Status of $p_T(W)$ Modelling.

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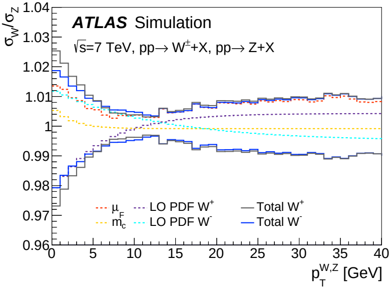
Summary of last few days' discussion
Thanks to everybody, in particular to Alessandro, Giancarlo, Ludovica, Maarten, Stefano for all the plots

Extrapolating from Z to W

Focus on low $p_T^W \lesssim 30\,\mathrm{GeV}$ relevant for m_W

- $\simeq 2\%$ uncertainties in p_T^W translate into $\simeq 10\,\mathrm{MeV}$ uncertainty in m_W

$\Rightarrow$ Use precise Z measurement to get best possible prediction for W



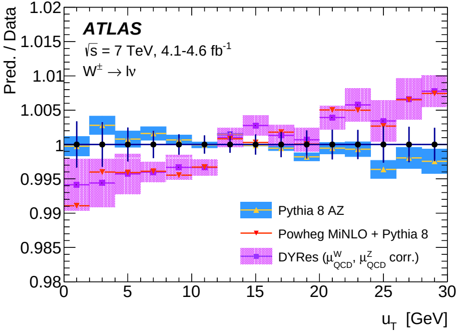
- One way to think about it

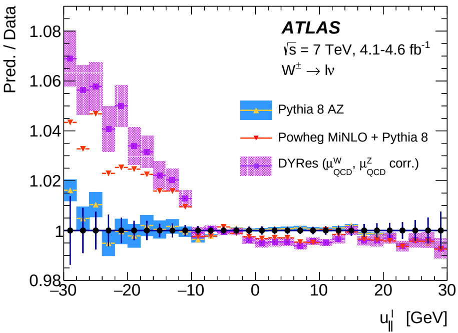
$$\frac{\mathrm{d}\sigma(W)}{\mathrm{d}p_T} = \left[\frac{\mathrm{d}\sigma(W)/\mathrm{d}p_T}{\mathrm{d}\sigma(Z)/\mathrm{d}p_T}\right]_{\mathrm{theory}} \times \left[\frac{\mathrm{d}\sigma(Z)}{\mathrm{d}p_T}\right]_{\mathrm{measured}}$$

  - There is no direct resummation for ratio, it is always a derived quantity
  - Relies on ratio being more precise than individual processes, which relies on theory uncertainties being strongly correlated between processes

- More general: Use common theory framework and fit to Z data
  - Not restricted to a specific combination (like ratio)
  - Tuning Pythia on Z data is one example of this
  - Requires explicit information on correlations between processes

Extrapolating from Z to W





Extrapolation hinges on two ingredients

- Precise cancellation of dominant common terms
  - Residual uncertainties entirely depend on precisely knowing correlations of theory uncertainties between $\mathrm{d}\sigma(Z)/\mathrm{d}p_T$ and $\mathrm{d}\sigma(W)/\mathrm{d}p_T$
- Precise understanding of (normally irrelevant) subdominant effects
  - At sub-% level many things can matter

$\Rightarrow$ Currently plain Pythia tuned to Z data works best

Overview.

	Uncertainty or size	Analytic resummation	Pythia	Leftover effect on W/Z
Leading-power resummation	5-10%	$\surd\surd\surd$	$\surd$	$\sim$ % ?
Power corrections	few %	$(\times)$	$(\surd)$?	?
Nonperturbative	few %	$(\surd)$	$(\surd)$	$\lesssim$ % ?
Massive quarks	few %	$\times$ $(\surd)$	$(\surd)$	few % (?)
QED (ISR)	$\lesssim$ %	$\times$	$\surd$ (?)	sub % (?)
PDFs	2%	$\surd$	$\surd$	$\surd$
$\alpha_s(m_Z)$	up to 5%??	$\surd$	$\surd$	$\surd$

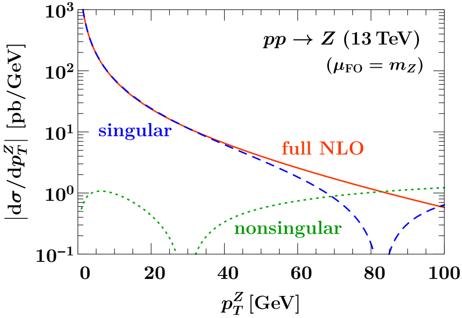
- Though it is a bit unsettling it is not unbelievable that plain Pythia currently describes the W/Z ratio best
  - Trying to understand and go beyond that
  - Still need to provide a robust uncertainty when used as prediction for W

Power Corrections.

