Port-Hamiltonian Description of Electro-Thermal Field-Circuit models

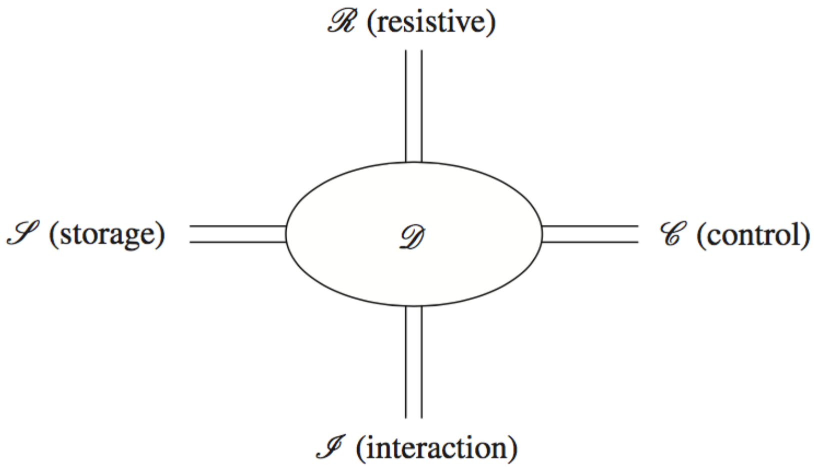
M. Maciejewski, *B. Auchmann*

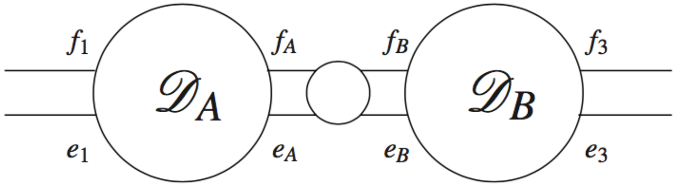
Outline

- Motivation
- Introduction
- Electromagnetic Field
- Irreversible Thermodynamics
- Conclusions

Motivation

- General framework for the multi-physics modelling
- Computational causality
- Analysis of the coupling strength between physical systems
- Distributed and lumped element phenomena

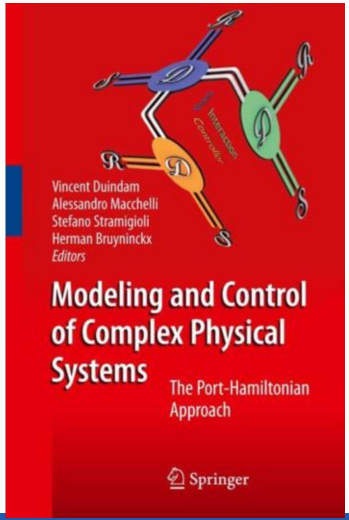




1.12 Future Trends

The following general future trends in bond graphs and port-based modeling can be distinguished:

- . . .
- ~~use of the port-based approach for *co-simulation*.~~



V. Duindam, et al. Modelling and Control of Complex Physical Systems
The Port-Hamiltonian Approach, Springer, 2009.

Modelling with bond graphs

Effort and Flow

effort*flow = Power

Causality assignment

1. Fixed
2. Arbitrary
3. Preferred
4. Restricted

Flow and effort

$\mathrm{Sf} \vdash\!\!\!\rightharpoonup \qquad \mathrm{Se} \rightharpoonup\!\!\!\dashv$

Resistor

$J \rightharpoonup\!\!\!\dashv R \qquad J \vdash\!\!\!\rightharpoonup R$

Inductor and Capacitor

$J \vdash\!\!\!\rightharpoonup C$

$J \rightharpoonup\!\!\!\dashv I$

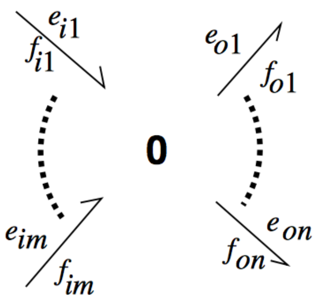
$$f(t) = \frac{1}{I}\int e(t)\,\mathrm{d}t$$

$$e(t) = \frac{1}{C}\int f(t)\,\mathrm{d}t$$

	f flow	e effort	$q = \int f\,\mathrm{d}t$ generalized displacement	$p = \int e\,\mathrm{d}t$ generalized momentum
electro-magnetic	i current	u voltage	$q = \int i\,\mathrm{d}t$ charge	$\lambda = \int u\,\mathrm{d}t$ magnetic flux linkage
mechanical translation	v velocity	F force	$x = \int v\,\mathrm{d}t$ displacement	$p = \int F\,\mathrm{d}t$ momentum
mechanical rotation	ω angular velocity	T torque	$\theta = \int \omega\,\mathrm{d}t$ angular displacement	$b = \int T\,\mathrm{d}t$ angular momentum
hydraulic pneumatic	ϕ volume flow	p pressure	$V = \int \phi\,\mathrm{d}t$ volume	$\Gamma = \int p\,\mathrm{d}t$ momentum of a flow tube
thermal	T temperature	f_S entropy flow	$S = \int f_S\,\mathrm{d}t$ entropy	

Modelling with bond graphs

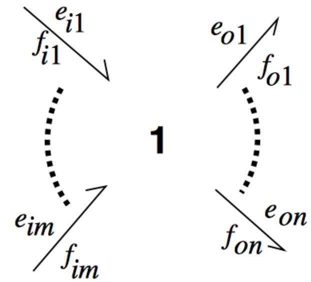
0-junction



$$\sum_{k=1}^{m} f_{ik} = \sum_{k=1}^{n} f_{ok}$$

$$e_{i1} = \cdots = e_{im} = e_{o1} \cdots = e_{on}$$

1-junction



$$\sum_{k=1}^{m} e_{ik} = \sum_{k=1}^{n} e_{ok}$$

$$f_{i1} = \cdots = f_{im} = f_{o1} \cdots = f_{on}$$

Transformer

$$\xrightarrow[f_1]{e_1} \underset{n}{\overset{\cdot\cdot}{\text{TF}}} \xrightarrow[f_2]{e_2}$$

$$\begin{bmatrix} e_1 \\ f_2 \end{bmatrix} = \begin{bmatrix} 0 & n \\ n & 0 \end{bmatrix} \begin{bmatrix} f_1 \\ e_2 \end{bmatrix}$$

