$\text{GUT} \,\&\, (\text{TD} + \text{BAU} \oplus \text{DM})$

Joydeep Chakrabortty

IIT Kanpur

WHEPP, 2017— IISER Bhopal

Keywords

*SM : Standard Model

Well Established Theory of Particle Physics

*GUT : Grand Unified Theory

Unification of Fundamental Forces

(Strong, Weak, Electromagnetic) apart from Gravity

*SUSY : Supersymmetry

Symmetry among Fermions and Bosons

*SUGRA : Supergravity

Local Supersymmetry

Beyond SM : GUT

- The new Physics may contain a larger symmetry as SM is expected to be embedded in that.
- Many unanswered questions in the SM can be explained within this framework.
- It is believed to have unified theory at some high scale and all the low energy Physics will evolve from that.
- The new framework is expected to be constrained and contains lesser number of free parameters –

Attractive and Natural.

What is GUT?

Unified framework —

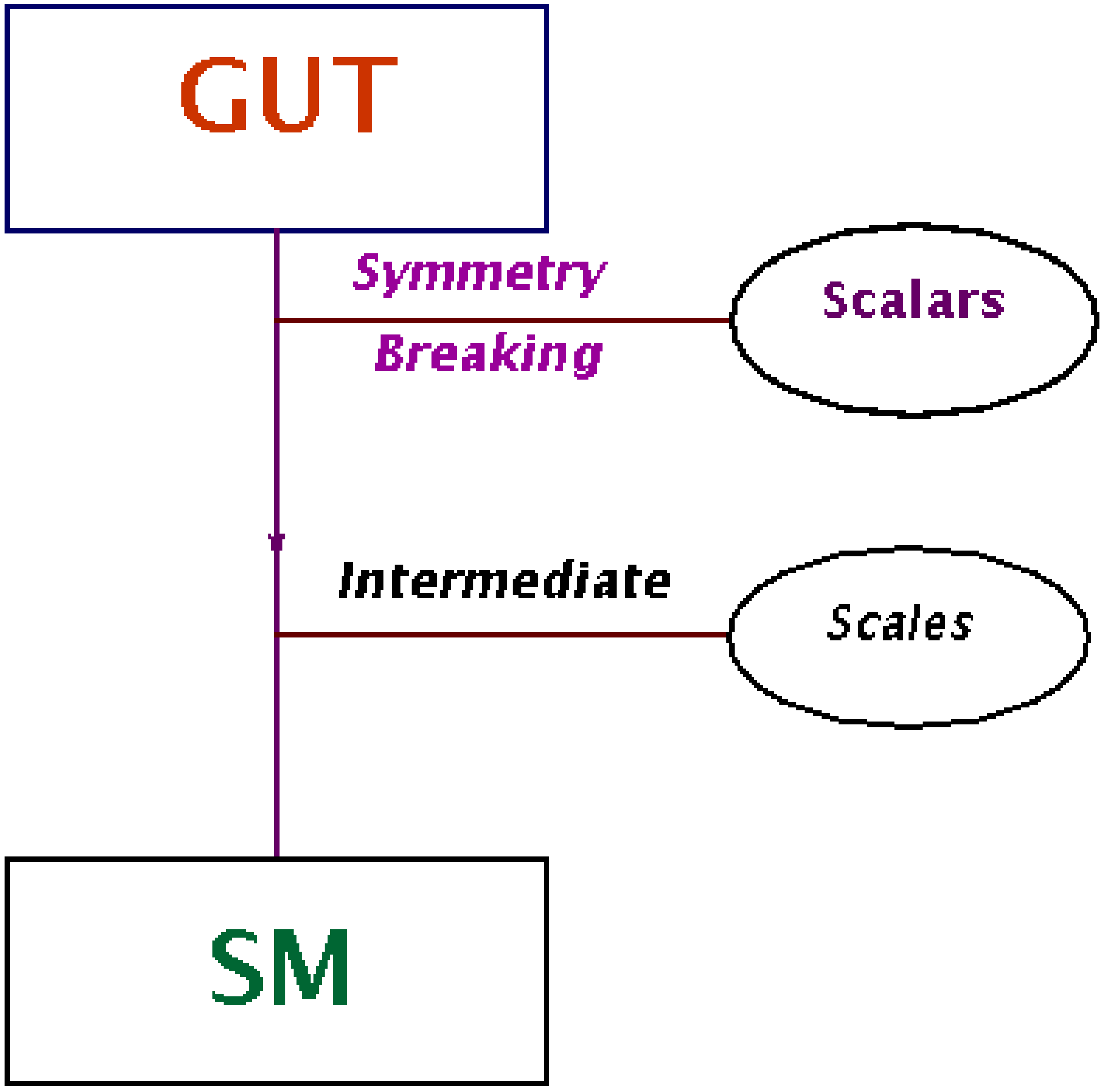
Gauge couplings are unified

Leptons and Quarks belong to same representations

Predicts Proton decay ~ GUT Scale !

Other BSM requirements are expected to be fulfilled !

Schematic view



Some aspects of GUT

D-parity

Extended Survival Hypothesis (ESH)

RGEs and Abelian Mixing of gauge couplings

Matching conditions

Effective gauge kinetic term — Planck scale effect

Vacuum manifold and Topological Defects

D-parity and ESH

D-Parity : connecting the "L" and "R" sectors

$$SU(N)_L \otimes SU(N)_R \otimes \mathcal{G}$$

$$(R_N, 1, R_G) \leftrightarrow (1, \overline{R_N}, \overline{R_G})$$

ESH :

only those scalars are light which are "Relevant"

$$SO(10) \to SU(2)_L \otimes SU(2)_R \otimes SU(4)_C \to SM$$

$$10 \equiv (2,2,1) \oplus (1,1,6)$$

Renormalisation Group Evolutions..

$$\mu\frac{dg_i}{d\mu} = \frac{g_i^3}{(4\pi)^2}\left[\frac{4\kappa}{3}T(F_i)D(F_j) + \frac{1}{3}T(S_i)D(S_j) - \frac{11}{3}C_2(G_i)\right] + \frac{1}{(4\pi)^4}g_i^5$$
$$\times\left[\left(\frac{10}{3}C_2(G_i) + 2C_2(F_i)\right)T(F_i)D(F_j) + \left(\frac{2}{3}C_2(G_i) + 4C_2(S_i)\right)T(S_i)D(S_j)\right.$$
$$\left.-\frac{34}{3}(C_2(G_i))^2\right] + \frac{1}{(4\pi)^4}g_i^3g_j^2\left[2C_2(F_j)T(F_i)D(F_j) + 4C_2(S_j)T(S_i)D(S_j)\right]$$

$$C_2(R) = T(R)d/D(R) \qquad T(R) = Tr(\lambda_i\lambda_j)$$

$$D(R) = \text{ dimensionality of representation}$$

Ex:

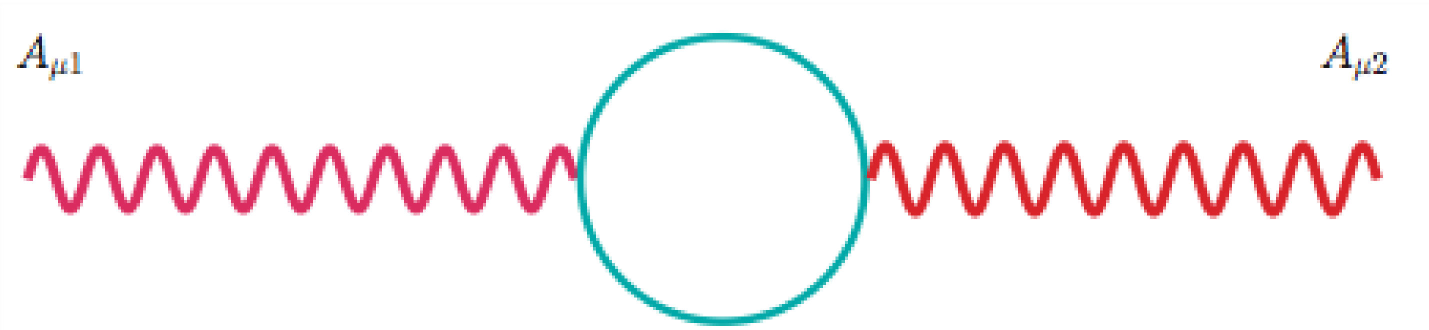
$$SO(10) \to SU(2)_L \otimes SU(2)_R \otimes SU(4)_C$$

$$10 \equiv (2,2,1) \oplus (1,1,6)$$

For $SU(2)_L:\ T(2) = 1/2\ D(R) = 2,\ d = 3,\ C_2(R) = 3/4$

For Abelian groups $C_2(G) = 0, T(R) = \sum_i q_i^2$

Two U(1) factors : demand special attention



$$\mathcal{L}_{kin} = -\frac{1}{4c} Tr(F_{\mu\nu} F^{\mu\nu}) \qquad \mathcal{L}_{kin} = -\frac{1}{4c} Tr(F_{\mu\nu} F'^{\mu\nu})$$

