QUANTUM DECOHERENCE IN NEUTRINO OSCILLATIONS

Poonam Mehta

School of Physical Sciences,
Jawaharlal Nehru University, New Delhi

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Recent works

Revisiting the quantum decoherence scenario as an explanation for the LSND anomaly

Pouya Bakhti,[1,*] Yasaman Farzan,[1,†] and Thomas Schwetz[2,‡]

[1] *Institute for research in fundamental sciences (IPM), PO Box 19395-5531, Tehran, Iran*

[2] *Oskar Klein Centre for Cosmoparticle Physics, Department of Physics, Stockholm University, SE-10691 Stockholm, Sweden*

Abstract

We propose an explanation for the LSND anomaly based on quantum decoherence, postulating an exponential behavior for the decoherence parameters as a function of the neutrino energy. Within this ansatz decoherence effects are suppressed for neutrino energies above 200 MeV as well as around and below few MeV, restricting deviations from standard three-flavour oscillations only to the LSND energy range of 20–50 MeV. The scenario is consistent with the global data on neutrino oscillations, alleviates the tension between LSND and KARMEN, and predicts a null-result for MiniBooNE. No sterile neutrinos are introduced, conflict with cosmology is avoided, and no tension between short-baseline appearance and disappearance data arises. The proposal can be tested at planned reactor experiments with baselines of around 50 km, such as JUNO or RENO-50.

$$d_i = \sqrt{\gamma_0}\ \exp\left[-\left(\frac{E}{E_i}\right)^n\right], \qquad (7)$$

γ_0 is a constant parameter with dimension of mass, universal for all mass eigenstates.

1503.05374 [hep-ph]

Recent works

PRL **118,** 221801 (2017) PHYSICAL REVIEW LETTERS week ending 2 JUNE 2017

Nonmaximal θ_{23} Mixing at NOvA from Neutrino Decoherence

João A. B. Coelho,[1,2,*] W. Anthony Mann,[1,†] and Saqib S. Bashar[1]

[1]*Physics Department, Tufts University, Medford, Massachusetts 02155, USA*

[2]*APC, Université Paris Diderot, CNRS/IN2P3, Sorbonne Paris Cité, F-75205 Paris, France*

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In a study of a muon-neutrino disappearance at 810 km, the NOvA experiment finds flavor mixing of the atmospheric sector to deviate from maximal ($\sin^2\theta_{23} = 0.5$) by 2.6σ. The result is in tension with the 295-km baseline measurements of T2K, which are consistent with maximal mixing. We propose that θ_{23} is in fact maximal, and that the disagreement is a harbinger of environmentally induced decoherence. The departure from maximal mixing can be accounted for by an energy-independent decoherence of strength $\Gamma = (2.3 \pm 1.1) \times 10^{-23}$ GeV.

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Recent works

Decoherence, matter effect, ν hierarchy signature in long-baseline experiments

João A. B. Coelho[1,2,*] and W. Anthony Mann[1,†]

[1] *Physics Department, Tufts University, Medford, Massachusetts 02155, USA*
[2] *APC, Université Paris Diderot, CNRS/IN2P3, Sorbonne Paris Cité, F-75205 Paris, France*

Environmental decoherence of oscillating neutrinos of strength $\Gamma = (2.3 \pm 1.1) \times 10^{-23}$ GeV can explain how maximal θ_{23} mixing observed at 295 km by T2K appears to be non-maximal at longer baselines. As shown recently by R. Oliveira, the MSW matter effect for neutrinos is altered by decoherence: In normal (inverted) mass hierarchy, a resonant enhancement of $\nu_\mu(\bar{\nu}_\mu) \to \nu_e(\bar{\nu}_e)$ occurs for $6 < E_\nu < 20$ GeV. Thus decoherence at the rated strength may be detectable as an excess of charged-current ν_e events in the full ν_μ exposures of MINOS+ and OPERA.

