Thoughts on V p_T Spectrum and W/Z Ratio.

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Outline

Overview.

Extrapolating from Z to W

Goal: Use measurement for Z to get an improved prediction for W

- One way to think about it

$$\frac{\mathrm{d}\sigma(W)}{\mathrm{d}p_T} = \left[\frac{\mathrm{d}\sigma(W)/\mathrm{d}p_T}{\mathrm{d}\sigma(Z)/\mathrm{d}p_T}\right]_{\mathrm{theory}} \times \left[\frac{\mathrm{d}\sigma(Z)}{\mathrm{d}p_T}\right]_{\mathrm{measured}}$$

  - Requires theory prediction for W/Z ratio to be more precise than for individual processes, which is equivalent to theory uncertainties being strongly correlated between processes
- More general: Use common theory framework and fit to Z data
  - Tuning Pythia on Z data is one example
  - Requires explicit information on correlations between processes

⇒ Either way, extrapolation hinges entirely on correlations of theory uncertainties between $\mathrm{d}\sigma(Z)$ and $\mathrm{d}\sigma(W)$

- Correlations, hm? Often we don't even know what our theory uncertainties really mean ...

⇒ At sub-% level many things matter

- Dominant and well-understood parts (mostly) drop out in W/Z ratio, exposing all the things we normally like to sweep under the rug

Outline and Summary.

	Uncertainty or size	Analytic resummation	Pythia	Leftover effect on W/Z
Leading-power resummation	5-10%	√√√	√	$\lesssim$ % (?)
Power corrections	few %	(×)	(√)?	?
Nonperturbative	few %	(√)	(√)	?
Massive quarks	few % (?)	× (√)	(√)?	few % (?)
QED	$\lesssim$ % (?)	×	√ (?)	$\lesssim$ % (?)
PDFs	2%	√	√	√
$\alpha_s(m_Z)$	up to 5%??	√	√	√

- Most ? could be addressed (and some just mean that I don't know ...)
- Though it is a bit unsettling it is not unbelievable that plain Pythia currently seems to describe the W/Z ratio better
  - We should of course try to get beyond that
  - Question of the uncertainty when used as prediction for W remains

Perturbative Structure – Singular vs. Nonsingular.

Consider both differential and cumulative distribution

- Define scaling variable $\tau \equiv p_T^2/Q^2$ and $\sigma(\tau^{\mathrm{cut}}) = \int^{\tau^{\mathrm{cut}}} \mathrm{d}\tau \, \frac{\mathrm{d}\sigma}{\mathrm{d}\tau}$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\tau} = \sum_k \alpha_s^k \left\{ c_{k,-1}\,\delta(\tau) + \sum_{n=0}^{2k-1} c_{kn} \left[\frac{\ln^n \tau}{\tau}\right]_+ \qquad + \quad f_k^{\mathrm{nons}}(\tau) \right\}$$

$$\sigma(\tau^{\mathrm{cut}}) = \sum_k \alpha_s^k \left\{ \underbrace{c_{k,-1} + \sum_{n=0}^{2k-1} c_{kn} \frac{\ln^{n+1} \tau^{\mathrm{cut}}}{n+1}}_{\text{“singular”}} \qquad + \quad \underbrace{F_k^{\mathrm{nons}}(\tau^{\mathrm{cut}})}_{\text{“nonsingular”}} \right\}$$