Gyrator

$$\xrightarrow[f_1]{e_1} \underset{n}{\overset{\cdot\cdot}{\text{TF}}} \xrightarrow[f_2]{e_2}$$

$$\begin{bmatrix} e_1 \\ f_2 \end{bmatrix} = \begin{bmatrix} 0 & n \\ n & 0 \end{bmatrix} \begin{bmatrix} f_1 \\ e_2 \end{bmatrix}$$

D. Jeltsema, A. Schaft, Port-Hamiltonian Systems, lecture, TU Delft.

Input-state-output Port Hamiltonian Model

$$\partial_t x = [J(x) - R(x)]\delta_x \mathbb{H}(x) + g(x)u$$
$$y = g^\top(x)\delta_x \mathbb{H}(x) + S(x)u,$$

where $x \in \mathbb{R}^n$ is the state vector,

$J(x) = -J(x)^\top \in \mathbb{R}^n \times \mathbb{R}^n$ is a skew-symmetric interconnection matrix

$R(x) = R(x)^\top \in \mathbb{R}^n \times \mathbb{R}^n$ is a symmetric matrix corresponding to the resistive port.

$g(x) \in \mathbb{R}^n \times \mathbb{R}^m$ is the input vector field

$S(x) \in \mathbb{R}^m \times \mathbb{R}^m$ is a direct feed-through matrix

B. Maschke and A. van der Schaft, “From conservation laws to port-hamiltonian representations of distributed-parameter systems,” in *16th IFAC World Congress*, Elsevier, 2005.

Modelling of Magnetoquasistatic Domain

For a superconducting magnet model we consider a magnetoquasistatic model including

1. energy stored in the magnetic field,
2. resistive voltage due to a quench,
3. induced voltage influenced by the eddy currents and cable coupling currents.

Electromagnetic Field – Topological Diagram

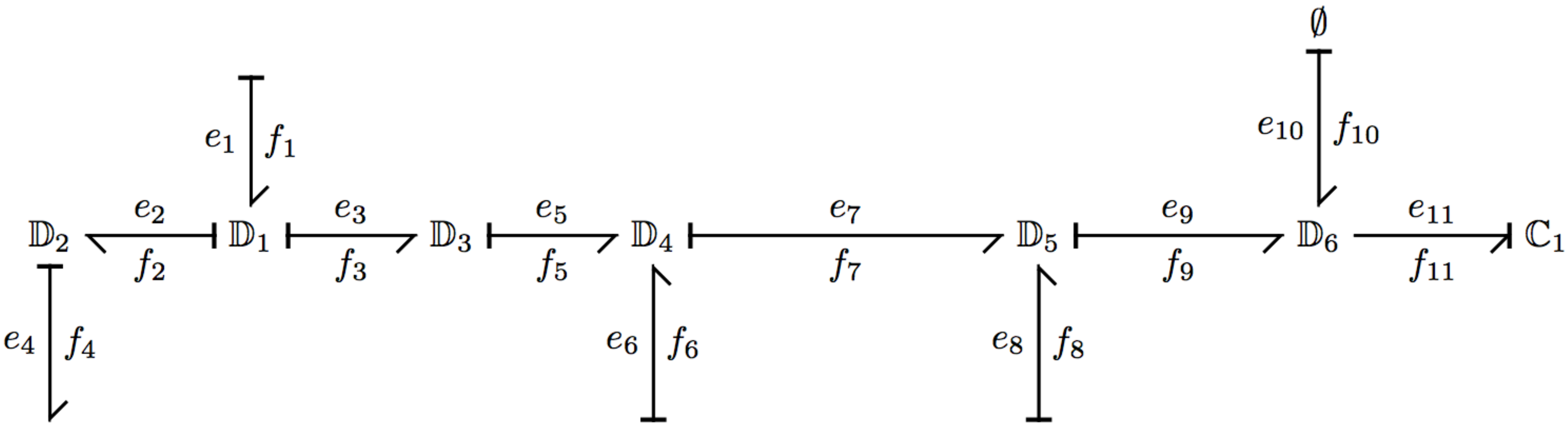
Symbol	Form order	Unit	Description
φ	$\Lambda^0(\Omega)$	V	Electric scalar potential
Ψ	$\Lambda^0(\Omega)$	A	Magnetic scalar potential
A	$\Lambda^1(\Omega)$	Wb	Magnetic vector potential
E	$\Lambda^1(\Omega)$	V	Electric field strength
H	$\Lambda^1(\Omega)$	A	Magnetic field strength
D	$\Lambda^2(\Omega)$	C	Electric flux density
B	$\Lambda^2(\Omega)$	Wb	Magnetic flux density
$\jmath$	$\Lambda^2(\Omega)$	A	Electric current density
ρ	$\Lambda^3(\Omega)$	C	Electric charge density

$$\begin{tikzcd}
(0) & \varphi \arrow[dr, "-d"] & & \\
(1) & A \arrow[r, "-\partial_t"] \arrow[dr, "d"] & E \arrow[dr, "-d"] & \\
(2) & & B \arrow[r, "-\partial_t"] \arrow[dr, "d"] & 0 \\
(3) & & & 0
\end{tikzcd}$$

$$\begin{tikzcd}
& \rho \arrow[r, "\partial_t"] & 0 & (3) \\
D \arrow[ur, "d"] \arrow[r, "-\partial_t"] & \jmath \arrow[ur, "d"] & & (2) \\
H \arrow[ur, "d"] & & & (1)
\end{tikzcd}$$

$$\jmath = *_\sigma E, \quad D = *_\epsilon E, \quad H = *_\nu B$$

Electromagnetic Field – Bond Graph Model (1/x)



For a superconducting magnet model we consider a magnetoquasistatic model including

1. energy stored in the magnetic field,
2. resistive voltage due to a quench,
3. induced voltage influenced by the eddy currents and cable coupling currents.