0708.05495 [hep-ph]

Recent works

Revisiting quantum decoherence in the matter neutrino oscillation framework

J.A. Carpio,[1,2] E. Massoni,[1] and A.M. Gago[1]

[1] *Sección Física, Departamento de Ciencias, Pontificia Universidad Católica del Perú, Apartado 1761, Lima, Perú*
[2] *Department of Physics, The Pennsylvania State University, University Park, Pennsylvania 16802, USA*

We re-examine the matter neutrino oscillation probabilities considering the decoherence phenomenon as a sub-leading effect. In this paper we point out the relevance of having the correct interpretation of the decoherence matrix in the different quantum bases, within the framework of neutrino oscillation probabilities in matter. Based on this treatment we develop an analytical formula for matter neutrino oscillation probabilities for three generations, with a range of application up to the decoherence parameter $\Gamma \sim 10^{-23}$ GeV. We observe that, due to decoherence, the amplitudes of the neutrino/antineutrino oscillation probabilities increase in an energy independent way. We also find that decoherence can reduce the absolute value of the CP asymmetry, relative to its value at the pure oscillation case, up to 25% and 35% for the CP violation phases $\delta = \pi/2$ and $3\pi/2$, respectively. As a side effect we have the introduction of a degeneracy between the decoherence parameter Γ and the CP violation phase δ.

1711.03680 [hep-ph]

Interesting work

Equivalence between Gaussian averaged neutrino oscillations and neutrino decoherence

Tommy Ohlsson*

Institut für Theoretische Physik, Physik Department, Technische Universität München, James-Franck-Straße, DE-85748 Garching bei München, Germany and Division of Mathematical Physics, Theoretical Physics, Department of Physics, Royal Institute of Technology, SE-100 44 Stockholm, Sweden

(Dated: February 1, 2008)

In this paper, we show that a Gaussian averaged neutrino oscillation model is equivalent to a neutrino decoherence model. Without loss of generality, the analysis is performed with two neutrino flavors. We also estimate the damping (or decoherence) parameter for atmospheric neutrinos and compare it to earlier obtained results.

0012272 [hep-ph]

Computing probability using Liouville-Lindblad formalism

$$\boxed{\frac{\partial \rho}{\partial t} = -i\,[H,\,\rho] + \mathcal{D}\,[\rho]}$$

H, is responsible for the usual unitary evolution,

$\mathcal{D}\,[\rho]$, for non-unitary evolution, i.e., decoherence.

Two flavour case

$$
\begin{aligned}
\rho &= \frac{1}{2}\left[p_\mu \lambda_\mu\right] = \frac{1}{2}\left[p_0 I + p_i \lambda_i\right] \\
H &= \frac{1}{2}\left[h_\mu \lambda_\mu\right] = \frac{1}{2}\left[h_0 I + h_i \lambda_i\right] \\
\mathcal{D}[\rho] &= \frac{1}{2}\left[\lambda_\mu d_{\mu\nu} \rho_\nu\right]
\end{aligned}
$$

$$\dot{p}_\mu = (h_{\mu\nu} + d_{\mu\nu}) p_\nu$$

Two flavour case

$$\boxed{\dot{p}_\mu = (h_{\mu\nu} + d_{\mu\nu})p_\nu}$$

$$\frac{d}{dt}\begin{pmatrix} p_0 \\ p_1 \\ p_2 \\ p_3 \end{pmatrix} \quad = \quad (-2)\left[\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & a & b+\omega & d \\ 0 & b-\omega & \alpha & \beta \\ 0 & d & \beta & \delta \end{pmatrix}\right]\begin{pmatrix} p_0 \\ p_1 \\ p_2 \\ p_3 \end{pmatrix}$$

$$\omega \;=\; \delta m^2/4E \;\text{ with }\; \delta m^2 \;=\; m_i^2 - m_j^2$$

$$\rho_{\nu_\alpha}(0) \quad = \quad \begin{pmatrix} U_{\alpha 1}^2 & U_{\alpha 1}U_{\alpha 2} \\ U_{\alpha 2}U_{\alpha 1} & U_{\alpha 2}^2 \end{pmatrix}$$