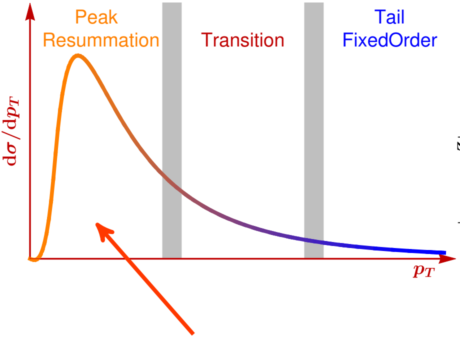
singular: leading-power terms

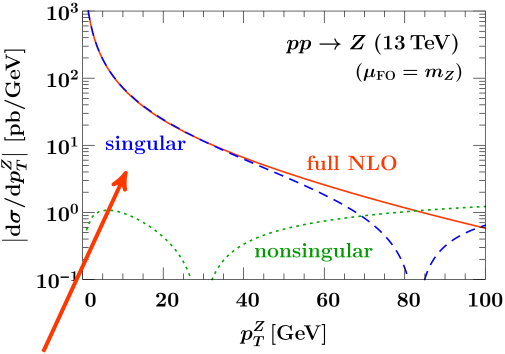
- to be resummed
- $c_{k,-1}$ contains k-loop virtuals
  (i.e. finite remainder after real-virtual cancellation)

nonsingular: power corrections

- suppressed by relative $\mathcal{O}(\tau)$
- $\tau f_k^{\mathrm{nons}}(\tau)$ and $F_k^{\mathrm{nons}}(\tau^{\mathrm{cut}})$ vanish for $\tau^{(\mathrm{cut})} \to 0$

Different Perturbative Regions.

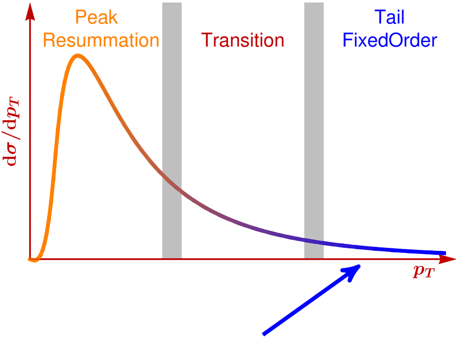


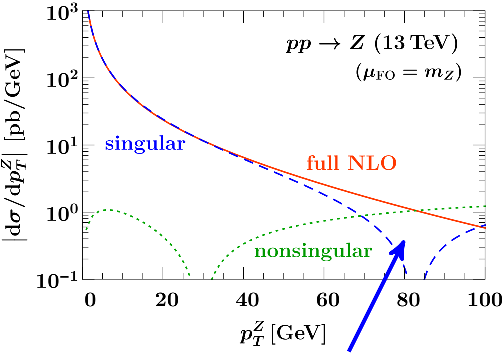


Resummation region

- Spectrum at low $p_T \ll Q$ and cross section with cut $p_T^{\mathrm{cut}} \ll Q$
  - Singular dominate and must be resummed (nonsingular are power-suppressed)
  - Fixed-order by itself becomes meaningless here
  - In MC: Parton shower regime

Different Perturbative Regions.

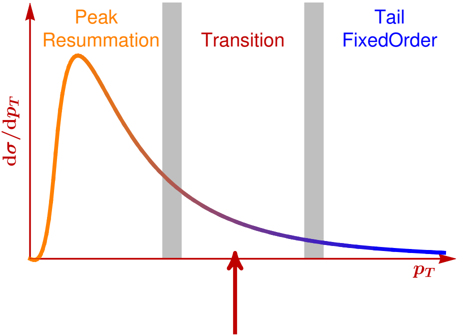


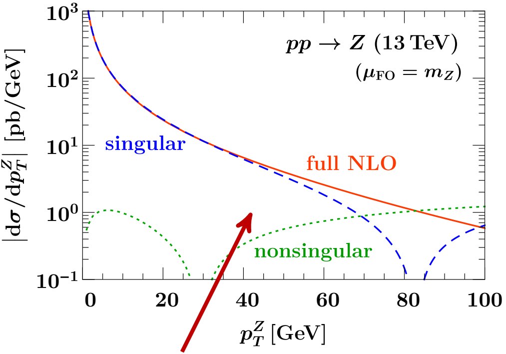


Fixed-order region

- Spectrum at high $p_T \sim Q$
  - Fixed-order calculation for inclusive V+1-jet process
  - In MC: Fixed-order matrix elements
  - Power expansion breaks down and resummation must be turned off

Different Perturbative Regions.