- **Incomplete light multiplets leading to Abelian mixing even at the 1-loop level.**

- **Both Scalars and Fermion multiplets contribute to this.**

RGEs in presence of Abelian mixing

$$\mu \frac{dg_{kb}}{d\mu} = \beta_{ab} g_{ka} \qquad \beta_{ab} = \frac{1}{(4\pi)^2} g_{sa} \Sigma_{sr} g_{rb}$$

$$\Sigma_{sr} = \sigma_{sr}^{(one-loop)} + \frac{1}{(4\pi)^2} \sigma_{sr}^{(two-loop)}$$

$$\sigma_{sr}^{(one-loop)} \equiv \tilde{b}_{sr} = \frac{2}{3} n_g \{y_s(F) y_r(F) D(F)\} + \frac{1}{3} \{y_s(S) y_r(S) D(S)\}$$

$$g = \begin{bmatrix} g_{11} & g_{12} & g_{13} & \cdots & g_{1n} \\ g_{21} & g_{22} & g_{23} & \cdots & g_{2n} \\ g_{31} & g_{32} & g_{33} & \cdots & g_{3n} \\ \cdots & \cdots\cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots. & \cdots & \cdots.. \\ g_{n1} & g_{n2} & g_{n3} & \cdots. & g_{nn} \end{bmatrix},$$

Matching condition

Unification criteria

$$\frac{1}{\alpha_i} - \frac{C_2(\mathcal{G}_i)}{12\pi} = \frac{1}{\alpha_j} - \frac{C_2(\mathcal{G}_j)}{12\pi} \text{=constant}$$

$$\alpha_i = g_i^2/(4\pi)$$

$$C_2(\mathcal{G}_i) = Tr(\Lambda_i \Lambda_j)$$

Λ_i ***generators in adj. rep.***

Intermediate scale matching criteria

$$\frac{1}{\alpha_X} = \left[Q.\frac{4\pi}{(g.g^T)}.Q^T + \sum_n w_n^2 \left(\frac{1}{\alpha_n} - \frac{C_2(\mathcal{G}_n)}{12\pi} \right) \right]$$

$$Q.Q^T + \sum_n w_n^2 = 1$$

$$1/\alpha_X = Q.[4\pi/(g.g^T)].Q^T \text{ with } Q.Q^T = 1$$

Vacuum Manifold

$$\mathcal{G} \rightarrow \mathcal{H}$$

$$T_i \in \mathcal{G},\ \tilde{T}_j \in \mathcal{H},\ \text{where } i > j.$$

$\tilde{T}_j \Rightarrow$ unbroken generators

$T_i - \tilde{T}_j \Rightarrow$ broken generators

Vacuum manifold, $\mathcal{G}/\mathcal{H}$, is spanned by broken generators

This determines the characteristics of symmetry breaking

Homotopy and Topological Defects

$$\Pi_0(\mathcal{G}) \neq \mathcal{I} \Rightarrow Domain\ \ Wall$$

$$\Pi_1(\mathcal{G}) \neq \mathcal{I} \Rightarrow Cosmic\ \ String$$

$$\Pi_2(\mathcal{G}) \neq \mathcal{I} \Rightarrow Monopole$$

$$\Pi_3(\mathcal{G}) \neq \mathcal{I} \Rightarrow Texture$$

There could be "hybrid" *structure - Walls bounded by Strings*

Texture decays, thus they are not stable.

Domain Walls and Monopoles are "Bad guys" if they are ***stable***

Need to worry about only Stable TDs!

SSB and Topological Defects

$$\Pi_{[2,1]}(\mathcal{G}) = \Pi_{[1,0]}(\mathcal{G}_i) = \Pi_{[1,0]}(\mathcal{G}_j) = \mathcal{I}$$

case-I $\mathcal{G} \rightarrow \mathcal{G}_i \otimes \mathcal{G}_j \otimes U(1) \rightarrow \mathcal{G}_i \otimes \mathcal{G}_j$

$$\Pi_1(\mathcal{G}/(\mathcal{G}_i \otimes \mathcal{G}_j \otimes U(1))) = \Pi_0(\mathcal{G}_i \otimes \mathcal{G}_j \otimes U(1)) = \mathcal{I}, \text{(No domain walls and cosmic strings)}$$

$$\Pi_2(\mathcal{G}/(\mathcal{G}_i \otimes \mathcal{G}_j \otimes U(1))) = \Pi_1(\mathcal{G}_i \otimes \mathcal{G}_j \otimes U(1)) = Z, \text{(monopoles will be there)}.$$

$$\Pi_1(\mathcal{G}_i \otimes \mathcal{G}_j \otimes U(1)/(\mathcal{G}_i \otimes \mathcal{G}_j)) = Z, \text{(presence of cosmic strings)}$$

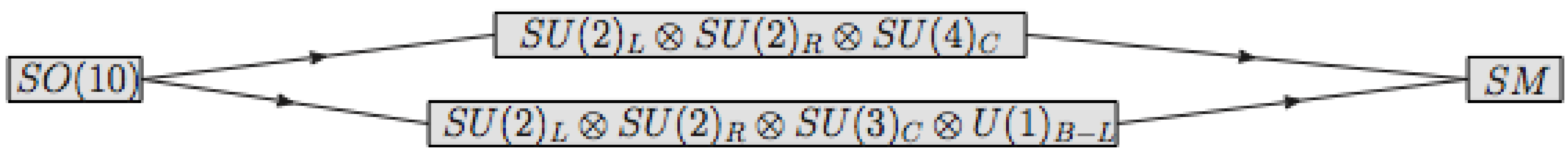
$$\Pi_2(\mathcal{G}_i \otimes \mathcal{G}_j \otimes U(1)/(\mathcal{G}_i \otimes \mathcal{G}_j)) = \Pi_2(U(1)) = \mathcal{I}, \text{(No monopole)}$$

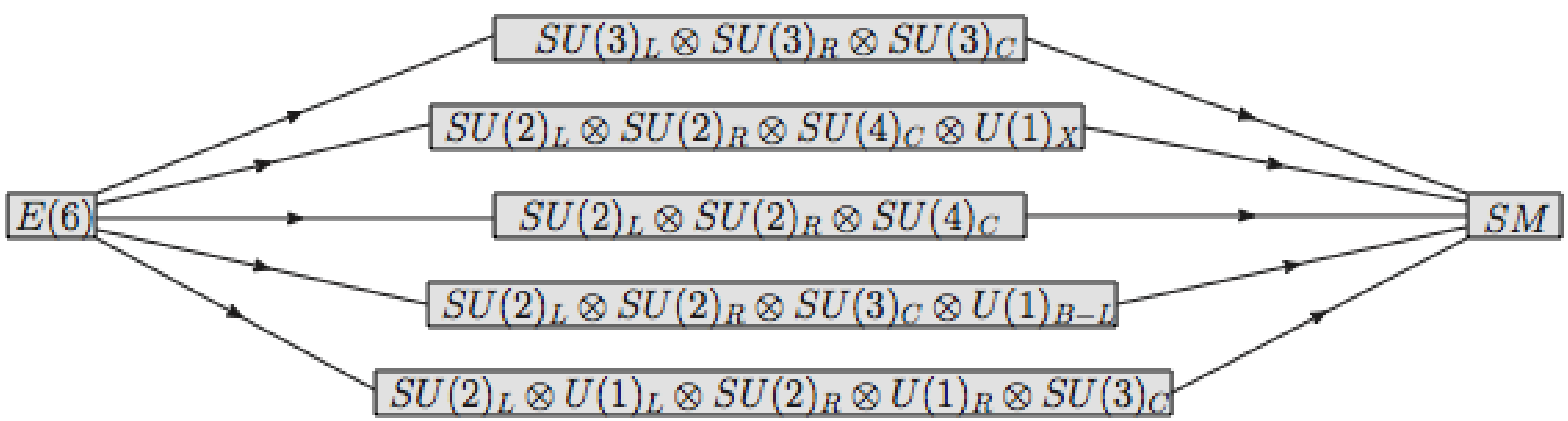
case-II $\mathcal{G} \rightarrow \mathcal{G}_i \otimes \mathcal{G}_j \otimes Z_2 \rightarrow \mathcal{G}_i \otimes \mathcal{G}_j$