The neutrino flavor density matrix for "pure" flavor states $|\,\nu_e\,\rangle = (\cos\theta, \sin\theta)$ and $|\,\nu_\mu\,\rangle = (-\sin\theta, \cos\theta)$ at the initial time $t=0$ are given by

$$\begin{aligned}
\rho_{\nu_e}(0) &= |\,\nu_e\,\rangle\langle\,\nu_e\,| = \begin{pmatrix} \cos^2\theta & \cos\theta\sin\theta \\ \cos\theta\sin\theta & \sin^2\theta \end{pmatrix} \\
\rho_{\nu_\mu}(0) &= |\,\nu_\mu\,\rangle\langle\,\nu_\mu\,| = \begin{pmatrix} \sin^2\theta & -\cos\theta\sin\theta \\ -\cos\theta\sin\theta & \cos^2\theta \end{pmatrix} \; .
\end{aligned} \tag{6}$$

Now we know that

$$\rho(t) = \frac{1}{2}\left[p_0 I + \vec{p}(t)\cdot\vec{\sigma}\right] = \frac{1}{2}\begin{pmatrix} 1+p_3(t) & p_1(t)-ip_2(t) \\ p_1(t)+ip_2(t) & 1-p_3(t) \end{pmatrix} \; , \tag{7}$$

where we have used the condition $Tr[\rho]=1$ to get $p_0=1$. We readily find p_i for the state $|\,\nu_e\,\rangle$ as

$$p_1(0) = \sin 2\theta, \quad p_2(0) = 0, \quad p_3(0) = \cos 2\theta \; , \tag{8}$$

while for $|\,\nu_\mu\,\rangle$ the components p_i are

$$p_1(0) = -\sin 2\theta, \quad p_2(0) = 0, \quad p_3(0) = -\cos 2\theta \; . \tag{9}$$

Using Eq. (8) and Eq. (9), the initial pure density matrices (Eq. (6)) can be expressed as

$$\begin{aligned}
\rho_{\nu_e}(0) &= \begin{pmatrix} \frac{1}{2}+\cos 2\theta & \sin 2\theta \\ \sin 2\theta & \frac{1}{2}-\cos 2\theta \end{pmatrix} \\
\rho_{\nu_\mu}(0) &= \begin{pmatrix} \frac{1}{2}-\cos 2\theta & -\sin 2\theta \\ -\sin 2\theta & \frac{1}{2}+\cos 2\theta \end{pmatrix} = 1-\rho_{\nu_e}(0)
\end{aligned} \tag{10}$$

The diagonal elements of $\rho(t)$ are referred to as populations while the off-diagonal elements as coherences. The phase information is contained in the coherences of the density matrix. Let us define the 3×3 block in Eq. (4) connecting the components $p_i (i \neq 0)$ by $\mathcal{L}$

$$\mathcal{L} = \begin{pmatrix} a & b-\omega & d \\ b+\omega & \alpha & \beta \\ d & \beta & \delta \end{pmatrix} \tag{11}$$

and denote $\mathcal{M} = e^{-2\mathcal{L}t}$ and then write the solutions $p_i(t)$ in terms of the elements of exponentiated matrix $\mathcal{M}$,

$$\begin{aligned} p_1(t) &= \mathcal{M}_{11}p_1(0) + \mathcal{M}_{12}p_2(0) + \mathcal{M}_{13}p_3(0) \\ p_2(t) &= \mathcal{M}_{21}p_1(0) + \mathcal{M}_{22}p_2(0) + \mathcal{M}_{23}p_3(0) \\ p_3(t) &= \mathcal{M}_{31}p_1(0) + \mathcal{M}_{32}p_2(0) + \mathcal{M}_{33}p_3(0) \ , \end{aligned} \tag{12}$$

where $p_i(0)$ are the components of the initial density matrix given in Eq. (5) and Eq. (6).

