

Robust optimization of the size of permanent magnets in a synchronous machine using deterministic and stochastic approaches



TECHNISCHE
UNIVERSITÄT
DARMSTADT

Zeger Bontinck¹, Oliver Lass², Sebastian Schöps¹

¹ Graduate School CE, TEMF, Technische Universität Darmstadt

² Chair of Nonlinear Optimization, Technische Universität Darmstadt

September 22, 2017



GRADUATE SCHOOL
computational engineering

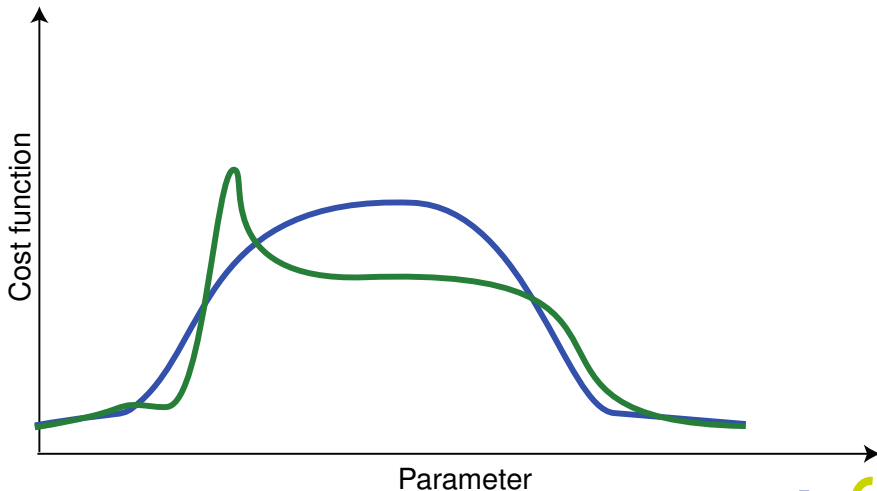
Overview

- 1 Introduction
- 2 Modeling of machines
- 3 UQ and MOR
- 4 Optimization
- 5 Conclusion and Prospects

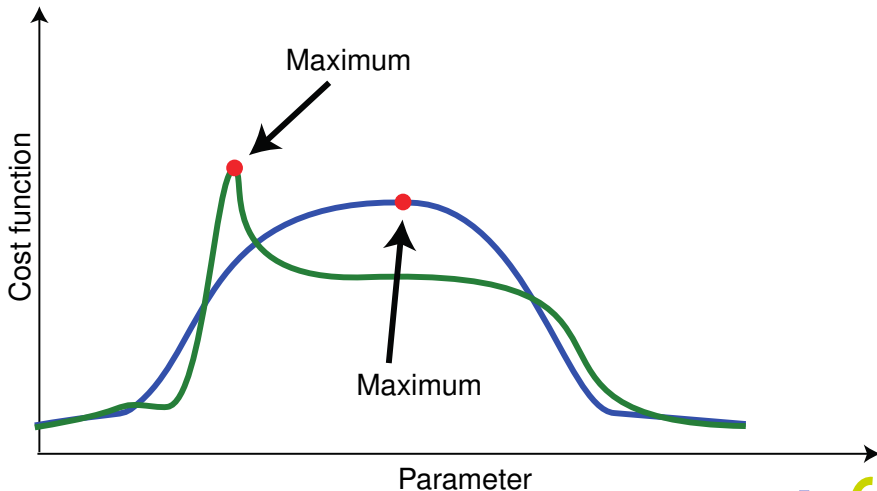
Table of Contents

- 1** Introduction
- 2 Modeling of machines
- 3 UQ and MOR
- 4 Optimization
- 5 Conclusion and Prospects

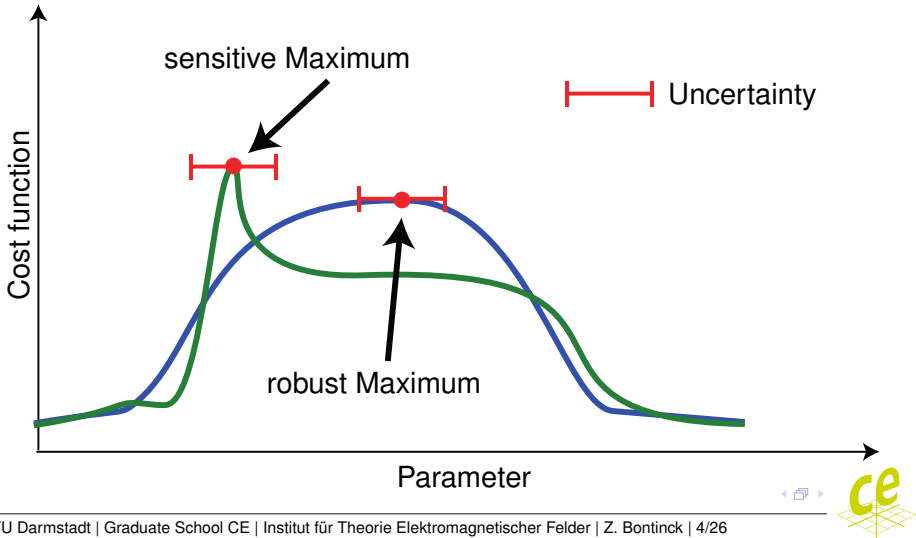
Why (robust) optimization with stochastic setting?