$$\Pi_1(\mathcal{G}/(\mathcal{G}_i \otimes \mathcal{G}_j \otimes Z_2)) = \Pi_0(\mathcal{G}_i \otimes \mathcal{G}_j \otimes Z_2) = Z_2, \text{(cosmic strings)}$$

$$\Pi_0(\mathcal{G}_i \otimes \mathcal{G}_j \otimes Z_2/(\mathcal{G}_i \otimes \mathcal{G}_j)) = Z_2, \text{(presence of domain walls)}$$

$$\mathcal{G}_{GUT} \to SU(N)_L \otimes SU(N)_R \otimes \mathcal{G} \to SM$$





SSB and Topological Defects

Intermediate Symmetry		Topological defects
$\mathcal{G}_{224}$	D-broken	monopoles
	D-conserved	domain wall + monopoles + Z_2-strings
$\mathcal{G}_{2231}$	D-broken	monopoles + embedded strings
	D-conserved	domain wall + monopoles + embedded strings
$\mathcal{G}_{2241}$	D-broken	monopoles + embedded strings
	D-conserved	domain walls + monopoles + embedded strings
$\mathcal{G}_{333}$	D-broken	textures
	D-conserved	domain walls + textures

These conclusions don't depend on whether scenarios are Non-SUSY or SUSY

To avoid the catastrophe due to the stable TDs, we expect the intermediate scale to be very large….

$$M_R \geq 10^{12}\ \mathrm{GeV}$$

Comment on Intermediate Scale

To evade the stable Topological Defects

Decay of leptoquark bosons leading to BAU with reasonable Yukawa coupling

UHE proton and neutrino spectra from collapse or annihilation of TDs

$$M_R \geq 10^{12} \text{ GeV}$$

$$SO(10) \to SU(2)_L \otimes SU(2)_R \otimes SU(4)_C$$

$$\frac{1}{\alpha_{3C}(M_R)} = \frac{1}{\alpha_{4C}(M_R)} - \frac{1}{12\pi},$$
$$\frac{1}{\alpha_{1Y}(M_R)} = \frac{3}{5}\left(\frac{1}{\alpha_{2R}(M_R)} - \frac{1}{6\pi}\right) + \frac{2}{5}\left(\frac{1}{\alpha_{4C}(M_R)} - \frac{1}{3\pi}\right)$$

D-parity not conserved

$$\text{Non-SUSY}: \quad b_{2L} = -3,\ b_{4C} = -\frac{23}{3},\ b_{2R} = \frac{11}{3}; \quad b_{ij} = \begin{pmatrix} 8 & \frac{45}{2} & 3 \\ \frac{9}{2} & \frac{643}{6} & \frac{153}{2} \\ 3 & \frac{765}{2} & \frac{584}{3} \end{pmatrix}$$

$$\text{SUSY}: \quad b_{2L} = 1,\ b_{2R} = 21,\ b_{4C} = 3; \quad b_{ij} = \begin{pmatrix} 25 & 3 & 45 \\ 3 & 265 & 405 \\ 9 & 81 & 231 \end{pmatrix}.$$