Power-continuity is the key.

Electromagnetic Field – Bond Graph Model (1/x)

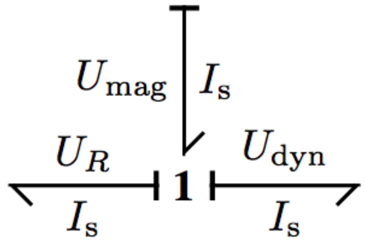
The voltage across a magnet is composed of a resistive and dynamic component

$$U_{\mathrm{mag}} = U_R + U_{\mathrm{dyn}}$$

which forms a 0-junction

$$\mathcal{D}_1 = \{(f_1, f_2, f_3, e_1, e_2, e_3) \in \mathcal{F}_1 \times \mathcal{E}_1 : f_1 = f_2 = f_3, e_1 + e_2 + e_3 = 0\}$$

$$\text{with } (f_1, f_2, f_3, e_1, e_2, e_3) = (I_{\mathrm{s}}, I_{\mathrm{s}}, I_{\mathrm{s}}, U_{\mathrm{mag}}, -U_R, -U_{\mathrm{dyn}})$$



Electromagnetic Field – Bond Graph Model (1/4)

The winding density function distributes current in the domain and forms a transformer

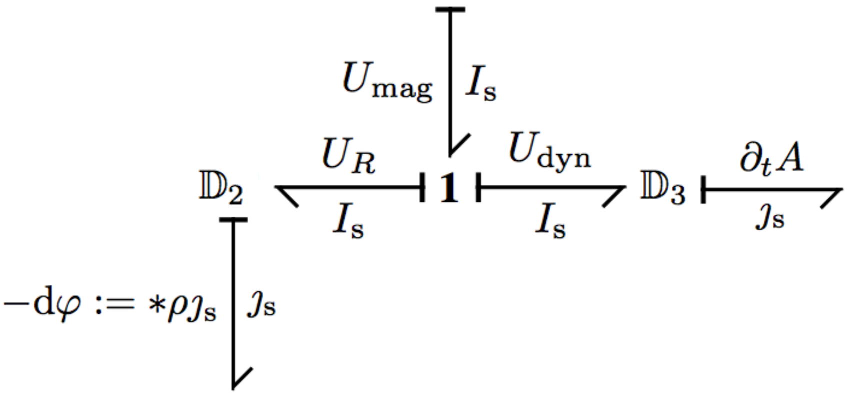
$$\begin{bmatrix} e_a \\ f_b \end{bmatrix} = \begin{bmatrix} 0 & -\chi^\top \\ \chi & 0 \end{bmatrix} \begin{bmatrix} f_a \\ e_b \end{bmatrix} \qquad f^\top e = 0$$

For the resistive voltage, the conjugate power pair takes form

$$\begin{bmatrix} e_a \\ f_a \end{bmatrix} = \begin{bmatrix} U_\mathrm{R} \\ I_\mathrm{s} \end{bmatrix}, \quad \begin{bmatrix} e_b \\ f_b \end{bmatrix} = \begin{bmatrix} \mathrm{d}\varphi \\ \jmath_\mathrm{s} \end{bmatrix}$$

and for the inductive voltage

$$\begin{bmatrix} e_a \\ f_a \end{bmatrix} = \begin{bmatrix} U_\mathrm{dyn} \\ I_\mathrm{s} \end{bmatrix}, \quad \begin{bmatrix} e_b \\ f_b \end{bmatrix} = \begin{bmatrix} \partial_t A \\ \jmath_\mathrm{s} \end{bmatrix}$$



Electromagnetic Field – Bond Graph Model (2/4)

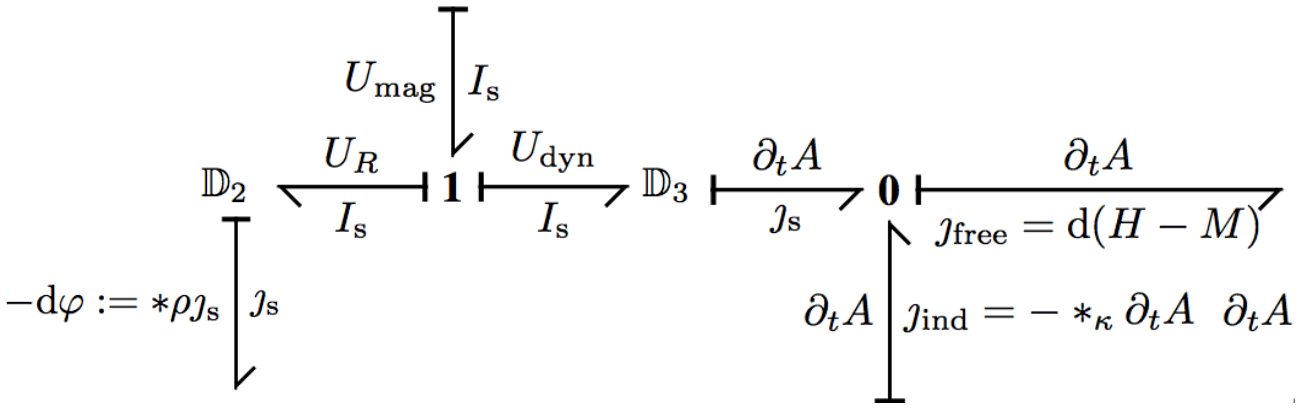
We consider eddy currents in copper wedges

