EFTs for finite temperature and finite density matter

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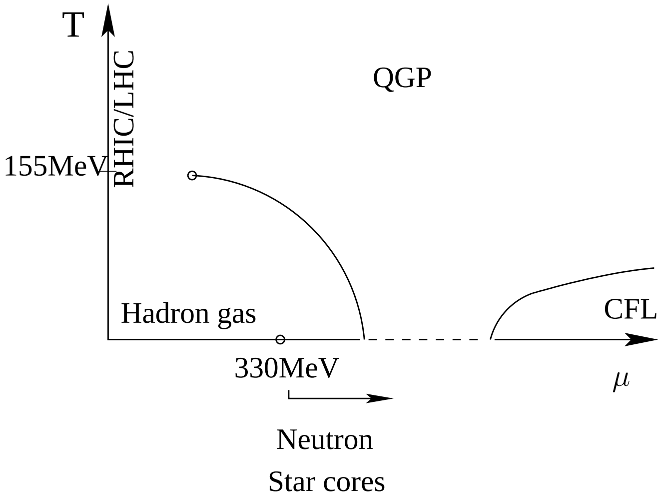
Plan of the talk

- Broad overview of the QCD phase diagram in the temperature (T) and chemical potential (μ) plane
- Steps towards an Effective Field Theory (EFT) based approach at low density and intermediately high T *[arXiv:1710.05345 [hep-ph]] Sourendu Gupta, RS*
- Examples of EFTs at high density and low T
- Application to the calculation of shear viscosity on a phase of dense quark matter *Phys. Rev. D (2017), Sreemoyee Sarkar, RS*

The phase diagram of QCD

- The lagrangian — QCD — is known
- Finite chemical potential: $\mathcal{L} = \mathcal{L}_{\mathrm{QCD}} + \mu\bar{\psi}\gamma^0\psi$
- Finite temperature: $e^{-H/T}$

Phase diagram of QCD



The phase diagram of QCD

- At low T and μ hadronic matter
- At very high T and/or high μ deconfined quarks and gluons
- Perturbative calculations show some control for $T > 1$GeV *eg. Mustafa et. al. (2015)*
- Perturbative calculations show some control for $\mu > 1.5$GeV *Kurkela et. al. (2009)*
- The intermediate region is challenging, but of physical interest

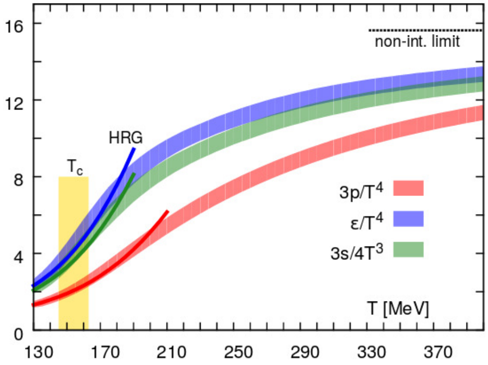
Warm QCD

QCD thermodynamics at $\mu = 0$

- m_q is small, and there is an approximaate chiral symmetry. $\psi \to e^{i\tau^a\theta^a}\psi$ $[SU(2)_V]$, $\psi \to e^{i\gamma^5\tau^a\theta^a}\psi$ $[SU(2)_A]$. τ^a are Pauli matrices that mix u and d spinors
- Relevant current $J_5^{a\mu}(x) = \bar{\psi}t^a\gamma^\mu\gamma^5\psi$
- Partial conservation: $\partial_\mu J_5^{a\mu} = 2m_q P^a$. $P^a(x) = \bar{\psi}t^a\gamma^5\psi$
- At low T, the symmetry is spontaneously broken by the condensate $\langle\bar{\psi}\psi\rangle$. $SU(2)_L \times SU(2)_R \to SU(2)_V$
- 3 light Goldstone bosons, π^a's
- At high enough T chiral symmetry is restored
- For $m_q = 0$, chiral symmetry restoration is a second order phase transition. For finite pion mass, the transition from the symmetry broken phase at small T to the restored phase at large T is a crossover

Calculating the QCD crossover

- Lattice QCD is a rigorous technique to compute the thermodynamics of QCD
- We know quantitatively that for the physical m_q (u, d and heavier s) the transition from hadronic matter at low T to the QGP at high T is a crossover around 145 – 165MeV *[Brookhaven/HotQCD, TIFR, Wuppertal-Budapest, Bielefeld ... collaborations]*. Eg. below *[Bazavov et. al. (1407.6387)]*



Calculating the QCD crossover

- Multiple observables computed on the lattice (eg. speed of sound, susceptibilities)
- But it is challenging to compute transport properties on the lattice
- Finite μ is also challenging

Towards an “effective” field theory (EFT) near crossover

- It will be useful to have an effective theory (EFT) whose parameters are fit using the static lattice calculations
- The expansion parameter is not the coupling constant but the ratio of the energy scale to a cutoff scale
- Matching can be done for static quantities that are measured in experiment or on the lattice
- This can then be used to compute dynamical quantities

The NJL model

- It is a simple, and widely studied model that captures the physics of the chiral crossover (*[Nambu, Jona-Lasinio (1961)]*)
- It is based on the assumption that quarks are light degree of freedom near the crossover
- The parameters of the model are fixed by using the vacuum properties, $f_\pi = 92.3$MeV, and the chiral condensate $(\langle\bar{\psi}\psi\rangle)^{(1/3)} = 251$MeV
- In the chiral limit, this gives T_c (defined by the point of inflection of the chiral condensate) as ~ 175MeV
- On the other hand lattice data (*Bazavov et. al.*, *Gupta et. al.*) gives $T_c \approx 155$MeV
- More complicated fields/energy functionals may be considered, but is there a systematic way?