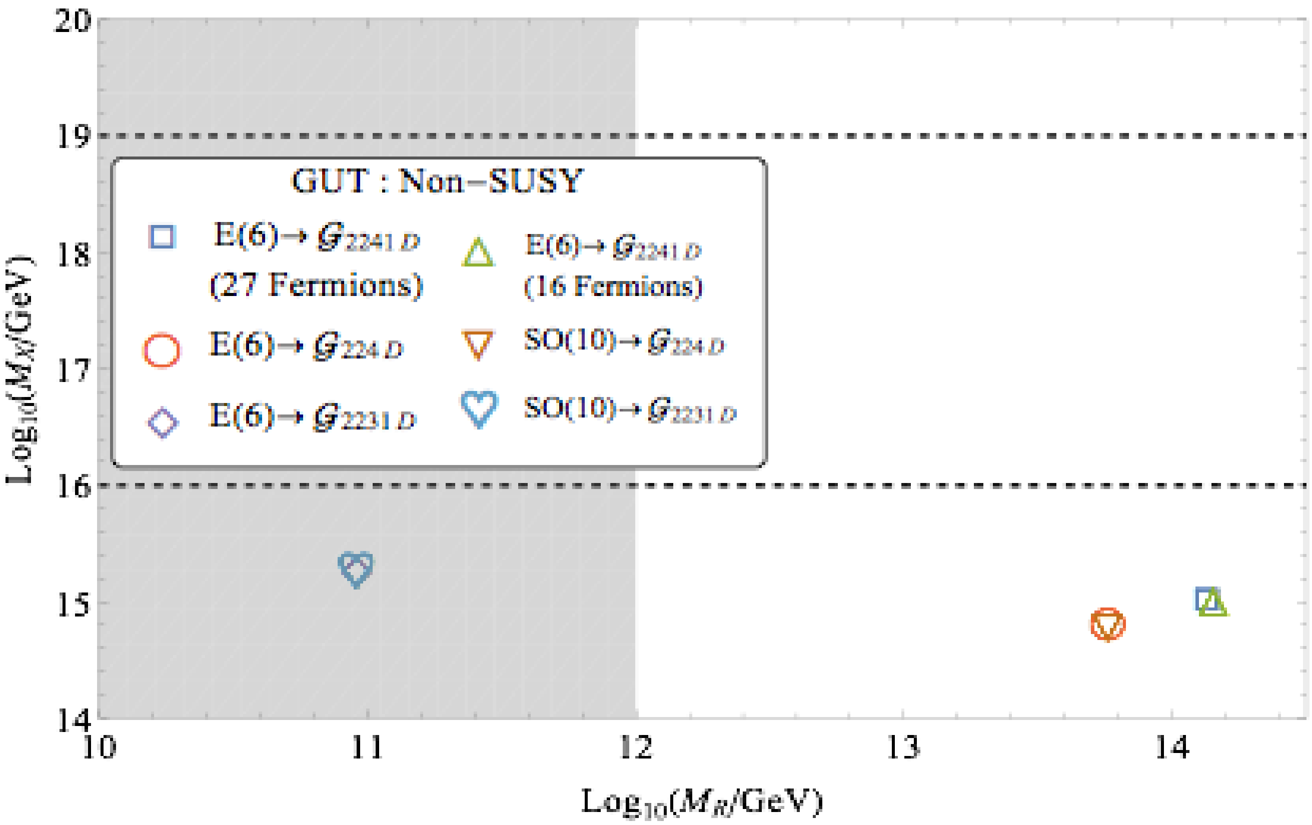
D-parity conserved

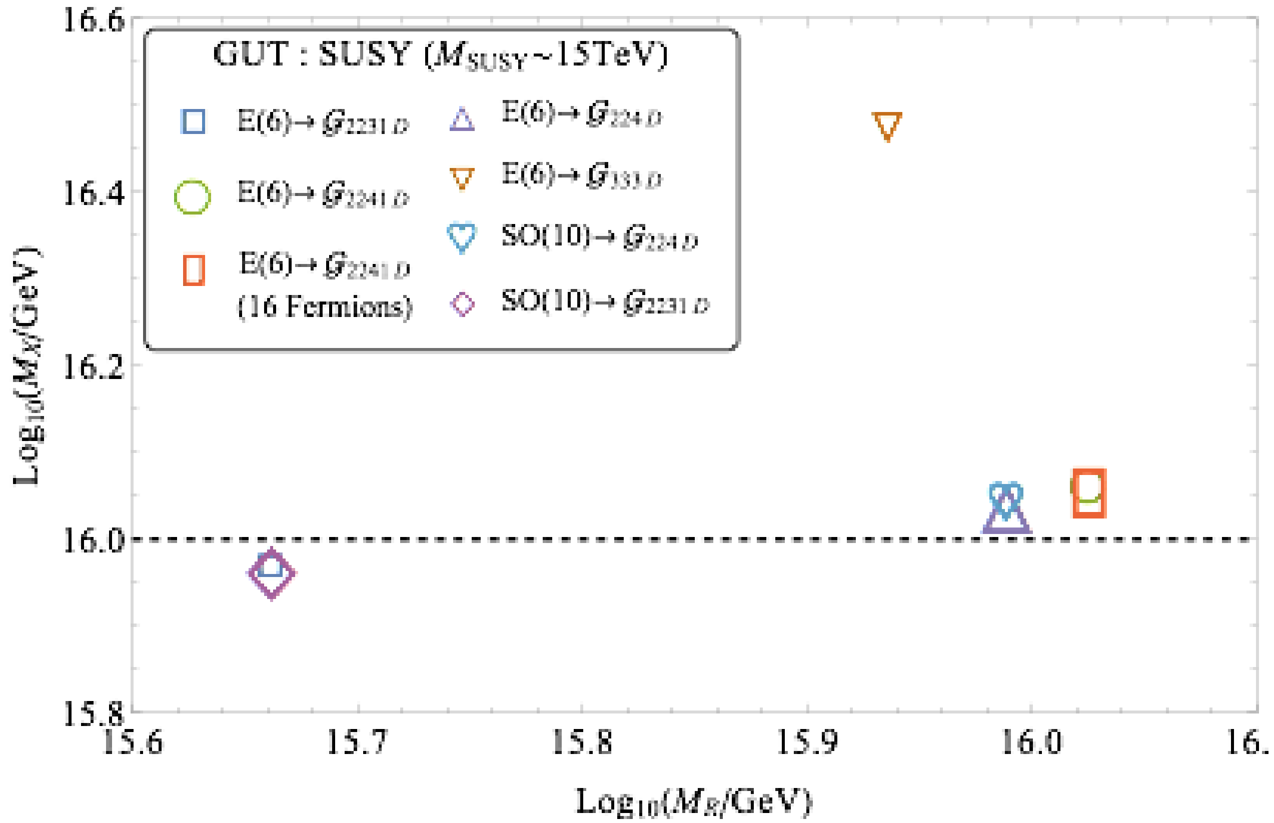
$$\text{Non-SUSY}: \quad b_{2L} = \frac{11}{3},\ b_{4C} = -\frac{14}{3},\ b_{2R} = \frac{11}{3}; \quad b_{ij} = \begin{pmatrix} \frac{584}{3} & \frac{765}{2} & 3 \\ \frac{153}{2} & \frac{1759}{6} & \frac{153}{2} \\ 3 & \frac{765}{2} & \frac{584}{3} \end{pmatrix}$$

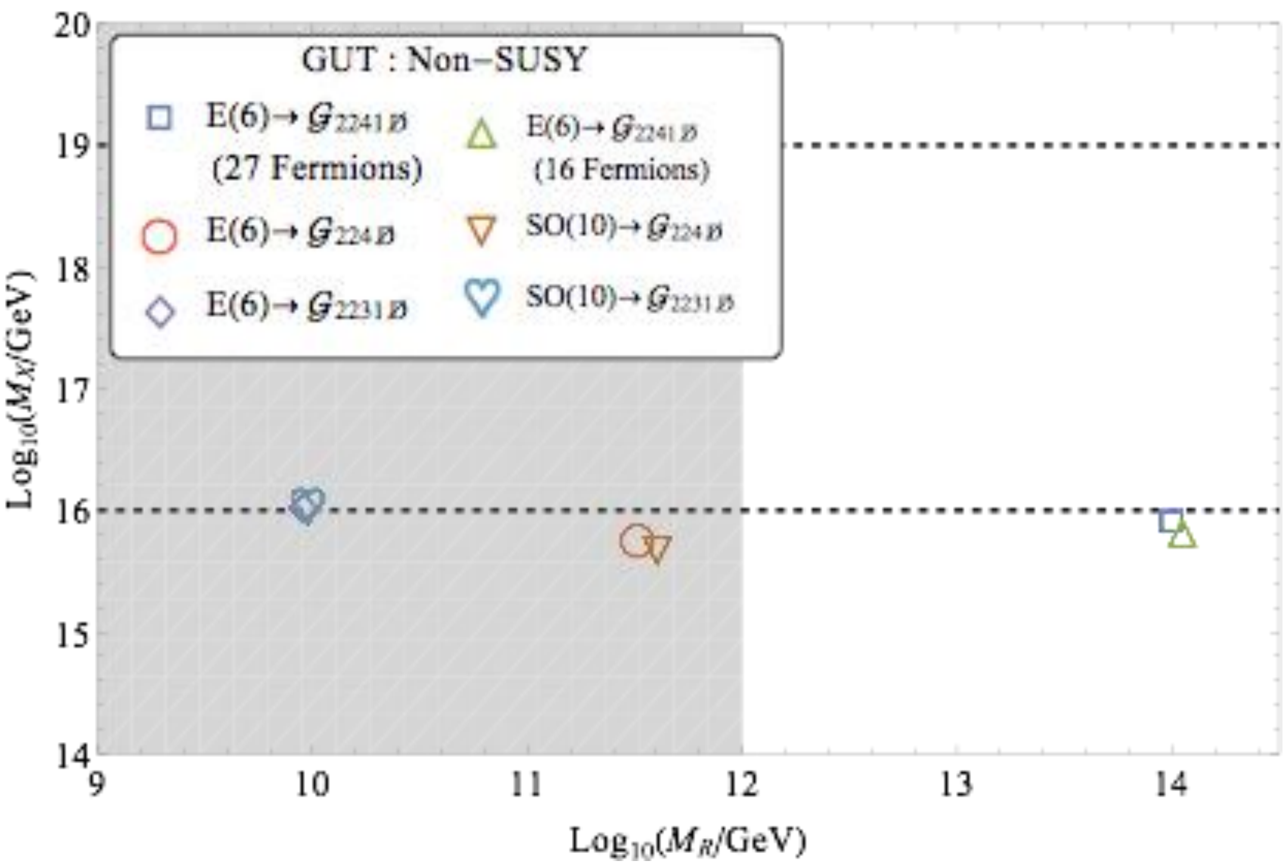
$$\text{SUSY}: \quad b_{2L} = 21,\ b_{2R} = 21,\ b_{4C} = 12; \quad b_{ij} = \begin{pmatrix} 265 & 3 & 405 \\ 3 & 265 & 405 \\ 81 & 81 & 465 \end{pmatrix}.$$