Finally the neutrino oscillation probability $P_{\alpha\beta}$ can be computed using

$$P_{\alpha\beta}(t) \quad = \quad \mathrm{Tr}[\rho_{\nu_\alpha}(t)\,\rho_{\nu_\beta}(0)]\ , \tag{13}$$

where $\rho_{\nu_\beta}(0)$ is the "pure" neutrino density matrix corresponding to flavor ν_β at $t = 0$ and $\rho_{\nu_\alpha}(t)$ is the density matrix at t for flavor ν_α.

$$\begin{aligned} P &= \mathrm{Tr}[\rho_{\nu_\mu}(0)\,\rho(t)] \\ &= \mathrm{Tr}\left[\begin{pmatrix} \sin^2\theta & -\sin\theta\cos\theta \\ -\sin\theta\cos\theta & \cos^2\theta \end{pmatrix} \frac{1}{2} \begin{pmatrix} 1+p_3 & p_1 - ip_2 \\ p_1 + ip_2 & 1-p_3 \end{pmatrix}\right] \\ &= \frac{1}{2}\left[1 - p_3(t)\cos(2\theta) - p_1(t)\sin(2\theta)\right]\ . \end{aligned} \tag{14}$$

$$P \quad = \quad \frac{1}{2} + \frac{1}{2}\left[-\cos^2(2\theta)\mathcal{M}_{33} - \sin^2(2\theta)\mathcal{M}_{11} - \frac{1}{2}\sin(4\theta)(\mathcal{M}_{13} + \mathcal{M}_{31})\right]\ . \tag{15}$$

1. All decoherence parameters $(a, b, d, \alpha, \beta, \delta)$ are set to zero.

$$\mathcal{L} = \begin{pmatrix} 0 & -\omega & 0 \\ \omega & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} , \tag{16}$$

for which we get

$$\begin{aligned} \mathcal{M}_{33} &= 1 \\ \mathcal{M}_{11} &= \cos(2\omega t) \\ \mathcal{M}_{13} + \mathcal{M}_{31} &= 0 \ . \end{aligned} \tag{17}$$

$$P = \frac{1}{2}\left[(1 + U_{e1}^2 - U_{e2}^2)U_{\mu 1}^2 + (1 - U_{e1}^2 + U_{e2}^2)U_{\mu 2}^2 + 4U_{e1}U_{e2}U_{\mu 1}U_{\mu 2}\cos 2\omega t\right] . \tag{18}$$

Eq. (15) leads to

$$\begin{aligned} P &= \frac{1}{2} + \frac{1}{2}\left[-\cos^2(2\theta) - \sin^2(2\theta)\cos(2\omega t)\right] \\ &= \frac{1}{2}\sin^2(2\theta)[1 - \cos(2\omega t)] \ . \end{aligned} \tag{19}$$

This matches with the standard expression for probability for pure state evolution. Averaging over time, we get averaged oscillations,

$$P \rightarrow \sum_i |U_{ei}|^2 \, |U_{\mu i}|^2 = \frac{1}{2}\sin^2(2\theta) \ . \tag{20}$$

2. Minimal Decoherence Scenario : final row and column of $\mathcal{L}$ have zero entries i.e., $d = \beta = \delta = 0$

$$\mathcal{L} \;=\; \begin{pmatrix} a & b-\omega & 0 \\ b+\omega & \alpha & 0 \\ 0 & 0 & 0 \end{pmatrix} . \tag{21}$$

In particular if we consider the form discussed in [20],

$$(-2)\mathcal{L} \;=\; \begin{pmatrix} -d^2 & k & 0 \\ -k & -d^2 & 0 \\ 0 & 0 & 0 \end{pmatrix} , \tag{22}$$

we obtain

$$\mathcal{M} \;=\; \begin{pmatrix} e^{-d^2 t}\cos(kt) & e^{-d^2 t}\sin(kt) & 0 \\ -e^{-d^2 t}\sin(kt) & e^{-d^2 t}\cos(kt) & 0 \\ 0 & 0 & 1 \end{pmatrix} . \tag{23}$$

We have

$$\begin{aligned}\mathcal{M}_{33} &= 1 \\ \mathcal{M}_{11} &= e^{-d^2 t}\cos(kt) \\ \mathcal{M}_{13} + \mathcal{M}_{31} &= 0\ . \end{aligned} \tag{24}$$