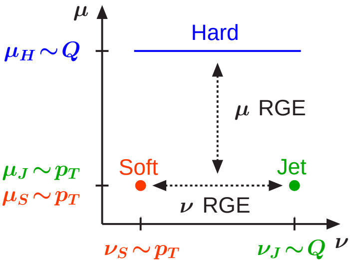
Transition region

- Anything in between (there are no fixed boundaries)
- Resummation still makes sense, fixed-order expansion also still works
  - Most precise predictions are obtained from consistent combination of resummation and fixed-order
  - In MC: This is where ME+PS matching/merging comes in

Resummation.

Leading-power p_T spectrum factorizes into hard, collinear, and soft contributions

$$\frac{\mathrm{d}\sigma^{\mathrm{sing}}}{\mathrm{d}\vec{p}_T} = \sigma_0\, H(Q,\mu) \int \mathrm{d}^2\vec{k}_a\, \mathrm{d}^2\vec{k}_b\, \mathrm{d}^2\vec{k}_s \\ \times B_a(\vec{k}_a,\mu,\nu)\, B_b(\vec{k}_b,\mu,\nu) \\ \times S(\vec{k}_s,\mu,\nu)\, \delta(\vec{p}_T - \vec{k}_a - \vec{k}_b - \vec{k}_s)$$



All-order structure of leading-power terms is fully determined by coupled system of differential equations (including their boundary conditions)

- in virtuality scale μ

$$\mu \frac{\mathrm{d}H(Q,\mu)}{\mathrm{d}\mu} = \gamma_H(Q,\mu)\, H(Q,\mu)$$

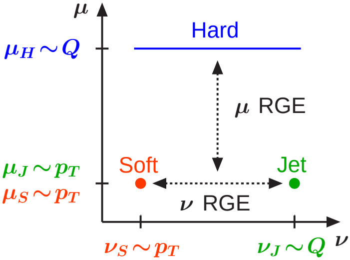
$$\mu \frac{\mathrm{d}B(\vec{p}_T,\mu,\nu)}{\mathrm{d}\mu} = \gamma_B(\mu,\nu)\, B(\vec{p}_T,\mu,\nu)$$

$$\mu \frac{\mathrm{d}S(\vec{p}_T,\mu,\nu)}{\mathrm{d}\mu} = \gamma_S(\mu,\nu)\, S(\vec{p}_T,\mu,\nu)$$

Resummation.

Leading-power p_T spectrum factorizes into hard, collinear, and soft contributions

$$\begin{aligned}\frac{\mathrm{d}\sigma^{\mathrm{sing}}}{\mathrm{d}\vec{p}_T} = \sigma_0\, H(Q,\mu) \int \mathrm{d}^2\vec{k}_a\, \mathrm{d}^2\vec{k}_b\, \mathrm{d}^2\vec{k}_s \\ \times B_a(\vec{k}_a,\mu,\nu)\, B_b(\vec{k}_b,\mu,\nu) \\ \times S(\vec{k}_s,\mu,\nu)\, \delta(\vec{p}_T - \vec{k}_a - \vec{k}_b - \vec{k}_s)\end{aligned}$$



All-order structure of leading-power terms is fully determined by coupled system of differential equations (including their boundary conditions)

- and rapidity scale ν (or ζ)

$$\nu \frac{\mathrm{d}B(\vec{p}_T,\mu,\nu)}{\mathrm{d}\nu} = -\frac{1}{2} \int \mathrm{d}^2\vec{k}_T\, \gamma_\nu(\vec{k}_T,\mu)\, B(\vec{p}_T - \vec{k}_T,\mu,\nu)$$

$$\nu \frac{\mathrm{d}S(\vec{p}_T,\mu,\nu)}{\mathrm{d}\nu} = \int \mathrm{d}^2\vec{k}_T\, \gamma_\nu(\vec{k}_T,\mu)\, S(\vec{p}_T - \vec{k}_T,\mu,\nu)$$

$$\mu \frac{\mathrm{d}}{\mathrm{d}\mu}\gamma_\nu(\vec{k}_T,\mu) = \nu \frac{\mathrm{d}}{\mathrm{d}\nu}\gamma_S(\mu,\nu)\delta(\vec{k}_T) = -4\Gamma_{\mathrm{cusp}}[\alpha_s(\mu)]\delta(\vec{k}_T)$$

Resummation Orders.