The NJL model

- NJL is a not bad model for chiral dynamics, but it doesn't contain the correct degrees of freedom at low and high temperatures
- At low energies, the effective theory describing the system is chiral perturbation theory where πs are the degrees of freedom
- At high energies, the effective theory describing the system should include dynamical gluonic degrees of freedom
- The NJL models, trying to match both these regimes, miss important physics near the crossover region
- Can one write a low energy effective theory of fermions valid in the crossover region?
- Need to write all terms consistent with the symmetries of the theory

The Euclidean action

- $$\mathcal{L} = d^{(0)} + \overline{\psi}\partial\!\!\!/_4\psi - \mu\overline{\psi}\gamma_4\psi + d^{43}\overline{\psi}\partial\!\!\!/_i\psi + d^3 T_0\overline{\psi}\psi + \mathcal{L}_6$$
- $$\begin{aligned}\mathcal{L}_6 = &+\frac{d^{65}}{T_0^2}\left[(\overline{\psi}\psi)^2 + (\overline{\psi}i\gamma^5 t^a\psi)^2\right] + \frac{d^{66}}{T_0^2}\left[(\overline{\psi}t^a\psi)^2 + (\overline{\psi}i\gamma^5\psi)^2\right]\\ &+\frac{d_t^{67}}{T_0^2}(\overline{\psi}\gamma_4\psi)^2 + \frac{d_s^{67}}{T_0^2}(\overline{\psi}i\gamma_i\psi)^2 + \frac{d_t^{68}}{T_0^2}(\overline{\psi}\gamma_5\gamma_4\psi)^2 + \frac{d_s^{68}}{T_0^2}(\overline{\psi}i\gamma_5\gamma_i\psi)^2\\ &+\frac{d_t^{69}}{T_0^2}\left[(\overline{\psi}\gamma_4 t^a\psi)^2 + (\overline{\psi}\gamma_5\gamma_4 t^a\psi)^2\right] + \frac{d_s^{69}}{T_0^2}\left[(\overline{\psi}\gamma^i t^a\psi)^2 + (\overline{\psi}\gamma^5\gamma^i t^a\psi)^2\right]\\ &+\frac{d^{61}}{T_0^2}\left[(\overline{\psi}i\Sigma_{i4}\psi)^2 + (\overline{\psi}i\gamma^5\Sigma_{ij}t^a\psi)^2\right] + \frac{d^{62}}{T_0^2}\left[(\overline{\psi}i\Sigma_{i4}t^a\psi)^2 + (\overline{\psi}\Sigma_{ij}\psi)^2\right]\\ &+\mathcal{O}\left(\frac{1}{T_0^5}(\overline{\psi}\psi)^3\right),\end{aligned}$$
- There are no dimension 5 terms (for eg. $\bar{\psi}(\partial)^2\psi$) consistent with the $SU(2)_A$ symmetry
- Dimension 6 terms with derivatives in the mean field approximation $\bar{\psi}(\partial\!\!\!/)^3\psi$ give higher order corrections than we study here. This is because we make a mean field approximation

Spatial momentum cutoff

- Take the energy cutoff to be of the order of T or slightly larger. We will instead use dim-reg
- T_0 is not a parameter; rather to be thought of as a scale

Parameters of the theory

- $m_q = d^3 T_0$ acts as the bare quark mass, but is not fitted to π mass at $T = 0$
- Time and space distinguished: $SO(3,1) \to SO(3)$. For example, the kinetic term is

$$\overline{\psi}\partial\!\!\!/_4\psi + d^{43}\overline{\psi}\partial\!\!\!/_i\psi$$

- Similarly, all vector interaction terms can have different spatial and temporal coefficients
- All interaction terms with chiral symmetry written down
- Seems hopeless, 12 unknown parameters

Mean field approximation

- But sectors of observables with only specific linear combinations of ds emerge
- For example, in the mean field approximation

$$\bar{\psi}_\alpha \psi_\beta \to \delta_{\alpha\beta} \langle \bar{\psi}\psi \rangle$$

-

$$\mathcal{L}_{\mathrm{MFT}} = -\mathcal{N} \frac{T_0^2}{4\lambda} \Sigma^2 + \overline{\psi} \partial\!\!\!/_4 \psi - \mu \overline{\psi} \gamma_4 \psi + d^{43} \overline{\psi} \partial\!\!\!/_i \psi + m_q \overline{\psi} \psi + d^{(0)}$$

- Including all the Fierz transformations,

$$\lambda = (\mathcal{N}+2) d^{65} - 2d^{66} - d_t^{67} + d_s^{67} + d_t^{68} - d_s^{68} + d_t^{60} - d_s^{60}$$

- $m = m_q + \Sigma$

Free energy

- $$\Omega = -\frac{\mathcal{N} T_0^2 \Sigma^2}{4\lambda} - \mathcal{N} I_0$$

- $$I_0 = \frac{T}{2} \sum_{p^4=(2n+1)\pi T} \int \frac{d^3\mathbf{p}}{(2\pi)^3} \log(\frac{m^2 + \mathbf{p}^2 + (p^4)^2}{T^2})$$
$$= \int \frac{d^3\mathbf{p}}{(2\pi)^3} \left(E_p + \log[1 + \exp(-E_p/T)]\right)$$