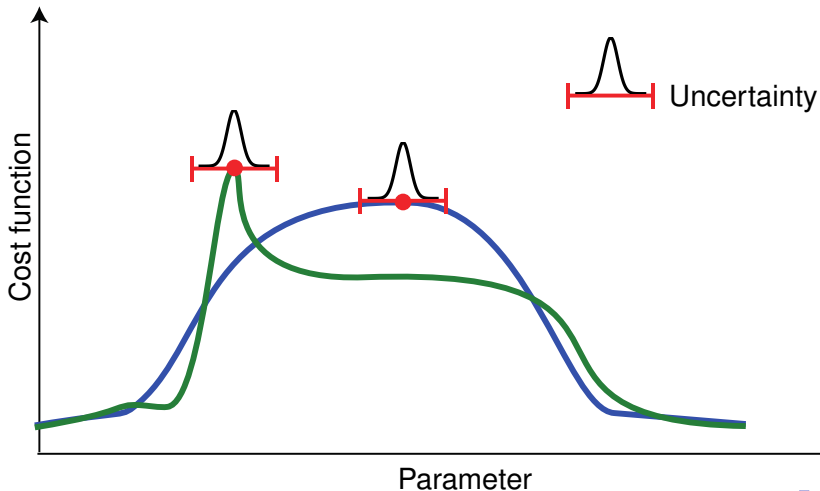
Why (robust) optimization with stochastic setting?



Why (robust) optimization with stochastic setting?

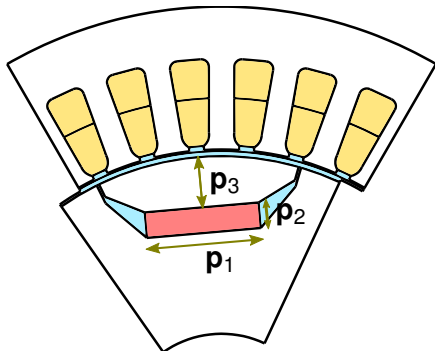


Why (robust) optimization with stochastic setting?



Problem description: Conceptual

- Reduce the size of the permanent magnet: $\mathbf{p}_1\mathbf{p}_2$
- Reposition the magnet in the rotor: \mathbf{p}_3
- Considering some design constraints and bounds for $\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3$: $G_1(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3) \leq 0$
- Such that you maintain a prescribed electromotive force (EMF) E_d
- Reduce vibrations



Cross-section of one pole of the PMSM

Table of Contents

- 1 Introduction
- 2 Modeling of machines**
- 3 UQ and MOR
- 4 Optimization
- 5 Conclusion and Prospects

Partial Differential Equation (PDE)

From Ampère's law

- Parabolic PDE

$$\sigma \frac{\partial \vec{A}}{\partial t} + \vec{\nabla} \times (\nu \vec{\nabla} \times \vec{A}) = \vec{J}_{\text{src}} - \vec{\nabla} \times \vec{H}_{\text{pm}}$$

Partial Differential Equation (PDE)

- Semi-Elliptical PDE ("curl-curl equation")

$$\sigma \frac{\partial \vec{A}}{\partial t} + \vec{\nabla} \times (\nu \vec{\nabla} \times \vec{A}) = \vec{J}_{\text{src}} - \vec{\nabla} \times \vec{H}_{\text{pm}}$$

- Model disregards eddy currents.

