Wire model in SixTrack

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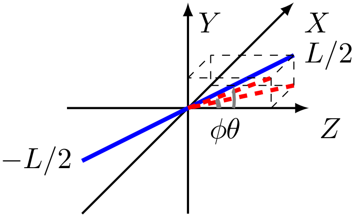
Outline

1. Vector potential of straight current carrying wire
2. Wire model in SIXTRACK - implementation and usage
3. The model verification
4. Application to the LHC

Finite wire in Cartesian system

Wire is centered in Cartesian system and fully described by 3 par.:



Direction cosines of the wire be expressed through ϕ and θ as:

$$cos(c_x) = \frac{tg(\phi)}{\sqrt{(tg^2(\phi) + tg^2(\theta) + 1)}} \tag{1a}$$

$$cos(c_y) = \frac{tg(\theta)}{\sqrt{(tg^2(\phi) + tg^2(\theta) + 1)}} \tag{1b}$$

$$cos(c_z) = \frac{1}{\sqrt{(tg^2(\phi) + tg^2(\theta) + 1)}} \tag{1c}$$

Finite wire - Vector potential derivation

From BiotSavart law (in SI):

$\mathbf{A} = \frac{I\mu_0}{4\pi} \oint \frac{\boldsymbol{dl}}{r}$

using natural parametrization :

$$r = \sqrt{(z - cos(c_z)t)^2 + (x - cos(c_x)t)^2 + (y - cos(c_y)t)^2}$$
$$\boldsymbol{dl} = i * cos(c_x)dt + j * cos(c_y)dt + k * cos(c_z)dt$$

and integration limits $[-L/2, L/2]$
The generic formula for vector potential of straight wire with length L:

$$A(x, y, z)_i = \frac{I\mu_0 cos(c_i)}{4\pi}(asinh\frac{L/2 - a}{\sqrt{b - a^2}} - asinh\frac{-L/2 - a}{\sqrt{b - a^2}}) \quad (3)$$

where: index i corresponds to x, y, z components;
$a = xcos(c_x) + ycos(c_y) + zcos(c_z)$ and $b = x^2 + y^2 + z^2$.
(If $cos(c_z) = 1$ and $x^2 + y^2 << L$ that $A_z -> log(1/r)$)

Hamiltonian for the wire

Wire parallel to the longitudinal axis:

$$A_x(x,y,z) = 0$$
$$A_y(x,y,z) = 0$$
$$A_z(x,y,z) = \frac{I\mu_0}{4\pi} \cdot \left(asinh\left(\frac{L/2 - z}{\sqrt{x^2+y^2}}\right) - asinh\left(\frac{-L/2 - z}{\sqrt{x^2+y^2}}\right)\right)$$

The standard Hamiltonian for the wire is then simply
$H = -A_s = -A_z$.

Wire Kick

'Kick' for thin element:

$$\Delta p_x = \int_{-L_{\text{emb}}/2}^{+L_{\text{emb}}/2} \frac{\partial A_z(x, y, s)}{\partial x} ds,$$

$$\Delta p_y = \int_{-L_{\text{emb}}/2}^{+L_{\text{emb}}/2} \frac{\partial A_z(x, y, s)}{\partial y} ds,$$

$$\Delta x = \Delta y = \Delta\delta = \Delta\sigma = 0$$

Wire 'kick' in SIXTRACK:

$$p_x \to p_x - 10^{-7} \cdot I \frac{e}{P_0} \frac{r_x}{r^2} \left(d^+ - d^-\right) - p_{co,wire}$$

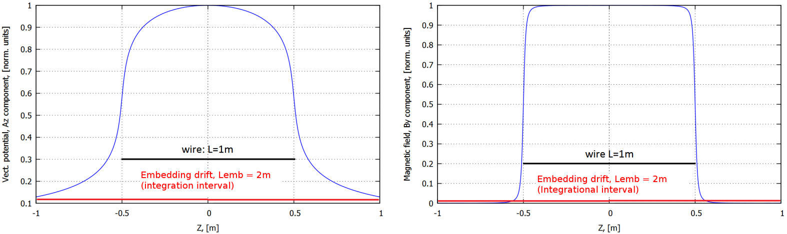
$$p_y \to p_y - 10^{-7} \cdot I \frac{e}{P_0} \frac{r_y}{r^2} \left(d^+ - d^-\right) - p_{co,wire}$$

with d^+ and d^- defined as:

$$d^+ = \sqrt{(L_{\text{emb}} + L)^2 + 4r^2}$$

$$d^- = \sqrt{(L_{\text{emb}} - L)^2 + 4r^2}$$

Wire Kick - Fringe field



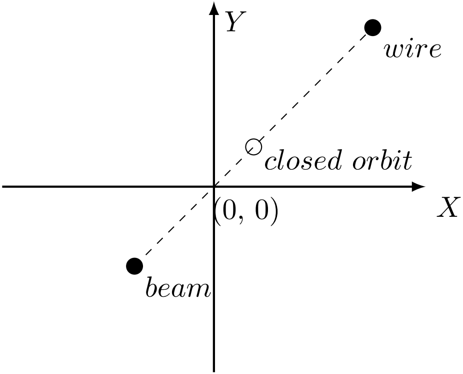
Vector potential (left) and magnetic field (right) of a current bearing wire with physical length $L = 1$ m and embedded drift (integrational length) $L_{\mathrm{emb}} = 2$ m.