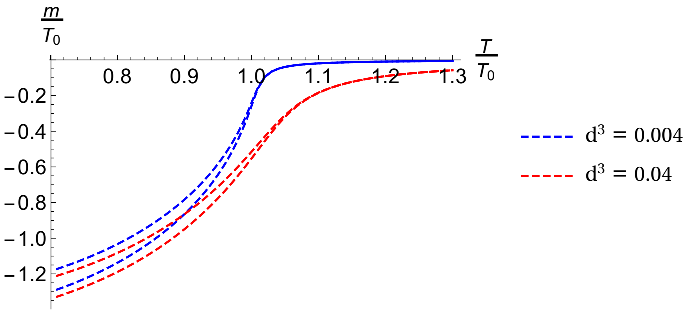
- $E_p = \sqrt{(d^4)^2 \mathbf{p}^2 + m^2}$
- $I_0 =$ $\frac{m^4}{64\pi^2 (d^4)^3} [-\frac{3}{2} - \log(\frac{(d^4)^2 M^2}{m^2})] + \frac{1}{2\pi^2} \int dp p^2 \log[1 + \exp(-E_p/T)]$
- M is the renormalization scale in the $\overline{MS}$ scheme

Transition and the order parameter

- In the chiral limit the second order phase transition point is denoted by T_c. We obtain, $\frac{(d^4)^3}{\lambda} = \frac{1}{12}\frac{T_c^2}{T_0^2}$
- For convenience, T_0 is chosen as the value for the critical point in the chiral limit
- All quantities in units of T_0
- $\frac{(d^4)^3}{\lambda} = \frac{1}{12}$
- Out of the three parameters, $m_q = d^3 T_0$, λ, d^4 one combination is fixed

Order parameter

- By minimizing the free energy we can find the order parameter m
- In the plot the width is associated with varying $M \in (1.25\pi T_0, 1.75\pi T_0)$

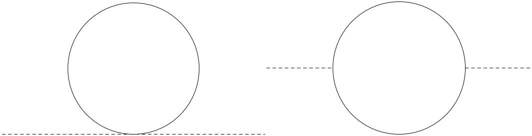


Fluctuations of the order parameter

- In mean field $\bar{\psi}_\beta \psi_\alpha \to \langle \psi \bar{\psi} \rangle \delta_{\alpha\beta}$
- Fluctuations $\psi \to e^{i\pi^a \tau^a \gamma^5/(2f)} \psi$, $\bar{\psi} \to \bar{\psi} e^{i\pi^a \tau^a \gamma^5/(2f)}$
- Therefore, $\psi_\alpha \bar{\psi}_\beta \to e^{i\pi^a \tau^a \gamma^5/(2f)}_{\beta'\beta} \langle \psi_\beta \bar{\psi}_\alpha \rangle e^{i\pi^a \tau^a \gamma^5/(2f)}_{\alpha\alpha'}$
- At very long wavelengths an effective lagrangian for the π's is applicable
- $\mathcal{L}_f = \frac{c^2 T_0^2}{2} \pi^2 + \frac{1}{2} (\partial_0 \pi)^2 + \frac{c^4}{2} (\nabla \pi)^2 + \frac{c^{41}}{8} \pi^4 + \cdot\cdot$

π lagrangian

- We start with the two point function
- $\mathcal{L}_f = \frac{c^2 T_0^2}{2}\pi^2 + \frac{1}{2}(\partial_0\pi)^2 + \frac{c^4}{2}(\nabla\pi)^2$



Correlation functions

- Correlation relations of currents related to π properties
- Two illustrative examples
- $\lim_{q^4 \to 0} \int d^4x e^{iqx} \langle P^a(x) P^b(0) \rangle = (\frac{f}{2m_q})^2 c^4 \frac{\delta^{ab} \mathbf{q}^4}{\mathbf{q}^2 + M_\pi^2}$
- $\lim_{q^4 \to 0} \int d^4x e^{iqx} \langle J_5^{ai}(x) J_5^{bi}(0) \rangle = ((2f)^2) c^4 \frac{\delta^{ab} \mathbf{q}^2}{\mathbf{q}^2 + M_\pi^2}$
- $M_\pi^2 = c^2 T_0^2 / c^4$ related to the screening length
- Static $\pi - \pi$ correlator decays as $\sim e^{-M_\pi r}$
- $u = \sqrt{c^4}$ is the π "speed"
- From a combination of the correlators one can extract f, c^4, M_π
- *[Brandt, Francis, Meyer, Robaina (2014)]*

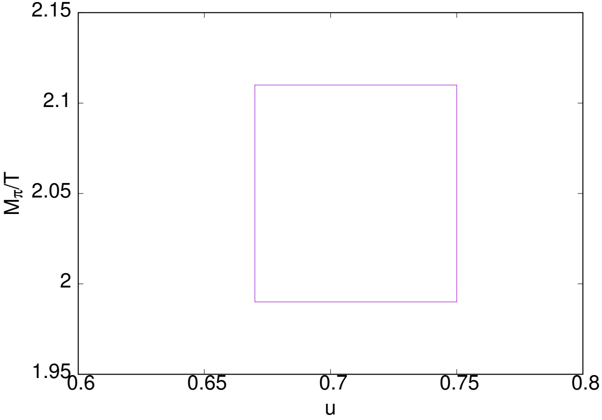
Correlation functions

- A finite temperature generalization of GOR relation is satisfied
- $c^2 T_0^2 = -\frac{\mathcal{N} m_q \langle\bar{\psi}\psi\rangle}{f^2}$
- *[Son, Stephanov (2002)]*
- We can compute f, c^4, M_π in the EFT and compare to the lattice data. I will describe this next