Scaling variable $\tau = p_T^2/Q^2$

$$\tau\frac{d\sigma}{d\tau} = \underbrace{\tau\frac{d\sigma^{\mathrm{resum}}}{d\tau}}_{\sim\mathcal{O}(1)} + \underbrace{\tau\frac{d\sigma^{\mathrm{nons}}}{d\tau}}_{\sim\mathcal{O}(\tau)}$$

- Resummation only captures $\mathcal{O}(1)$ leading-power corrections
- $\mathcal{O}(\tau)$ power corrections are only known and included at fixed order
  - In principle possible up to $\mathcal{O}(\alpha_s^3)$ using NNLO $V + j$



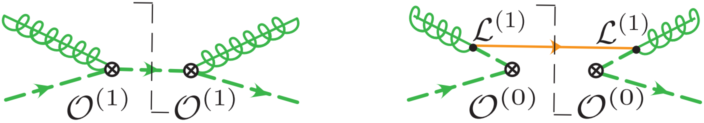
- Important caveat: They also contain large logs

$$\tau\frac{d\sigma^{\mathrm{nons}}}{d\tau} \sim \left[\alpha_s\tau(1+\ln\tau) + \alpha_s^2\tau(\;1\; + \ln\tau\; + \ln^2\tau + \ln^3\tau) + \cdots\right] + \mathcal{O}(\tau^2)$$

e.g. for $\tau = 0.01$ $\quad\sim \alpha_s^2(0.01 + 0.05 + 0.21\; + 0.98)$

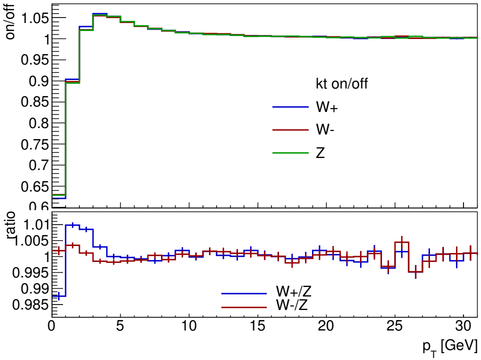
  - They are only $\mathcal{O}(\tau)$ power-suppressed if they are being resummed as well
  - p_T resummation at subleading power is much more complicated and currently not available even at LL

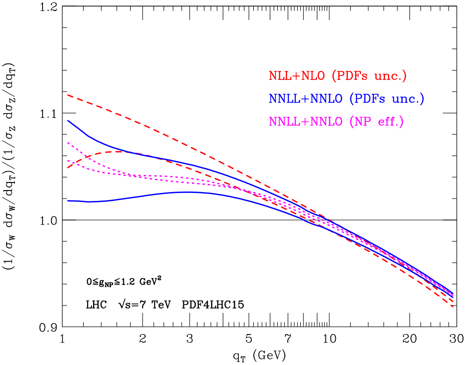
Structure of Power Corrections.



- New contributions appear at subleading power already at LL that have no leading-power analog (e.g. soft quarks)
  - gq channels contribute at LL, can be as large as $q\bar{q}$ channels
  - Different color structure at LL: C_F^2 vs. $T_F(C_F + C_A)$
  - Multiplying nonsingular by leading-power Sudakov exponent is not correct even at LL
- Numerically important type of contribution are “kinematic” power corrections that depend on PDF derivatives $x f_q'(x)$
  - Describe the effect that PDFs also need to provide small momentum components for p_T recoil
  - Might in fact be captured reasonably well in Pythia due to it enforcing momentum conservation at each splitting
  - Less likely to cancel in W/Z ratio

Nonperturbative Effects.



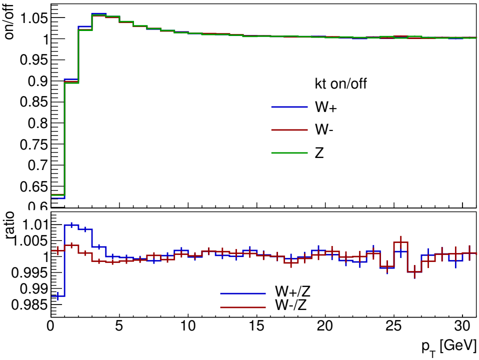


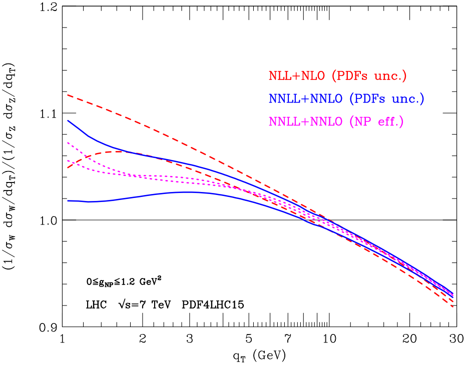
Formal scaling: $\Lambda^2_{\mathrm{QCD}}/p_T^2 \sim \Lambda^2_{\mathrm{QCD}} b^2$ (perhaps only $\Lambda_{\mathrm{QCD}}/p_T \sim \Lambda_{\mathrm{QCD}} b$)

- Flavor-independent pieces
  - Pythia: modelled via primordial/intrinsic k_T
  - DYRes: nonpert. form factor $S_{\mathrm{NP}} = \exp(-g_{\mathrm{NP}} b^2)$ with $0 < g_{\mathrm{NP}} < 1.2\,\mathrm{GeV}^2$

$\Rightarrow$ In both cases cancel to sub-%

Nonperturbative Effects.