$$\mathcal{J}_{\mathrm{s}} = \mathcal{J}_{\mathrm{free}} - \mathcal{J}_{\mathrm{ind}}.$$

forming a 0-junction

$$\mathcal{D}_4 = \{(f_5, f_6, f_7, e_5, e_6, e_7) \in \mathcal{F}_2 \times \mathcal{E}_2 : f_5 + f_6 + f_7 = 0, e_5 = e_6 = e_7\}$$

$$\text{with } (f_5, f_6, f_7, e_5, e_6, e_7) = (\mathcal{J}_{\mathrm{s}}, -*_\kappa\, \partial_t A, -\mathrm{d}(H - M), \partial_t A, \partial_t A, \partial_t A)$$



Electromagnetic Field – Bond Graph Model (3/4)

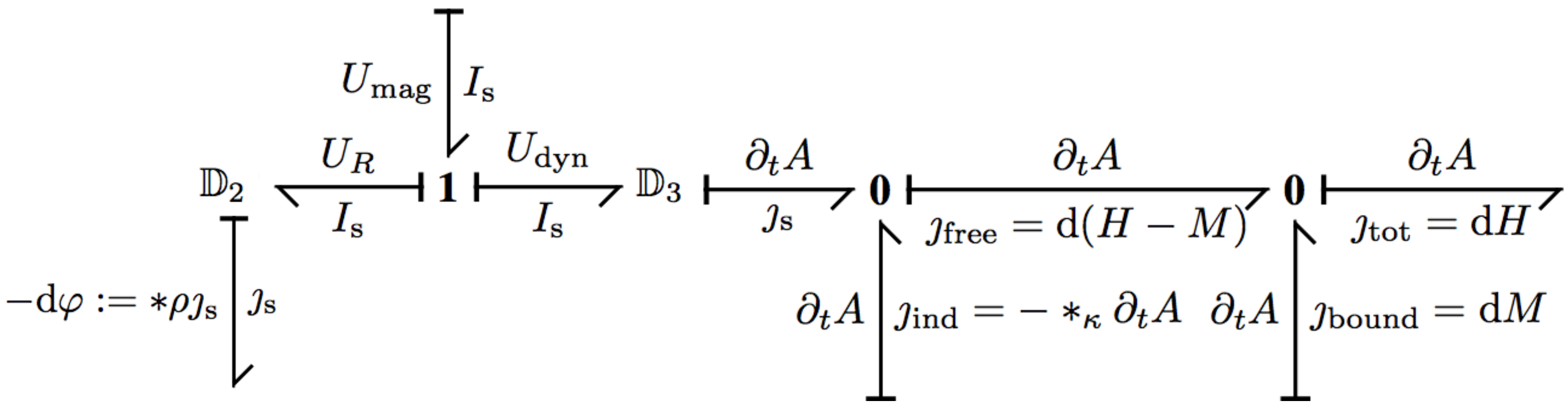
Coupling currents in the fully-transposed cables are accounted for as

$$\jmath_{\mathrm{tot}} = \jmath_{\mathrm{free}} + \jmath_{\mathrm{bound}} = \mathrm{d}H$$

forming a 0-junction

$$\mathcal{D}_5 = \{(f_7, f_8, f_9, e_7, e_8, e_9) \in \mathcal{F}_3 \times \mathcal{E}_3 : f_7 + f_8 + f_9 = 0, e_7 = e_8 = e_9\}$$

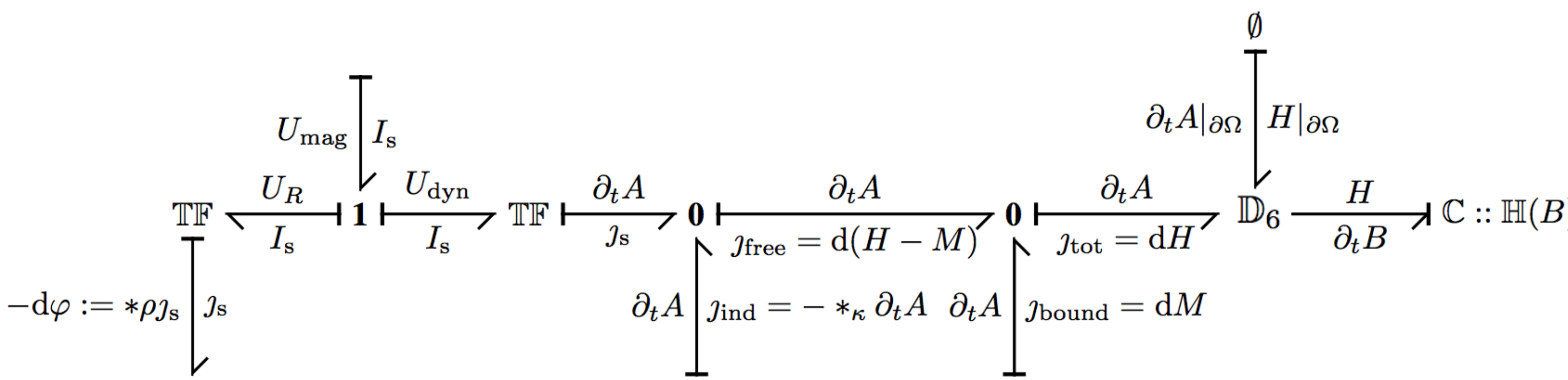
$$(f_7, f_8, f_9, e_7, e_8, e_9) = (-\mathrm{d}(H - M), -\mathrm{d}M, \mathrm{d}H, \partial_t A, \partial_t A, \partial_t A)$$