Partial Differential Equation (PDE)

- Semi-Elliptical PDE ("curl-curl equation")

$$\vec{\nabla} \times \left(\nu(\mathbf{p}) \vec{\nabla} \times \vec{A}(\mathbf{p}) \right) = \vec{J}_{\text{src}}(\mathbf{p}) - \vec{\nabla} \times \vec{H}_{\text{pm}}(\mathbf{p})$$

- Model disregards eddy currents.
- Dependencies on \mathbf{p}

Partial Differential Equation (PDE)

- Semi-Elliptical PDE ("curl-curl equation")

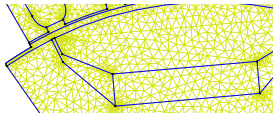
$$\vec{\nabla} \times \left(\nu(\mathbf{p}) \vec{\nabla} \times \vec{A}(\mathbf{p}) \right) = \vec{J}_{\text{src}}(\mathbf{p}) - \vec{\nabla} \times \vec{H}_{\text{pm}}(\mathbf{p})$$

- Model disregards eddy currents.
- Dependencies on \mathbf{p}
- Discretization of \vec{A} by edge shape functions \vec{w}_j related to the nodal finite elements $N_j(x, y)$ leads to

$$\vec{A}(\mathbf{p}) \approx \sum_j^N a_j(\mathbf{p}) \vec{w}_j = \sum_j^N a_j(\mathbf{p}) \frac{N_j(x, y)}{\ell_z} \vec{e}_z$$

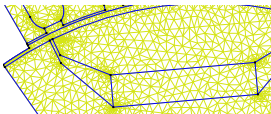
Reduction to 2D

- $\vec{J} = (0, 0, J_z)$ and $\vec{B} = (B_x, B_y, 0)$
- Triangulation of the machine's cross section



Reduction to 2D

- $\vec{J} = (0, 0, J_z)$ and $\vec{B} = (B_x, B_y, 0)$
- Triangulation of the machine's cross section



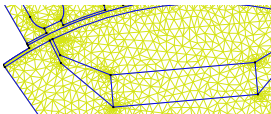
- Testing with \vec{w}_j yields the system of equations

$$\mathbf{K}(\mathbf{p})\mathbf{a}(\mathbf{p}) = \mathbf{j}_{\text{src}}(\mathbf{p}) + \mathbf{j}_{\text{pm}}(\mathbf{p})$$

- Applying the loading method (Rahman 1991) enables us to calculate E_0 from \mathbf{a} : $E_0(\mathbf{p}) = f(\mathbf{a}(\mathbf{p}))$

Reduction to 2D

- $\vec{J} = (0, 0, J_z)$ and $\vec{B} = (B_x, B_y, 0)$
- Triangulation of the machine's cross section



- Testing with \vec{w}_j yields the system of equations

$$\mathbf{K}(\mathbf{p})\mathbf{a}(\mathbf{p}) = \mathbf{j}_{\text{src}}(\mathbf{p}) + \mathbf{j}_{\text{pm}}(\mathbf{p})$$

- Applying the loading method (Rahman 1991) enables us to calculate E_0 from \mathbf{a} : $E_0(\mathbf{p}) = f(\mathbf{a}(\mathbf{p}))$
- Avoid remeshing and matrix assembling: **Affine Decomposition**

Affine decomposition

(e.g. Rozza 2008)

- Avoid remeshing, magnet region decomposed in L triangles
- System matrix:

$$\mathbf{K}(\mathbf{p}) = \mathbf{K}^{\text{out}} + \sum_{\ell=1}^L \vartheta^{\ell}(\mathbf{p}) \mathbf{K}^{\ell},$$

where

$$\begin{aligned} \vartheta^{\ell}(\mathbf{p}) \mathbf{K}^{\ell} := & \vartheta_1^{\ell}(\mathbf{p}) \mathbf{K}_{xx}^{\ell} + \vartheta_2^{\ell}(\mathbf{p}) \mathbf{K}_{yy}^{\ell} \\ & + \vartheta_3^{\ell}(\mathbf{p}) \mathbf{K}_{xy}^{\ell} + \vartheta_4^{\ell}(\mathbf{p}) \mathbf{K}_{yx}^{\ell}. \end{aligned}$$

- Analogous for $\mathbf{j}_{\text{src}}(\mathbf{p}) + \mathbf{j}_{\text{pm}}(\mathbf{p})$

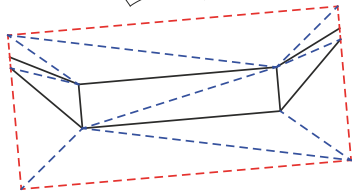
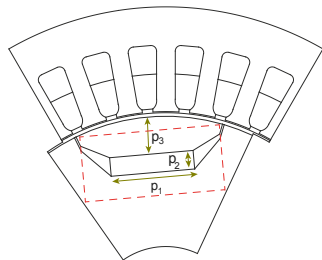


Table of Contents

- 1 Introduction
- 2 Modeling of machines
- 3 UQ and MOR**
- 4 Optimization
- 5 Conclusion and Prospects