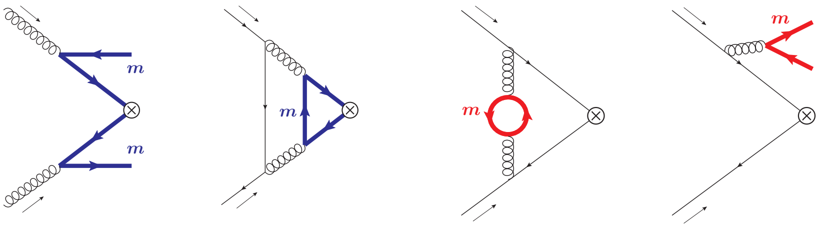


Formal scaling: $\Lambda^2_{\rm QCD}/p_T^2 \sim \Lambda^2_{\rm QCD} b^2$ (perhaps only $\Lambda_{\rm QCD}/p_T \sim \Lambda_{\rm QCD} b$)

- TODO: Need to investigate flavor-dependent effects (TMDPDFs)
  - Even small differences could leave noticeable remnant for $p_T \lesssim 5\,\mathrm{GeV}$
  - Even collinear PDFs already reach several %

Massive Quark Effects.

"Primary" mass effects at fixed order — "Secondary" mass effects at fixed order

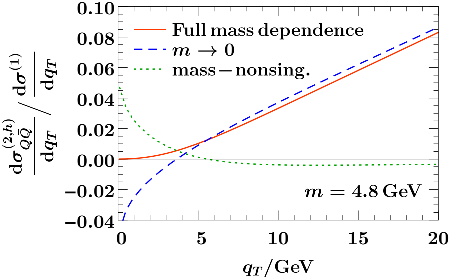


Multi-scale problem with several possible scale hierarchies

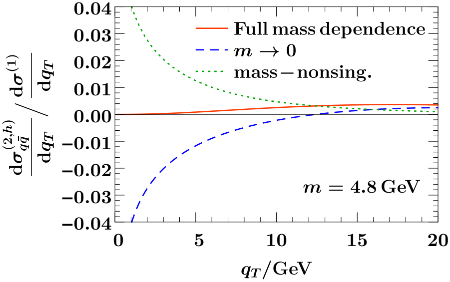
- p_T distribution goes through different regimes
  - $\Lambda_{\rm QCD} \ll p_T \ll m_b \ll Q$: heavy quark decouples (4FS for $m_b \sim Q$)
  - $\Lambda_{\rm QCD} \ll p_T \sim m_b \ll Q$: quark mass changes resummation structure (including nonperturbative effects)
  - $\Lambda_{\rm QCD} \ll m_b \ll p_T \ll Q$: massless limit (usual 5FS)
- Few-% level effects, primary mass effects do not cancel in W/Z ratio

Massive Quark Effects.

“Primary” mass effects at fixed order



“Secondary” mass effects at fixed order



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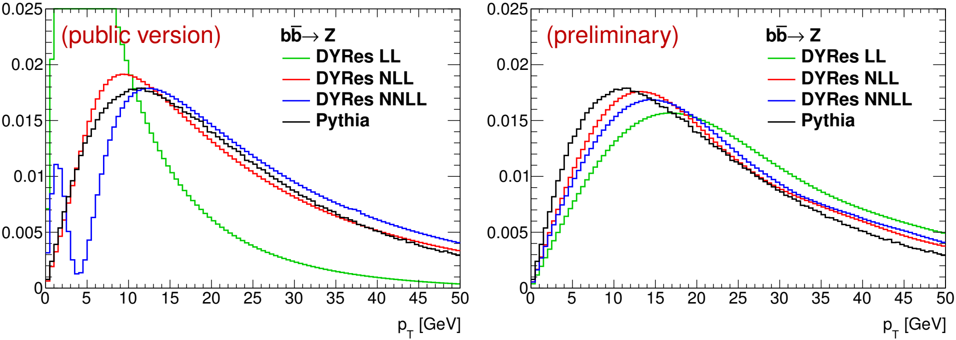
$b\bar{b} \to Z$ in DYRes and Pythia.

