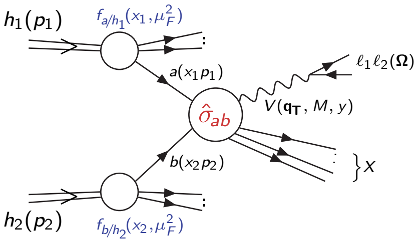
Transverse-momentum resummation for vector boson production

Giancarlo Ferrera

Milan University & INFN Milan

M_W working Workshop – Paris – October 2 2017

Drell–Yan $\mathbf{q_T}$ distribution



$$\mathbf{h_1(p_1) + h_2(p_2) \;\rightarrow\; V + X \;\rightarrow\; \ell_1 + \ell_2 + X}$$

where $\quad V = Z^0/\gamma^*, W^\pm$

QCD factorization formula:

$$\frac{d\sigma}{d^2\mathbf{q_T}\, dM^2\, dy\, d\mathbf{\Omega}} = \sum_{a,b} \int_0^1 dx_1 \int_0^1 dx_2\; f_{a/h_1}(x_1, \mu_F^2)\; f_{b/h_2}(x_2, \mu_F^2)\; \frac{d\hat{\sigma}_{ab}}{d^2\mathbf{q_T}\, dM^2\, d\hat{y}\, d\mathbf{\Omega}}(\hat{s}; \alpha_S, \mu_R^2, \mu_F^2).$$

Fixed-order perturbative expansion reliable

only for $q_T \sim M$. When $q_T \ll M$:

$$\int_0^{q_T^2} d\bar{q}_T^2 \frac{d\hat{\sigma}_{q\bar{q}}}{d\bar{q}_T^2} \sim 1 + \alpha_S \left[c_{12} L_{q_T}^2 + c_{11} L_{q_T} + \cdots \right]$$

$$+\alpha_S^2 \left[c_{24} L_{q_T}^4 + \cdots + c_{21} L_{q_T} + \cdots \right] + \mathcal{O}(\alpha_S^3)$$

with $\alpha_S^n L_{q_T}^m \equiv \alpha_S^n \log^m(M^2/q_T^2) \gg 1$.

Resummation of logarithmic corrections needed.

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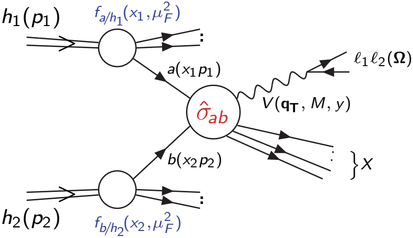

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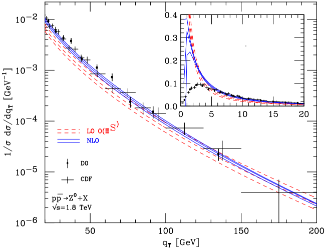
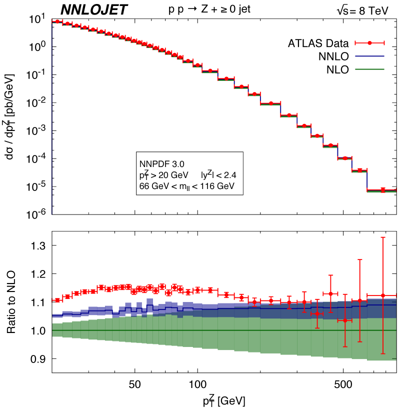


NNLO QCD predictions at large q_T



Z q_T spectrum ($q_T > 20$ GeV).

→ see A. Huss talk

- ATLAS data ($\sqrt{s} = 8$ TeV) [1512.02192] (2.8% luminosity uncertainty not shown).
- NNLO (i.e.$\mathcal{O}(\alpha_S^3)$) QCD predictions [G.-De Ridder,Gehrmann,Glover,Huss, Morgan('16)]. NNLO correction positive ($\sim$6-8%) and reduce scale dependence (factor 2 around $\mu = \sqrt{M^2 + q_T^2}$).
- Agreement between data and theory improves by considering normalized distributions.

