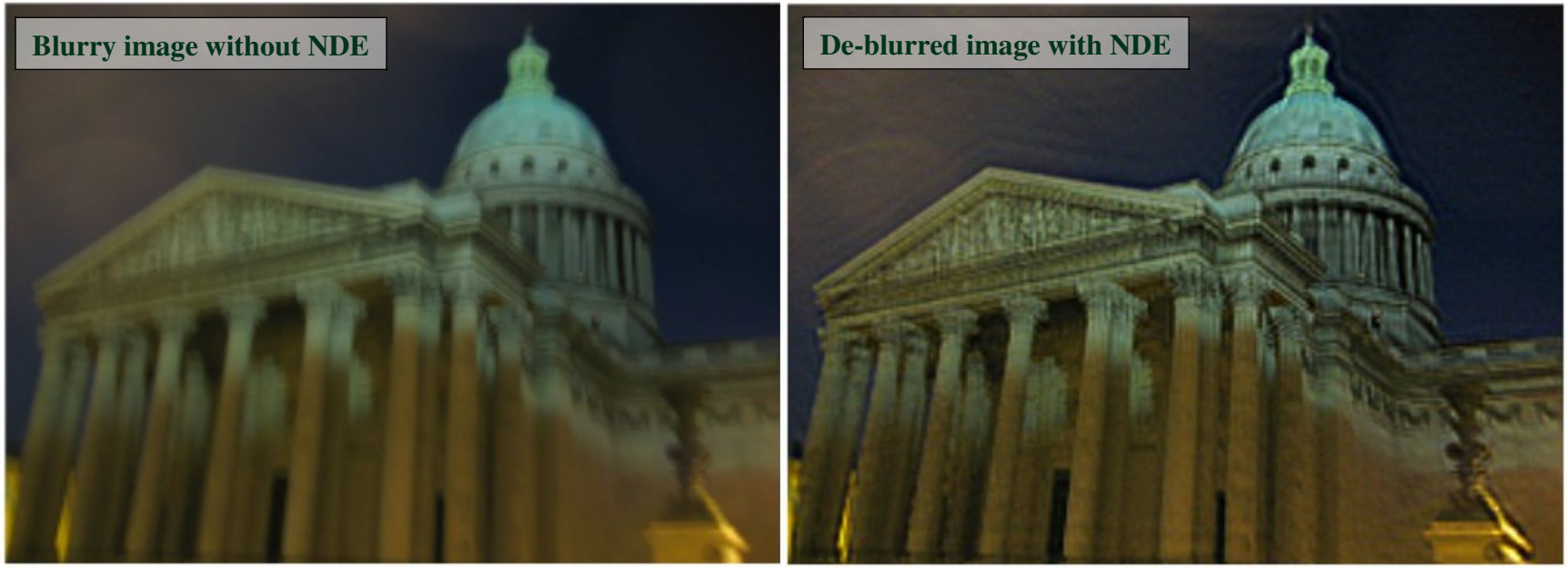
ILLINOIS INSTITUTE OF TECHNOLOGY

MICE

Non-Parametric Density Estimation (NDE) I

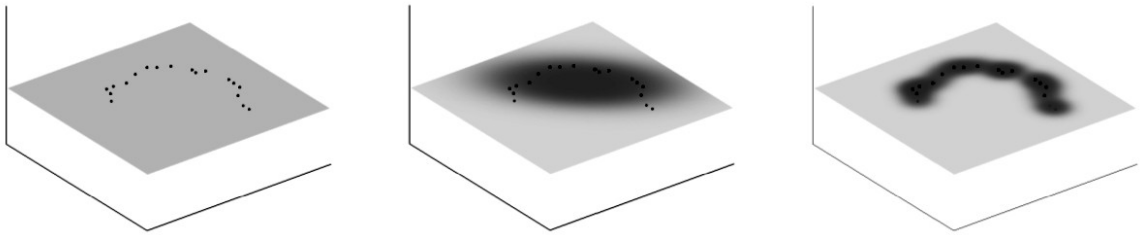
- Estimates the probability density function or density

Image Processing



D. Krishnan et al., "Blind Deconvolution Using a Normalized Sparsity Measure", DOI: 10.1109/CVPR.2011.5995521

Actual distribution RMS measurement NDE



M. Rousson et al., "Efficient Kernel Density Estimation of Shape and Intensity Priors for Level Set Segmentation", DOI:10.1007/978-0-387-68343-0_13

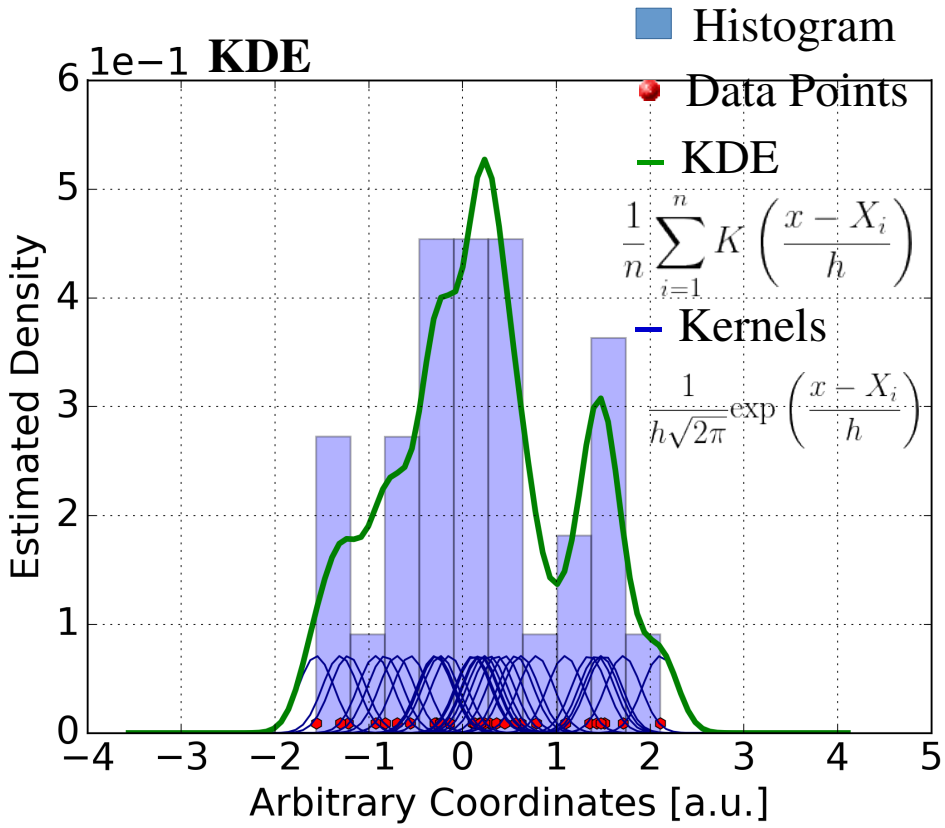
- "Data points speak for themselves"
- Powerful tool when density function is not known
- Estimates the density at each point
- A single particle measurement for a single particle experiment!

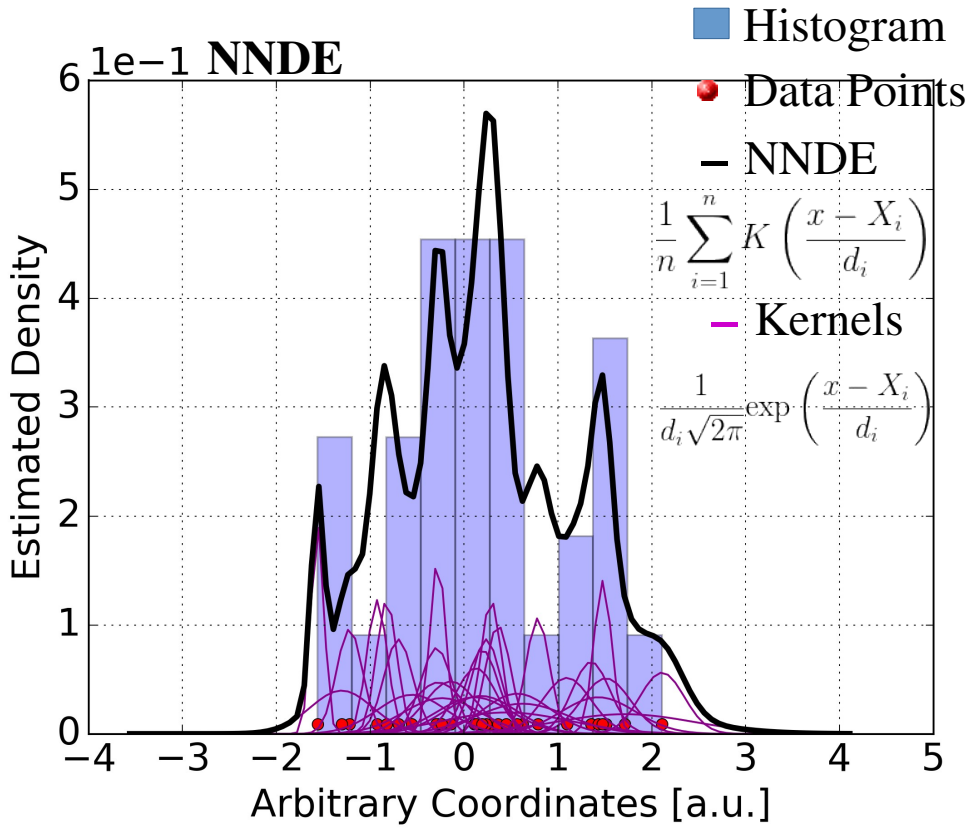
Power of NDE

Non-parametric Density Estimation II

- Oldest example: histogram
  - ★ Bins of certain widths
- Other examples: kernel density estimation (KDE), kth nearest neighbor (kernel-based NNDE):
  - ★ Kernels (smooth weight functions) of certain widths