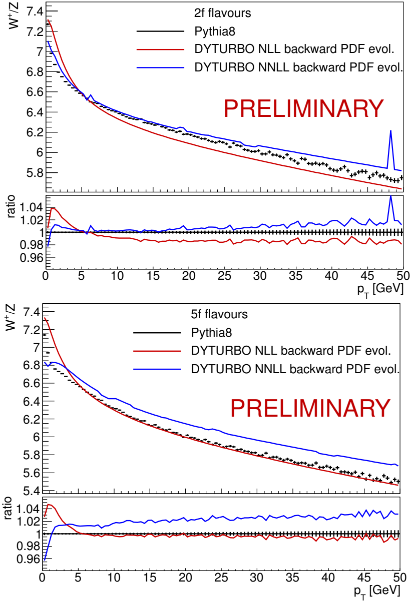
fixed 5FS (backward evolution from μ_F) — zero-mass VFS (forward evolution)



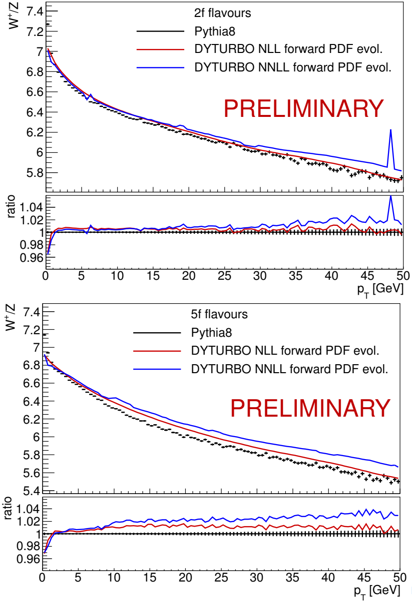
- VFS and Pythia only turn on b-PDF above matching scale $\mu_b \equiv m_b$
  - $f_b(\mu = 1/b_T < m_b) = 0$ leads to smooth turn off for $p_T < m_b$
  - Pythia models $g \to b\bar{b}$ splitting kinematics in p_T space with finite m_b
- TODO: Use PDF evolution allowing for general μ_b and its variation
- TODO: Perform full finite-mass multi-scale resummation

Impact on W/Z Ratio

fixed 5FS (backward evolution from μ_F)



zero-mass VFS (forward evolution)

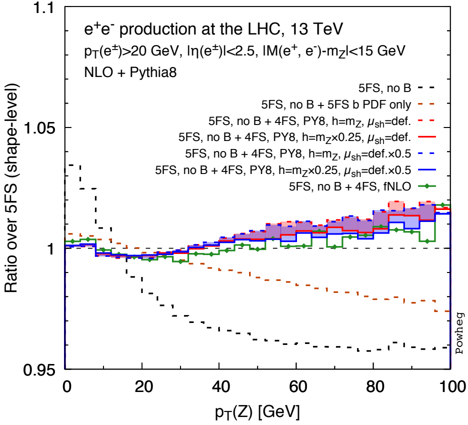


Combined 4FS/5FS at Hadron Level.

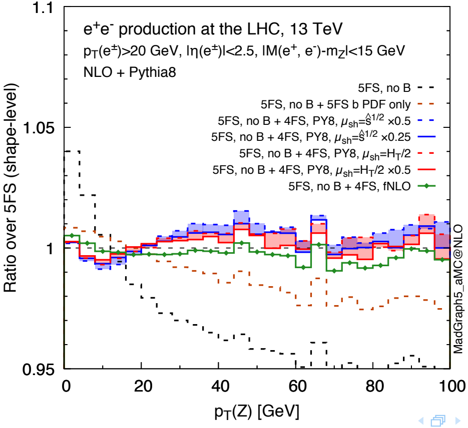
Combine NLO+PS 4FS for $pp \to \ell^+\ell^- b\bar{b}$ with NLO+PS 5FS for non-b

$$\frac{\mathrm{d}\sigma}{\mathrm{d}p_T}^{\mathrm{combined}} = \frac{\mathrm{d}\sigma}{\mathrm{d}p_T}^{\mathrm{5FS}}(\mathrm{B-veto}) + \frac{\mathrm{d}\sigma}{\mathrm{d}p_T}^{\mathrm{4FS}}(\ell^+\ell^- b\bar{b})$$

POWHEG



MC@NLO



Combined 4FS/5FS at Hadron Level: Impact on m_W

To evaluate possible impact on m_W compare templates in plain 5FS (brown) with distributions reweighted to combined results

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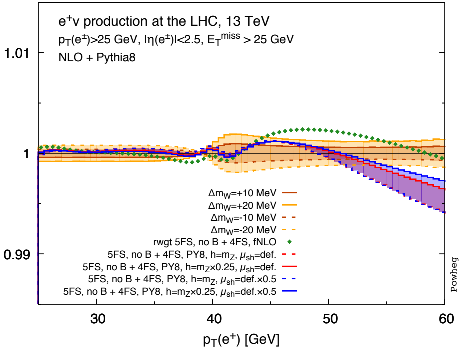