In the small q_T region effects of soft-gluon resummation are essential

At the LHC 90% of the $W^{\pm}$ and Z^0 are produced with $q_T \lesssim 20$ GeV

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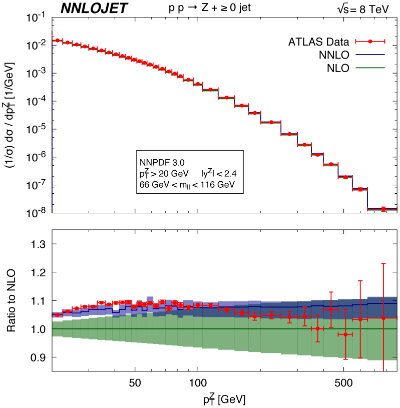

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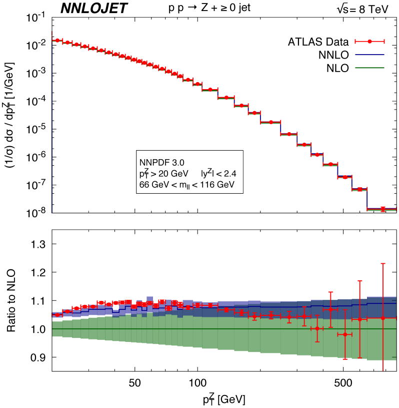

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$$\frac{d\hat{\sigma}}{d^2\mathbf{q_T}\, dM^2\, d\hat{y}\, d\mathbf{\Omega}} = \left[d\hat{\sigma}^{(res)}\right] + \left[d\hat{\sigma}^{(fin)}\right];$$

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$$\left[d\hat{\sigma}^{(res)}\right] = \frac{d\hat{\sigma}^{(0)}}{d\mathbf{\Omega}} \frac{1}{\hat{s}} \int \frac{d^2\mathbf{b}}{4\pi^2} e^{i\mathbf{b}\cdot\mathbf{q_T}}\, \mathcal{W}(b, M, \hat{y}, \hat{s}),$$

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$$g^{(1)}(\alpha_S L) = \frac{A^{(1)}}{\beta_0} \frac{\lambda + \ln(1-\lambda)}{\lambda} \quad ,$$

$$g_N^{(2)}\left(\alpha_S L; \frac{M^2}{\mu_R^2}, \frac{M^2}{Q^2}\right) = \frac{\overline{B}_N^{(1)}}{\beta_0} \ln(1-\lambda) - \frac{A^{(2)}}{\beta_0^2}\left(\frac{\lambda}{1-\lambda} + \ln(1-\lambda)\right)$$
$$+ \frac{A^{(1)}}{\beta_0}\left(\frac{\lambda}{1-\lambda} + \ln(1-\lambda)\right) \ln \frac{Q^2}{M^2}$$
$$+ \frac{A^{(1)}\beta_1}{\beta_0^3}\left(\frac{1}{2}\ln^2(1-\lambda) + \frac{\ln(1-\lambda)}{1-\lambda} + \frac{\lambda}{1-\lambda}\right) \quad ,$$

$$g_N^{(3)}\left(\alpha_S L; \frac{M^2}{\mu_R^2}, \frac{M^2}{Q^2}\right) = -\frac{A^{(3)}}{2\beta_0^2}\frac{\lambda^2}{(1-\lambda)^2} - \frac{\overline{B}_N^{(2)}}{\beta_0}\frac{\lambda}{1-\lambda} + \frac{A^{(2)}\beta_1}{\beta_0^3}\left(\frac{\lambda(3\lambda-2)}{2(1-\lambda)^2} - \frac{(1-2\lambda)\ln(1-\lambda)}{(1-\lambda)^2}\right)$$
$$+ \frac{\overline{B}_N^{(1)}\beta_1}{\beta_0^2}\left(\frac{\lambda}{1-\lambda} + \frac{\ln(1-\lambda)}{1-\lambda}\right) - \frac{A^{(1)}}{2}\frac{\lambda^2}{(1-\lambda)^2}\ln^2\frac{Q^2}{M^2}$$
$$+ \ln\frac{Q^2}{M^2}\left(\overline{B}_N^{(1)}\frac{\lambda}{1-\lambda} + \frac{A^{(2)}}{\beta_0}\frac{\lambda^2}{(1-\lambda)^2} + A^{(1)}\frac{\beta_1}{\beta_0^2}\left(\frac{\lambda}{1-\lambda} + \frac{1-2\lambda}{(1-\lambda)^2}\ln(1-\lambda)\right)\right)$$
$$+ A^{(1)}\left(\frac{\beta_1^2}{2\beta_0^4}\frac{1-2\lambda}{(1-\lambda)^2}\ln^2(1-\lambda) + \ln(1-\lambda)\left[\frac{\beta_0\beta_2 - \beta_1^2}{\beta_0^4} + \frac{\beta_1^2}{\beta_0^4(1-\lambda)}\right]\right.$$
$$\left. + \frac{\lambda}{2\beta_0^4(1-\lambda)^2}(\beta_0\beta_2(2-3\lambda) + \beta_1^2\lambda)\right) \quad ,$$