Analytic resummation amounts to solving this system of differential equations

- Formal resummation accuracy is fundamentally defined by perturbative input used for anomalous dimensions and boundary conditions
  - In Fourier space (as in standard CSS) solution is a pure exponential and resummation orders map onto common counting of logs in the exponent
- Current perturbative uncertainties at NNLL′+NNLO at 5-10% level
  - $\mathrm{N^3LL}$ is available but not full $\mathrm{N^3LL'}$+$\mathrm{N^3LO}$, hard to see it can go below 2%
  - Compare: Thrust spectrum in $e^+e^- \to q\bar{q}$ at $Q = m_Z$ has ≃ 2% precision at $\mathrm{N^3LL'}$+$\mathrm{N^3LO}$

	Boundary conditions	Anomalous dimensions		FO matching
	(singular)	$\gamma_{H,B,S,\nu}$	$\Gamma_{\mathrm{cusp}}, \beta$	(nonsingular)
NLL	1	1-loop	2-loop	-
$\mathrm{NLL^{(\prime)}}$+NLO	α_s	1-loop	2-loop	α_s
NNLL+NLO	α_s	2-loop	3-loop	α_s
$\mathrm{NNLL^{(\prime)}}$+NNLO	α_s^2	2-loop	3-loop	α_s^2
$\mathrm{N^3LL}$+NNLO	α_s^2	3-loop	4-loop	α_s^2
$\mathrm{N^3LL^{(\prime)}}$+$\mathrm{N^3LO}$	α_s^3	3-loop	4-loop	α_s^3

Variety of Resummation Approaches/Implementations.

[Collins, Soper, Sterman (CSS); Balazs, Berge, Isaacson, Nadolsky, Olness, Su, C-P Yuan, F Yuan (ResBos); Bozzi, Catani, de Florian, Ferrera, Grazzini (DYRes); Becher, Luebbert, Neubert, Wilhelm (CuTe); Neill, Rothstein, Vaidya; D'Alesio, Echevarria, Idilbi, Kang, Melis, Scimemi, Vladimirov, Vitev; Monni, Re, Torrielli; Ebert, Stewart, FT, Zhu; ...]

- Differences in precise choices (and/or additional approximations)
  - Boundary conditions to the solution (starting point of the evolution)
  - How resummation is turned off (endpoint of the evolution)
  - Treatment of power corrections (matching to full fixed order)
  - Treatment of nonperturbative corrections
- Choices are (mostly) beyond formal accuracy, but can matter for numerical results and perturbative uncertainties/precision
  - In the end, precision is given by the size of the perturbative uncertainties, *but only if* they are estimated to that purpose (i.e. to cover all-order result)

Known Unknowns.

Resummation for W/Z Ratio.

$$R(p_T) = \frac{\mathrm{d}\sigma(W)/\mathrm{d}p_T}{\mathrm{d}\sigma(Z)/\mathrm{d}p_T}$$

- There is no direct resummation formula for $R(p_T)$
    - $R(p_T)$ is always derived from individual resummed spectra
    - Need to know correlations anyway
    - There is no real advantage of using $R(p_T)$ over more general common theory framework that is not restricted to this specific combination

Intrinsic differences that already exist at leading power

- Vector and axial currents differ by singlet terms starting at NNLL′
    - Often neglected since they are tiny in inclusive cross section
- Different flavor mix for W and Z
- Different Q ranges
    - $m_W \neq m_Z$, nonnegligible photon contribution at low Q
    - $R(p_T)$ is actually not ideal, $R(p_T/Q)$ would already be better
    - In principle, common theory framework takes care of this