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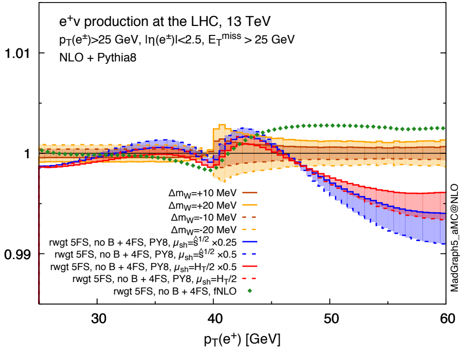
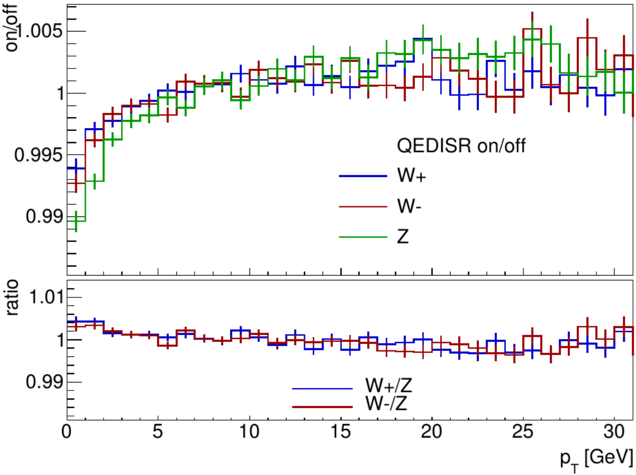


- Provides a qualitative statement, realistic estimate more involved
- Estimate up to $\mathcal{O}(5\ \mathrm{MeV})$ shift (fit range $p_T^\ell \in [32, 45]\ \mathrm{GeV}$)

QED (ISR) in Pythia.



- At most %-level effects, cancel to sub-% in W/Z ratio
  - Agrees with expected parametric size of $\mathcal{O}(\alpha_{\mathrm{em}}/\alpha_s) \sim \mathcal{O}(\%)$
  - Effects cancel better than perhaps expected
- TODO: Should be possible to double-check in analytic resummation

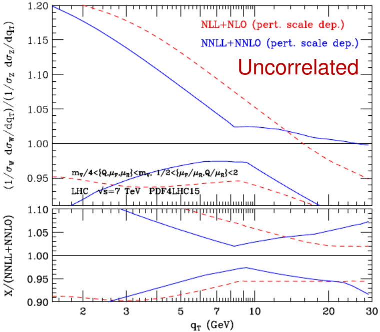
Theory Uncertainty Correlations.

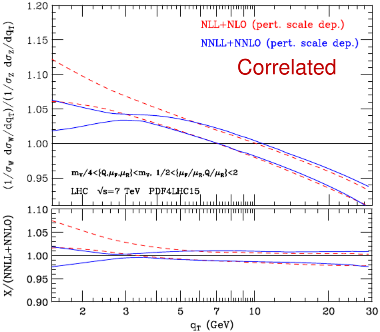
$$\mathrm{d}\sigma(W)/\mathrm{d}p_T = c_0(p_T) + \epsilon\, c_1(p_T) + (\epsilon^2\, c_2(p_T) + \cdots)$$

$$\mathrm{d}\sigma(Z)/\mathrm{d}p_T = d_0(p_T) + \epsilon\, d_1(p_T) + (\epsilon^2\, d_2(p_T) + \cdots)$$

QCD corrections for W and Z are *largely* the same but also *not entirely*

- Using correlated scale variations for both processes
  - Scale dependence largely cancels in their ratio
  - Possible differences between processes at higher order are precisely not probed by scale variations





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Correlations only come from common sources of uncertainties

- QCD scales are not physical parameters
  - They do not have an uncertainty that can be propagated
  - They also cannot be regarded as the fundamental sources of uncertainties, i.e. they cannot be used as nuisance parameters to imply correlations
  - A priori, they do not imply anything about correlations among different processes or different kinematic regions

⇒ Scale variations are intrinsically ill-suited for this

Beyond Scales: Parametric Theory Uncertainties.

[Disclaimer: very much work in progress ...]

Idea: Identify actual nuisance parameters for perturbative uncertainties

- Provides immediate solution to the two key problems
  - Provide true correlations between different processes and p_T values
  - Can be constrained by data, and therefore allows one to fully consistently use Z measurements to reduce theory uncertainties in W predictions
- For resummed leading-power contributions
  - Scale and p_T dependence is fully determined in terms of RGE ingredients (anomalous dimensions and boundary conditions)
  - Nuisance parameters can be unambigously identified with missing perturbative ingredients at the next higher resummed order (i.e. full N^4LL)
- TODOs and open questions to address
  - Must ensure that leftover scale dependence at higher order is small compared to parametric theory uncertainty at current order
  - Possible degeneracy between perturbative and nonperturbative parameters
  - Treatment of power corrections
  - Validation and feasibility study at known lower order

Summary and Outlook.