q_T resummation in QCD

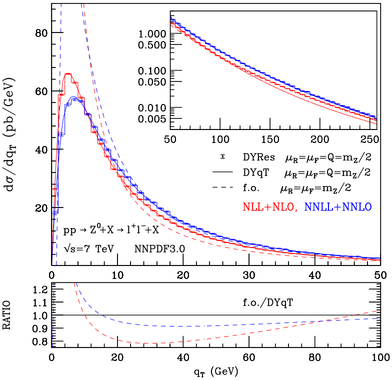
$$g^{(1)}(\alpha_S L) = \frac{A^{(1)}}{\beta_0} \frac{\lambda + \ln(1-\lambda)}{\lambda} \quad ,$$

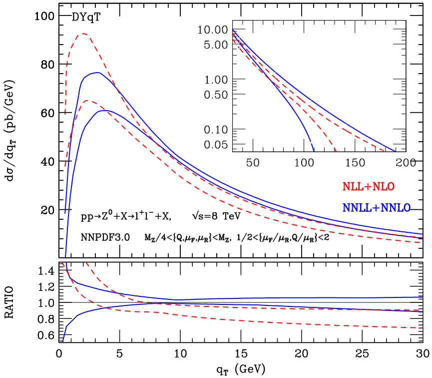
$$g_N^{(2)}\left(\alpha_S L; \frac{M^2}{\mu_R^2}, \frac{M^2}{Q^2}\right) = \frac{\overline{B}_N^{(1)}}{\beta_0} \ln(1-\lambda) - \frac{A^{(2)}}{\beta_0^2}\left(\frac{\lambda}{1-\lambda} + \ln(1-\lambda)\right)$$

$$+ \frac{A^{(1)}}{\beta_0}\left(\frac{\lambda}{1-\lambda} + \ln(1-\lambda)\right) \ln \frac{Q^2}{M^2}$$

$$+ \frac{A^{(1)}\beta_1}{\beta_0^3}\left(\frac{1}{2}\ln^2(1-\lambda) + \frac{\ln(1-\lambda)}{1-\lambda} + \frac{\lambda}{1-\lambda}\right) \quad ,$$

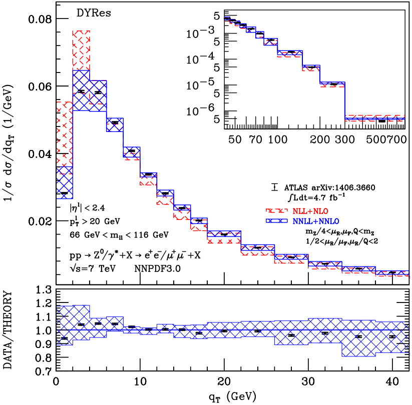
q_T spectrum of the Z boson

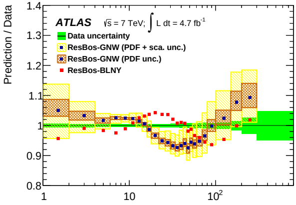


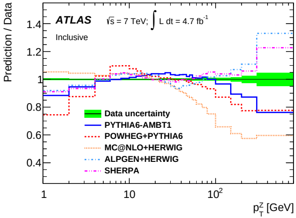


NLL+NLO and NNLL+NNLO Z q_T spectrum at the LHC at $\sqrt{s} = 7/8$ TeV.

q_T spectrum of Z boson: theory vs ATLAS data



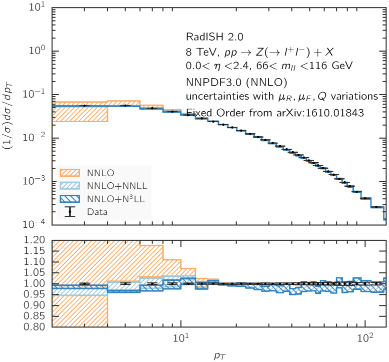




Left: NLL+NLO and NNLL+NNLO bands for Z/γ^* q_T spectrum compared with and ATLAS data (7 TeV).
Right Top: Ratios between ResBos predictions and ATLAS data.
Right Bottom: Ratios between various MC generators results and ATLAS data.