Power Corrections.

$$\tau \frac{d\sigma}{d\tau} = \underset{\sim \mathcal{O}(1)}{\tau \frac{d\sigma^{\text{resum}}}{d\tau}} + \underset{\sim \mathcal{O}(\tau)}{\tau \frac{d\sigma^{\text{nons}}}{d\tau}} \qquad \text{with} \quad \tau = \frac{p_T^2}{Q^2}$$

- Resummation only captures $\mathcal{O}(1)$ leading-power
- $\mathcal{O}(\tau)$ power corrections are only known and included at fixed order

Usually small at small p_T, but there are some caveats

- Could get enhanced in W/Z ratio when leading-power terms cancel
- They also contain large logs

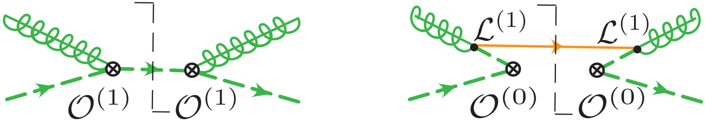
$$\tau \frac{d\sigma^{\text{nons}}}{d\tau} \sim \left[\alpha_s \tau (1 + \ln \tau) + \alpha_s^2 \tau (\; 1 \; + \ln \tau \; + \ln^2 \tau + \ln^3 \tau) + \cdots \right] + \mathcal{O}(\tau^2)$$

$$\text{e.g. for } \tau = 0.01 \qquad \sim \alpha_s^2 (0.01 + 0.05 + 0.21 \; + 0.98)$$

  - They are only $\mathcal{O}(\tau)$ power-suppressed if they are being resummed as well
  - p_T resummation at subleading power is much more complicated and currently not available even at LL

Power Corrections.

[Moult, Rothen, Stewart, FT, Zhu '16]



- New contributions appear at subleading power already at LL that have no leading-power analog (e.g. soft quarks)
  - gq channels contribute at LL, can be as large as $q\bar{q}$ channels
  - Different color structure at LL: C_F^2 vs. $T_F(C_F + C_A)$
  - Multiplying nonsingular by leading-power Sudakov exponent is not correct even at LL
- Numerically important type of contribution are "kinematic" power corrections that depend on PDF derivatives $x f_q'(x)$
  - Describe the effect that PDFs also need to provide small momentum components for p_T recoil
  - Might in fact be captured reasonably well in Pythia due to it enforcing momentum conservation at each splitting
  - Less likely to cancel in W/Z ratio

Nonperturbative Effects.

Nonperturbative corrections can be treated in field theory based on singular factorization theorem

- In principle there are flavor-independent and flavor-dependent effects (though the latter are often neglected)
  - Cause few-% uncertainty at $p_T = 5\,\mathrm{GeV}$, quickly increase below that
  - Should at least partially cancel in W/Z ratio
- For $\Lambda^2_{\mathrm{QCD}} \ll p_T^2$ (peak and above)
  - Can be expanded in powers of $\Lambda^2_{\mathrm{QCD}}/p_T^2 \sim \Lambda^2_{\mathrm{QCD}} b^2$ and parametrized by nonperturbative coefficients of first correction
  - Typically done in b space, but equivalently possible in physical p_T space
  - Parameters can be fitted from DY data, including low-energy data

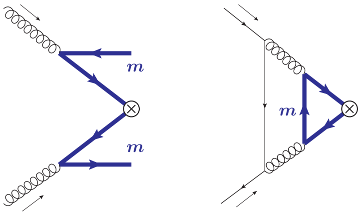
    [see e.g. Echevarria, Idilbi, Kang, Vitev '14; Su, Isaacson, C-P Yuan, F Yuan '14; D'Alesio, Echevarria, Melis, Scimemi '14; ...]
- For $\Lambda^2_{\mathrm{QCD}} \sim p_T^2$ (below peak)
  - Requires full shape of nonperturbative TMDPDF
- In Pythia modelled primarily through primordial/intrinsic k_T (flavor-blind)
  - Also nontrivial interplay with ISR shower parameters (cutoff, α_s^{ISR})

$\Rightarrow$ More work needed to draw firm conclusions for W/Z ratio

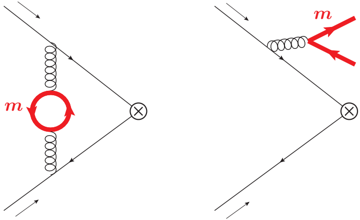
Massive Quark Effects.