Results

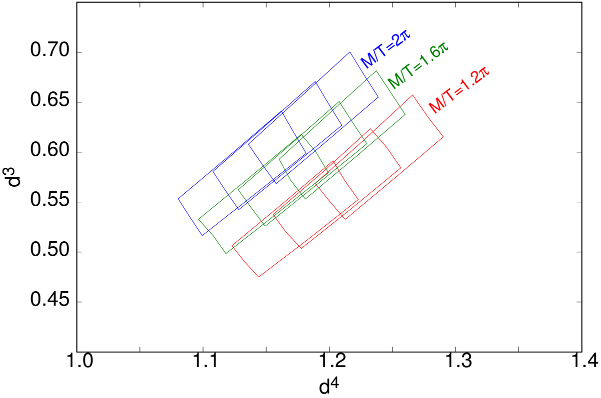
Inputs

- Matching u and M_π at $T = 0.84\,T_{co}$
- Error in T associated with $T_{co} = 211(5)\text{MeV}$
- *[Brandt, Francis, Meyer, Robaina (2014)]*



Outputs

- By fitting u and M_π parameters we obtain the fermionic parameters
- Uncertainty associated with M
- Different boxes associated with varying T_{co} in the error band
- Useful if the fermionic parameters do not vary rapidly with T

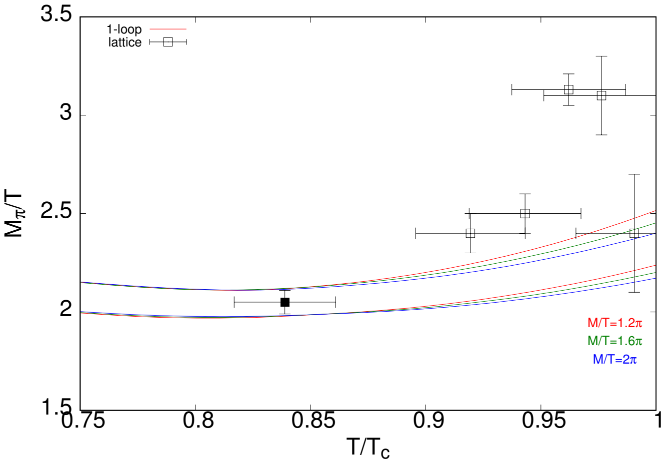


T_c and f

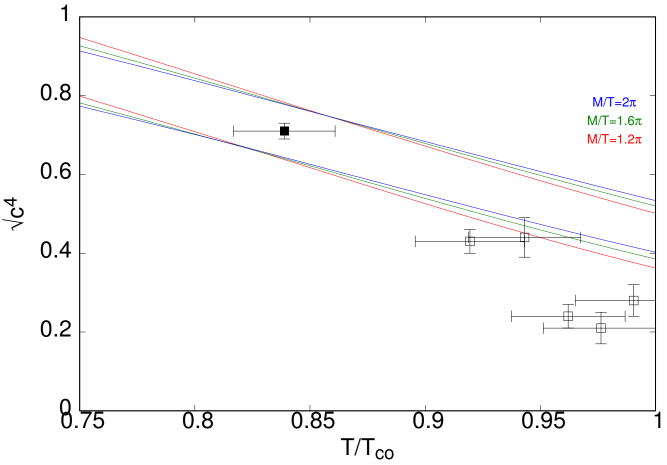
- The peak of the chiral susceptibility in the EFT occurs at $T_{co} = 1.24\,T_c$
- Taking $T_{co} = 211(5)$MeV, we get $T_c = 170 \pm 6$MeV
- This agrees with the lattice prediction *[Brandt et. al. (2013)]* for 2 flavors: $T_c \approx 170$MeV
- A little larger than the value of T_c from the lattice for $2 + 1$ flavors *[Bazavov et. al. (2014), Borsanyi et. al. (2013), Aoki et. al. (2009)]*
- $fu/T(0.84\,T_{co}) = 0.41(2)$ in *[Brandt et. al. (2013)]*
- Our calculated value $fu/T(0.84\,T_{co}) = [0.41]^{+1(\mathrm{input})}_{-1}|^{+3(\mathrm{scale})}_{-2}|^{+2(\mathrm{T})}_{-2}$

M_π

- Pion Debye screening mass

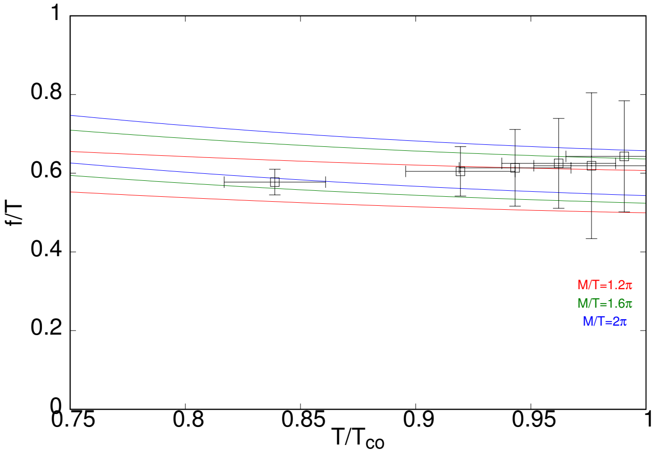


- Pion velocity



f

- Pion constant f
- An independent prediction



P

- Pressure of the π

$$P_\pi = -\frac{3(c^2 T_0^2)^2}{64\pi^2 (c^4)^{(3/2)}}[\log(\frac{c^2 T_0^2}{c^4 M^2}) - \frac{3}{2}]$$
$$- 3T \int \frac{d^3\mathbf{p}}{(2\pi)^3} \log(1 - e^{E^\pi/T})$$

- $E^\pi = \sqrt{c^4\mathbf{p}^2 + c^2 T_0^2}$
- If c^2 is small the pressure is large. Energetic cost is small