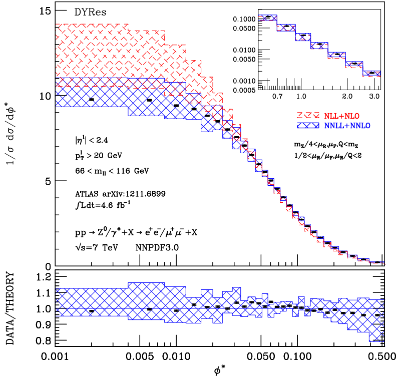
q_T spectrum of Z boson: theory vs ATLAS data

Results from W. Bizon, P. Monni, E. Re, L. Rottoli, P. Torrielli

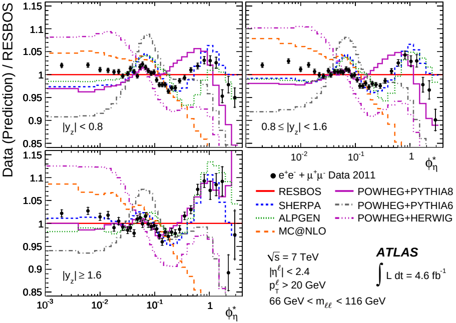


NNLO,NLL+NNLO and N3LL+NNLO bands for Z/γ^* q_T spectrum compared with and ATLAS data (7 TeV). Matching with $\mathcal{O}(\alpha_S^3)$ at large q_T.

ϕ^* spectrum of Z boson: theory vs ATLAS data

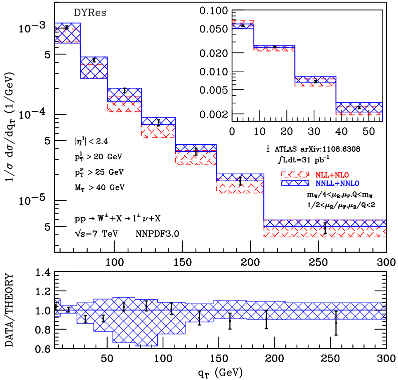


NLL+NLO and NNLL+NNLO bands for Z/γ^* ϕ^* spectrum compared with ATLAS data.

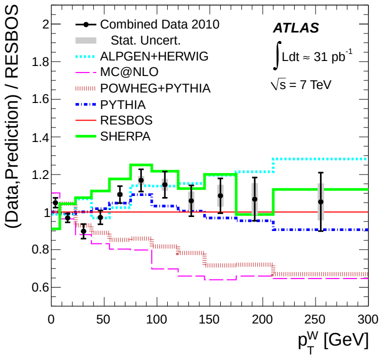


MC generators results and ATLAS data ratio to ResBos.

q_T spectrum of W: theory vs ATLAS data

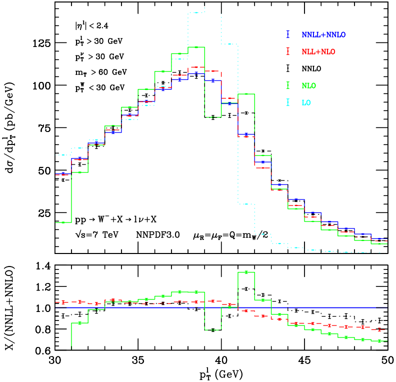


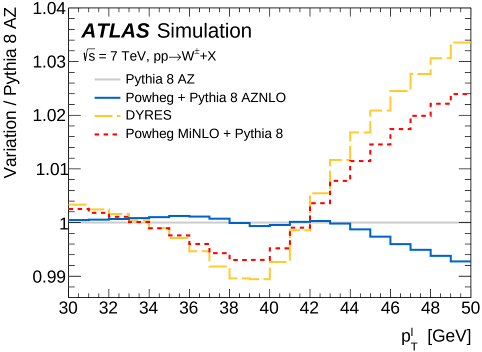
NLL+NLO and NNLL+NNLO bands for $W^\pm$ q_T spectrum compared with ATLAS data.