[Pietrulewicz, Samitz, Spiering, FT; arXiv:1703.09702]

“Primary” mass effects at fixed order



“Secondary” mass effects at fixed order



Multi-scale problem with several possible scale hierarchies

- p_T distribution goes through different regimes
  - $\Lambda_{\mathrm{QCD}} \ll p_T \ll m_b \ll Q$: heavy quark decouples (4FS for $m_b \sim Q$)
  - $\Lambda_{\mathrm{QCD}} \ll p_T \sim m_b \ll Q$: quark mass changes resummation structure (including nonperturbative effects)
  - $\Lambda_{\mathrm{QCD}} \ll m_b \ll p_T \ll Q$: massless limit (usual 5FS)
- Formally enter at NNLL′ for $b\bar{b} \to Z$ and at NLL′ for $c\bar{s} \to W$

$\Rightarrow$ Few-% level effects, primary mass effects do not cancel in W/Z ratio

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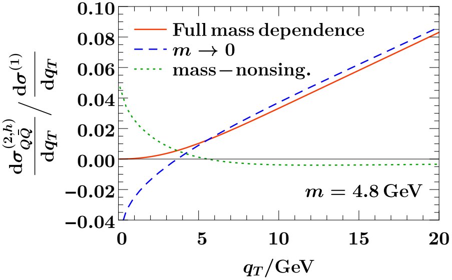

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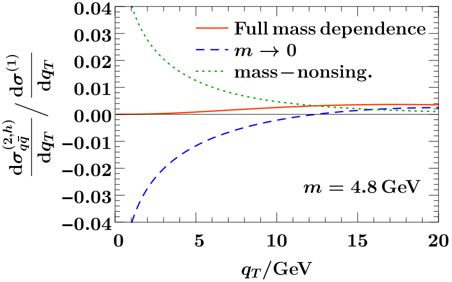

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QED Corrections.

Many studies on QED/EW and mixed QCD-QED/EW effects

I'm not actually aware of a dedicated study of QED effects in the context of analytic resummed calculations (could just be my own ignorance)

- All resummation ingredients (boundary conditions, anomalous dimensions) receive corrections from soft and collinear photon radiation
  - Relative parametric size of $\mathcal{O}(\alpha_{\mathrm{em}}/\alpha_s) \sim \mathcal{O}(\%)$
  - Effects will clearly not drop out of W/Z ratio
- Combined QCD+QED shower in Pythia should captures the dominant effect from this

$\Rightarrow$ Should be straightforward to evaluate/incorporate
(at least when one ignores initial-final-state interference)

Theory Uncertainties.

What is a Scale Variation?

It is an easy way to obtain (slightly) different expansions for the same quantity

$$\epsilon = \alpha_s(\mu) \qquad \to \qquad \sigma = c_0 + \epsilon\, c_1 + \epsilon^2\, c_2 + \cdots$$
$$\tilde\epsilon = \alpha_s(\tilde\mu) \qquad \to \qquad \sigma = c_0 + \tilde\epsilon\, \tilde c_1 + \tilde\epsilon^2\, \tilde c_2 + \cdots$$