Introducing Stochasticity

- \mathbf{p} can have random **deviations**, e.g., uniform:

$$\mathbf{p}(\omega) \sim \mathcal{U}(\bar{\mathbf{p}} - \Delta, \bar{\mathbf{p}} + \Delta)$$

- The PDE becomes stochastic

$$\mathbf{K}(\mathbf{p}(\omega))\mathbf{a}(\mathbf{p}(\omega)) = \mathbf{j}_{\text{src}}(\mathbf{p}(\omega)) + \mathbf{j}_{\text{pm}}(\mathbf{p}(\omega))$$

and thus $E_0(\omega) = f(\mathbf{a}(\mathbf{p}(\omega)))$

- Easily generalized for additional uncertainties that are not optimization variables

- Quadrature (e.g. Xiu 2010)

$$\mathbb{E}[f(\mathbf{a})] \approx \sum_{k=1}^M w_k f(\mathbf{a}(\mathbf{p}^{(k)}))$$

$$\text{Var}[f(\mathbf{a})] \approx \sum_{k=1}^M w_k f(\mathbf{a}(\mathbf{p}^{(k)}))^2 - \mathbb{E}[f(\mathbf{a})]^2$$

$$\text{Var}[\cdot] = \sum_i \text{Var}_{(\mathbf{p}_i)}[\cdot] + \sum_{i < j} \text{Var}_{(\mathbf{p}_i, \mathbf{p}_j)}[\cdot] + \text{Var}_{(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3)}[\cdot]$$

- Quadrature (e.g. Xiu 2010)

$$\mathbb{E}[f(\mathbf{a})] \approx \sum_{k=1}^M w_k f(\mathbf{a}(\mathbf{p}^{(k)}))$$

$$\text{Var}[f(\mathbf{a})] \approx \sum_{k=1}^M w_k f(\mathbf{a}(\mathbf{p}^{(k)}))^2 - \mathbb{E}[f(\mathbf{a})]^2$$

$$\text{Var}[\cdot] = \sum_i \text{Var}_{(\mathbf{p}_i)}[\cdot] + \sum_{i < j} \text{Var}_{(\mathbf{p}_i, \mathbf{p}_j)}[\cdot] + \text{Var}_{(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3)}[\cdot]$$

- Global Sensitivities S_i

$$S_i = \frac{\text{Var}_{(\mathbf{p}_i)}[\cdot]}{\text{Var}[\cdot]}$$

- Quadrature (e.g. Xiu 2010)

$$\mathbb{E}[f(\mathbf{a})] \approx \sum_{k=1}^M w_k f(\mathbf{a}(\mathbf{p}^{(k)}))$$

$$\text{Var}[f(\mathbf{a})] \approx \sum_{k=1}^M w_k f(\mathbf{a}(\mathbf{p}^{(k)}))^2 - \mathbb{E}[f(\mathbf{a})]^2$$

$$\text{Var}[\cdot] = \sum_i \text{Var}_{(\mathbf{p}_i)}[\cdot] + \sum_{i < j} \text{Var}_{(\mathbf{p}_i, \mathbf{p}_j)}[\cdot] + \text{Var}_{(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3)}[\cdot]$$

- Global Sensitivities S_i

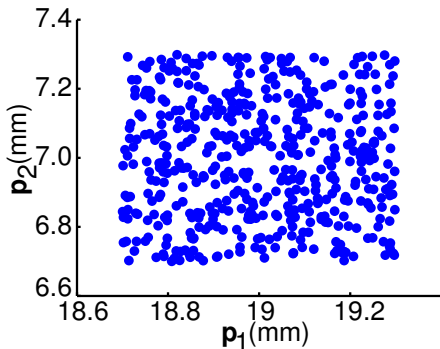
$$S_i = \frac{\text{Var}_{(\mathbf{p}_i)}[\cdot]}{\text{Var}[\cdot]}$$

- How to **determine** w_k ?