MC generators results and ATLAS data ratio to ResBos.

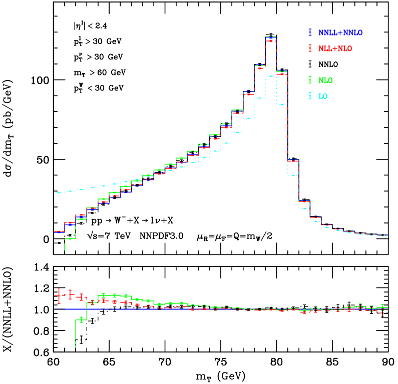
Lepton p_T distributions from W decay

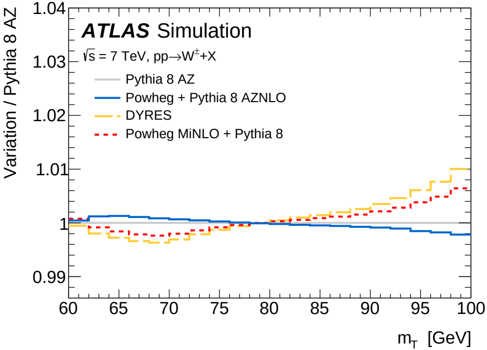




Ratios of the lepton p_T normalised distribution obtained using Powheg+Pythia 8 AZNLO, DYRES and Powheg MiNLO+Pythia 8 to the distribution obtained using PYTHIA 8 AZ.

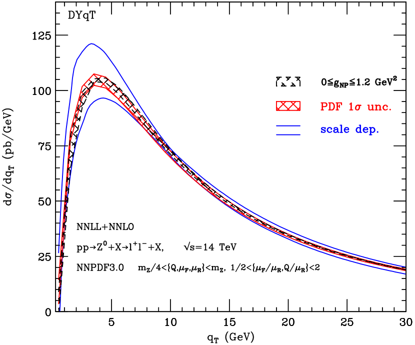
Transverse-mass distributions from W decay





Ratios of the m_T normalised distribution obtained using Powheg+Pythia 8 AZNLO, DYRES and Powheg MiNLO+Pythia 8 to the distribution obtained using PYTHIA 8 AZ.

PDF uncertainties and NP effects



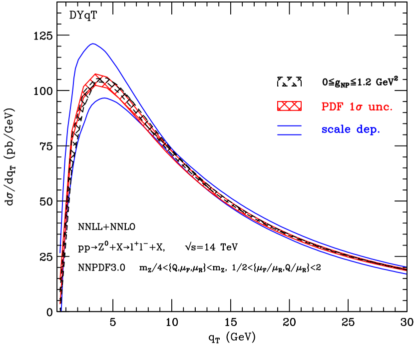
NNLL+NNLO result for Z q_T spectrum at the LHC. Perturbative scale dependence, PDF uncertainties and impact of NP effects.

- PDF uncertainty is smaller than the scale uncertainty and it is approximately independent on q_T (around the 3% level).
- Non perturbative *intrinsic* k_T effects parametrized by a NP form factor $S_{NP} = \exp\{-g_{NP} b^2\}$ with $0 < g_{NP} < 1.2\, GeV^2$:

$$\exp\{\mathcal{G}_N(\alpha_S, \widetilde{L})\} \quad \to \quad \exp\{\mathcal{G}_N(\alpha_S, \widetilde{L})\}\, S_{NP}$$

- NP effects increase the hardness of the q_T spectrum at small values of q_T.
- NNLL+NNLO result with NP effects very close to perturbative result except for $q_T < 3 GeV$ (i.e. below the peak).

PDF uncertainties and NP effects



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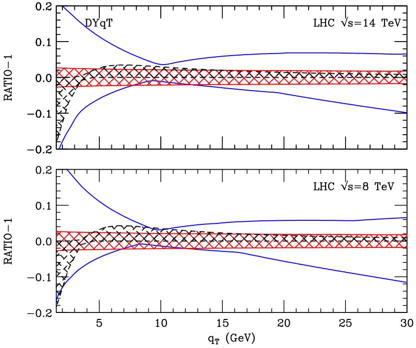

NNLL+NNLO result for Z q_T spectrum at the LHC. Perturbative scale dependence, PDF uncertainties and impact of NP effects normalized to central NNLL+NNLO prediction.