Kick is proportional to the surface under curve

Note: for the LHC there is no effect and could be $L = L_{emb}$

Wire Kick - Wire positioning

Wire can be defined in respect to the closed orbit or to the center of the beam pipe.



Wire transport MAP with arbitrary orientation

Steps:

1. Shift to the origin: We define the new variables

$$r_x := x - x_{co} + dx$$
$$r_y := y - y_{co} + dy$$

where x_{co} and y_{co} denotes the closed orbit at the wire's center.

2. Symplectic rotation of coordinate system
3. Wire 'kick'
4. Backward rotation

For details see:
E. Forest Beam dynamics: A New Attitude and Framework, 1998;
SIXTRACK physics manual

Wire in SIXTRACK - Example

Wire in SINGLE BLOCK): format: WIRENAME TYPE=15
Wire name in (STRUCTURE): format ... WIRENAME ...
Wire BLOCK (in fort.3):

Arguments	unit	Description
name	-	Name of wire. Must be the same as in list of single elements.
flag_co	-	flag to define the displacement of the wire in respect to the closed orbit or x=y=0. For *flag_co*=+1 *disp_** is the distance between x=y=0 and the wire. For *flag_co*=-1 *disp_** is the distance between the closed orbit and the wire.
current	A	wire current
int_length	m	integrated length of the wire
phys_length	m	physical length of the wire
disp_x	mm	hor. displacement of the wire
disp_y	mm	vert. displacement of the wire
tilt_x	degrees	hor. tilt of the wire $-90 < tilt\_x < 90$ (uses same defintion as DISP block)
tilt_y	degrees	vert. tilt of the wire $-90 < tilt\_y < 90$ (uses same defintion as DISP block)

Wire in SIXTRACK - Example

In fort.2
SINGLE elements:

```
wire1  15  0  0  0  0  0  0
wire2  15  0  0  0  0  0  0
```

In fort.2
STRUCTURE:

```
... BLOC1 wire1 ... BLOC101 wire2 ..
```

In fort.3 - WIRE block definition:

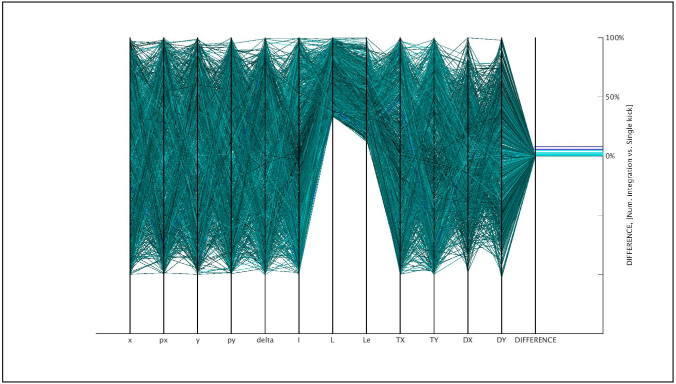
```
WIRE
wire1   1   100  2.0   1.0   2.73   4.99   0.0  0.0
wire2  -1  -100  2.0   1.0   5.00   5.00   0.0  0.0
NEXT
```

Model limitations

1. Wire is treated as thin element and approximated by singe kick
2. Particle trajectory should not cross the wire's plane (otherwise singe kick assumption doesn't hold)
3. Wire tilt angles should not be close to 90^0

Single kick model was compared with first order Euler symplectic integrator (also implemented in SIXTRACK (in test version only)) - next 2 slides:

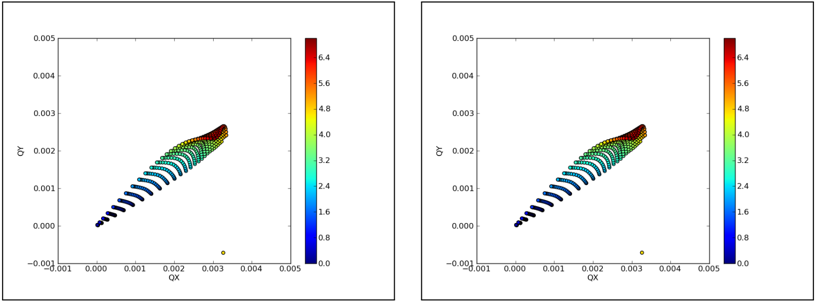
Parallel Coordinates diagram:



The axis on the right shows the difference between models (in %) for arbitrary combinations of the variables: x,px,y,py,δ, tilts: tx,ty; and displacements: dx,dy.
The limitations were: $x << L, px << 1, dx < L$
20000 of combinations are shown; 99.8 % of cases provides the difference $<$ 0.1 % ; in a few cases the difference is about 1-5 % and can be explained by numerical rounding, when the angles is close to 90 degrees.