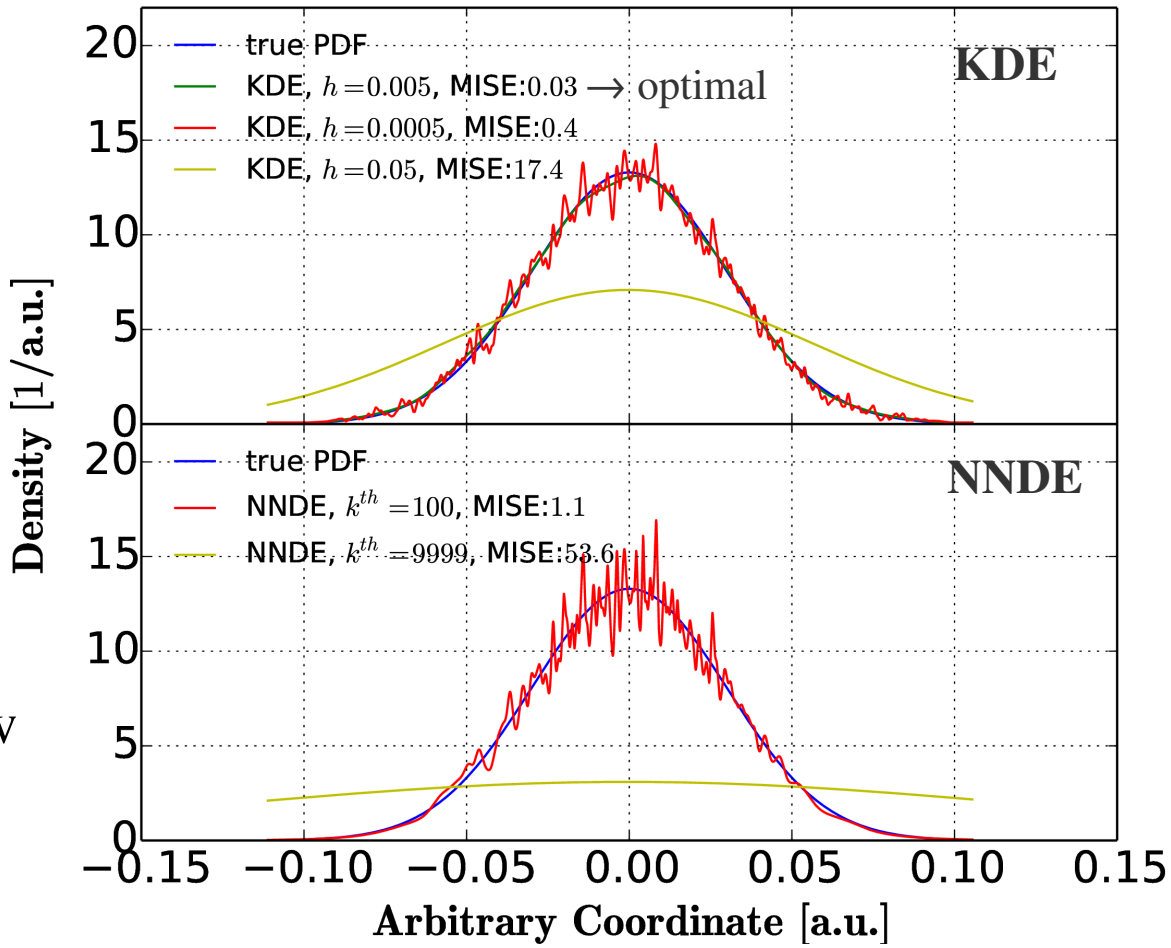
h and d_i (distances between points): widths of KDE and NNDE kernels, n: sample size





KDE vs NNDE

- Mean integrated squared error (MISE), common measure of error
- Optimal kernel width (bandwidth) minimizes MISE:
  - ★ In KDE, it minimizes MISE (green curve), is less noisy than NNDE's optimal bandwidth (red curve)
- Improvements to kernel-based NNDE are underway, but for now applied KDE to MICE with the optimal bandwidth.

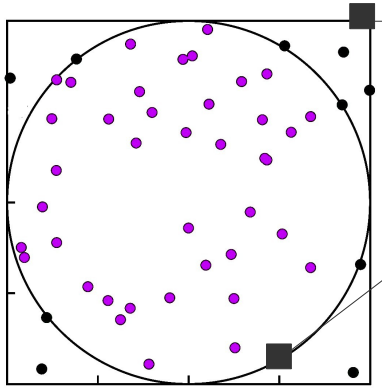


KDE Density and Volume Toy Measurements

MICE lattice with all coils operational

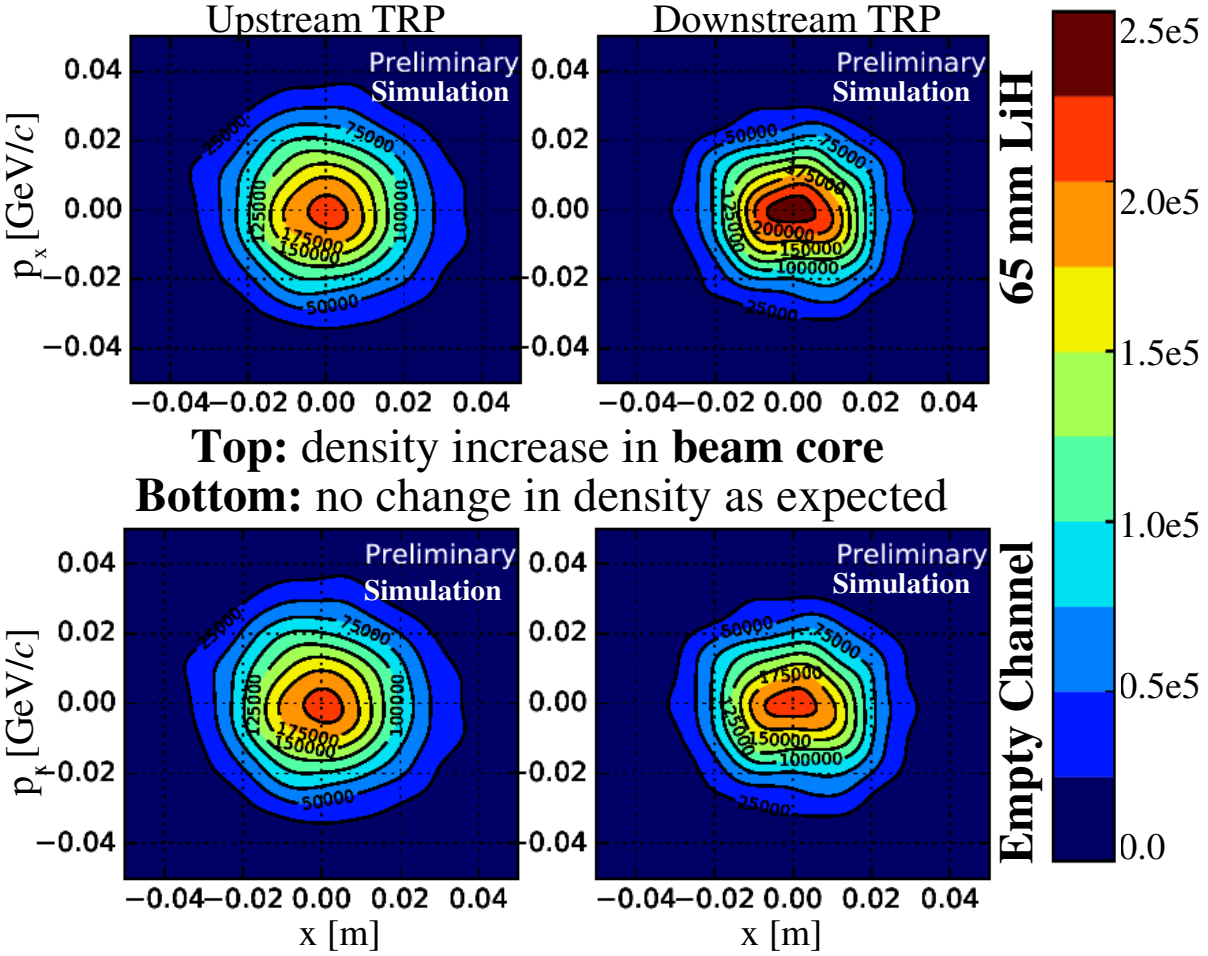
- Phase-space **density**:
  - Sum over 4D Gaussian kernels
  - Form contours in phase-space
  - Density increase at the core or the distribution 1σ (the 9th percentile contour in 4D)

2D volume measurement example

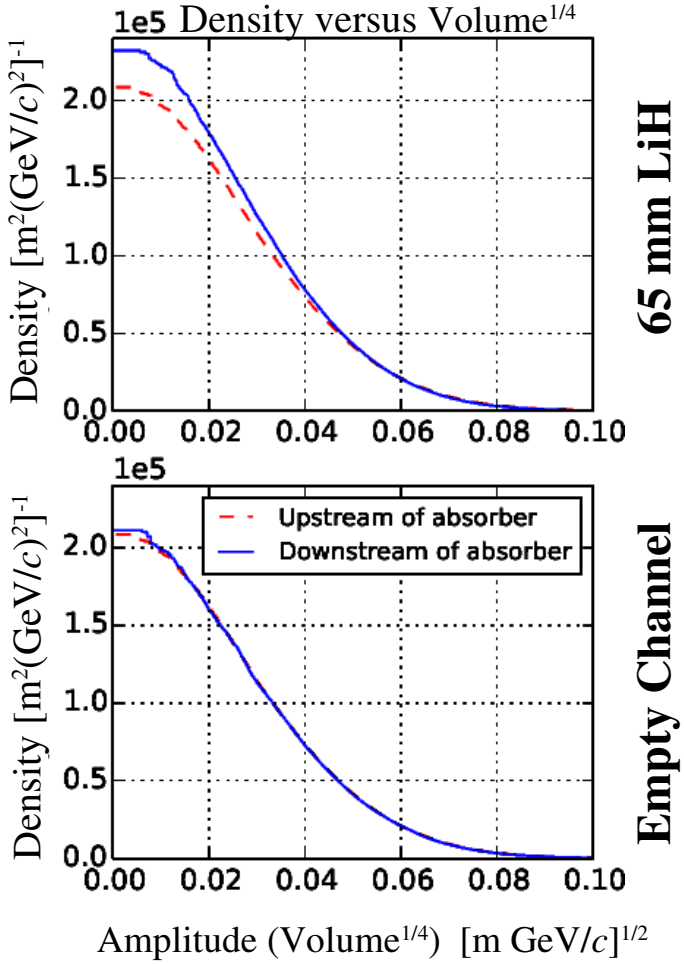


Bounding box

- Phase-space **volume**:
  - Generate MC (monte carlo) points inside a box bounding the 9th percentile contour
  - Multiply the box volume by MC points inside the box/total MC points