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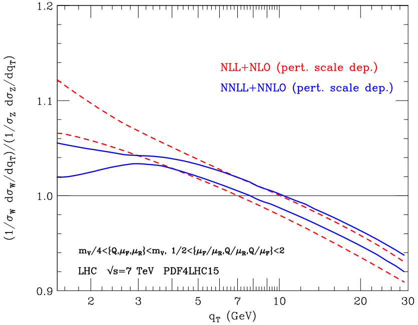
W/Z ratio: the q_T spectrum



Ratio of NNLL+NNLO and NLL+NLO results for W/Z q_T spectra at the LHC. Perturbative scale dependence.

- Ratio of W/Z observables substantially reduces both the experimental and theoretical systematic uncertainties [Giele,Keller('97)].
- *Correlated* ($\mu^W/M_W = \mu^Z/M_Z$) scale variations by factor 2 (avoiding ratios larger than 2) gives reasonable estimate of pert. uncertainty (nice overlap of scale variation bands for $q_T > 3\,GeV$).
- PDF uncertainty dominates at very small (q_T $q_T \lesssim 5\,GeV$).
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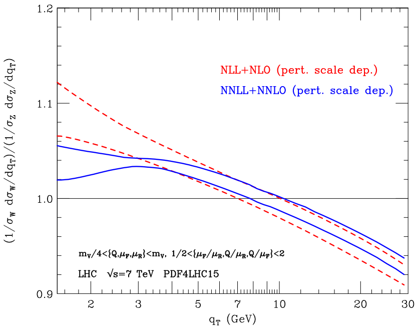

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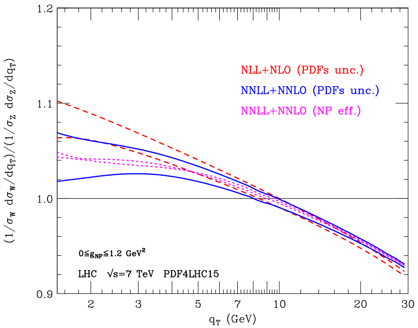

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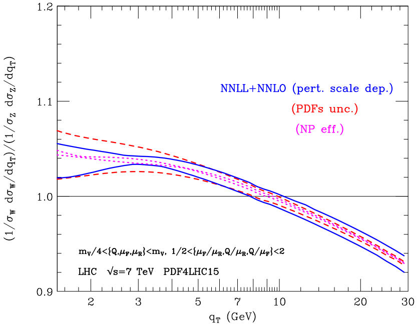

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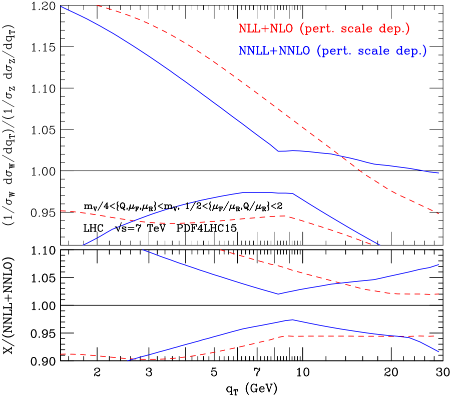
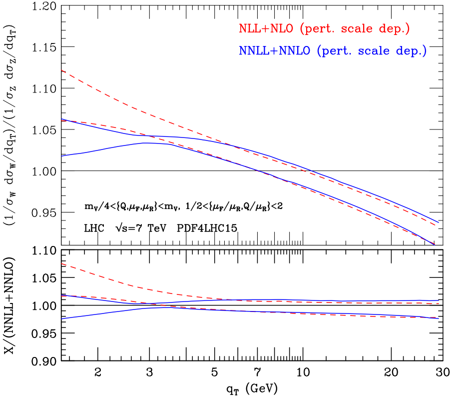
W/Z ratio q_T spectrum: perturbative scale uncertainty