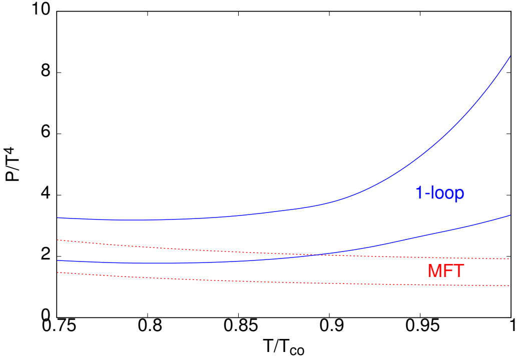
P

- Rise in the pressure of the π because of the thermal piece

$$-3T\int \frac{d^3\mathbf{p}}{(2\pi)^3}\log(1-e^{E^\pi/T}) \tag{1}$$

as u decreases

- Thermodynamic derivatives like entropy, and specific heat under weaker control



Towards finite μ

- If we use the standard modification $H \to H - \mu N$
- In dim-reg an interesting result that $T_c(\mu)^2 + \frac{3}{\pi^2}\mu^2 = T_0^2$ in the chiral limit
- In particular, implies that for small μ, $T_c(\mu) = T_c(0) - \frac{1}{2}\kappa\frac{\mu^2}{T_c(0)} + \mathcal{O}(\mu^3)$
- $T_c(0)\kappa = \frac{3}{\pi^2}$
- Thus the mean field prediction is roughly $5 - 10$ times the lattice prediction for $2 + 1$ flavors *[Bielefeld, HotQCD, collaborations]*
- Several corrections in the EFT required at finite μ

Future directions

- Can be extended by
  - Analyzing $2 + 1$ flavors so that comparison with other lattice calculations is possible
  - Include the role of σ fluctuations
  - Calculating transport properties
  - Going to finite μ

Dense quark matter

Quark matter at high density

- Physically interesting regime between dense hadronic matter and dense quark matter at around $\mu \sim [500, 700]$MeV
- Quantitative perturbative control is difficult but qualitative difference between hadronic matter may show up
- With this philosophy we study the properties of quark matter at high density perturbatively
- Starting point, weakly interacting, nearly massless light quarks (assuming the strange quark mass can be ignored), interacting weakly via gluons

Quark matter at high density: illustrative example

- Know from basic statistical physics that quarks will fill up energy levels up to a Fermi surface
- If the only other scale in the problem is T (unpaired quark matter), and we are interested in $\mu \gg T$, only excitations near the Fermi surface participate in dynamics
- $T \sim$ keV, $\mu \sim 1000$MeV
- This calls out for an effective theory with an expansion in T/μ
- Quarks well below the Fermi surface, and anti-quarks can be integrated out
- Systematic method: High Density Effective theory

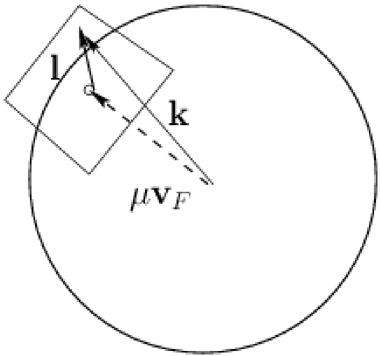
HDET lagrangian

- Instead of the full lagrangian

$$\mathcal{L} = \bar{\psi} i \not{D} \psi + \mu \bar{\psi} \gamma^0 \psi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

- The magnitude of the momentum is close to μ

Patches



Hong (1998, 1999); Casalbuoni, Gatto, Nardulli, (2001); Schaefer (2003)

HDET lagrangian

- Instead of the full quark lagrangian

$$\mathcal{L}_q = \bar{\psi} i \not{D} \psi + \mu \bar{\psi} \gamma^0 \psi$$

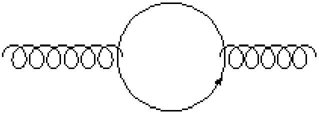
- An effective lagrangian

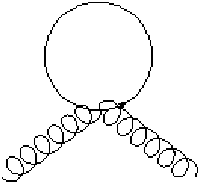
$$\mathcal{L}_q = \sum_{v_F} [\psi_+^\dagger iV \cdot D \psi_+ - \psi_+^\dagger D_\perp \frac{1}{2\mu} D_\perp \psi_+]$$

- $V^\mu = (1, \mathbf{v})$ and $D_\perp$ is the perpendicular derivative
- Additional contact terms suppressed by higher powers of μ
- Formal similarities to HQET
- Can not be used to compute the pressure but can be used to compute transport properties

Gluons

Gluon polarization diagrams





Gluon screening

- Longitudinal gluons are Debye screened

$$\Delta_L(q) = i\frac{\hat{q}^i\hat{q}^j}{(q^0)^2 - \mathbf{q}^2 - \Pi_L(q)} \tag{2}$$

- $\Pi_L(0) = m_D^2 = g^2 N_f g_S \frac{\mu^2}{2\pi^2}$
- Transverse gluons are Landau damped

$$\Delta_t(q) = i\frac{\delta_{ij} - \hat{q}^i\hat{q}^j}{(q^0)^2 - \mathbf{q}^2 - \Pi_t(q)} \tag{3}$$

- $\Pi_t(q^\mu \to 0) = i g^2 N_f g_S \frac{\pi}{4}\frac{q^0}{q}\frac{\mu^2}{2\pi^2}$

Shear viscosity in the unpaired phase

- Shear viscosity measures the ability to transport momentum between two layers of a fluid
- $\eta \sim n\langle p\rangle\langle\tau\rangle$
- $n = \frac{p_F^3}{3\pi^2}$
- $p \sim p_F$
- τ is inversely proportional to the scattering cross-section

$$\tau \propto \frac{1}{|\mathcal{M}|^2}$$

- $\mathcal{M} \sim \frac{g^2}{((q^0)^2 - \mathbf{q}^2 - \Pi)}$
- A simplification that the Landau damped transverse gluons dominate at small T *Heiselberg, Pethick (1993)*
- $\tau \sim \frac{\mu}{g^3 T^2}\left(\frac{T}{g\mu}\right)^{1/3}$, $\eta \sim \frac{\mu^5}{g^3 T^2}\left(\frac{T}{g\mu}\right)^{1/3}$
- Similarly one can calculate the bulk viscosity