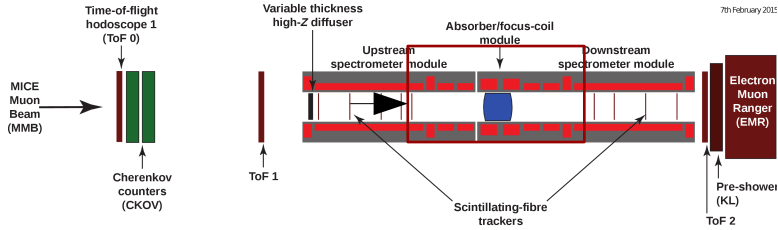


Top: density increase in **beam core**
Bottom: no change in density as expected

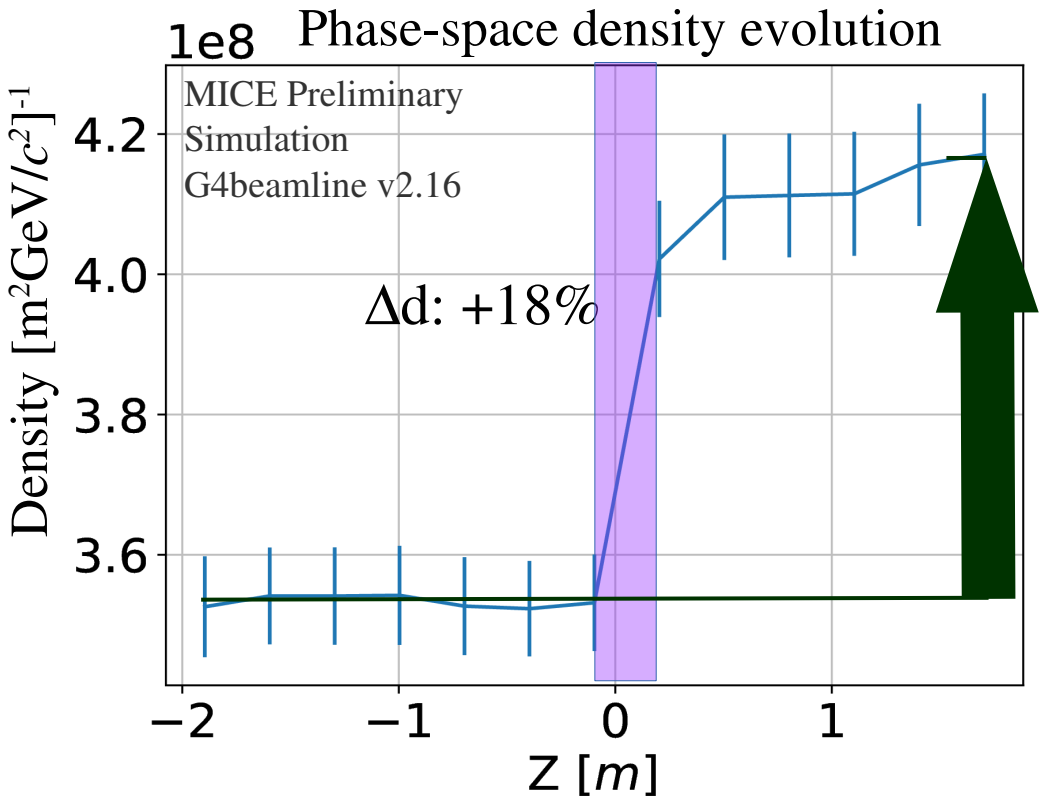


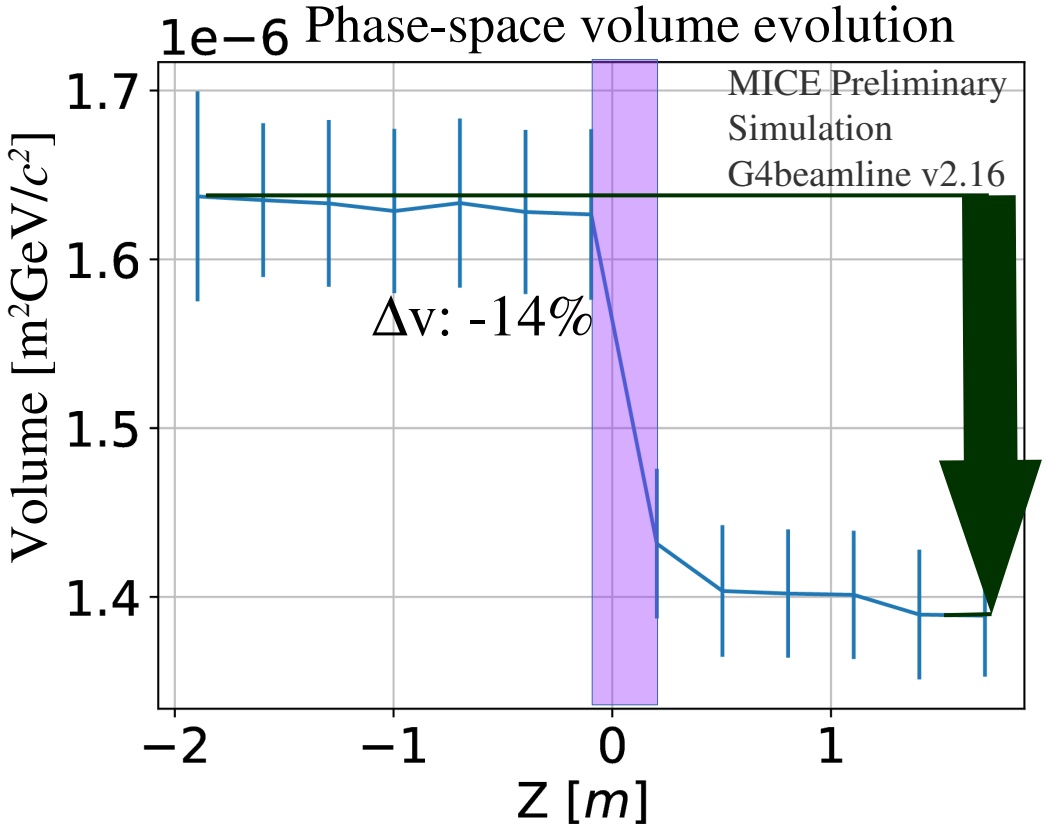
G4beamline Simulation – (6,140)

- Transmission: 84% (no transmission selection)
- Tracking 9th percentile contour from upstream to downstream tracker reference planes (US, DS TRPs), across LiH absorber
- Error bars: uncertainty of repeated (10 times) density and volume measurements with **different randomly generated input beams**



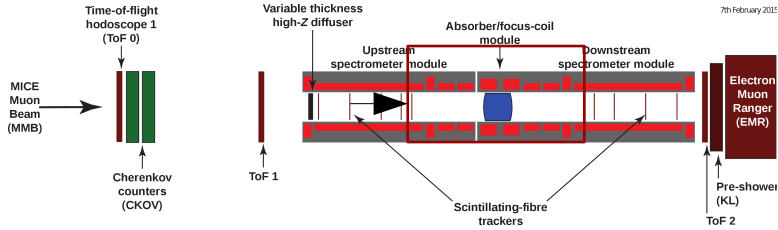
	US TRP	DS TRP
Sample Size	9,990	8,405



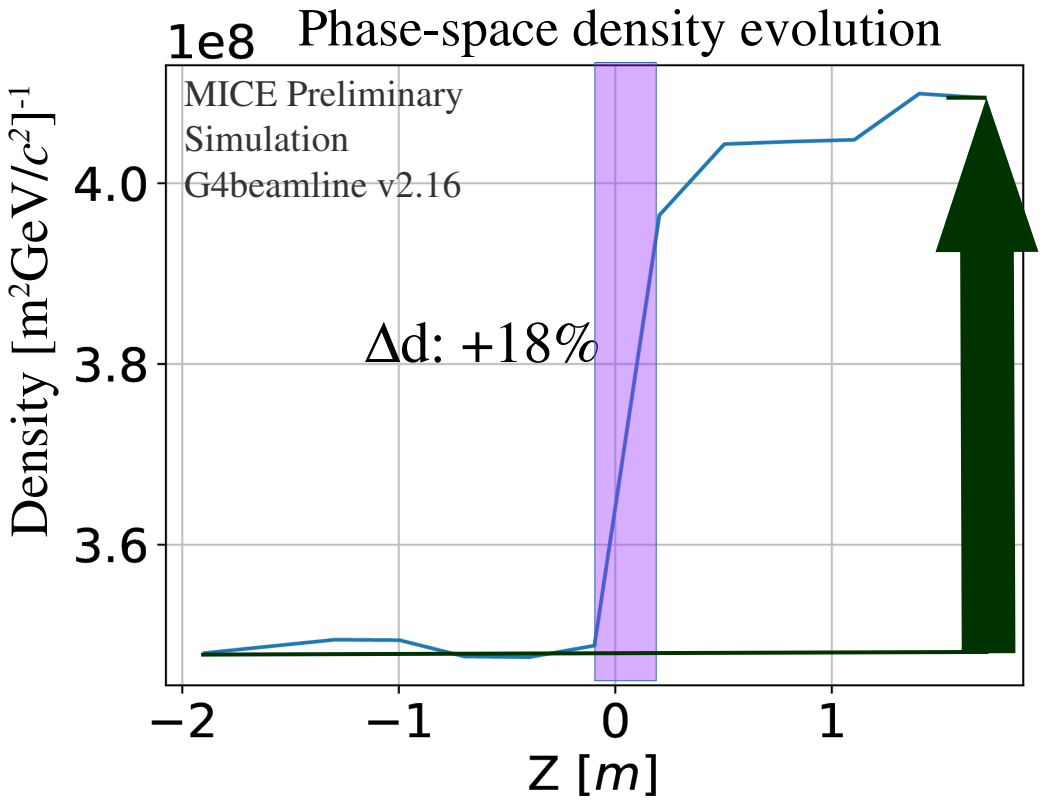


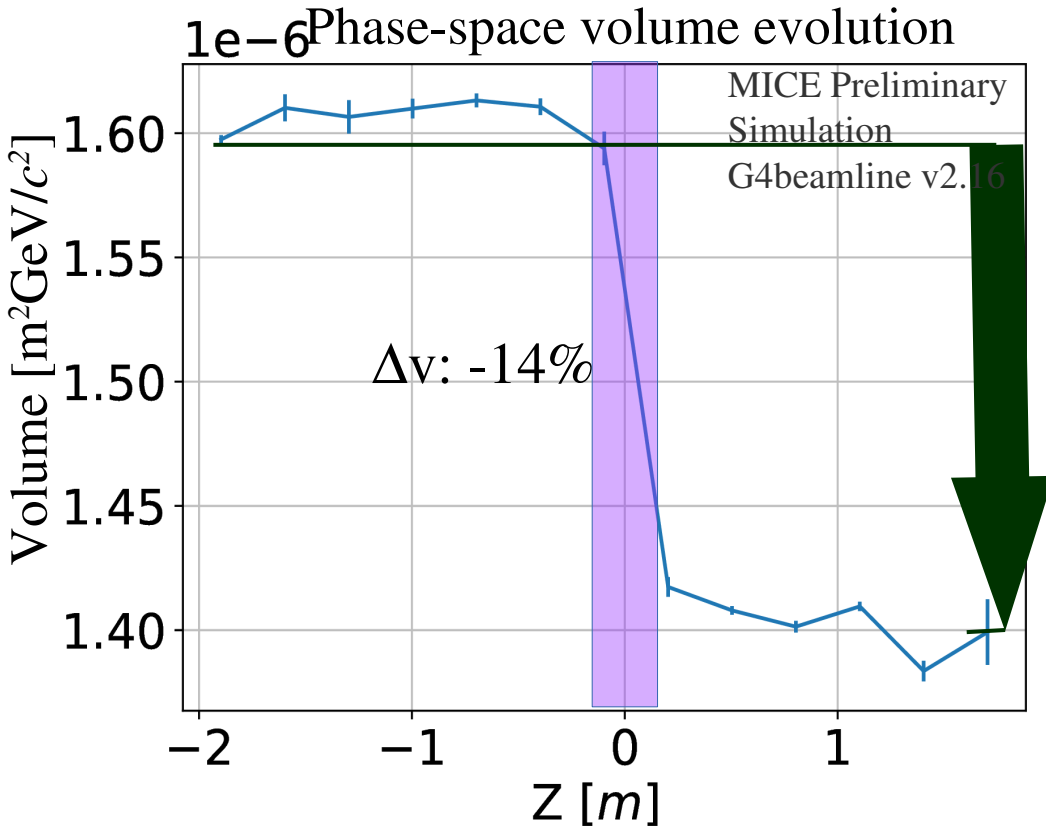
G4beamline Simulation – (6,140) cont.