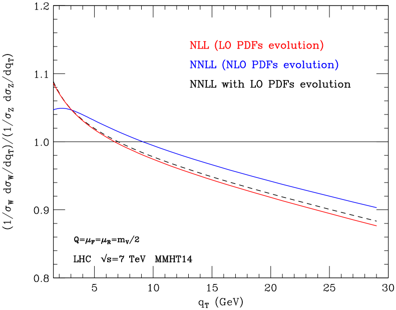


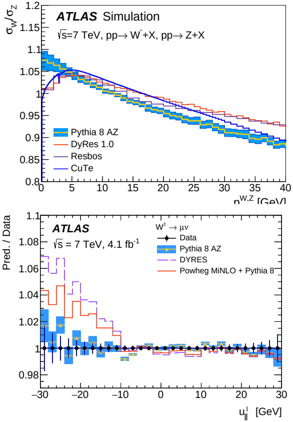
DYqT resummed predictions for the ratio of W/Z normalized q_T spectra. Uncorrelated perturbative scale variation band.



DYqT resummed predictions for the ratio of W/Z normalized q_T spectra. Correlated perturbative scale variation band.

W/Z ratio: the q_T spectrum





Left: Ratio of NNLL results for W/Z q_T spectra at the LHC, effect of PDFs evolution.
Right: Ratios of the normalised W/Z q_T spectra predicted by Pythia 8 and several resummation programs for W^+ and W^-.

Back up slides

Idea of (analytic) resummation

Idea of large logs (Sudakov) resummation: reorganize the perturbative expansion by all-order summation.

$\alpha_S L^2$	$\alpha_S L$	$\cdots$	$\cdots$	$\cdots$	$\mathcal{O}(\alpha_S)$
$\alpha_S^2 L^4$	$\alpha_S^2 L^3$	$\alpha_S^2 L^2$	$\alpha_S^2 L$	$\cdots$	$\mathcal{O}(\alpha_S^2)$
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
$\alpha_S^n L^{2n}$	$\alpha_S^n L^{2n-1}$	$\alpha_S^n L^{2n-2}$	$\cdots$	$\cdots$	$\mathcal{O}(\alpha_S^n)$
dominant logs	next-to-dominant logs	$\cdots$	$\cdots$	$\cdots$	$\cdots$

- Ratio of two successive rows $\mathcal{O}(\alpha_S L^2)$: fixed order expansion valid when $\alpha_S L^2 \ll 1$.
- Ratio of two successive columns $\mathcal{O}(1/L)$: resummed expansion valid when $1/L \ll 1$.

Soft gluon exponentiation

Sudakov resummation feasible when:
dynamics AND kinematics factorize
$\Rightarrow$ exponentiation.

- Dynamics factorization: general propriety of QCD matrix element for soft emissions.

$$dw_n(q_1, \ldots, q_n) \simeq \frac{1}{n!} \prod_{i=1}^{n} dw_i(q_i)$$

- Kinematics factorization: not valid in general. For q_T distribution of DY process it holds in the impact parameter space (Fourier transform).

$$\int d^2\mathbf{q_T}\ \exp(-i\mathbf{b}\cdot\mathbf{q_T})\,\delta\left(\mathbf{q_T} - \sum_{j=1}^{n}\mathbf{q_{T_j}}\right) = \exp(-i\mathbf{b}\cdot\sum_{j=1}^{n}\mathbf{q_{T_j}}) = \prod_{j=1}^{n}\exp(-i\mathbf{b}\cdot\mathbf{q_{T_j}})\,.$$

- Exponentiation holds in the impact parameter space. Results have then to be transformed back to the physical space.

q_T resummation at NNLL+NNLO

- q_T resummation performed for Drell–Yan process up to NNLL+NNLO by using the formalism developed in [Catani,de Florian,Grazzini('01)], [Bozzi,Catani,de Florian,Grazzini('06,'08)]. We have included
  - NNLL logarithmic contributions to all orders (i.e. up to $\exp(\sim\alpha_S^n L^{n-1})$);
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  - NNLO result (i.e. up to $\mathcal{O}(\alpha_S^2)$) for the total cross section.
- Analytic resummation implemented in publicly available codes:

  DYqT: computes resummed q_T spectrum, inclusive over other kinematical variables [Bozzi,Catani,de Florian,G.F.,Grazzini('09,'11)]

  DYRes: computes resummed q_T spectrum and related distributions, it retains full kinematics of the vector boson and of its leptonic decay products (possible to apply arbitrary cuts on these variables, and to plot the corresponding distributions) [Catani,de Florian,G.F.,Grazzini('15)]

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