Implications: r-modes

- Rotating neutron stars ($\Omega = 2\pi f$) feature an unstable fluid dynamics mode *Andersson (1998), Friedman and Morsink (1998)*
- First treating the fluid as an ideal fluid one obtains in a rotating frame

$$\mathbf{v}(\mathbf{r}) \approx a\Omega f(r)\mathbf{Y}_{lm}e^{i(m\phi-\sigma_r t)}$$

- $\sigma_r \approx -\frac{2m\Omega}{l(l+1)} < 0$ for $m > 0$
- $\sigma_I = \sigma_r + m\Omega > 0$ for $m > 2$

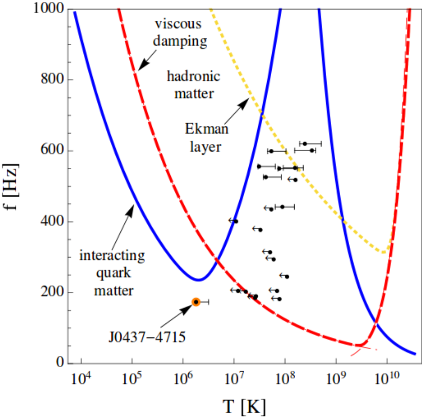
r-modes

- Including "damping" from gravitational waves: couple gravitons to the fluid motion
- $E \approx E_0 e^{-2t/\tau_{\mathrm{GR}}}$
- $\tau_{\mathrm{GR}} < 0$, implying instability
- The mode grows with time
- Note that an inertial observer far away sees the angular momentum as well as the energy of the star decrease
- $1/\tau_{\mathrm{GR}} \sim -(G_N)\Omega^{2l+2}$: instability increases with Ω
- ($l = m = 2$ is the dominant mode and is most studied)

r-modes

- Viscosities in the fluid indeed damp the fluid flow
- Including damping from gravitational waves, shear viscosity η, and bulk viscosity ζ
- $E \approx E_0 e^{-2(t/\tau_{\mathrm{GR}}+1/\tau_\eta+1/\tau_\zeta)}$
- $\frac{1}{\tau_\eta} \propto \int d^3x \eta \delta\sigma^{ab} \delta\sigma_{ab}$
- In the absence of microscopic damping mechanisms, the loss in angular momentum is very rapid (the rotational speed of about 500Hz drops by a substantial fraction in 1 year)
- The non-observation of such spin down constrains the microscopic properties of neutron stars

Quark matter versus hadronic matter



- Unpaired quark matter is consistent with non-obervation of rapid de-spinning of the fast rotating pulsars but not hadronic
- *Jaikumar, Rupak, Steiner (2008); Alford, Schwenzer (2014)*

Additional damping effects

- Caveat is that there could be additional damping effects
- Other condensates in hadronic matter
- Friction between the crust-core interface *Bildsten, Ushomirsky (1999); Lindblom, Ushomirsky (2000); Jaikumar, Rupak (2010)*
- Non-linear saturation of the $r-$modes to a small magnitude *Alford, Mahmoodifar, Schwenzer (2012); Alford, Han, Schwenzer (2012)*

Color superconductivity

- But quark matter is expected to be in a paired phase because the interaction between quarks is attractive in the color antisymmetric channel *Alford, Rajagopal, Wilczek and Shuryak, Schaefer, Rapp (1998)*
- At asymptotically high densities where the strange quark mass can be ignored, quark matter is in the CFL phase
- The di-quark condensate is antisymmetric in color and in spin, and therefore also in flavor

$$\langle\psi_{\alpha i}(p)(C\gamma^5)\psi_{\beta j}(-p)\rangle \propto \Delta \sum_I \epsilon_{I\alpha\beta}\epsilon_{Iij} \tag{4}$$

- $U(1) \times SU_c(3) \times SU_L(3) \times SU_R(3) \rightarrow Z_2 \times SU_{c+L+R}(3)$
- The ϵ tensors "lock" color and flavor, and hence CFL

Complete change in low energy excitations

- In the CFL phase all fermionic quasi-particle excitations are gapped due to pairing
- Energy scales $\mu > 500\text{MeV}$, $\Delta \sim 10\text{MeV}$, $T \sim 0.001 - 1\text{MeV}$ where Δ is proportional to the condensate and is the gap in the fermionic spectrum
- $E = \sqrt{(p-\mu)^2 + \Delta^2}$
- This is the analog of electronic superconductivity where the electrons form Cooper pairs, and to break a Cooper pair one needs to supply an energy Δ
- Therefore a hierarchy of scales $\mu \gg \Delta \gg T$

EFT for CFL

- Therefore the fermionic contribution to transport properties is exponentially suppressed $e^{-\Delta/T}$
- The gluons are also screened on length scales much shorter than $1/T$
- They can be integrated out and an effective theory based only on the Goldstone modes is sufficient to describe phenomena for $T \ll \Delta$
- $U(1) \times SU_c(3) \times SU_L(3) \times SU_R(3) \to Z_2 \times SU_{c+L+R}(3)$
- Ignoring the gauged part of the symmetry breaking, the breaking pattern of the continuous symmetry is $U(1) \times SU_L(3) \times SU_R(3) \to SU_{L+R}(3)$ *Alford, Rajagopal, Wilczek, (1998)*
- This pattern is familiar from chiral symmetry breaking in vacuum, except for the additional $U(1)_B$