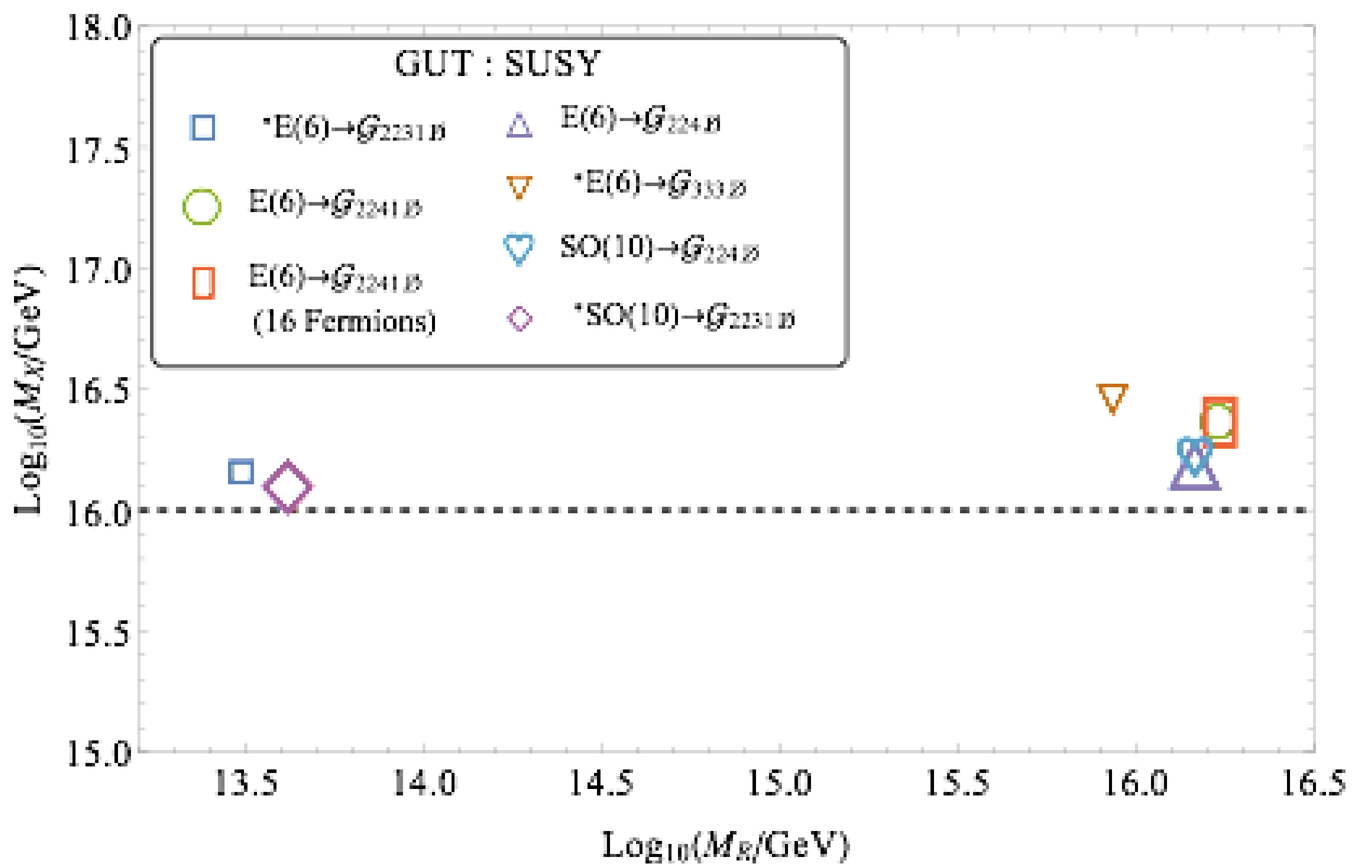
	$SO(10)$	$\mathcal{G}_{224}$	$\mathcal{G}_{213}$
Scalars	10	$(2,2,1)$	$(2,\pm\frac{1}{2},1)$
	126	$(1,3,10)$	-
		$(3,1,\overline{10})_D$	-
	$(54,\ 770)_D$	-	-
	$(210)_{\not D}$	-	-
Fermions	16	$(2,1,4)$	$(2,\frac{1}{6},3)$
			$(2,-\frac{1}{2},1)$
		$(1,2,\bar{4})$	$(1,\frac{1}{3},\bar{3})$
			$(1,-\frac{2}{3},\bar{3})$
			$(1,1,1)$
			$(1,0,1)$