Eq. (15) gives

$$\begin{aligned} P &= \frac{1}{2} + \frac{1}{2}\left[-\cos^2(2\theta) - \sin^2(2\theta) e^{-d^2 t}\cos(kt)\right] \\ &= \frac{1}{2}\sin^2(2\theta)\left[1 - e^{-d^2 t}\cos(kt)\right]\ . \end{aligned} \tag{25}$$

The above equation matches Eq. (46) of Ref. [20]. For $k = 2\omega = \delta m^2/2E$ and $t \simeq L$, we have

$$P = \frac{1}{2}\sin^2(2\theta)\left[1 - e^{-d^2 L}\cos\left(\frac{\delta m^2}{2E}L\right)\right]\ . \tag{26}$$

Averaging Eq. (25) over time, we get the same limit as that for the case of no decoherence (Eq. (20)) which depends upon the mixing angle θ. Note that the probability in the asymptotic limit is not the steady state value (1/2) since this case corresponds to energy being conserved in the neutrino system. This case is the simplest possible extension of standard oscillation probability with minimal decoherence parameters, $a = \alpha = -d^2/(-2)$, $b = 0$ and $\omega = -k/(-2)$.

3. Decoherence Scenario : $\beta \neq 0$

$$\mathcal{L} = \begin{pmatrix} 0 & -\omega & 0 \\ \omega & 0 & \beta \\ 0 & \beta & 0 \end{pmatrix} . \tag{27}$$

$$\begin{aligned} \mathcal{M}_{33} &= \frac{-\omega^2 + \beta^2 \cos\left(2\Omega_\beta t\right)}{-\Omega_\beta^2} \\ \mathcal{M}_{11} &= \frac{\beta^2 - \omega^2 \cos\left(2\Omega_\beta t\right)}{-\Omega_\beta^2} \\ \mathcal{M}_{13} + \mathcal{M}_{31} &= 0 \ , \end{aligned} \tag{28}$$

where $\Omega_\beta = \sqrt{\omega^2 - \beta^2}$. Eq. (15) leads to

$$P = \frac{1}{2} + \frac{1}{2}\left[\cos^2(2\theta)\left\{-\frac{\omega^2}{\Omega_\beta^2} + \frac{\beta^2}{\Omega_\beta^2}\cos(2\Omega_\beta t)\right\} + \sin^2(2\theta)\left\{\frac{\beta^2}{\Omega_\beta^2} - \frac{\omega^2}{\Omega_\beta^2}\cos(2\Omega_\beta t)\right\}\right] . \tag{29}$$

The above equation matches Eq. (13) of Ref. [7]. Averaging over time, we get

$$P \rightarrow \frac{1}{2} + \frac{1}{2}\left\{-\frac{\omega^2}{\Omega_\beta^2}\cos^2(2\theta) + \frac{\beta^2}{\Omega_\beta^2}\sin^2(2\theta)\right\} . \tag{30}$$

4. Decoherence Scenario : $d \neq 0$

$$\mathcal{L} = \begin{pmatrix} 0 & -\omega & d \\ \omega & 0 & 0 \\ d & 0 & 0 \end{pmatrix} . \tag{31}$$

$$\begin{aligned} \mathcal{M}_{33} &= \frac{\omega^2 - d^2 \cos(2\Omega_d t)}{\Omega_d^2} \\ \mathcal{M}_{11} &= \cos(2\Omega_d t) \\ \mathcal{M}_{13} + \mathcal{M}_{31} &= \frac{-2d}{\Omega_d} \sin(2\Omega_d t) , \end{aligned} \tag{32}$$

where $\Omega_d = \sqrt{\omega^2 - d^2}$. Also, this is the only case where we have $\mathcal{M}_{13} + \mathcal{M}_{31} \neq 0$. Eq. (15) gives

$$\begin{aligned} P &= \frac{1}{2} + \frac{1}{2}\left[\cos^2(2\theta)\left\{-\frac{\omega^2}{\Omega_d^2} + \frac{d^2}{\Omega_d^2}\cos(2\Omega_d t)\right\} + \sin^2(2\theta)\left\{-\cos(2\Omega_d t)\right\}\right. \\ &+ \left.\sin(4\theta)\left\{\frac{d}{\Omega_d}\sin(2\Omega_d t)\right\}\right] . \end{aligned} \tag{33}$$