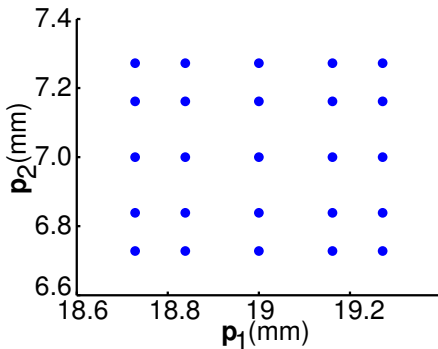
Methods for Uncertainty Quantification:

Stochastic Quadrature

Stochastic approach:
Monte Carlo



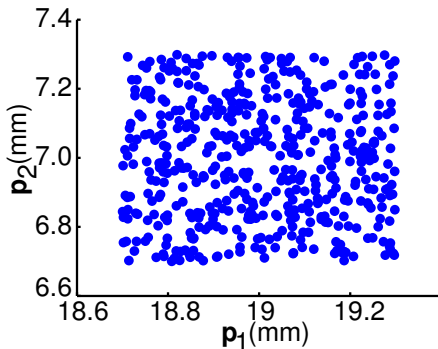
Deterministic approach:
Collocation



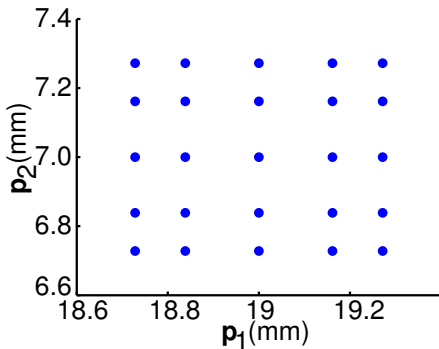
Methods for Uncertainty Quantification:

Stochastic Quadrature

Stochastic approach:
Monte Carlo



Deterministic approach:
Collocation



We increased the computational time → **Model order reduction**

Pseudocode: Basic algorithm (offline cost)

- 0) Initialize $j = 1$
- 1) Select a configuration $\mathbf{p}^{(j)} = [\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3]$ from the parameter domain
- 2) Solve the original PDE for $\mathbf{p}^{(j)}$ and obtain $\mathbf{a}^{(j)}$
- 3) Orthonormalize $\mathbf{a}^{(j)}$ w.r.t. $\mathbf{V}_{j-1} := [\mathbf{a}^{(1)}, \dots, \mathbf{a}^{(j-1)}]$
- 4) if $j < n$ goto 1)

Result: reduced basis $\mathbf{V} \in \mathbb{R}^{N \times n}$ where

- N corresponds to the original finite element dofs
- n dofs that are sufficient for the dynamics on the parameter space

Finally: project the (discretized) PDE on the lower dimensional subspace (online cost), with $\tilde{\mathbf{a}}(\mathbf{p}) \approx \mathbf{V}^\top \mathbf{a}(\mathbf{p})$

$$\mathbf{V}^\top \left(\mathbf{K}^{\text{out}} + \sum_{\ell=1}^L \vartheta^\ell(\mathbf{p}) \mathbf{K}^\ell \right) \mathbf{V} \tilde{\mathbf{a}}(\mathbf{p}) = \mathbf{V}^\top \left(\mathbf{j}^{\text{out}} + \sum_{\ell=1}^L \vartheta^\ell(\mathbf{p}) \mathbf{j}^\ell \right)$$

Pseudocode: Basic algorithm (offline cost)

- 0) Initialize $j = 1$
- 1) Select a configuration $\mathbf{p}^{(j)} = [\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3]$ from the parameter domain
- 2) Solve the original PDE for $\mathbf{p}^{(j)}$ and obtain $\mathbf{a}^{(j)}$
- 3) Orthonormalize $\mathbf{a}^{(j)}$ w.r.t. $\mathbf{V}_{j-1} := [\mathbf{a}^{(1)}, \dots, \mathbf{a}^{(j-1)}]$
- 4) if $j < n$ goto 1)

Result: reduced basis $\mathbf{V} \in \mathbb{R}^{N \times n}$ where

- N corresponds to the original finite element dofs
- n dofs that are sufficient for the dynamics on the parameter space

Finally: project the (discretized) PDE on the lower dimensional subspace (online cost), with $\tilde{\mathbf{a}}(\mathbf{p}) \approx \mathbf{V}^T \mathbf{a}(\mathbf{p})$

$$\left(\mathbf{V}^T \mathbf{K}^{\text{out}} \mathbf{V} + \sum_{\ell=1}^L \vartheta^\ell(\mathbf{p}) \mathbf{V}^T \mathbf{K}^\ell \mathbf{V} \right) \tilde{\mathbf{a}}(\mathbf{p}) = \mathbf{V}^T \mathbf{j}^{\text{out}} + \sum_{\ell=1}^L \vartheta^\ell(\mathbf{p}) \mathbf{V}^T \mathbf{j}^\ell$$

Pseudocode: Basic algorithm (offline cost)