- The full result is the same and independent of the choice of ϵ vs. $\tilde\epsilon$
  - We only know the first few orders, which do depend on the choice
  - Comparing both expansions *might* provide a way to estimate the typical size of the missing $\epsilon^3 c_3 + \cdots$ terms
  - It also *might not*, because it only knows about the structures present in c_1 and c_2 and so cannot estimate the effect of possible new structures appearing in c_3 and beyond
- Differential spectra complicate things further
  - Resummation scales often have quadratic dependence from double logs
  - Scale variations typically cross each other or the central result at some point in the spectrum

Correlations.

$$\mathrm{d}\sigma(W)/\mathrm{d}p_T = c_0(p_T) + \epsilon\, c_1(p_T) + (\epsilon^2\, c_2(p_T) + \cdots)$$

$$\mathrm{d}\sigma(Z)/\mathrm{d}p_T = d_0(p_T) + \epsilon\, d_1(p_T) + (\epsilon^2\, d_2(p_T) + \cdots)$$

QCD corrections for W and Z are *largely* the same but also *not entirely*

- Using correlated scale variations for both processes
  - Scale dependence will largely cancel in their ratio (easily factor 10 or more)
  - Possible differences between processes at higher order are precisely not probed by scale variations
  - ⇒ Left-over scale dependence has little to no meaning in terms of uncertainties

Correlations only come from common sources of uncertainties

- QCD scales are not physical parameters
  - They do not have an uncertainty that can be propagated
  - They also cannot be regarded as the fundamental sources of uncertainties, i.e. they cannot be used as nuisance parameters to imply correlations
  - A priori, they do not imply anything about correlations among different processes or different kinematic regions

⇒ In short, scale variations are intrinsically ill-suited for this

A New Idea.

[Disclaimer: very much work in progress ...]

Imagine we had actual nuisance parameters for perturbative uncertainties

- Would provide immediate solution to the two key problems
  - Provide true correlations between different processes
  - Can be constrained by data, and therefore allows one to fully consistently use Z measurements to reduce theory uncertainties in W predictions
- I think this is possible with a small number of unambiguous parameters (at least for resummed leading-power contributions)
- Disadvantages and open issues
  - Going to be much more involved to implement
  - Feasibility for fitting (flat directions, ...)
  - Must be thoroughly tested/validated

⇒ I'd be happy to discuss this and get your feedback during next days

Backup Slides

Perturbative Accuracy (Oversimplified).

Terms in the cross section that are reproduced at some resummation order (not the definition of the order) with $\tau = p_T^2/Q^2$, $L = \ln\tau$, $L_{\rm cut} = \ln\tau^{\rm cut}$

$$\begin{array}{lcccccccccr}
 & \text{LL} & & \text{NLL} & & \text{NLL}' & & \text{NNLL} & & & \\
\dfrac{\sigma(\tau^{\rm cut})}{\sigma_B} = & 1 & & & & & & & & & \text{LO} \\
+\ \alpha_s\big[& \frac{c_{11}}{2}L_{\rm cut}^2 & + & c_{10}L_{\rm cut} & + & c_{1,-1} & + & & & F_1^{\rm nons}(\tau^{\rm cut})\big] & \text{NLO} \\
+\ \alpha_s^2\big[& \vdots & + & \vdots & + & \vdots & + & \vdots & & & \\
 & & & & & & & & & & \\
\dfrac{1}{\sigma_B}\dfrac{d\sigma}{d\tau} = \alpha_s/\tau\big[& c_{11}L & + & c_{10} & + & & & & & \tau f_1^{\rm nons}(\tau)\big] & \text{LO}_1 \\
+\,\alpha_s^2/\tau\big[& c_{23}L^3 & + & c_{22}L^2 & + & c_{21}L & + & c_{20} & + & \tau f_2^{\rm nons}(\tau)\big] & \text{NLO}_1 \\
+\,\alpha_s^3/\tau\big[& \vdots & + & \vdots & + & \vdots & + & \vdots & & &
\end{array}$$

- Lowest perturbative accuracy at all p_T requires (N)LL+LO$_1$
  - Provided by LO ME+PS, also plain Pythia (has full ME for first emission)
  - LO is naturally part of LL and so automatically included