- Transmission: 84% (no transmission selection)
- Tracking 9th percentile contour from US TRP to DS TRP, across LiH absorber
- Error bars: uncertainty of repeated (10 times) density and volume measurements with **same input beam**
- Note density error bars are too small to show



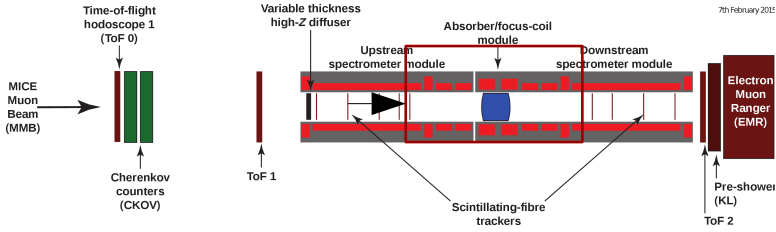
	US TRP	DS TRP
Sample Size	9,990	8,405



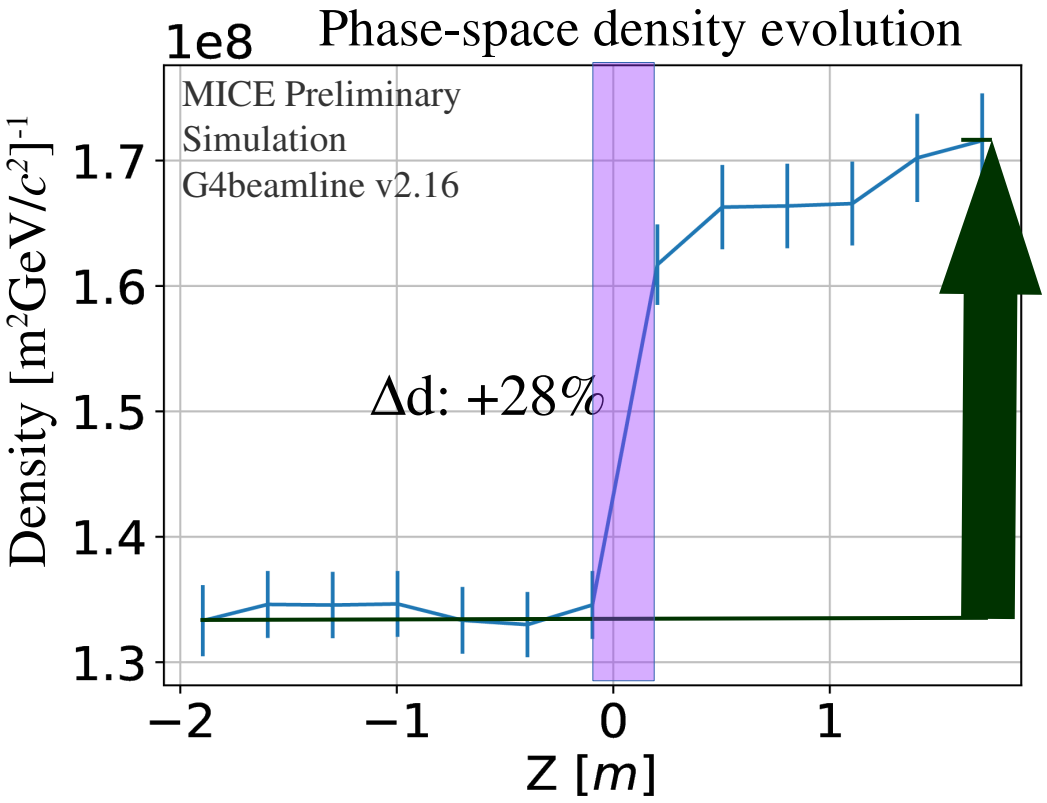


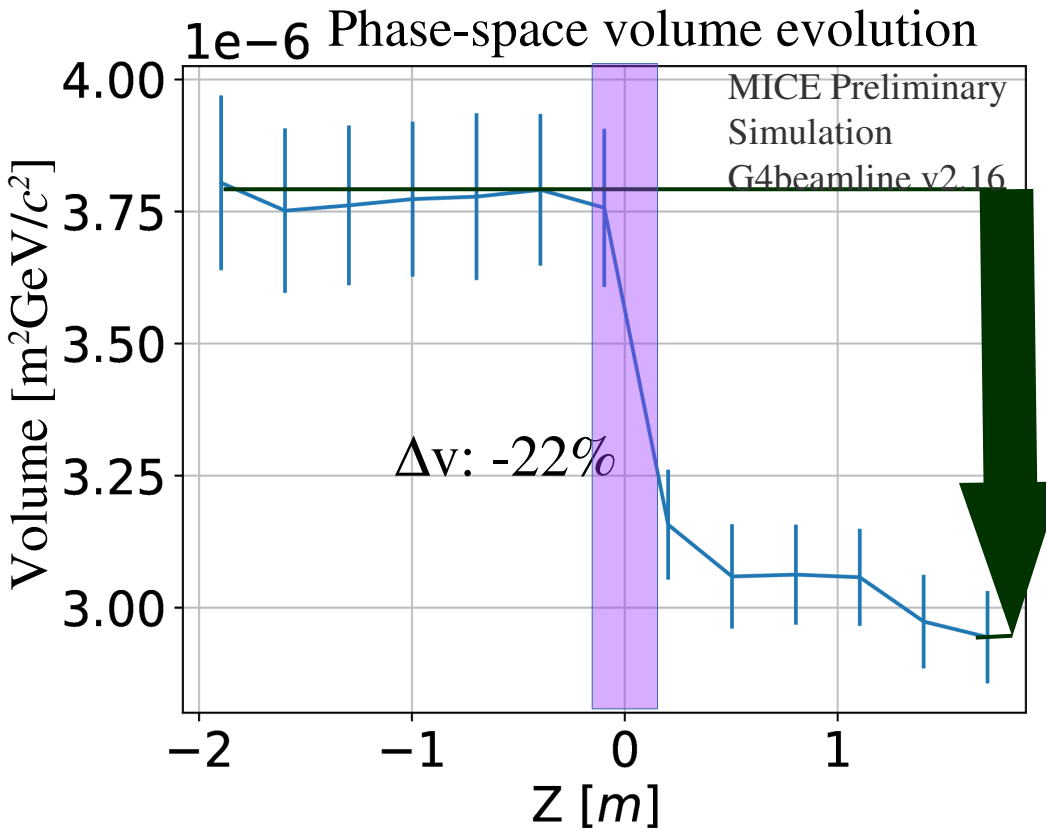
G4beamline Simulation – (10,140)

- Transmission: 62% (no transmission selection)
- Tracking 9th percentile contour from US TRP to DS TRPs, across LiH absorber
- Error bars: uncertainty of repeated (10 times) density and volume measurements with **different randomly generated input beams**



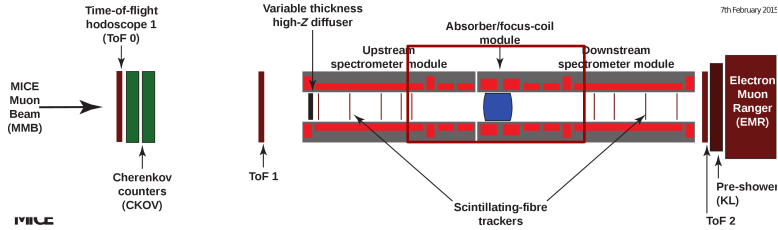
	US TRP	DS TRP
Sample Size	9,786	6,145



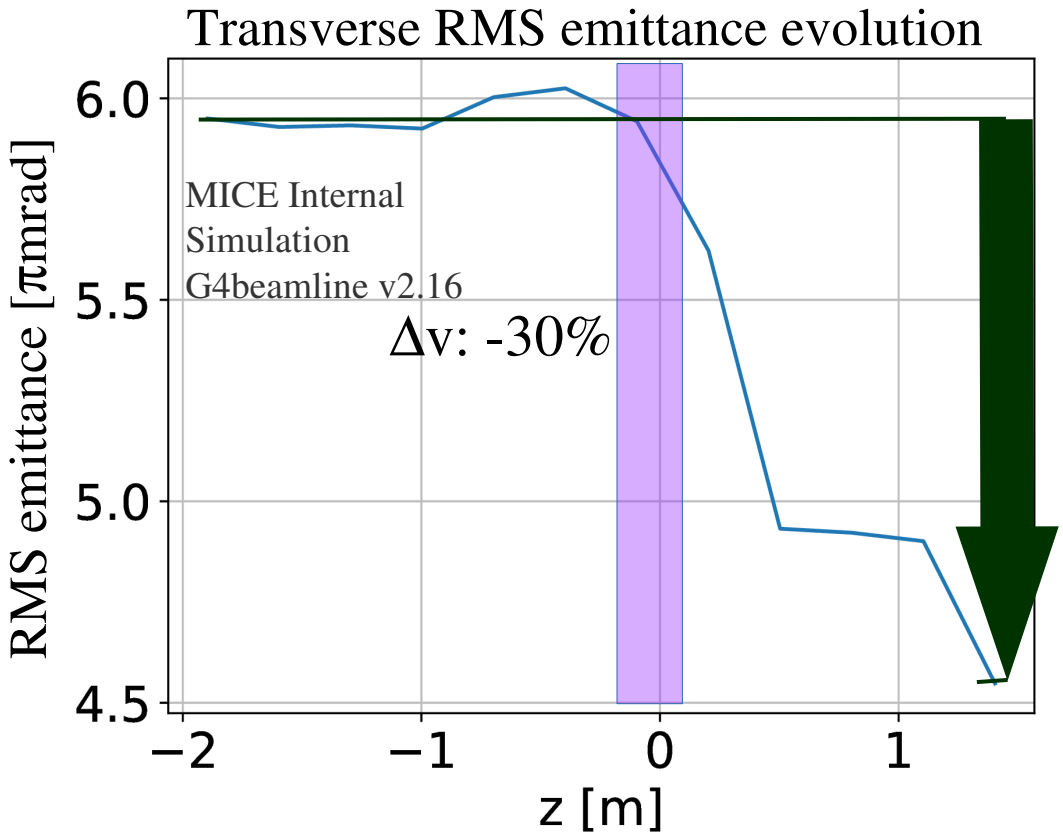


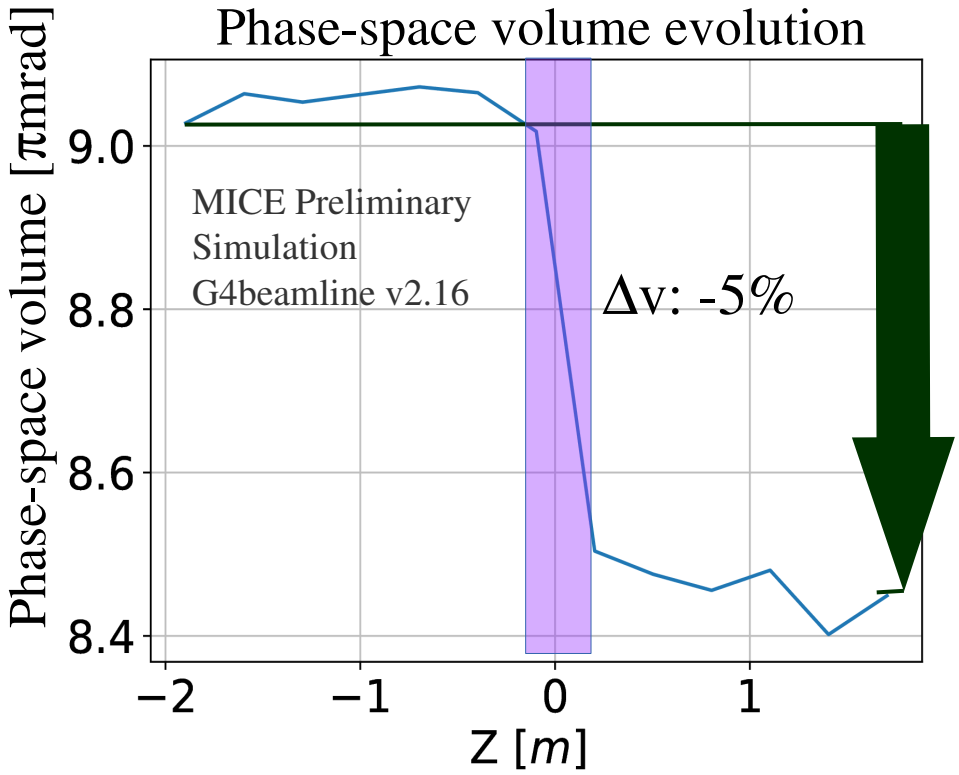
G4beamline Simulation – (6,140) Volume vs Emittance