Mesonic EFT

-

$$\begin{aligned}\mathcal{L} = &\frac{1}{4f_\pi^2}\mathrm{tr}[\partial_0\Sigma\partial_0\Sigma] - v_\pi^2\frac{1}{4f_\pi^2}\mathrm{tr}[\partial_i\Sigma\partial_i\Sigma] \\ &+ \frac{1}{2f_\phi^2}[\partial_0\phi\partial_0\phi] - v_\phi^2\frac{1}{2f_\phi^2}[\partial_i\phi\partial_i\phi] \\ &+ c_4[(\partial_0\phi)^4 + (\partial_i\phi)^4 - 2(\partial_i\phi)^2(\partial_0\phi)^2] \\ &+ c_3(\partial_i\phi)^2(\partial_0\phi) + ...\end{aligned} \tag{5}$$

- ϕ associated with $U_B(1)$ breaking
- $\Sigma = \exp(\frac{it^a\pi^a}{f_\pi})$ associated with $L - R$
- *Son, Stephanov (1999), Casalbuoni, Gatto (1999, 2000), Schaefer (2000)*

Mesonic EFT coefficients

- In perturbation theory to lowest order in g
- $f_\pi^2 = \frac{21-8\log(2)}{18}\frac{\mu^2}{2\pi^2},\ v_\pi = 1/3$
- $f_\phi^2 = 9\frac{\mu^2}{2\pi^2},\ v_\phi = 1/3$
- $c_4 = \frac{3}{4\pi^2}$
- $c_3 = \frac{3\mu}{\pi^2}$
- Can include small quark mass corrections in the standard manner *Son, Stephanov (1999), Casalbuoni, Gatto (1999, 2000), Schaefer (2000)*

Scattering of mesons

- An important feature is that mesons only interact via derivative interactions
- Consequently at least $|\mathcal{M}| \propto T^4$ for ϕ
- *Manuel, Dobado, Estrada (2005); Mannarelli, Manuel, 'Saad (2008); Mannarelli, Manuel (2010)*
- A detailed calculation gives $\tau \propto \mu^4/T^5$
- This corresponds to mean free path larger than the size of the neutron star: no damping

Constraints on CFL

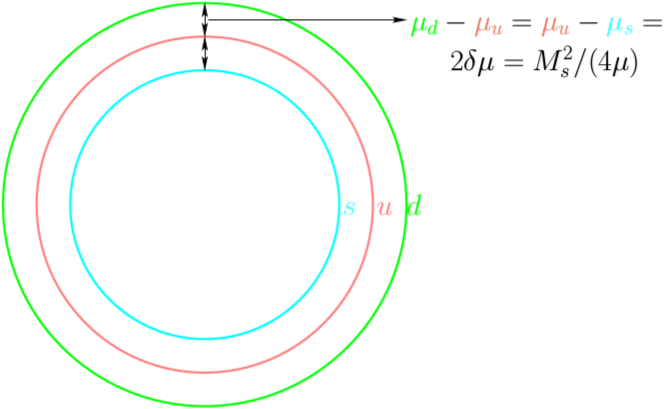
- CFL phase is inconsistent with r-mode stability constraints
  Manuel, Mannarelli, S'ad (2008), Jaikumar, Rupak (2010)

Strange quark mass and neutrality

- Ignoring M_s is not a good approximation if μ is not very large
- $\sqrt{M_s^2 + (p_s^F)^2} = \mu \implies p_s^F \approx \mu - M_s^2/(2\mu)$, but this leaves an unbalanced positive charge.
- Need to introduce a chemical potential, μ_e, to restore neutrality.
- Weak equilibrium implies $\mu_d - \mu_s = 0$, $\mu_d - \mu_u = \mu_e$
- Electrical neutrality is imposed by demanding $\frac{\partial \Omega}{\partial \mu_e} = 0$.
- Similarly, color neutrality by desiring $\frac{\partial \Omega}{\partial \mu_{3,8}} = 0$

Neutral unpaired quark matter

- For unpaired quark matter we obtain $\mu_e = M_s^2/(4\mu)$, $\mu_3 = \mu_8 = 0$.



Alford, Burgess, Rajagopal (1999)

Inhomogenoue pairing phases

- Focus on the two flavors u and d
- CFL involves pairing between different flavors

$$\langle u(\mathbf{p})d(-\mathbf{p})\rangle \propto \Delta$$

or in position space

$$\langle u(x)d(x)\rangle \propto \Delta$$

- This is preferred if the Fermi surfaces are equal in size
- An inhomogeneous pairing pattern may be preferred if $\delta\mu$ is large enough

$$\langle u(\mathbf{p}+\mathbf{q})d(-\mathbf{p}+\mathbf{q})\rangle \propto \Delta$$

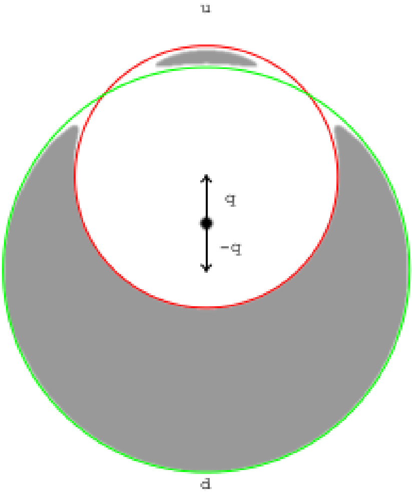
or in position space

$$\langle u(x)d(x)\rangle \propto \Delta e^{i2\mathbf{q}\cdot\mathbf{r}}$$

Alford, Bowers, Rajagopal (2001). Favoured for $\mu \sim$ 500MeV for a range of parameters *Rajagopal, RS (2005)*