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+ & \alpha_s^2\big[& \vdots & + & \vdots & + & \vdots & + & \vdots & & & \\
\dfrac{1}{\sigma_B}\dfrac{\mathrm{d}\sigma}{\mathrm{d}\tau} = & \alpha_s/\tau\big[& c_{11}L & + & c_{10} & + & & & & & \tau f_1^{\mathrm{nons}}(\tau)\big] & \text{LO}_1 \\
+ & \alpha_s^2/\tau\big[& c_{23}L^3 & + & c_{22}L^2 & + & c_{21}L & + & c_{20} & + & \tau f_2^{\mathrm{nons}}(\tau)\big] & \text{NLO}_1 \\
+ & \alpha_s^3/\tau\big[& \vdots & + & \vdots & + & \vdots & + & \vdots & & &
\end{array}
$$

- NLO+PS matching (MC@NLO, POWHEG) adds full NLO to $\sigma(\tau^{\mathrm{cut}})$
  - Improves accuracy for $\sigma(\tau^{\mathrm{cut}} \sim 1)$ (incl. cross section) to NLO
  - Does not automatically improve formal accuracy of spectrum beyond ME+PS

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 & \text{LL} & & \text{NLL} & & \text{NLL}' & & \text{NNLL} & & & \\
\dfrac{\sigma(\tau^{\rm cut})}{\sigma_B} = & 1 & & & & & & & & & \text{LO} \\
+\ \alpha_s\big[& \frac{c_{11}}{2}L_{\rm cut}^2 & + & c_{10}L_{\rm cut} & + & c_{1,-1} & + & & & F_1^{\rm nons}(\tau^{\rm cut})\big] & \text{NLO} \\
+\ \alpha_s^2\big[& \vdots & + & \vdots & + & \vdots & + & \vdots & & & \\
\dfrac{1}{\sigma_B}\dfrac{d\sigma}{d\tau} = \alpha_s/\tau\big[& c_{11}L & + & c_{10} & + & & & & & \tau f_1^{\rm nons}(\tau)\big] & \text{LO}_1 \\
+\alpha_s^2/\tau\big[& c_{23}L^3 & + & c_{22}L^2 & + & c_{21}L & + & c_{20} & + & \tau f_2^{\rm nons}(\tau)\big] & \text{NLO}_1 \\
+\alpha_s^3/\tau\big[& \vdots & + & \vdots & + & \vdots & + & \vdots & & &
\end{array}$$

- NLL′ and NNLL fully incorporate 1-loop virtuals ($c_{1,-1}$) into resummation and therefore naturally match to NLO
- Similarly NNLL′ and N^{3}LL incorporate 2-loop virtuals and match to NNLO

PDFs and α_s

PDFs

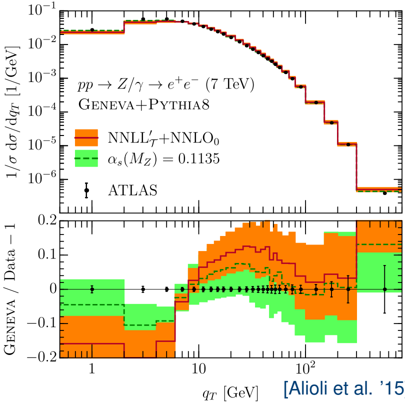
- $\sim$ 2% uncertainty at low p_T, mostly affect normalization and not shape

$\Rightarrow$ Physical parameters so in principle straightforward to take into account correlations for W/Z ratio

To be aware of: $\alpha_s(m_Z)$

- p_T tail is $\sim \alpha_s$ and α_s also appears in resummation
- Various extractions clearly favor much lower values than PDG average
  - In particular thrust in e^+e^- with high resummation
- Changing $\alpha_s(m_Z) = 0.118 \to 0.114$ has $\sim$ 5% effect on p_T spectrum

$\Rightarrow$ Should drop out of W/Z ratio (and also easy to propagate through)



[Alioli et al. '15]