- Transmission: 84% (no transmission selection)
- RMS emittance:
  - ★ Demonstrates large cooling (cooling + scraping); not expected for a single cooling channel like MICE (across LiH absorber)
  - ★ Greatly affected by scraping and optical heating in the beam



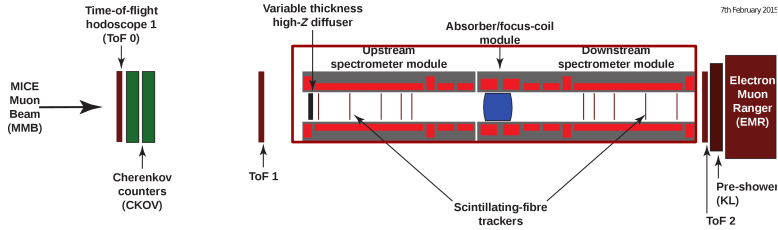
	US TRP	DS TRP
Sample Size	9,990	8,405





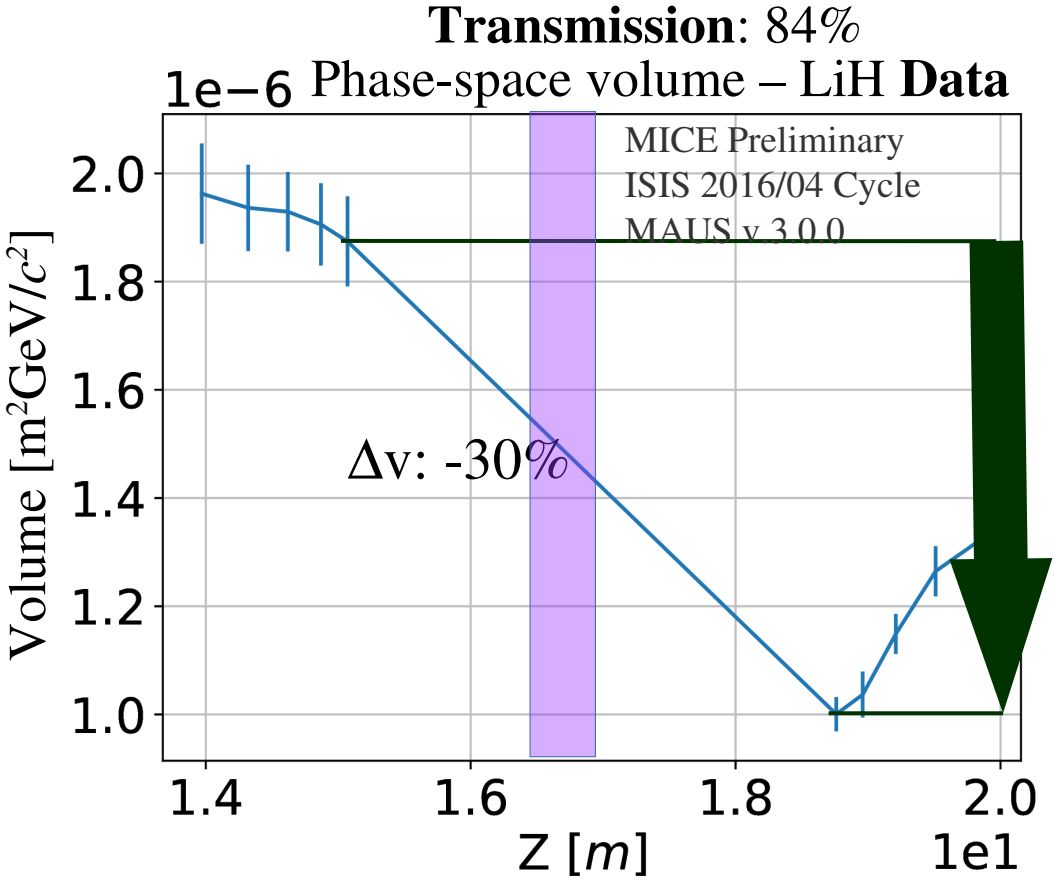
Data – (6,140)

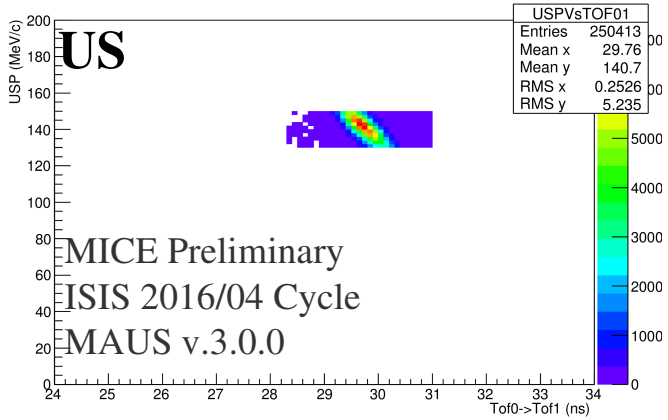
- **KDE reran on beam core,** then core tracked from US TRP to DS TRPs. Error bars are uncertainties of subsample density and volume measurements (no transmission selection).

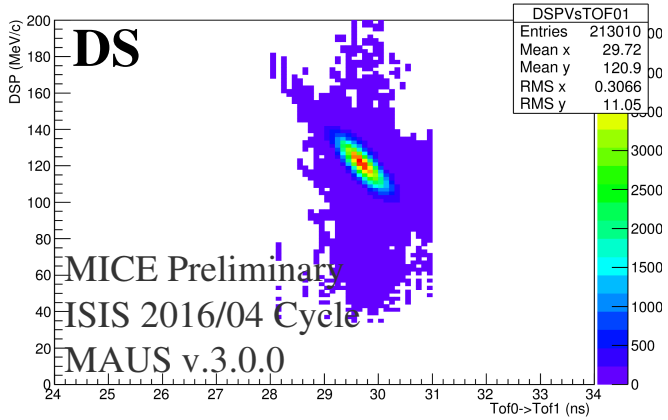


List of data quality cuts:

Data quality cuts	Tof01	TKU P	# TKU tracks	TKU p-value
	28-31	130-150 (MeV/c)	1	> 0/02



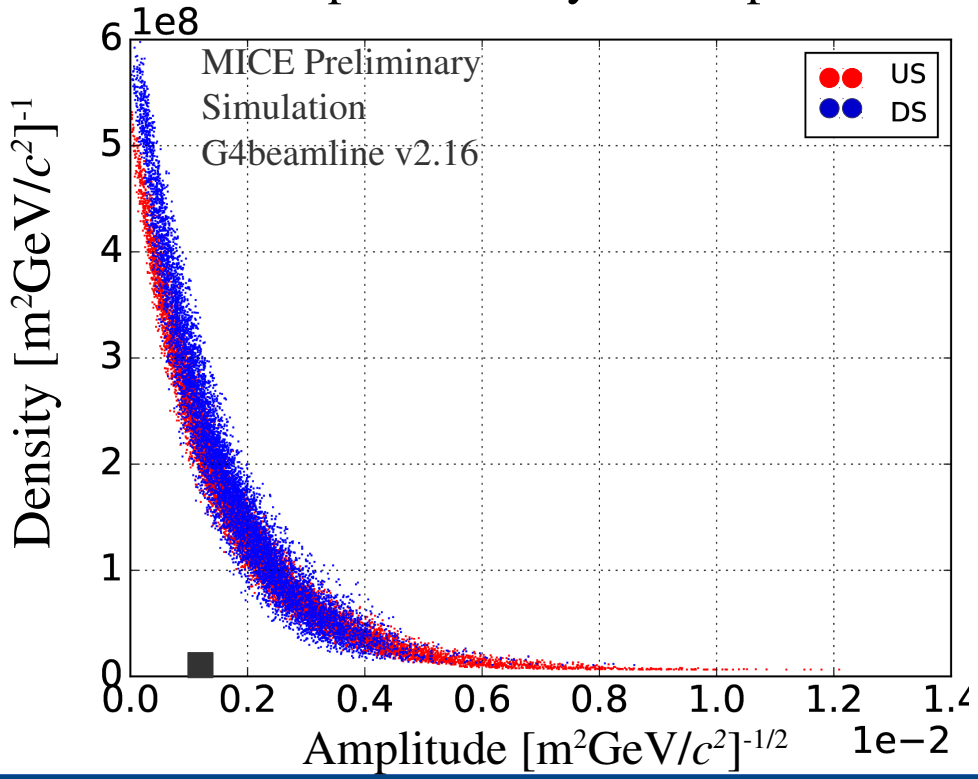




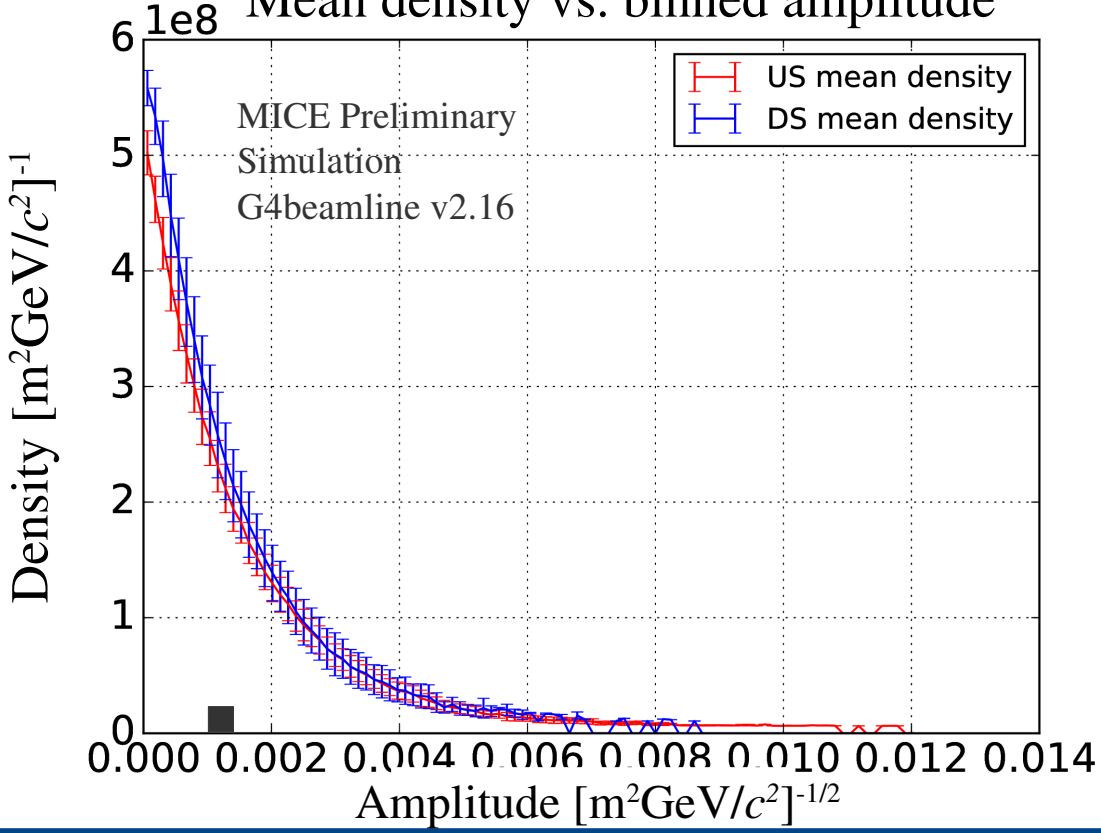
Density vs Amplitude – G4beamline Simulation – (6-140)