The above equation matches Eq. (14) of Ref. [7] (as well as Eq. (35) of Ref. [8]). Averaging over time, we get

$$P \to \frac{1}{2} + \frac{1}{2}\left\{-\frac{\omega^2}{\Omega_d^2}\cos^2(2\theta)\right\} . \tag{34}$$

Note that even though in case (3) and (4), we have violation of conservation of energy in the neutrino system (since one of the elements in the final row or column in $\mathcal{L}$ matrix is non-zero), yet the asymptotic limit is not exactly (1/2) (steady state value) (see Eq. (30) and Eq. (34)). This is because the terms of $\mathcal{M}$ are not exponentially suppressed, but are oscillatory functions (Eq. (28) and Eq. (32)).

5. Decoherence Scenario : d and β are zero

$$\mathcal{L} = \begin{pmatrix} a & b-\omega & 0 \\ b+\omega & \alpha & 0 \\ 0 & 0 & \delta \end{pmatrix} . \tag{35}$$

Setting $\Omega_b = \sqrt{\omega^2 - b^2}$ and $a = \alpha$ for which we get

$$\begin{aligned} \mathcal{M}_{33} &= e^{-2\delta t} \\ \mathcal{M}_{11} &= e^{-2at} \cos(2\Omega_b t) \\ \mathcal{M}_{13} + \mathcal{M}_{31} &= 0 . \end{aligned} \tag{36}$$

We note from Eq. (36) that $\mathcal{M}$ has either zero or exponentially suppressed entries. From Eq. (15), we have

$$P = \frac{1}{2} + \left[\cos^2(2\theta) \left\{ -e^{-2\delta t} \right\} + \sin^2(2\theta) \left\{ -e^{-2at} \cos(2\Omega_b t) \right\} \right] . \quad (37)$$

matches Eq. (34) of [8]. The asymptotic limit ($t \to \infty$) in this case is clearly

$$P \to \frac{1}{2} . \quad (38)$$

So, the crucial requirement to get the steady state value is that along with violation of conservation of energy in the neutrino system ($\delta \neq 0$), we must have some other decoherence parameters non-zero (such as a or $\alpha \neq 0$). This leads to the exponentially suppressed elements of $\mathcal{M}$ which go to zero in long t limit. Even if $b = 0$, we have the same steady state limit. If $b = 0$ and if we impose energy conservation ($\delta = 0$), we get back case (2) for which the asymptotic limit is different (Eq. (20)). This means non-conservation of energy is only a necessary but not sufficient condition to get steady state limit, 1/2.

Decoherence parameters

D (E) ~ exp(-2 k L E^n)

- E independent (n=0)
- n=2 [String inspired models]
- n=-1 [Lorentz invariant models]

3 flavour case - averaged oscillations

$$P_{\alpha\beta} = \frac{1}{3} + \frac{1}{2}(U_{\alpha 1}^2 - U_{\alpha 2}^2)(U_{\beta 1}^2 - U_{\beta 2}^2)D_\Psi + \frac{1}{6}(U_{\alpha 1}^2 + U_{\alpha 2}^2 - 2U_{\alpha 3}^2)(U_{\beta 1}^2 + U_{\beta 2}^2 - 2U_{\beta 3}^2)D_\delta \,, \quad (20)$$

where D_Ψ and D_δ are the damping factors (corresponding to the eigenvalues of λ_3 and λ_8, respectively, of the decoherence matrix described in Appendix A) given by [6]

$$D_\kappa(E) = \exp\left(-2\,\kappa\, L\, E^n\right) \,. \quad (21)$$

Here $D_\kappa(E)$ parameterizes effects due to quantum decoherence and $U_{\alpha i}$ are the elements of the standard neutrino mixing matrix. Here n carries the energy dependent imprint of a specific model. In the literature, $n = -1, 0, 2$ have been used (see also Ref. [69]). In principle, Ψ and δ can take different values, however, in what follows we will assume the same energy dependence for the two parameters.