Electromagnetic Field – Bond Graph Model (4/4)

$$\mathcal{D}_6 = \left\{ \begin{array}{c} (f_9, f_{10}, f_{11}, e_9, e_{10}, e_{11} \in \mathcal{F} \times \mathcal{E}) \\ \begin{bmatrix} f_9 \\ f_{10} \end{bmatrix} = \begin{bmatrix} 0 & -\mathrm{d} \\ \mathrm{d} & 0 \end{bmatrix} \begin{bmatrix} e_9 \\ e_{10} \end{bmatrix} \\ \begin{bmatrix} f_{11} \\ e_{11} \end{bmatrix} = \begin{bmatrix} e_9|_{\partial\Omega} \\ f_9|_{\partial\Omega} \end{bmatrix} \end{array} \right\}.$$

$$(-\partial_t B, -\partial_t D, f_{11}, H, \partial_t A, e_{11} \in \mathcal{F} \times \mathcal{E}) \in \mathcal{D}_6$$



$$\mathbb{H}(B, D) = \frac{1}{2} \int_\Omega *_\nu B \wedge B + *_{\frac{1}{\epsilon}} D \wedge D$$

$$\delta_B \mathcal{H} = H \qquad \delta_D \mathcal{H} = \partial_t A$$

Electromagnetic Field – port-Hamiltonian Model

Combining the Dirac structures together, the state and output equations are given as

$$-\partial_t \begin{bmatrix} B \\ 0 \end{bmatrix} = \left(\begin{bmatrix} 0 & -\mathrm{d} \\ \mathrm{d} & 0 \end{bmatrix} - \begin{bmatrix} 0 & 0 \\ 0 & *_\kappa + \mathrm{d} *_{\nu\tau} \mathrm{d} \end{bmatrix} \right) \begin{bmatrix} H \\ \partial_t A \end{bmatrix} + \begin{bmatrix} 0 \\ \chi \end{bmatrix} I_\mathrm{s}$$

$$U_\mathrm{mag} = \begin{bmatrix} 0 & \chi^\top \end{bmatrix} \begin{bmatrix} H \\ \partial_t A \end{bmatrix} + \begin{bmatrix} 0 & \chi^\top \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 0 & *_\rho \end{bmatrix} \begin{bmatrix} 0 \\ \chi \end{bmatrix} I_\mathrm{s},$$

which forms an input-state-output port-Hamiltonian system

$$\partial_t x = \left(J(x) - R(x)\right) \delta_x \mathbb{H}(x) + gu$$
$$y = g^\top \delta_x \mathbb{H}(x) + Su,$$

The power of the system is bounded by the product of input and output

$$\partial_t \mathbb{H}(x) = \delta_x \mathbb{H}(x)^\top \partial_t x = \delta_x \mathbb{H}(x)^\top \left(\left(J(x) - R(x)\right) \delta_x \mathbb{H}(x) + gu \right)$$

$$\partial_t \mathbb{H}(x) = -\delta_x \mathbb{H}(x)^\top R(x) \delta_x \mathbb{H}(x) + y^\top u - u^\top S^\top u \le y^\top u.$$

Electromagnetic Field – port-Hamiltonian Model

Considering the input-state-output equations representing the magnetoquasistatic mode, the variation of the Hamiltonian function reads

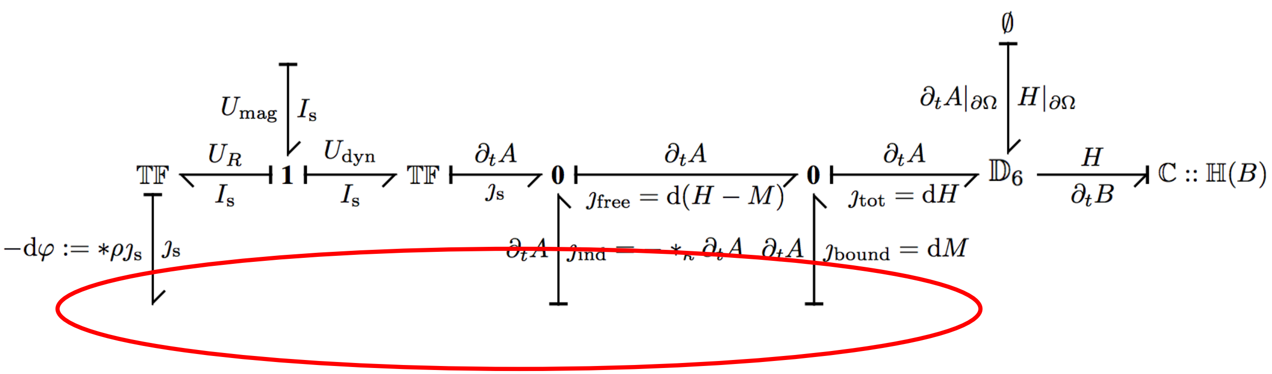
$$
\begin{aligned}
\partial_t \mathbb{H}(x) &= -\delta_x \mathbb{H}(x)^\top R(x) \delta_x \mathbb{H}(x) + y^\top u - u^\top S^\top u \\
&= -\begin{bmatrix} H & \partial_t A \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 0 & *_\kappa + \mathrm{d} *_{\nu\tau} \mathrm{d} \end{bmatrix} \begin{bmatrix} H \\ \partial_t A \end{bmatrix} + \left(\chi^\top \partial_t A + \chi *_\rho \chi^\top I_\mathrm{s}\right) I_\mathrm{s} - I_\mathrm{s} \chi *_\rho \chi^\top I_\mathrm{s} \\
&= \partial_t A \chi I_\mathrm{s} - \partial_t A (*_\kappa + \mathrm{d} *_{\nu\tau} \mathrm{d}) \partial_t A
\end{aligned}
$$