- Transmission: 84% (no transmission selection)
- Amplitude: $A_\perp = \varepsilon_\perp \vec{u}^T V^{-1} \vec{u}$
- Density: phase-space density from KDE (slide 2)
- Density vs. binned amplitude plot:
  - ★ Divide amplitude space into equal bins, take the mean of the density values within each bin, error bars: standard deviation of densities within each bin

Phase-space density vs. amplitude



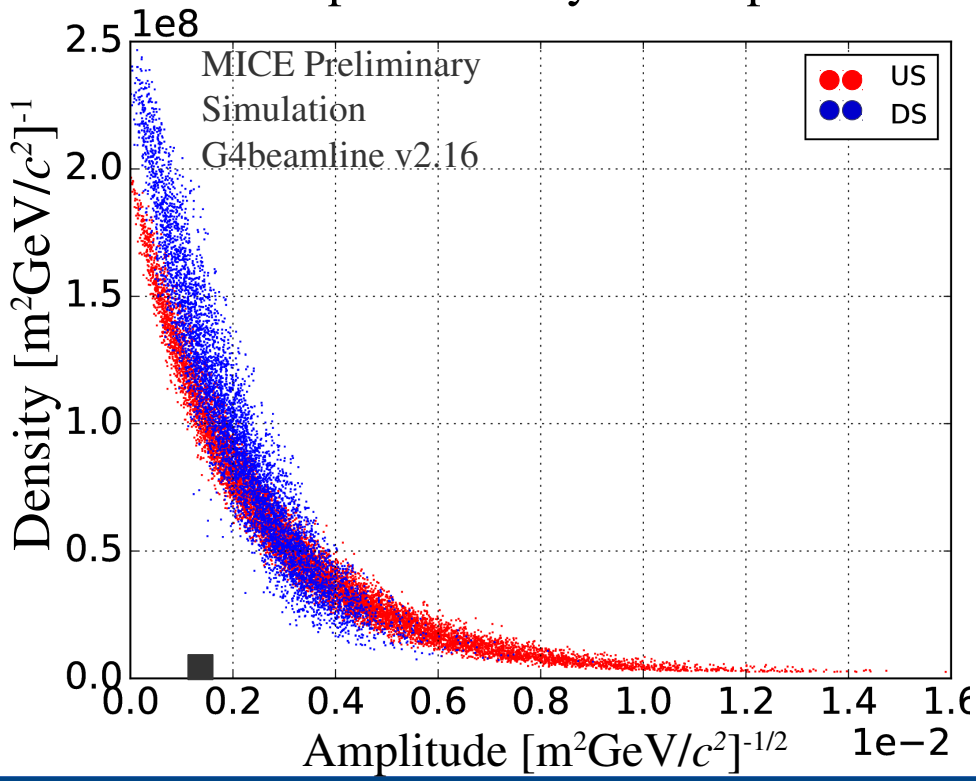
Mean density vs. binned amplitude



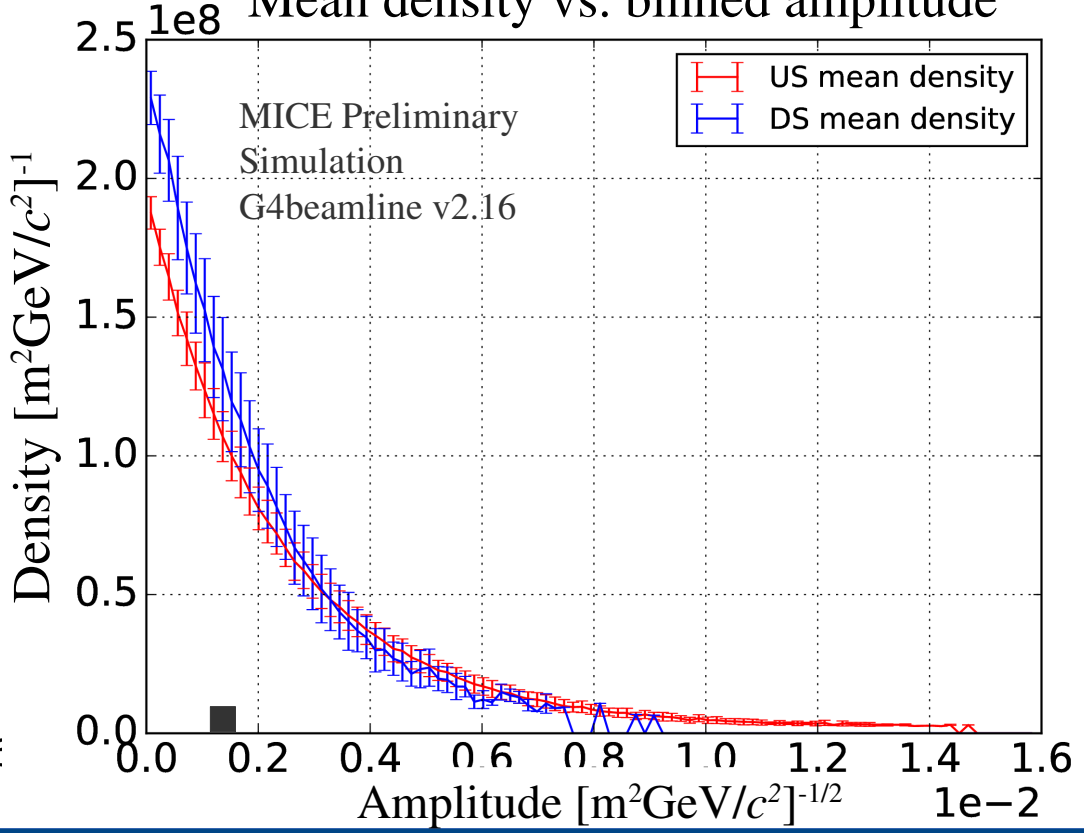
Density vs Amplitude – G4beamline Simulation – (10-140)

- Transmission: 62% (no transmission selection)
- Amplitude: $A_\perp = \varepsilon_\perp \vec{u}^T V^{-1} \vec{u}$
- Density: phase-space density from KDE (slide 2)
- Density vs. binned amplitude plot:
  - ★ Divide amplitude space into equal bins, take the mean of the density values within each bin, error bars: standard deviation of densities within each bin

Phase-space density vs. amplitude



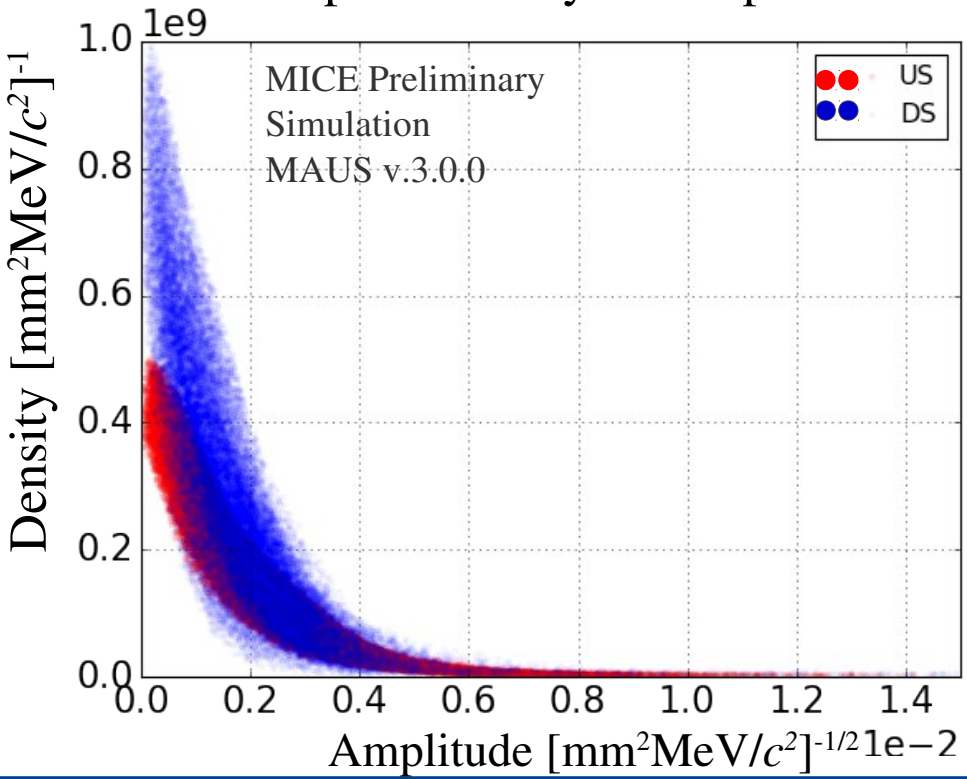
Mean density vs. binned amplitude



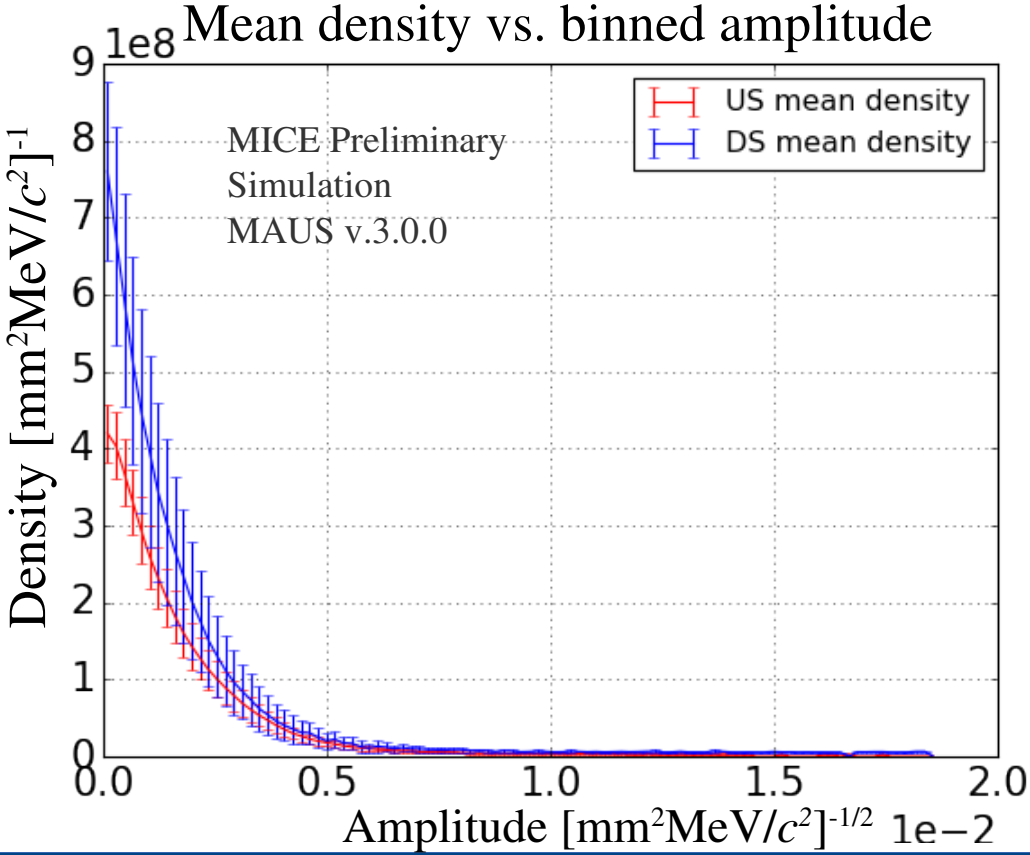
Density vs Amplitude – Data – (6-140)