	Uncertainty or size	Analytic resummation	Pythia	Leftover effect on W/Z
Leading-power resummation	5-10%	$\surd\surd\surd$	$\surd$	$\sim$ % ?
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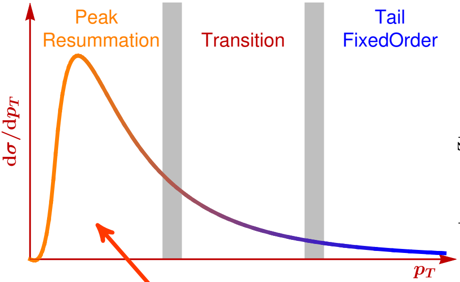
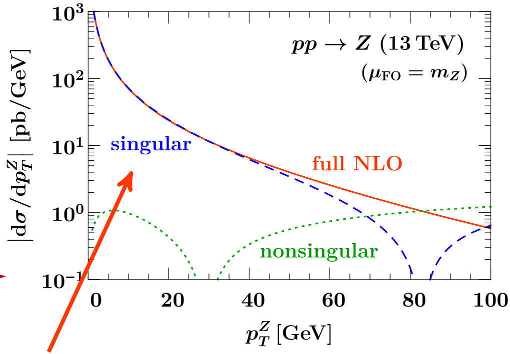
QED (ISR)	$\lesssim$ %	$\times$	$\surd$ (?)	sub % (?)
PDFs	2%	$\surd$	$\surd$	$\surd$
$\alpha_s(m_Z)$	up to 5%??	$\surd$	$\surd$	$\surd$

- Progress on all fronts
- Seems to me that all ? can in principle be addressed
  - Need robust uncertainties (small is not enough ...)
  - Requires nontrivial effort

Backup Slides

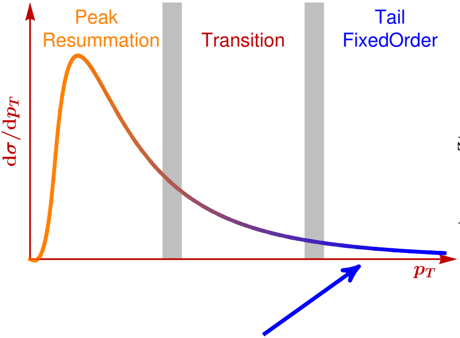
Different Perturbative Regions.

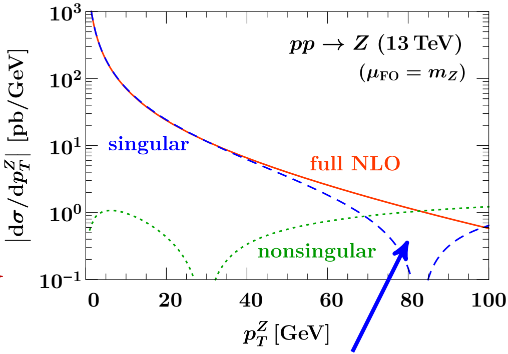




Resummation region

- Spectrum at low $p_T \ll Q$ and cross section with cut $p_T^{cut} \ll Q$
  - Singular dominate and must be resummed (nonsingular are power-suppressed)
  - Fixed-order by itself becomes meaningless here
  - In MC: Parton shower regime

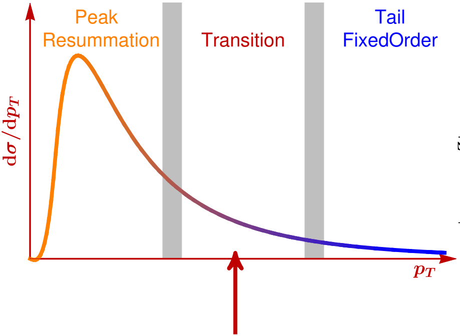


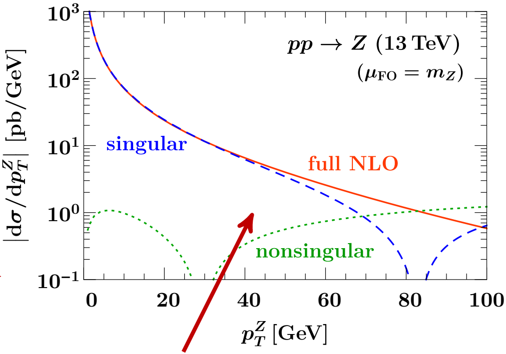


Fixed-order region

- Spectrum at high $p_T \sim Q$
  - Fixed-order calculation for inclusive V+1-jet process
  - In MC: Fixed-order matrix elements
  - Power expansion breaks down and resummation must be turned off

Different Perturbative Regions.





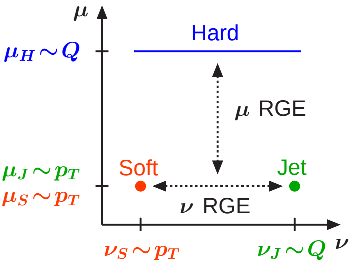
Transition region

- Anything in between (there are no fixed boundaries)
- Resummation still makes sense, fixed-order expansion also still works
  - Most precise predictions are obtained from consistent combination of resummation and fixed-order
  - In MC: This is where ME+PS matching/merging comes in

Leading-Power Resummation.