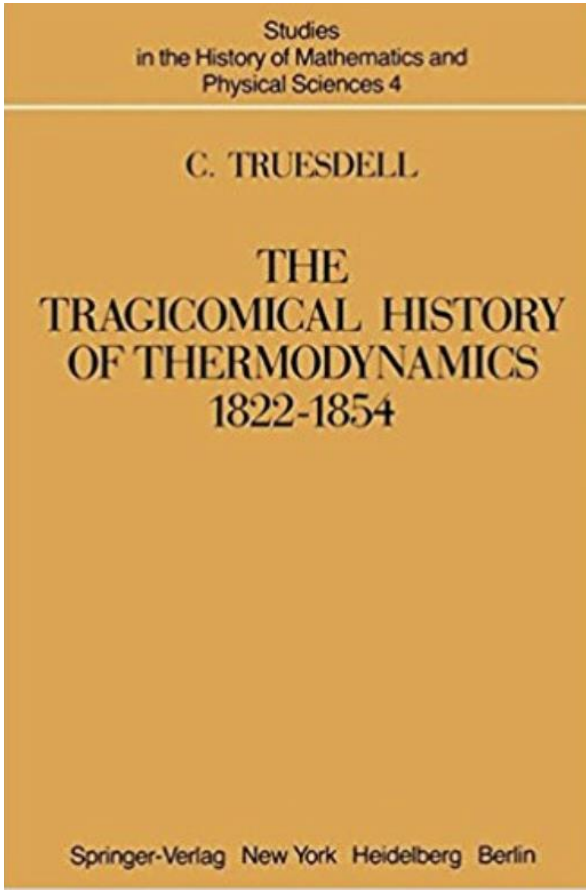
The magnetic energy **decreases only due** to the eddy and coupling current losses and **is not directly** influenced by the Joule losses after a quench.

Modelling of Irreversible Thermodynamics

Irreversible thermodynamics modelling considers

1. storage of the thermal energy
2. heat conduction
3. heat generation in the EM domain





Irreversible Thermodynamics – Topological Diagram

Symbol	Form order	Unit	Description
T	$\Lambda^0(\Omega)$	K	Temperature
J_S	$\Lambda^2(\Omega)$	W/K	Entropy flux
J_Q	$\Lambda^2(\Omega)$	W	Heat flux
$\mathcal{U}$	$\Lambda^3(\Omega)$	J	Internal energy density
S	$\Lambda^3(\Omega)$	J/K	Entropy density
σ	$\Lambda^3(\Omega)$	W/K	Irreversible entropy flow
f_s	$\Lambda^3(\Omega)$	W/K	Reversible entropy flow

$$
\begin{tikzcd}
& \sigma_{\mathrm{ext}} \arrow[d] & & & -q_0 \arrow[d] \\
S \arrow[r, "\partial_t"] & \sigma & T \arrow[r, dashed, bend left, "\mathrm{d}\mathcal{U} = c_p \mathrm{d}T"] \arrow[d, "\mathrm{d}"'] & \mathcal{U} \arrow[r, "\partial_t"] & 0 \\
& J_{\mathrm{S}} \arrow[u, "\mathrm{d}"] & \mathrm{d}T \arrow[l, dashed, bend right, "-*_{k(t)}\ \frac{1}{T}"'] \arrow[rr, dashed, bend left, "-*_{k(T)}"] & & J_{\mathrm{Q}} \arrow[u, "\mathrm{d}"']
\end{tikzcd}
$$

D. Eberard and B. Maschke, “Port hamiltonian systems extended to irreversible systems: The example of heat conduction,” in *IFAC Nonlinear Control Congress*, Elsevier, 2004.

Irreversible Thermodynamics – Bond Graph

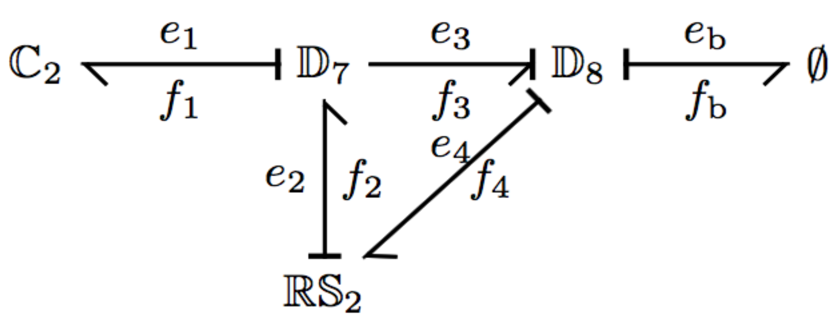
Jaumann's entropy balance forms a 0-junction

$$\mathcal{D}_7 = \{(f_{13}, f_{14}, f_{15}, e_{13}, e_{14}, e_{15}) \in \mathcal{F}_\mathrm{s} \times \mathcal{E}_\mathrm{s} : f_{13} + f_{14} + f_{15} = 0, e_{13} = e_{14} = e_{15}\}$$

$$\text{with } ((f_{13}, f_{14}, f_{15}, e_{13}, e_{14}, e_{15}) = (\partial_t s, -\sigma_\mathrm{s}, \mathrm{d}J_\mathrm{s}, T, T, T).$$

Stokes-Dirac structure (symplectic gurator) accounts for the interaction through a boundary