- Transmission: 84% (no transmission selection)
- Amplitude: $A_\perp = \varepsilon_\perp \vec{u}^T V^{-1} \vec{u}$
- Density: phase-space density from KDE (slide 2)
- Density vs. binned amplitude plot:
  - ★ Divide amplitude space into equal bins, take the mean of the density values within each bin, error bars: standard deviation of densities within each bin

Phase-space density vs. amplitude



Mean density vs. binned amplitude



Conclusion and Future Prospects

- KDE-based density and volume evolution curves show cooling in MC and Data
- KDE-based volume measurement is more robust against beam loss than RMS emittance
- KDE density-amplitude curves give an insight into the behavior of the beam core and halo in the presence of nonlinearities and beam loss
- KDE is also useful in locating the core of the beam (what was tracked on slide 9)
- Once available, will apply KDE analysis to the official 6-140/10-140 MC production.
- The effects of systematics on the density and volume measurements are under investigation.

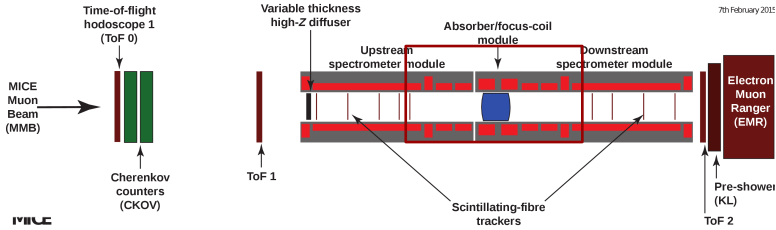
References

1. T. A. Mohayai, P. Snopok, D. Neuffer, C. Rogers, “Novel Application of of Non-Parametric Density Estimation in MICE ”, APS-DPF (2017) in progress
2. T. A. Mohayai, et al., “Novel Implementation of Non-Parametric Density Estimation in MICE”, IPAC’17, IPAC-2017-WEPAB135 (2017)
3. T. A. Mohayai, “Measurements Of Beam Cooling In Muon Ionization Cooling Experiment”, University of Mississippi invited colloquium talk (2017)
4. T. A. Mohayai, “Novel Application of Kernel Density Estimation in MICE”, MICE-Note-506 (2017)
5. T. A. Mohayai, P. Snopok, D. Neuffer, C. Rogers, “Measurements of Beam Cooling in the Muon Ionization Cooling Experiment”, APS April Meeting'17 (2017)
6. T. A. Mohayai, P. Snopok, D. Neuffer, C. Rogers, “Simulated Measurements of Beam Cooling in Muon Ionization Cooling Experiment”, Proc. NA-PAC’16, NA-PAC-2016-WEPOA36 (2016)
7. T. A. Mohayai, P. Snopok, D. Neuffer, C. Rogers, “Simulated Measurements of Cooling in Muon Ionization Cooling Experiment”, Proc. IPAC’16, IPAC-2016-TUPMY011 (2016)

Additional Slides

G4beamline Simulation – (6,140) – RMS Volume

- Transmission: 84%
- Tracking 9th percentile contour from US TRP to DS TRP
- Volume now in units of RMS emittance:
  - ★ We get the expected 5% volume reduction with a 65 mm LiH absorber and 3mm equilibrium emittance.



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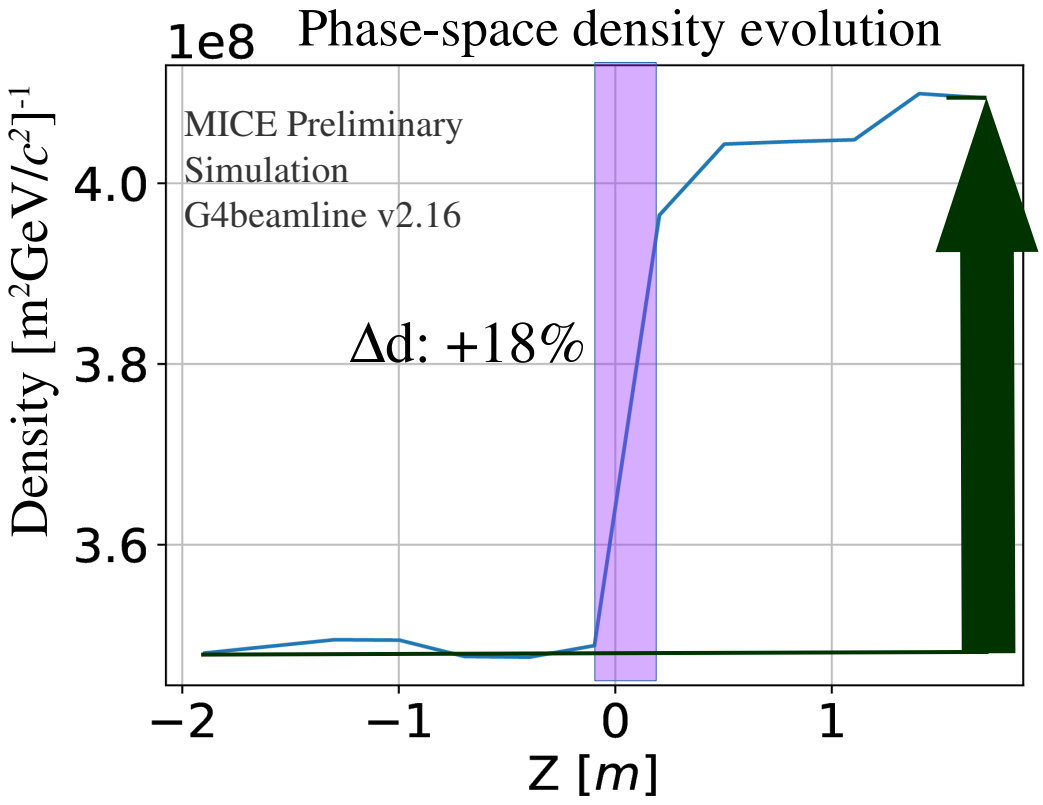
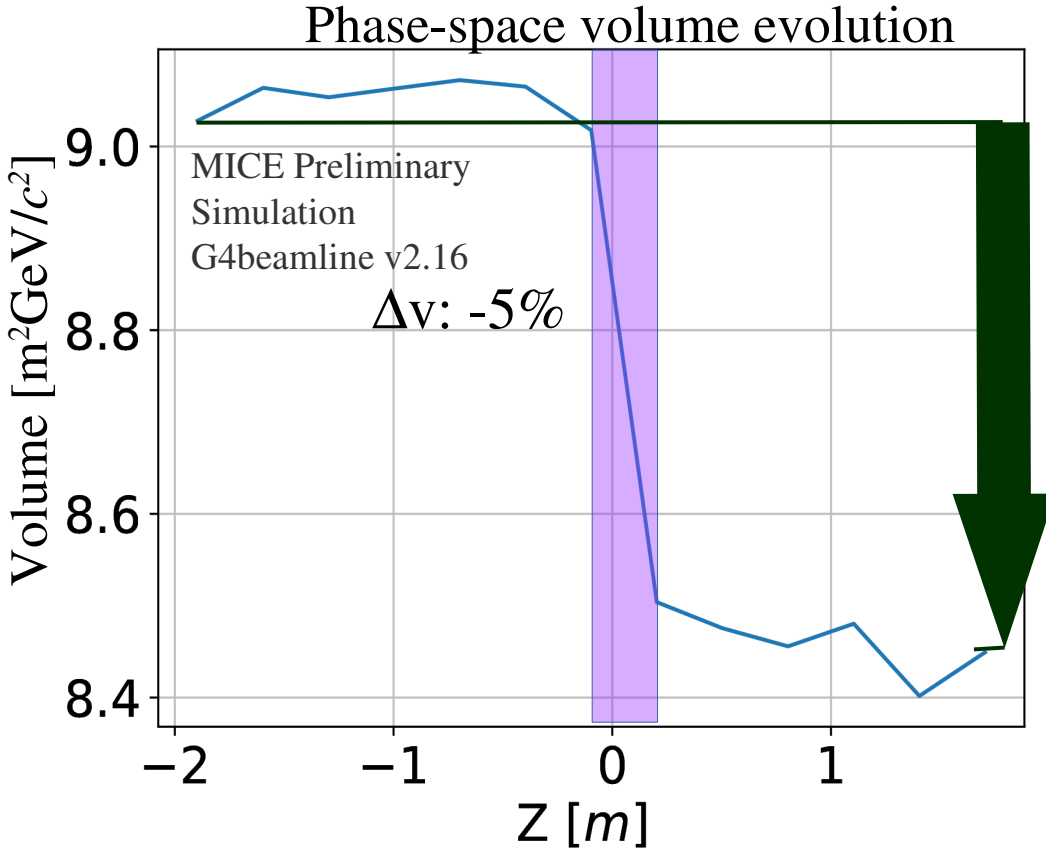
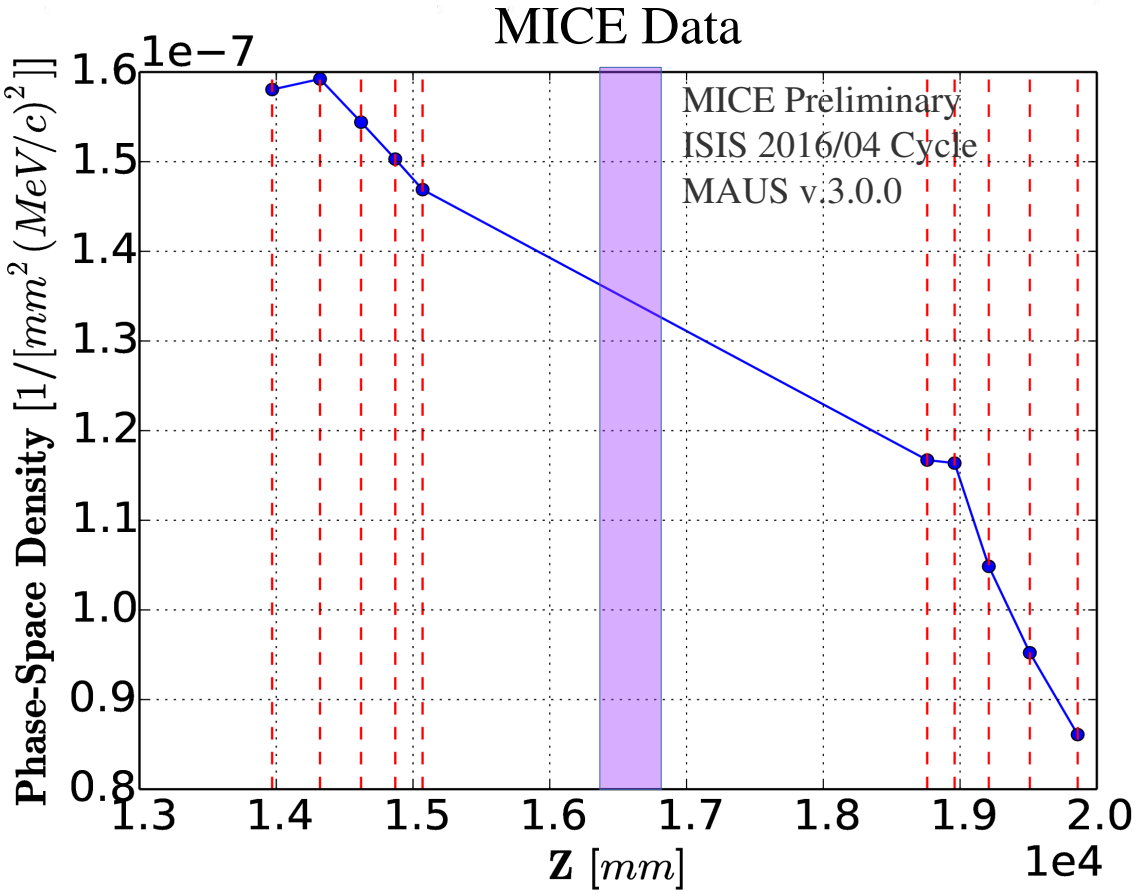
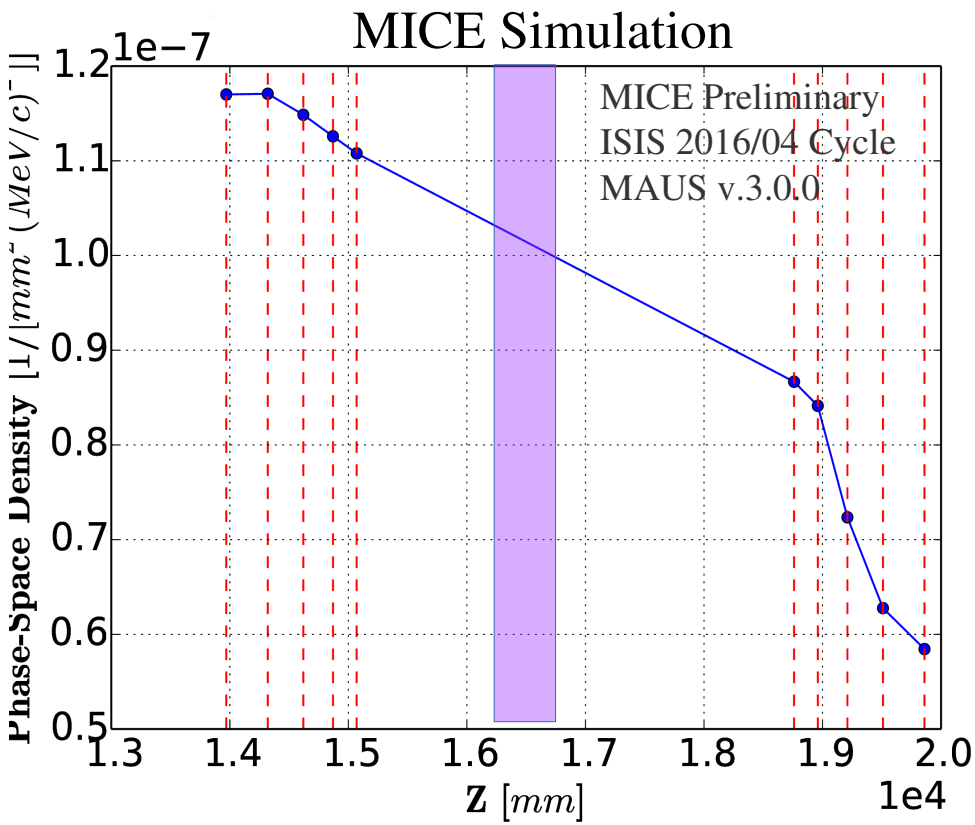