Test on the real example:
LHC tune footprint calculation.

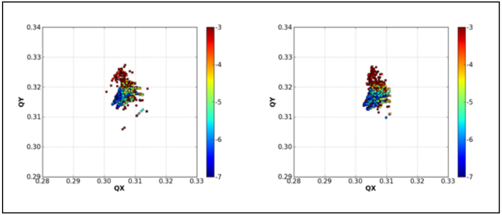
LEFT - Single kick model; RIGHT - Euler integrator (1000 steps)



Wire vs. Electron lens

SIXTRACK simulations: wire vs. beam-beam element
Beam-beam kick from one long-range beam-beam encounter and equivalent current: $e \cdot c \cdot N_p = L \cdot I$ (L-length, I current)

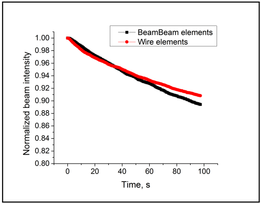
Footprints: wires vs. beam-beam



LHC optics: MD2016, Collisions at IP1 5 Crossing angle 180 murad.; Emit. = 2.5. 6 sigma separation for compensators 4 wires per beam (2 per IP), I=55 Amps 4 beam-beam elements per beam, S=8 (eq. current 46 Amps) Beam decay constants: 3.98 for wires, 4.4 for beam-beam.

For 10 sigma separation - No difference

Beam intensity



Conclusions and Notes:

1. Wire model has been developed and verified with numerical methods.
2. Numerical tests show that the model is applicable for the LHC.
3. Wire is equivalent to Beam-Beam element if wire's separation $>>$ beam sigma and wire's length $>>$ separation.

Notes :

1. For practical usage - fringe field of the wire is negligible (if separation $<<$ wire's length), and Embedded drift could be equal (or almost equal) to wire's length.
2. Particle should not cross the wire's plane - otherwise the single kick model is incorrect
3. Wire's tilt should not be close to 90^0 due to numerical inaccuracy
4. In SIXTRACK: IBECO option effects on WIRE in the same way as on Beam-Beam element (closed orbit subtraction is ON/OFF - see manuals for details)

Thank you for your attention

Additional slides 1. First order Euler Integrator for SIXTRACK

From Euler method (and for the Hamiltonian which is used in SixTrack) the explicit map can be obtained:

$$p_{xn} = \frac{b_1 p_y + p_x a_1 - b_1 c_2 - a_2 c_1}{a_1 a_2 - b_1 b_2} \tag{5a}$$

$$p_{yn} = \frac{a_1 p_y - a_1 c_2 + b_2 p_x - b_2 c_1}{a_1 a_2 - b_1 b_2} \tag{5b}$$

$$p_{zn} = pz \tag{5c}$$

$$x_n = x + ds \frac{p_{xn} - A_x}{1 + \delta} \tag{5d}$$

$$y_n = y + ds \frac{p_{yn} - A_y}{1 + \delta} \tag{5e}$$

$$z_n = z + ds[1 - \frac{\beta_0}{\beta} - \frac{\beta_0 (p_{xn} - A_x)^2 + (p_{yn} - A_y)^2}{2\beta(1 + \delta)}] \tag{5f}$$

Ax; Ay and Az - any analytical functions (with the first derivative)

Additional slides 2. First order Euler Integrator for SIXTRACK

Where:

$$a_1 = 1 - \frac{ds * dA_x/dx}{1+\delta} \qquad (6a)$$

$$a_2 = 1 - \frac{ds * dA_y/dy}{1+\delta} \qquad (6b)$$

$$b_1 = \frac{ds * dA_y/dx}{1+\delta} \qquad (6c)$$

$$b_2 = \frac{ds * dA_x/dy}{1+\delta} \qquad (6d)$$

$$c_1 = -ds * dA_z/dx + \frac{ds * A_x * dA_x/dx + ds * A_y * dA_y/dx}{1+\delta} \qquad (6e)$$

$$c_2 = -ds * dA_z/dy + \frac{ds * A_x * dA_x/dy + ds * A_y * dA_y/dy}{1+\delta} \qquad (6f)$$

Ax; Ay and Az - any analytical functions (with the first derivative)