Status of Unification

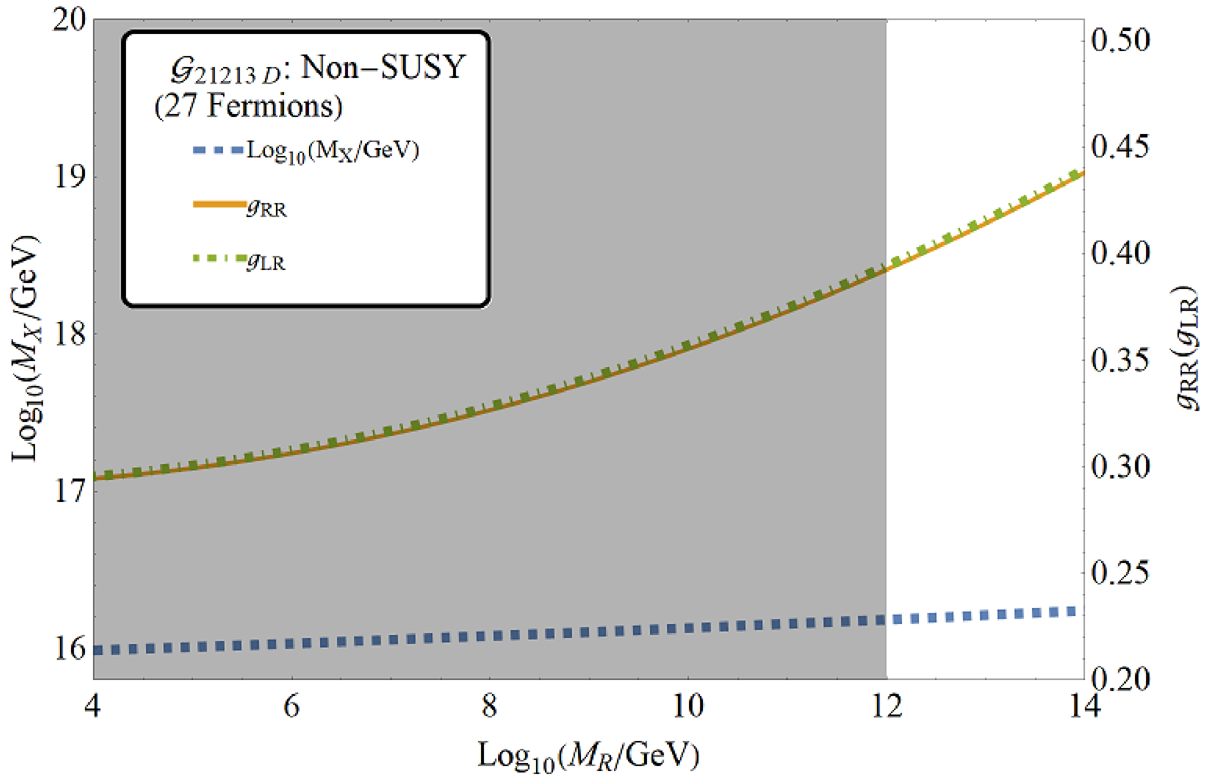


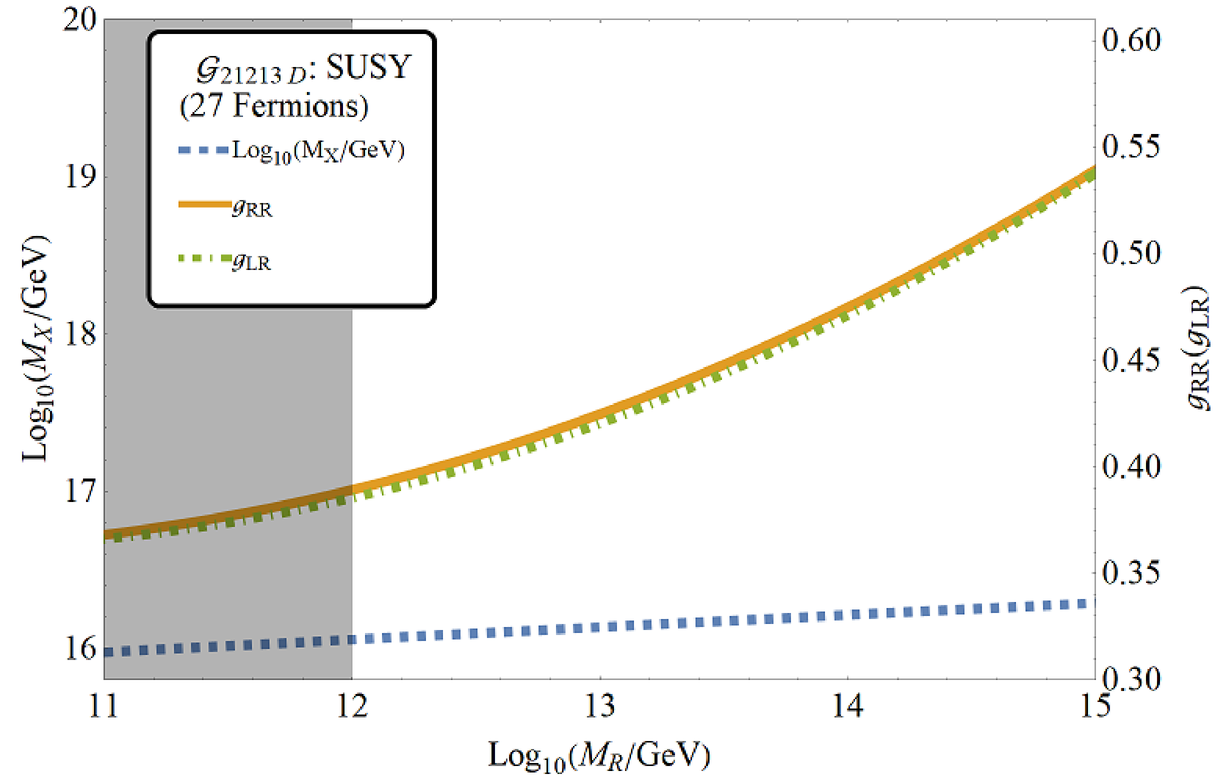


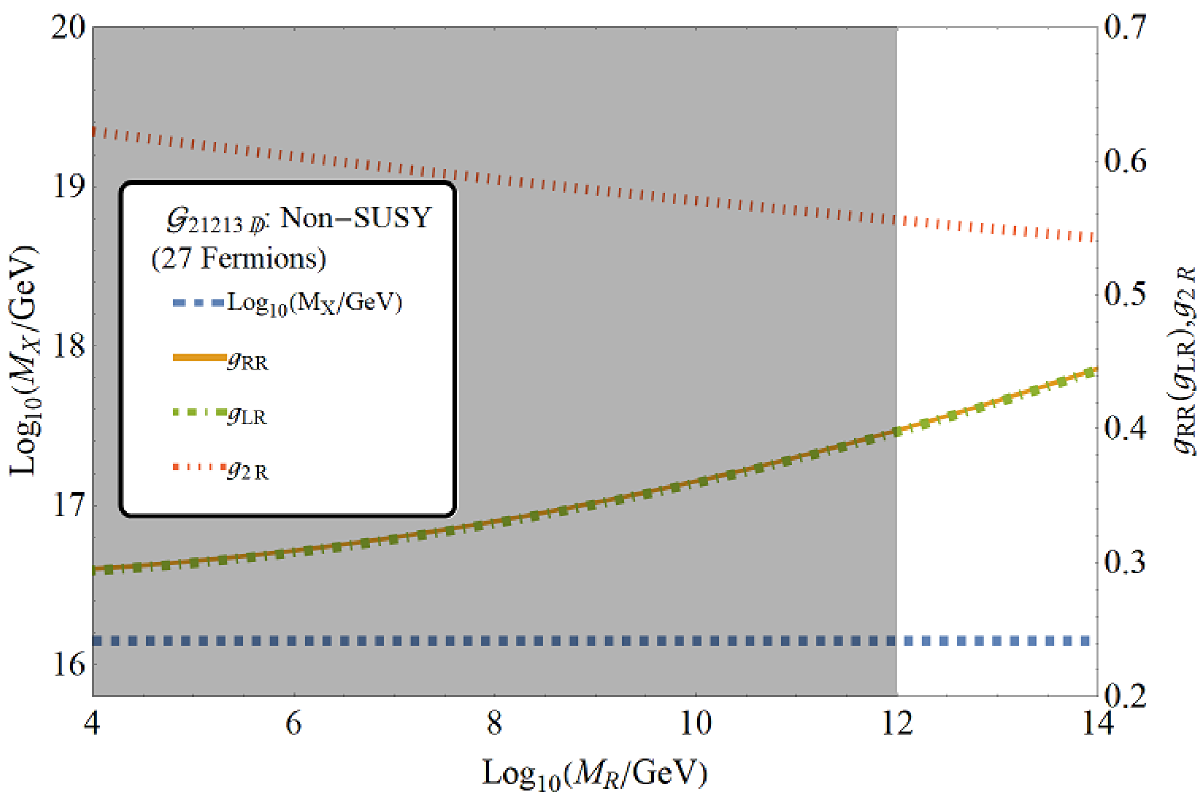


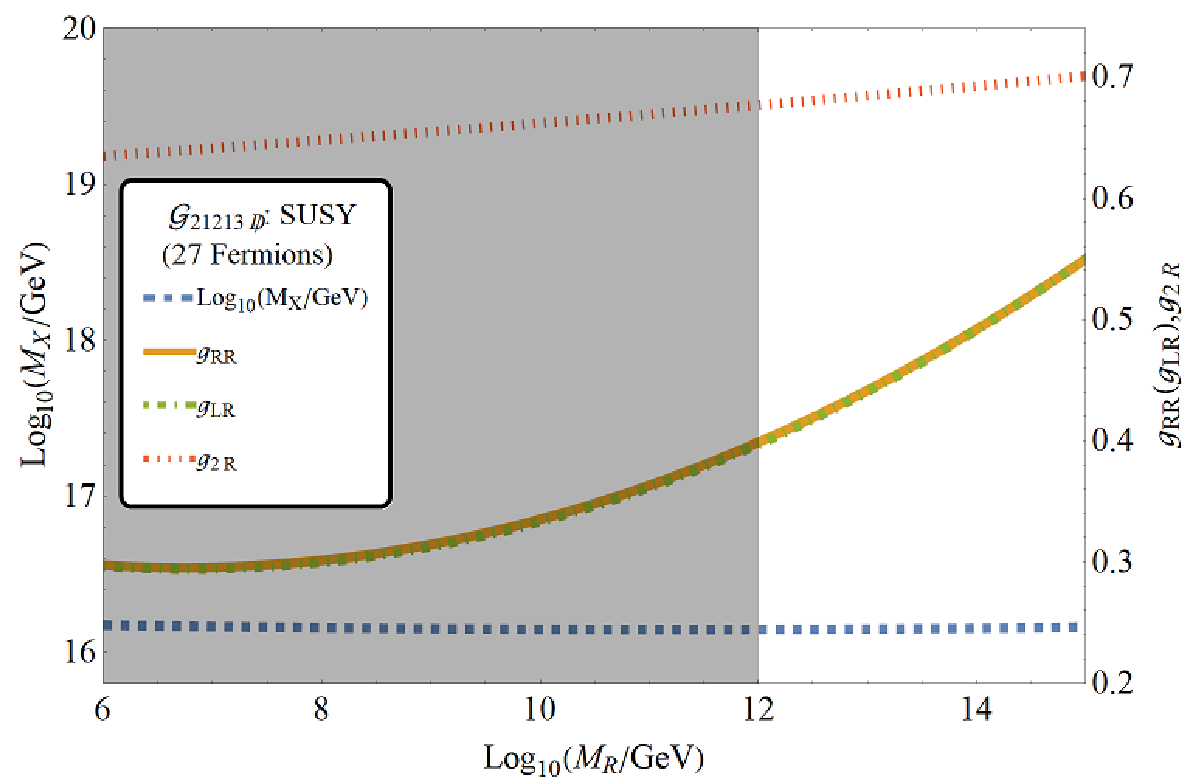


Status of Unification (with Abelian mixing)









- **Gauge Kinetic term:** $\mathcal{L}_{kin} = -\frac{1}{4c} Tr(F_{\mu\nu} F^{\mu\nu})$

$F_{\mu\nu} = \sum \lambda_i . F_i^{\mu\nu}$: matrix form of gauge field strength tensor

λ_i are the generators,

Normalised as: $Tr(\lambda_i \lambda_j) = c\delta_{ij}$

$c = \frac{1}{2}$ when λ_i are in fundamental representation

- **Dimension-5 Operator:** $\mathcal{L}_{dim-5} = -\frac{\eta}{M_{Pl}} [\frac{1}{4c} Tr(F_{\mu\nu} \Phi_D F^{\mu\nu})]$

- Φ_D is the D-dimensional Scalar, belongs to the symmetric product of adjoint representations.