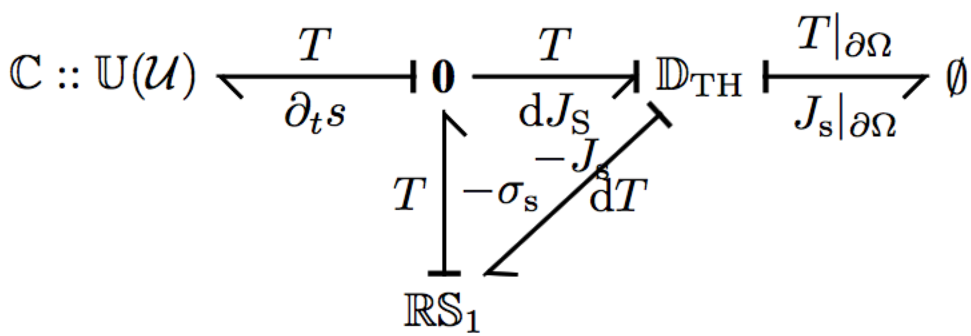
$$\mathcal{D}_2 = \left\{ \begin{matrix} (f_3, f_4, f_\mathrm{b}, e_3, e_4, e_\mathrm{b} \in \mathcal{F} \times \mathcal{E}) \\ \begin{bmatrix} f_3 \\ f_4 \end{bmatrix} = \begin{bmatrix} 0 & -\mathrm{d} \\ \mathrm{d} & 0 \end{bmatrix} \begin{bmatrix} e_3 \\ e_4 \end{bmatrix} \\ \begin{bmatrix} f_\mathrm{b} \\ e_\mathrm{b} \end{bmatrix} = \begin{bmatrix} e_\mathrm{th}|_{\partial\Omega} \\ f_\mathrm{th}|_{\partial\Omega} \end{bmatrix} \end{matrix} \right\} \qquad (\mathrm{d}J_\mathrm{S}, \mathrm{d}T, J_\mathrm{S}|_{\partial\Omega}, T, -J_\mathrm{S}, T|_{\partial\Omega} \in \mathcal{F} \times \mathcal{E})$$



Irreversible Thermodynamics – Bond Graph

Heat conduction (the Fourier's law) in the thermal domain is represented as an RS element

$$\mathcal{I}_3 = \left\{ \begin{array}{c} (f_2, f_4, e_2, e_4 \in \mathcal{F} \times \mathcal{E}) \\ \begin{bmatrix} e_4 \\ f_2 \end{bmatrix} = \begin{bmatrix} 0 & e_R \\ -e_R & 0 \end{bmatrix} \begin{bmatrix} f_4 \\ e_2 \end{bmatrix} \end{array} \right\} \qquad e_R = \frac{*\lambda}{T^2} \mathrm{d}T \wedge$$



H. Ramirez, B. Maschke, and D. Sbarbaro, "Irreversible port-hamiltonian systems: A general formulation of irreversible processes with application to the cstr," *Chemical Engineering Science*, vol. 89,

Irreversible Port-Hamiltonian Model

Hamiltonian function is obtained with the internal energy

$$\mathbb{U} = \int_\Omega \mathcal{U}.$$

Dynamics of the thermal model can be represented as

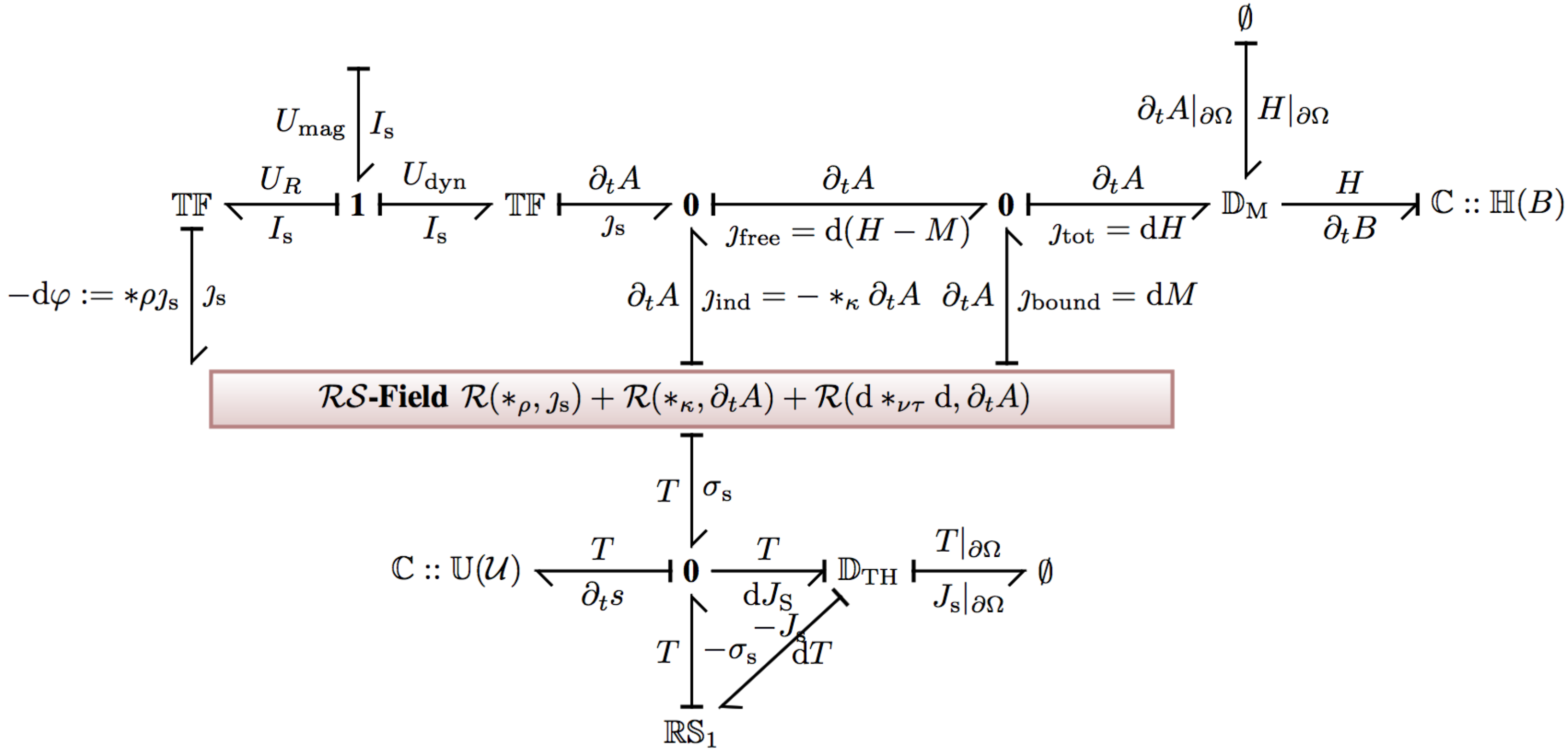
$$\begin{aligned} \partial_t x &= (\mathrm{d}e_R + e_R \mathrm{d})\delta_x \mathcal{U} + u \\ y &= \delta_x \mathcal{U}, \end{aligned}$$