Gapless fermionic modes

- $E = -\delta\mu - q\cos\theta + \sqrt{(p-\mu)^2 + \Delta^2}$
- This dispersion relation has gapless surfaces (if $|\delta\mu + q| < \Delta$)



Low energy degrees of freedom

- Gapless modes of the u and d quarks
- In general, lattice phonons associated with translational symmetry breaking
- Gauge bosons of which only transverse gluons, t^1, t^2, and t^3 are relevant because they are long ranged
- The polariziation tensor for these was calculated in *RS EPJA (2017)*

Low energy lagrangian

$$\begin{aligned}\mathcal{L} = &\frac{1}{2}\sum_{v_F}\Psi^\dagger_{Lv_F}\begin{pmatrix} V\cdot\partial - q\cos\theta - \delta\mu & \Delta \\ \Delta & \tilde{V}\cdot\partial - q\cos\theta - \delta\mu \end{pmatrix}\Psi_{Lv_F} \\ &+ \frac{1}{2}\sum_{v_F} g A^a_\mu \Psi^\dagger_{Lv_F}\begin{pmatrix} V^\mu t^a & 0 \\ 0 & -\tilde{V}^\mu t^{a*} \end{pmatrix}\Psi_{Lv_F} \\ &+ \frac{c_\mu}{f_\varphi}\partial_\mu\varphi^a\bar{\psi}_{Lv_F}\gamma^\mu\psi_{Lv_F} + (L\to R)\end{aligned} \tag{6}$$

Gluonic and Goldstone contribution

- Gluons have a short mean free path and their contribution to viscosity is subdominant
- Because of scattering off gapless quarks, the contribution of the Goldstone mode is also sub-dominant

$$\eta_\phi \sim \frac{1}{v_\phi^3}\frac{f_\phi^2}{\mu^2}T^3 \tag{7}$$

- Therefore the dominant contribution comes from quarks
- The dominant scattering mechanism is the exchange of transverse t^1, t^2, t^3 gluons

Shear viscosity in the FF phase

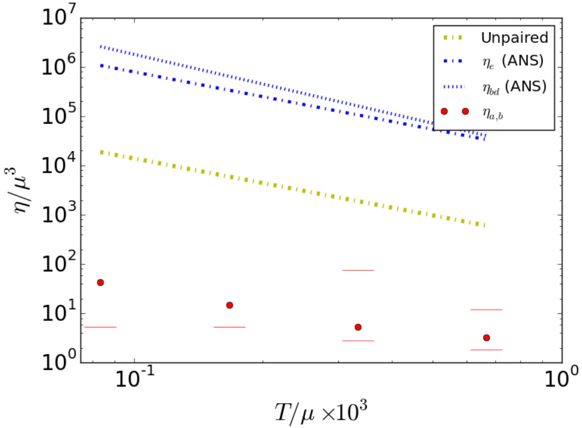
- The modification of the density of states is simple — geometric
- $\eta^{(0)} \approx \frac{\mu^4}{5\pi^2}(1 - \frac{\Delta}{q})\tau^{(0)}$
- $\tau^{(0)}$ is related to the collision integral

$$\frac{1}{\tau^{(0)}} \propto \frac{1}{T} \int \frac{d^3p_1}{(2\pi)^3} \frac{d^3p_2}{(2\pi)^3} \frac{d^3p_3}{(2\pi)^3} \frac{d^3p_4}{(2\pi)^3}$$
$$|\mathcal{M}(12 \to 34)|^2$$
$$(2\pi)^4 \delta(\sum p^\mu)[f_1 f_2 (1 - f_3)(1 - f_4)]$$
$$\phi_i^{ab}.\Pi_{abcd}^{(0)}.\phi_i^{cd}$$

with $\phi_i^{ab} = v^a p^b$, $\Pi_{ijkl} = \frac{3}{2}(\hat{e}_i \hat{e}_j - \delta_{ij})(\hat{e}_k \hat{e}_l - \delta_{kl})$

- Complicated because the distribution functions f depend on the angles in addition to the magnitude of the momentum. Needs to be done numerically

Results for the FF phase



Sarkar, RS (2017); *Alford Nishimura Sedrakian [ANS] (2014)*

Conclusions and future work

- Data on the angular velocity of neutron stars puts constraints on the viscosity of the matter the cores of neutron stars: possibly suggesting the presence of a (1) deconfined phase with (2) gapless fermionic excitations
- Crystalline color superconducting phases are natural candidates for a paired quark matter phase with gapless excitations. The shear viscosity is even larger compared to unpaired quark matter in the two flavor case
- Will be interesting to see if results of the full three flavor problem consistent with the data

Backup slides

Bulk viscosities

- Similarly one can calculate the bulk viscosity
- Bulk viscosity is related to particle production during compression and expansion
- For example expansion will break the weak equilibrium between u and d. Electro-weak processes changing u to d re-establish the equilibrium
- $\zeta = A\frac{\Gamma}{\Omega^2+\Gamma^2}$. Has a Lorentzian shape with the peak at $\Gamma = \Omega$
- $\Gamma \sim G_F^2 T^2 \mu^3$ *Madsen (1998)*

Favourability of LOFF phases

- The inhomogeneous (FF) phase thermodynamically preferred state compared to isotropic states for $\delta\mu \sim [0.707, 0.754]\Delta$, where Δ is the gap for $\delta\mu = 0$
- A detailed analysis (*Mannarelli, Rajagopal, RS (2005), Ippolito, Nardulli, Ruggieri (2007)*) suggests that for three flavors $440 \lesssim \mu \lesssim 520$MeV an inhomogenous state might be the ground state. This is the relevant region for neutron star cores
- We take the simplest phase with only one momentum direction $\mathbf{q}$
- We only consider two flavors of quarks *u* and *d* in this first analysis

Intuition for favoured inhomogeneous pairing