Mehta and Winter, JCAP (2011), see the Appendix

3 flavour case - averaged oscillations

$$\rho_{\nu_\alpha}(0) = \begin{pmatrix} U_{\alpha 1}^2 & U_{\alpha 1}U_{\alpha 2} & U_{\alpha 1}U_{\alpha 3} \\ U_{\alpha 2}U_{\alpha 1} & U_{\alpha 2}^2 & U_{\alpha 2}U_{\alpha 3} \\ U_{\alpha 3}U_{\alpha 1} & U_{\alpha 3}U_{\alpha 2} & U_{\alpha 3}^2 \end{pmatrix}$$

$$\rho(t) = \frac{1}{2}\begin{pmatrix} \frac{2}{3} + p_3 + \frac{p_8}{\sqrt{3}} & p_1 - ip_2 & p_4 - ip_5 \\ p_1 + ip_2 & \frac{2}{3} - p_3 + \frac{p_8}{\sqrt{3}} & p_6 - ip_7 \\ p_4 + ip_5 & p_6 + ip_7 & \frac{2}{3} - \frac{2p_8}{\sqrt{3}} \end{pmatrix}$$

Mehta and Winter, JCAP (2011), see the Appendix

3 flavour case - averaged oscillations

$$\mathcal{L} = \begin{pmatrix} A & B+\omega_{21} & 0 & 0 & 0 & 0 & 0 & 0 \\ B-\omega_{21} & \Lambda & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \Psi & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & x & y+\omega_{31} & 0 & 0 & 0 \\ 0 & 0 & 0 & y-\omega_{31} & z & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & a & b+\omega_{32} & 0 \\ 0 & 0 & 0 & 0 & 0 & b-\omega_{32} & \alpha & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \delta \end{pmatrix}$$

$$P_{\alpha\beta}(t) = \mathrm{Tr}[\rho_{\nu_\alpha}(t)\,\rho_{\nu_\beta}(0)] \ ,$$

3 flavour case - averaged oscillations

- $\Psi \neq 0, \delta = 0$:

$$P_{\alpha\beta} = \frac{1}{3} + \frac{1}{6}(U_{\alpha 1}^2 + U_{\alpha 2}^2 - 2U_{\alpha 3}^2)(U_{\beta 1}^2 + U_{\beta 2}^2 - 2U_{\beta 3}^2)\,,$$

- $\Psi = 0, \delta \neq 0$:

$$P_{\alpha\beta} = \frac{1}{3} + \frac{1}{2}(U_{\alpha 1}^2 - U_{\alpha 2}^2)(U_{\beta 1}^2 - U_{\beta 2}^2)\,,$$

- $\Psi = \delta = 0$:

$$P_{\alpha\beta} = \sum_{i=1}^{3} |U_{\beta i}|^2\, |U_{\alpha i}|^2\ .$$

- $\Psi = \delta \neq 0$:

$$P_{\alpha\beta} = \frac{1}{3}\,,$$

Mehta and Winter, JCAP (2011), see the Appendix

Decoherence formalism - useful references

- **Banks, Susskind and Peskin**, Difficulties for evolution of pure states into mixed states, Nucl. Phys. B 244, 125-134 (1984)
- **Ellis, Hagelin, Nanopoulos and Srednicki**, Search for violations of quantum mechanics, Nucl. Phys. B 241, 381-405 (1984)
- **Liu**, Evolution of pure states into Mixed states, hep-th/9301082
- **Raffelt, Sigl and Stodolsky**, Non-abelian Boltzmann Equation for Mixing and Decoherence, hep-ph/9209276
- **Benatti and Floreanini**, Open system approach to neutrino oscillations, JHEP02, 032 (2000)