Leading-power p_T spectrum factorizes into hard, collinear, and soft contributions

$$\frac{\mathrm{d}\sigma^{\mathrm{sing}}}{\mathrm{d}\vec{p}_T} = \sigma_0\, H(Q,\mu) \int \mathrm{d}^2\vec{k}_a\, \mathrm{d}^2\vec{k}_b\, \mathrm{d}^2\vec{k}_s \times B_a(\vec{k}_a,\mu,\nu)\, B_b(\vec{k}_b,\mu,\nu) \times S(\vec{k}_s,\mu,\nu)\, \delta(\vec{p}_T - \vec{k}_a - \vec{k}_b - \vec{k}_s)$$



All-order structure of leading-power terms is fully determined by coupled system of differential equations (including their boundary conditions)

- in virtuality scale μ

$$\mu \frac{\mathrm{d}H(Q,\mu)}{\mathrm{d}\mu} = \gamma_H(Q,\mu)\, H(Q,\mu)$$

$$\mu \frac{\mathrm{d}B(\vec{p}_T,\mu,\nu)}{\mathrm{d}\mu} = \gamma_B(\mu,\nu)\, B(\vec{p}_T,\mu,\nu)$$

$$\mu \frac{\mathrm{d}S(\vec{p}_T,\mu,\nu)}{\mathrm{d}\mu} = \gamma_S(\mu,\nu)\, S(\vec{p}_T,\mu,\nu)$$

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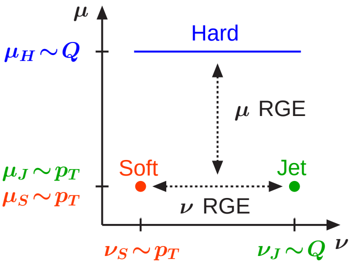

All-order structure of leading-power terms is fully determined by coupled system of differential equations (including their boundary conditions)

- and rapidity scale ν (or ζ)

$$\nu \frac{\mathrm{d}B(\vec{p}_T,\mu,\nu)}{\mathrm{d}\nu} = -\frac{1}{2} \int \mathrm{d}^2\vec{k}_T\, \gamma_\nu(\vec{k}_T,\mu)\, B(\vec{p}_T - \vec{k}_T,\mu,\nu)$$

$$\nu \frac{\mathrm{d}S(\vec{p}_T,\mu,\nu)}{\mathrm{d}\nu} = \int \mathrm{d}^2\vec{k}_T\, \gamma_\nu(\vec{k}_T,\mu)\, S(\vec{p}_T - \vec{k}_T,\mu,\nu)$$

$$\mu \frac{\mathrm{d}}{\mathrm{d}\mu}\gamma_\nu(\vec{k}_T,\mu) = \nu \frac{\mathrm{d}}{\mathrm{d}\nu}\gamma_S(\mu,\nu)\delta(\vec{k}_T) = -4\Gamma_{\mathrm{cusp}}[\alpha_s(\mu)]\delta(\vec{k}_T)$$

Resummation Orders.

Analytic resummation amounts to solving this system of differential equations

- Formal resummation accuracy is fundamentally defined by perturbative input used for anomalous dimensions and boundary conditions
  - In Fourier space (as in standard CSS) solution is a pure exponential and resummation orders map onto common counting of logs in the exponent
- Current perturbative uncertainties at NNLL′+NNLO at 5-10% level
  - N^{3}LL is available but not full N^{3}LL′+N^{3}LO, hard to see it can go below 2%
  - Compare: Thrust spectrum in $e^+e^- \to q\bar{q}$ at $Q = m_Z$ has $\simeq$ 2% precision at N^{3}LL′+N^{3}LO

	Boundary conditions (singular)	Anomalous dimensions $\gamma_{H,B,S,\nu}$	Γ_{cusp}, β	FO matching (nonsingular)
NLL	1	1-loop	2-loop	-
NLL$^{(\prime)}$+NLO	α_s	1-loop	2-loop	α_s
NNLL+NLO	α_s	2-loop	3-loop	α_s
NNLL$^{(\prime)}$+NNLO	α_s^2	2-loop	3-loop	α_s^2
N^{3}LL+NNLO	α_s^2	3-loop	4-loop	α_s^2
N^{3}LL$^{(\prime)}$+N^{3}LO	α_s^3	3-loop	4-loop	α_s^3

Perturbative Accuracy (Oversimplified).