- 0) Initialize $j = 1$
- 1) Select a configuration $\mathbf{p}^{(j)} = [\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3]$ from the parameter domain
- 2) Solve the original PDE for $\mathbf{p}^{(j)}$ and obtain $\mathbf{a}^{(j)}$
- 3) Orthonormalize $\mathbf{a}^{(j)}$ w.r.t. $\mathbf{V}_{j-1} := [\mathbf{a}^{(1)}, \dots, \mathbf{a}^{(j-1)}]$
- 4) if $j < n$ goto 1)

Result: reduced basis $\mathbf{V} \in \mathbb{R}^{N \times n}$ where

- N corresponds to the original finite element dofs
- n dofs that are sufficient for the dynamics on the parameter space

Finally: project the (discretized) PDE on the lower dimensional subspace (online cost), with $\tilde{\mathbf{a}}(\mathbf{p}) \approx \mathbf{V}^T \mathbf{a}(\mathbf{p})$

$$\left(\tilde{\mathbf{k}}^{\text{out}} + \sum_{\ell=1}^L \vartheta^\ell(\mathbf{p}) \tilde{\mathbf{k}}^\ell \right) \tilde{\mathbf{a}}(\mathbf{p}) = \tilde{\mathbf{j}}^{\text{out}} + \sum_{\ell=1}^L \vartheta^\ell(\mathbf{p}) \tilde{\mathbf{j}}^\ell$$

Table of Contents

- 1 Introduction
- 2 Modeling of machines
- 3 UQ and MOR
- 4 Optimization**
- 5 Conclusion and Prospects

- Sequential Quadratic Programming requires deriving the EMF

$$\frac{\partial E_0(\mathbf{p}, \mathbf{a}(\mathbf{p}))}{\partial \mathbf{p}_i} = E_0(\mathbf{p}, \mathbf{s}_i),$$

with sensitivity $\mathbf{s}_i = \partial \mathbf{a}(\mathbf{p}) / \partial \mathbf{p}_i$ ($i = 1, \dots, 3$)

- Solve

$$\mathbf{K}(\mathbf{p})\mathbf{s}_i(\mathbf{p}) = \mathbf{j}_i,$$

with

$$\mathbf{j}_i = (\mathbf{j}_{\text{src}} + \mathbf{j}_{\text{pm}})_{\mathbf{p}_i} - \mathbf{K}_{\mathbf{p}_i}(\mathbf{p})\mathbf{a}(\mathbf{p})$$

- Sequential Quadratic Programming requires deriving the EMF

$$\frac{\partial E_0(\mathbf{p}, \mathbf{a}(\mathbf{p}))}{\partial \mathbf{p}_i} = E_0(\mathbf{p}, \mathbf{s}_i),$$

with sensitivity $\mathbf{s}_i = \partial \mathbf{a}(\mathbf{p}) / \partial \mathbf{p}_i$ ($i = 1, \dots, 3$)

- Solve

$$\mathbf{K}(\mathbf{p})\mathbf{s}_i(\mathbf{p}) = \mathbf{j}_i,$$

with

$$\mathbf{j}_i = (\mathbf{j}_{\text{src}} + \mathbf{j}_{\text{pm}})_{\mathbf{p}_i} - \mathbf{K}_{\mathbf{p}_i}(\mathbf{p})\mathbf{a}(\mathbf{p})$$

- Exploit affine decomposition for $\mathbf{K}_{\mathbf{p}_i} \rightarrow \vartheta_{\mathbf{p}_i}^k$

Deterministic Optimization

Nominal and robust Optimization

- Nominal optimization (“D Opt”)

$$\begin{cases} \min_{\bar{\mathbf{p}} \in \mathbb{R}^3} J(\bar{\mathbf{p}}) := \bar{\mathbf{p}}_1 \bar{\mathbf{p}}_2, \\ \text{s.t. } G(\bar{\mathbf{p}}) := [G_1(\bar{\mathbf{p}}), E_d - E_0(\bar{\mathbf{p}})] \leq 0 \end{cases}$$

- Worst case robust optimization

$$\begin{cases} \min_{\bar{\mathbf{p}} \in \mathbb{R}^3} \max_{\delta} J(\bar{\mathbf{p}} + \delta) \\ \text{s.t. } \max_{\delta} G(\bar{\mathbf{p}} + \delta) \leq 0 \end{cases}$$

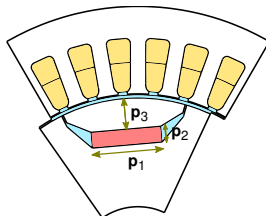
- Linearization with $D = \text{diag}(\delta)$ (“D Rob 1”):

$$J(\bar{\mathbf{p}} + \delta) \approx J(\bar{\mathbf{p}}) + \|D \nabla_{\bar{\mathbf{p}}} J(\bar{\mathbf{p}})\|_1$$

and

$$G(\bar{\mathbf{p}} + \delta) \approx G(\bar{\mathbf{p}}) + \|D \nabla_{\bar{\mathbf{p}}} G(\bar{\mathbf{p}})\|_1$$