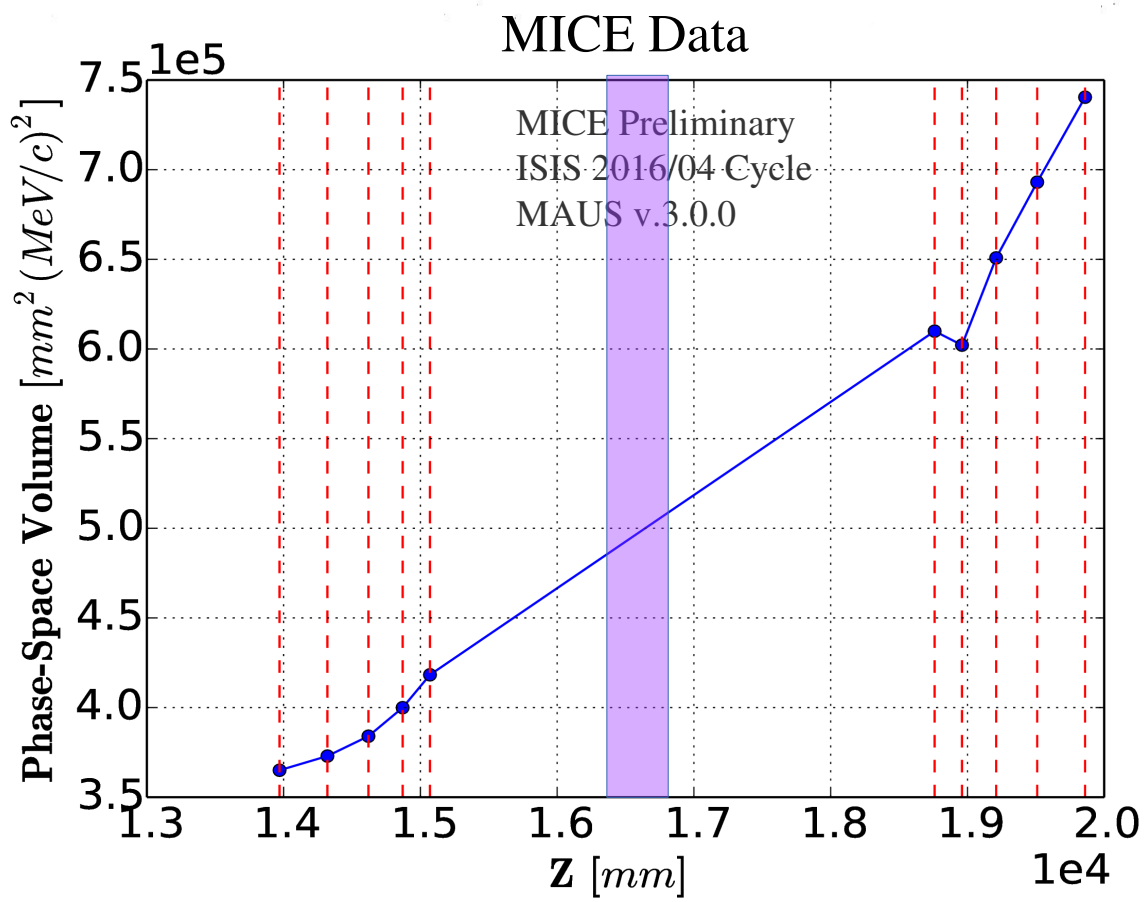
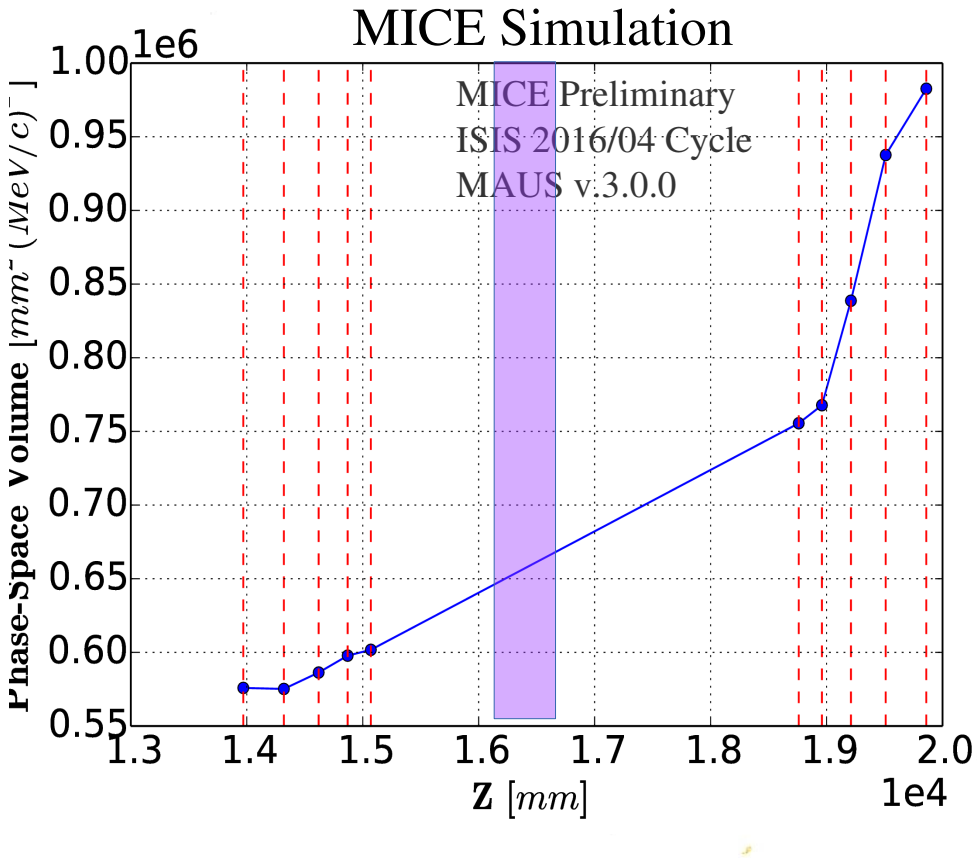


KDE Application to MICE Data vs Simulation

- Run 8681, Data versus Simulation (no transmission selection but this is the PDF):
  - 3-140, emittance-momentum setting, LiH absorber, downstream Match coils off.
  - Dashed red lines: tracker stations.
  - $28 < \text{TOF01} < 30.5$ ns, $125 < p_{upstream} < 162$ MeV/c, fixed contour evolution.





KDE Application to MICE Data vs Simulation

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