The variation of the internal energy reads

$$\begin{aligned} \partial_t U &= \delta_x U \partial_t x = T(\mathrm{d}R \wedge T - R \wedge \mathrm{d}T) + Tu \\ &= T(\mathrm{d}R \wedge T - R \wedge \mathrm{d}T) + *_\rho j_\mathrm{s} \wedge j_\mathrm{s} + \partial_t A \wedge *_\kappa \partial_t A - \partial_t A \wedge \mathrm{d} *_{\nu\tau} \mathrm{d}\partial_t A \end{aligned}$$

CERN

Magneto-Thermal Model of an SC Magnet



Irreversible entropy creation

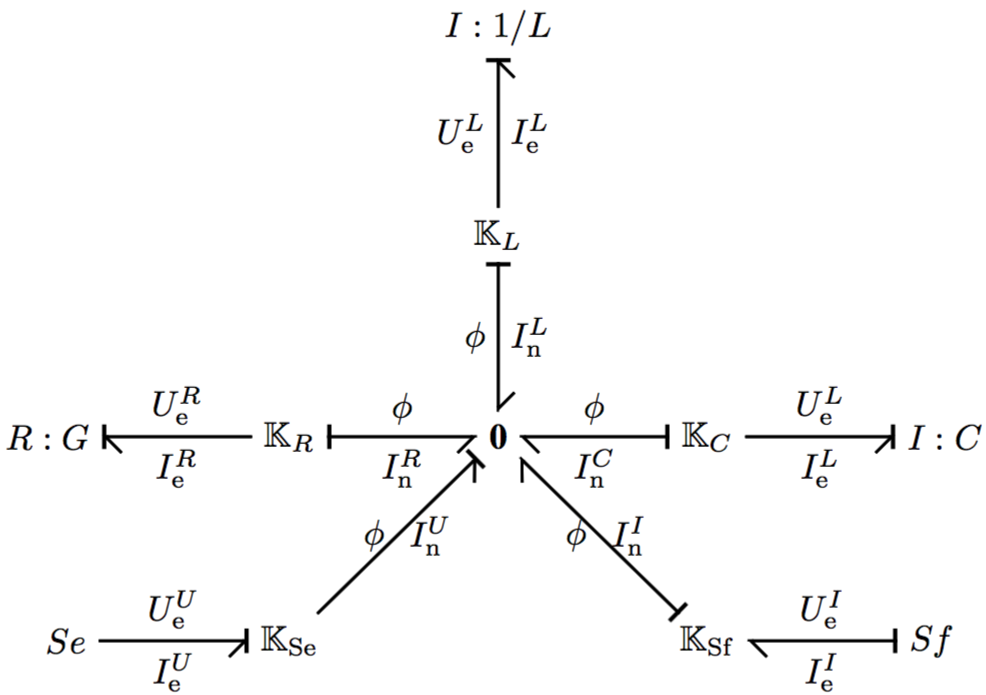
$$\sigma_{\mathrm{em}} = \frac{1}{T}\left(*_\rho j_{\mathrm{s}} \wedge j_{\mathrm{s}} + \partial_t A \wedge *_\kappa \partial_t A - \partial_t A \wedge \mathrm{d} *_{\nu\tau} \mathrm{d}\partial_t A\right)$$

$$\mathcal{R} = \left\{ \begin{array}{c} (f_1, e_1, f_2, e_2, f_3, e_3, f_4, e_4) \in \Lambda^3(\Omega) \times \Lambda^0(\Omega) \\ \times \Lambda^2(\Omega) \times \Lambda^1(\Omega) \times \Lambda^2(\Omega) \times \Lambda^1(\Omega) \times \Lambda^2(\Omega) \times \Lambda^1(\Omega) \\ e_2 = *_\rho f_2, f_3 = *_\kappa e_3, f_4 = \mathrm{d} *_{\nu\tau} \mathrm{d}e_4, \sum_{i=1}^4 e_i \wedge f_i = 0 \end{array} \right\}$$

Port-based Model of an SC Circuit

Starting with the Modified Nodal Analysis

$$
\begin{aligned}
A_{\mathrm{C}}\partial_t Q + A_{\mathrm{L}} I_{\mathrm{L}} + A_{\mathrm{R}} g_{\mathrm{R}}(A_{\mathrm{R}}^\top \phi, t) + A_{\mathrm{U}} I_{\mathrm{U}} + A_{\mathrm{I}} I(t) &= 0, \\
L\partial_t I_{\mathrm{L}} - A_{\mathrm{L}}^\top \phi &= 0 \\
A_{\mathbf{S}}^\top \phi &= U_{\mathbf{S}}
\end{aligned}
$$



Summary

1. Port-based modelling allows for a generic representation of multi-physical systems (both, SC magnets, and circuits)
2. Variation of the magnetic and thermal energy is studied
3. Computational causality can be derived from a bond-graph model

- ~~use of the port-based approach for *co-simulation*.~~

www.cern.ch