- Requires even 2nd derivatives



Counterparts for

- the nominal optimization

$$\begin{cases} \min_{\mathbf{p}(\omega) \in \mathbb{R}^3} \mathbb{E}[J(\mathbf{p}(\omega))] \\ \text{s.t. } \mathbb{E}[G(\mathbf{p}(\omega))] \leq 0 \end{cases}$$

- robustification

$$\begin{cases} \min_{\mathbf{p} \in \mathbb{R}^3} \mathbb{E}[J(\mathbf{p}(\omega))] + \lambda \text{std}[J(\mathbf{p}(\omega))] \\ \text{s.t. } \mathbb{E}[G(\mathbf{p}(\omega))] + \lambda \text{std}[G(\mathbf{p}(\omega))] \leq 0. \end{cases}$$

- Remember: $\mathbf{p}(\omega) = \bar{\mathbf{p}} + \delta'$ with $\delta' = \delta'(\omega) \sim \mathcal{U}(-\Delta, \Delta)$
- Linearization for e.g. $\mathbb{E}[\mathbf{J}(\bar{\mathbf{p}} + \delta')]$

$$\mathbb{E}[\mathbf{J}(\bar{\mathbf{p}} + \delta')] = \mathbf{J}(\bar{\mathbf{p}}) + \int_{-\Delta}^{\Delta} (\delta' \cdot \nabla_{\bar{\mathbf{p}}} \mathbf{J}(\bar{\mathbf{p}})) \varrho(\omega) d\omega + \mathcal{O}(\Delta^2)$$

- Linearization for e.g. $\text{Var}[\mathbf{J}(\bar{\mathbf{p}} + \delta')]$

$$\begin{aligned} \text{Var}[\mathbf{J}(\bar{\mathbf{p}} + \delta')] &= \text{Var}[\delta' \cdot \nabla_{\bar{\mathbf{p}}} \mathbf{J}(\bar{\mathbf{p}})] + \mathcal{O}(\Delta^3) \\ &= \sum_{i=1}^3 \text{Var}[\delta'_i] \left(\frac{\partial \mathbf{J}(\bar{\mathbf{p}})}{\partial \mathbf{p}_i} \right)^2 + \mathcal{O}(\Delta^3) \end{aligned}$$

Comparison Robust Optimizations

- Linearized standard deviation

$$\text{std} [J(\bar{\mathbf{p}}) + \delta'] \approx \|\text{std}[\delta'] \circ \nabla_{\bar{\mathbf{p}}} J(\bar{\mathbf{p}})\|_2$$

- Choose $\lambda := \text{diag}(D_{ii}/\text{std}[\delta_i])$

- Stochastic (“UQ Rob Opt”)

$$\mathbb{E} [J(\mathbf{p}(\omega))] + \lambda \text{std} [J(\mathbf{p}(\omega))] \approx \bar{\mathbf{p}}_1 \bar{\mathbf{p}}_2 + \|D \nabla_{\bar{\mathbf{p}}} J(\bar{\mathbf{p}})\|_2$$

- Deterministic

$$J(\bar{\mathbf{p}} + \delta) \approx \bar{\mathbf{p}}_1 \bar{\mathbf{p}}_2 + \|D \nabla_{\bar{\mathbf{p}}} J(\bar{\mathbf{p}})\|_1$$

Comparison Robust Optimizations

- Linearized standard deviation

$$\text{std} [J(\bar{\mathbf{p}}) + \delta'] \approx \|\text{std}[\delta'] \circ \nabla_{\bar{\mathbf{p}}} J(\bar{\mathbf{p}})\|_2$$

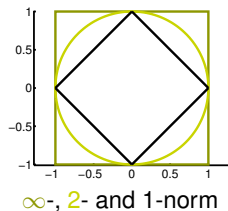
- Choose $\lambda := \text{diag}(D_{ii}/\text{std}[\delta_i])$

- Stochastic (“UQ Rob Opt”)

$$\mathbb{E} [J(\mathbf{p}(\omega))] + \lambda \text{std} [J(\mathbf{p}(\omega))] \approx \bar{\mathbf{p}}_1 \bar{\mathbf{p}}_2 + \|D \nabla_{\bar{\mathbf{p}}} J(\bar{\mathbf{p}})\|_2$$