Terms in the cross section that are reproduced at some resummation order (not the definition of the order) with $\tau = p_T^2/Q^2$, $L = \ln\tau$, $L_{\rm cut} = \ln\tau^{\rm cut}$

$$
\begin{array}{llccccl}
 & & \text{LL} & \text{NLL} & \text{NLL}' & \text{NNLL} & \\
\dfrac{\sigma(\tau^{\rm cut})}{\sigma_B} = & & 1 & & & & \text{LO} \\
 & + \alpha_s\big[& \frac{c_{11}}{2}L_{\rm cut}^2 & + c_{10}L_{\rm cut} & + c_{1,-1} & + \qquad F_1^{\rm nons}(\tau^{\rm cut})\big] & \text{NLO} \\
 & + \alpha_s^2\big[& \vdots & + \vdots & + \vdots & + \vdots & \\
\dfrac{1}{\sigma_B}\dfrac{d\sigma}{d\tau} & = \alpha_s/\tau\big[& c_{11}L & + c_{10} & + & \tau f_1^{\rm nons}(\tau)\big] & \text{LO}_1 \\
 & + \alpha_s^2/\tau\big[& c_{23}L^3 & + c_{22}L^2 & + c_{21}L & + c_{20} + \tau f_2^{\rm nons}(\tau)\big] & \text{NLO}_1 \\
 & + \alpha_s^3/\tau\big[& \vdots & + \vdots & + \vdots & + \vdots &
\end{array}
$$

- Lowest perturbative accuracy at all p_T requires (N)LL+LO$_1$
  - Provided by LO ME+PS, also plain Pythia (has full ME for first emission)
  - LO is naturally part of LL and so automatically included

Perturbative Accuracy (Oversimplified).

Terms in the cross section that are reproduced at some resummation order (not the definition of the order) with $\tau = p_T^2/Q^2$, $L = \ln\tau$, $L_{\rm cut} = \ln\tau^{\rm cut}$

$$
\begin{array}{rlllllllll}
 & & \text{LL} & & \text{NLL} & & \text{NLL}' & & \text{NNLL} & \\
\dfrac{\sigma(\tau^{\rm cut})}{\sigma_B} = & & 1 & & & & & & & \text{LO} \\
 & + \alpha_s\big[& \frac{c_{11}}{2}L_{\rm cut}^2 & + & c_{10}L_{\rm cut} & + & c_{1,-1} & + & \qquad F_1^{\rm nons}(\tau^{\rm cut})\big] & \text{NLO} \\
 & + \alpha_s^2\big[& \vdots & + & \vdots & + & \vdots & + & \vdots & \\
\dfrac{1}{\sigma_B}\dfrac{d\sigma}{d\tau} = & \alpha_s/\tau\big[& c_{11}L & + & c_{10} & + & & & \qquad \tau f_1^{\rm nons}(\tau)\big] & \text{LO}_1 \\
 & + \alpha_s^2/\tau\big[& c_{23}L^3 & + & c_{22}L^2 & + & c_{21}L & + & c_{20} \; + \; \tau f_2^{\rm nons}(\tau)\big] & \text{NLO}_1 \\
 & + \alpha_s^3/\tau\big[& \vdots & + & \vdots & + & \vdots & + & \vdots &
\end{array}
$$

- NLO+PS matching (MC@NLO, POWHEG) adds full NLO to $\sigma(\tau^{\rm cut})$
  - Improves accuracy for $\sigma(\tau^{\rm cut} \sim 1)$ (incl. cross section) to NLO
  - Does not automatically improve formal accuracy of spectrum beyond ME+PS

Perturbative Accuracy (Oversimplified).

Terms in the cross section that are reproduced at some resummation order (not the definition of the order) with $\tau = p_T^2/Q^2$, $L = \ln\tau$, $L_{\rm cut} = \ln\tau^{\rm cut}$

$$
\begin{array}{rlcccccccccl}
 & & \text{LL} & & \text{NLL} & & \text{NLL}' & & \text{NNLL} & & & \\
\dfrac{\sigma(\tau^{\rm cut})}{\sigma_B} = & & 1 & & & & & & & & & \text{LO} \\
+ & \alpha_s\Big[& \frac{c_{11}}{2}L_{\rm cut}^2 & + & c_{10}L_{\rm cut} & + & c_{1,-1} & + & & & F_1^{\rm nons}(\tau^{\rm cut})\Big] & \text{NLO} \\
+ & \alpha_s^2\Big[& \vdots & + & \vdots & + & \vdots & + & \vdots & & & \\
\dfrac{1}{\sigma_B}\dfrac{d\sigma}{d\tau} = & \alpha_s/\tau\Big[& c_{11}L & + & c_{10} & + & & & & & \tau f_1^{\rm nons}(\tau)\Big] & \text{LO}_1 \\
+ & \alpha_s^2/\tau\Big[& c_{23}L^3 & + & c_{22}L^2 & + & c_{21}L & + & c_{20} & + & \tau f_2^{\rm nons}(\tau)\Big] & \text{NLO}_1 \\
+ & \alpha_s^3/\tau\Big[& \vdots & + & \vdots & + & \vdots & + & \vdots & & &
\end{array}
$$

- NLL′ and NNLL fully incorporate 1-loop virtuals ($c_{1,-1}$) into resummation and therefore naturally match to NLO
- Similarly NNLL′ and N^{3}LL incorporate 2-loop virtuals and match to NNLO