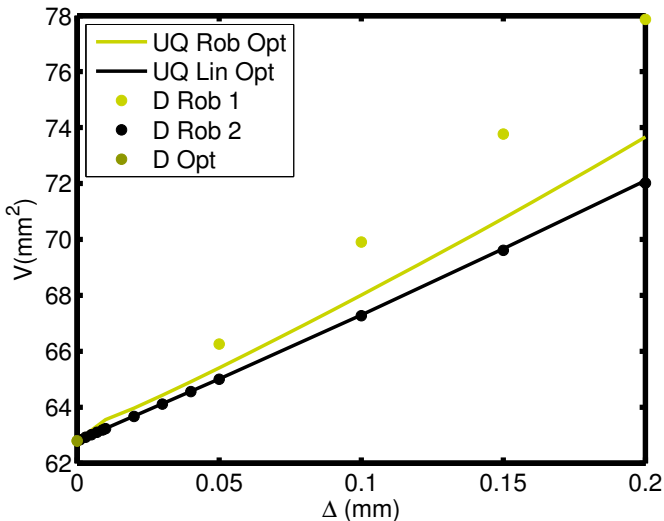
- Deterministic

$$J(\bar{\mathbf{p}} + \delta) \approx \bar{\mathbf{p}}_1 \bar{\mathbf{p}}_2 + \|D \nabla_{\bar{\mathbf{p}}} J(\bar{\mathbf{p}})\|_1$$



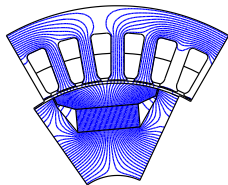
Results

Influence of the deviation

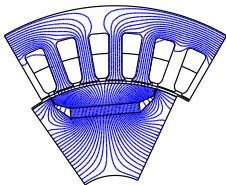


Results

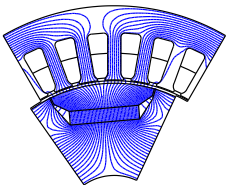
$\Delta = 0.2 \text{ mm}$



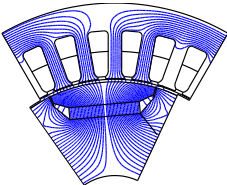
Initial



D opt



D Rob 1



UQ Rob opt

	p(mm)	V (mm²)	E₀ (V)	time (s)	time w. MOR (s+s)
Initial	(19.00, 7.00, 7.00)	133	30.370	-	-
D Opt	(21.07, 2.98, 6.61)	62.80	30.370	2.0	-
UQ Nom Opt (Col)	(21.07, 2.98, 6.61)	62.80	30.370	224	241 + 12
UQ Nom Opt (MC)	(21.06, 2.98, 6.61)	62.83	30.371	9100	241 + 461
D Rob 1	(20.88, 3.73, 6.82)	77.87	31.086	5.9	-
UQ Rob Opt (Col)	(20.87, 3.53, 6.80)	73.66	30.815	239	241 + 13
UQ Rob Opt (MC)	(20.86, 3.53, 6.80)	73.68	30.814	9700	241 + 503

Table of Contents

- 1 Introduction
- 2 Modeling of machines
- 3 UQ and MOR
- 4 Optimization
- 5 Conclusion and Prospects**

Conclusion and Prospects

Conclusion

- Successful reduction of the size of the PM
- Computational efficient procedure due to
 - Affine decomposition
 - Model order reduction
- Equivalence of local and global sensitivities in linearized optimization setting
- Need of fewer derivatives → less intrusive

Prospects

- Error estimator for linearization
→ switch adaptively from linear to nonlinear formulation
- To add uncertain parameters that are not optimization parameters

Thank you for your attention!

Acknowledgments

- This research project is funded by the BMBF **SIMUROM** Verbundprojekt with grant numbers 05M2013.
- and partially supported by the ‘**Excellence Initiative**’ of the German Federal and State Governments and the Graduate School of Computational Engineering at TU Darmstadt.

Please ask questions now or contact

Zeger Bontinck
bontinck@gsc.tu-darmstadt.de
<http://www.gsc.tu-darmstadt.de>

GEFÖRDERT VOM



Bundesministerium
für Bildung
und Forschung

References

- Rahman et al., “Determination of saturated parameters of PM motors using loading magnetic fields”, *IEEE Trans. Magn.*, vol 27, no 5, pp 229-275, 1991.
- Rozza et al., “Reduced Basis approximation and a posteriori error estimation for affinely parametrized elliptic coercive partial differential equations”, *Arch. Comput. Methods. Eng.*, vol 15, pp 229-275, 2008.
- D. Xiu, *Numerical methods for stochastic computations: A spectral method application*, Princeton University Press, 2010.