η parametrises the *strength* of this interaction.

Unification Condition

- **In absence of dim-5 operators the coupling unification condition :**

$$\frac{1}{\alpha_i(M_X)} - \frac{C_i}{12\pi} = \text{constant}$$

where C_i is the quadratic Casimir for the i-th subgroup.

- **Modified gauge coupling unification condition:**

$$\frac{1}{\alpha_i(M_X)(1+\epsilon\delta_i)} - \frac{C_i}{12\pi} = \text{constant}$$

- δ_i **are the corrections arise from dim-5 operators, and**

$$\epsilon = \eta < \Phi_D > /2M_{Pl} \sim \mathcal{O}(M_X/M_{Pl})$$

Comments on TD+GUT : arXiv:1711.11391

Intermediate Symmetry (Non-SUSY)		Topological defects $M_R \gtrsim 10^{12}$ GeV		Proton life time $M_X \gtrsim 10^{16}$ GeV	
		No dim-5	dim-5	No dim-5	dim-5
$\mathcal{G}_{224}$	D-conserved	✓	✓	×	✓
	D-broken	×	×	×	✓
$\mathcal{G}_{2231}$	D-conserved	×	×	×	✓
	D-broken	×	×	✓	✓
$\mathcal{G}_{2241}$	D-conserved	✓	✓	×	✓
	D-broken	✓	✓	✓	✓
$\mathcal{G}_{333}$	D-conserved	NS	✓	NS	✓
	D-broken	NS	✓	NS	✓

Intermediate Symmetry (SUSY)		Topological defects $M_R \gtrsim 10^{12}$ GeV		Proton life time $M_X \gtrsim 10^{16}$ GeV	
		No dim-5	dim-5	No dim-5	dim-5
$\mathcal{G}_{224}$	D-conserved	✓	✓	✓	✓
	D-broken	✓	✓	✓	✓
$\mathcal{G}_{2231}$	D-conserved	✓	✓	✓	✓
	D-broken	✓	✓	✓	✓
$\mathcal{G}_{2241}$	D-conserved	✓	✓	✓	✓
	D-broken	✓	✓	✓	✓
$\mathcal{G}_{333}$	D-conserved	✓	✓	✓	✓
	D-broken	✓	✓	✓	✓

Possible direction..

Classify all possible breaking patterns,
starting from SO(10), E(6)

Analyse the Vacuum Manifold for individual breaking chain
at every stages of symmetry breaking

Study the Homotopy of those vacuum manifolds
and identify the Topological Defects (Stable)

Analyse their consequences and constraints on
the Intermediate Symmetry breaking scale

Dark Matter within Unified framework

SUSY : Lightest SUSY Particle(LSP)

Non-SUSY : Stable (lightest) BSM particles

e.g. Non-abelian vector dark boson

N=1 Unified SUGRA framework

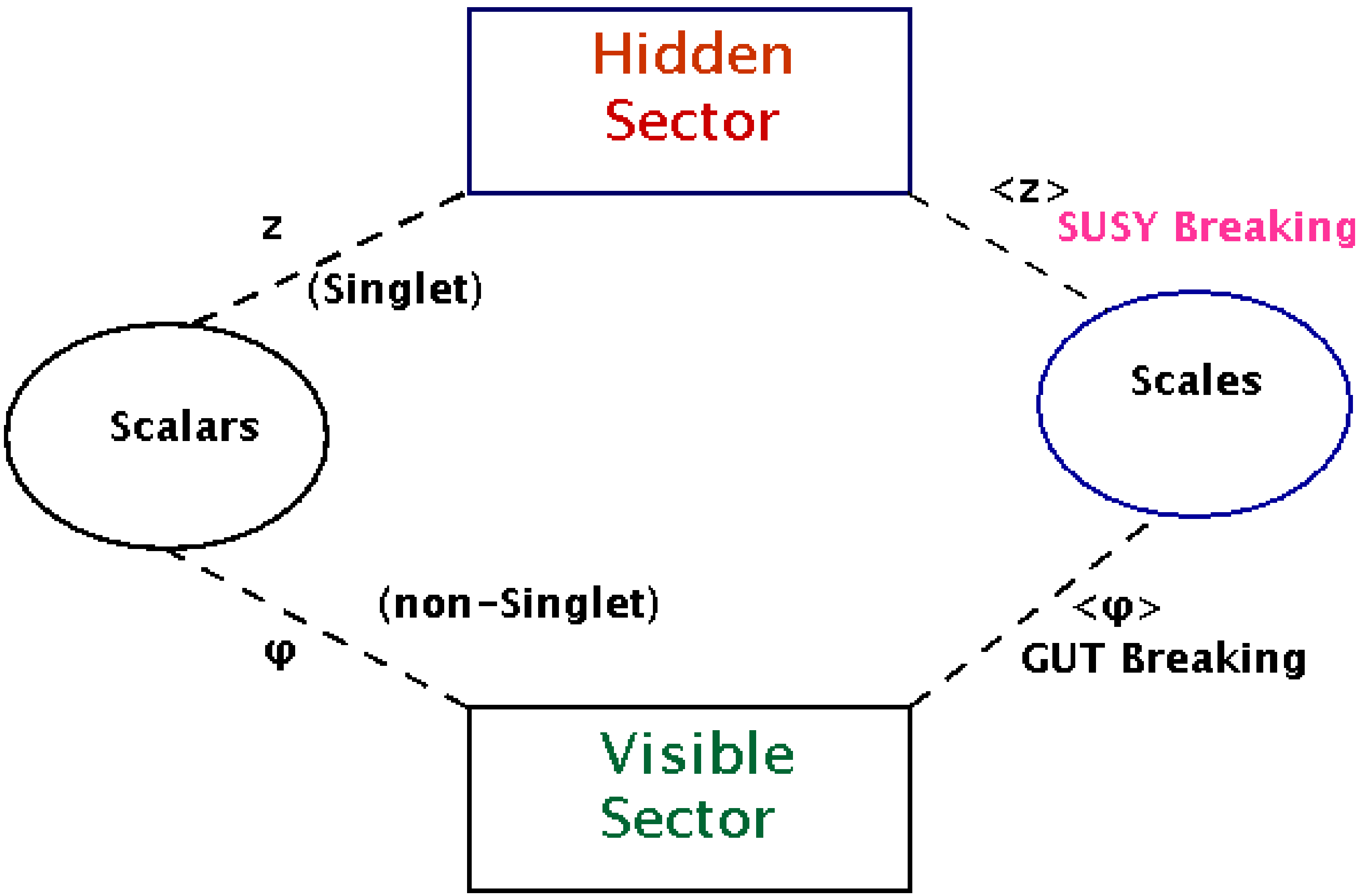
Gaugino-LSP:

Supersymmetric partner of Gauge bosons

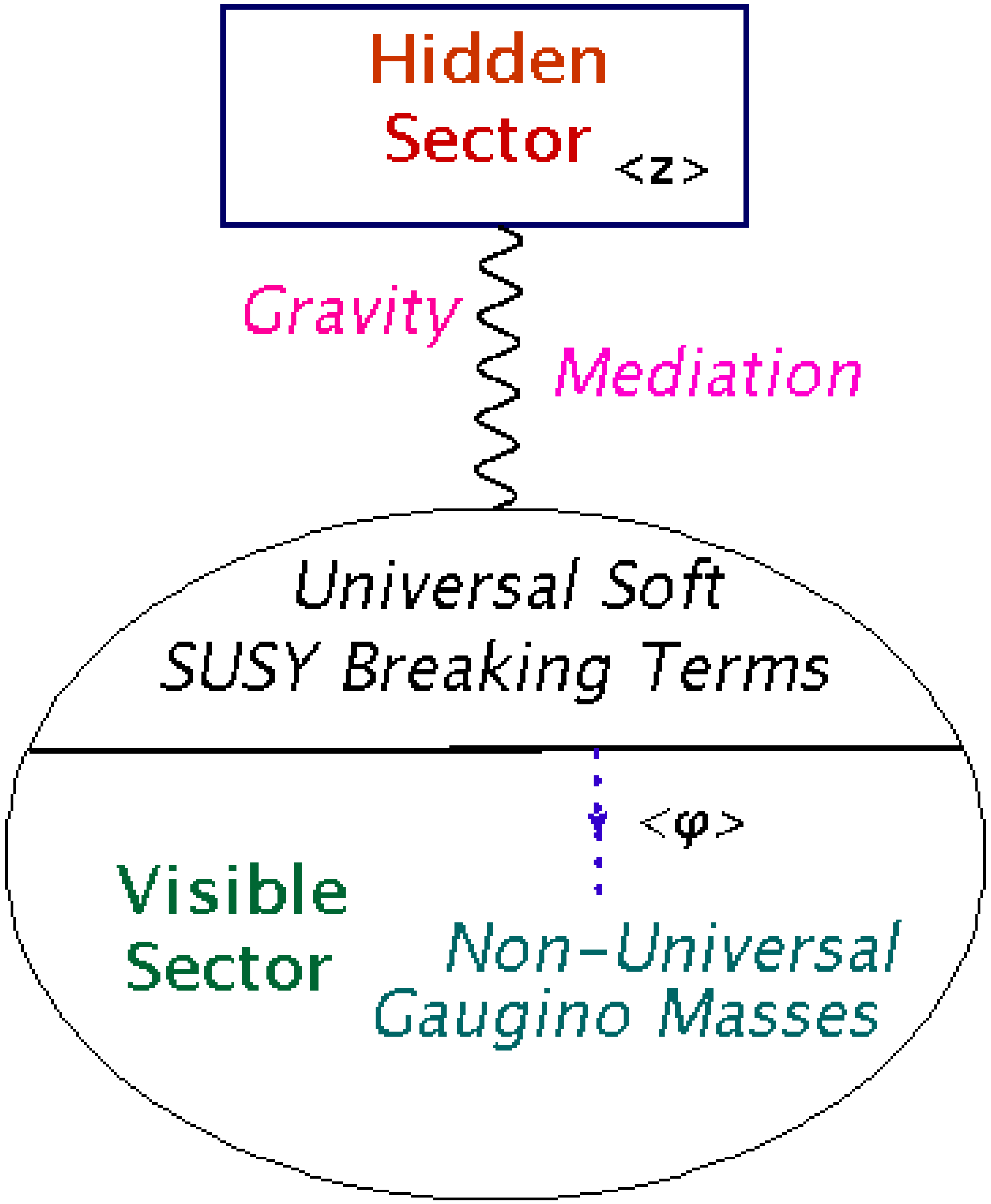
Gaugino Mass Operator is related to :

dimension-5 "Gauge Kinetic Operator"

N=1 Unified SUGRA framework



SUGRA Breaking and Generation of Soft Terms



Non-Universal Gaugino Masses

$$\mathcal{L}_{dim-5} = -\frac{\eta}{M_{Pl}}[\frac{1}{4c}Tr(F_{\mu\nu}\Phi_D F^{\mu\nu})]$$

$$\mathcal{F}_{\alpha\beta} = \mathcal{F}_1\ \delta_{\alpha\beta} + \mathcal{F}_2\ d_{\alpha\beta\gamma}\ \Phi_D^{\gamma}$$

$$M_1 : M_2 : M_3 = (A + \delta_1 B) : (A + \delta_2 B) : (A + \delta_3 B)$$

SU(5) Representations	δ_1	δ_2	δ_3
24	$1/\sqrt{15}$	$3/\sqrt{15}$	$-2/\sqrt{15}$
75	$4/\sqrt{3}$	$-12/5\sqrt{3}$	$-4/5\sqrt{3}$
200	$1/\sqrt{21}$	$1/5\sqrt{21}$	$1/10\sqrt{21}$

Impact of non-universal gaugino masses

- **High Scale Boundary Conditions:**

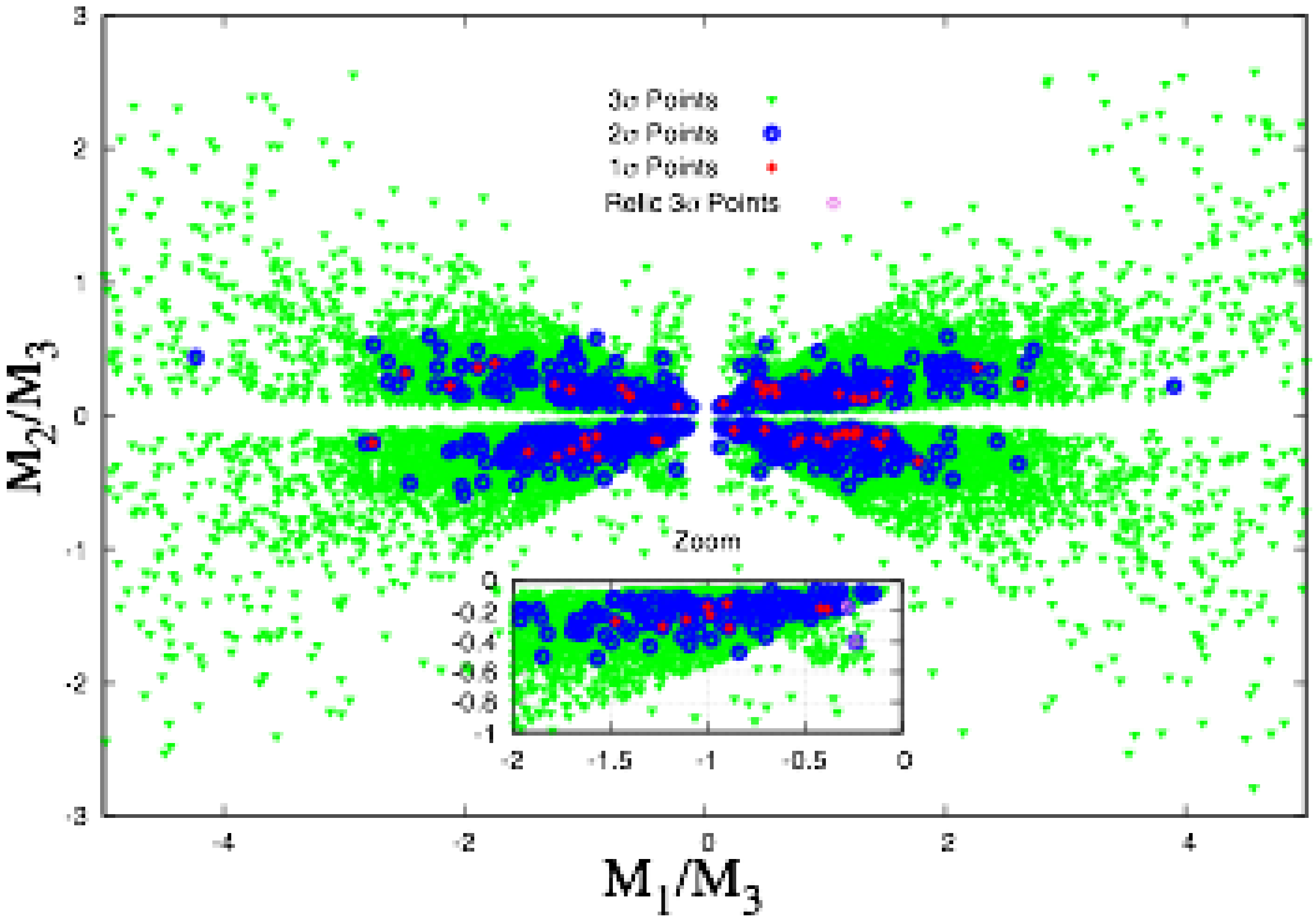
$$M_0, M_3, A_0, \tan\beta, sgn(\mu)$$

- **Different Non-universality in Gaugino Masses:**

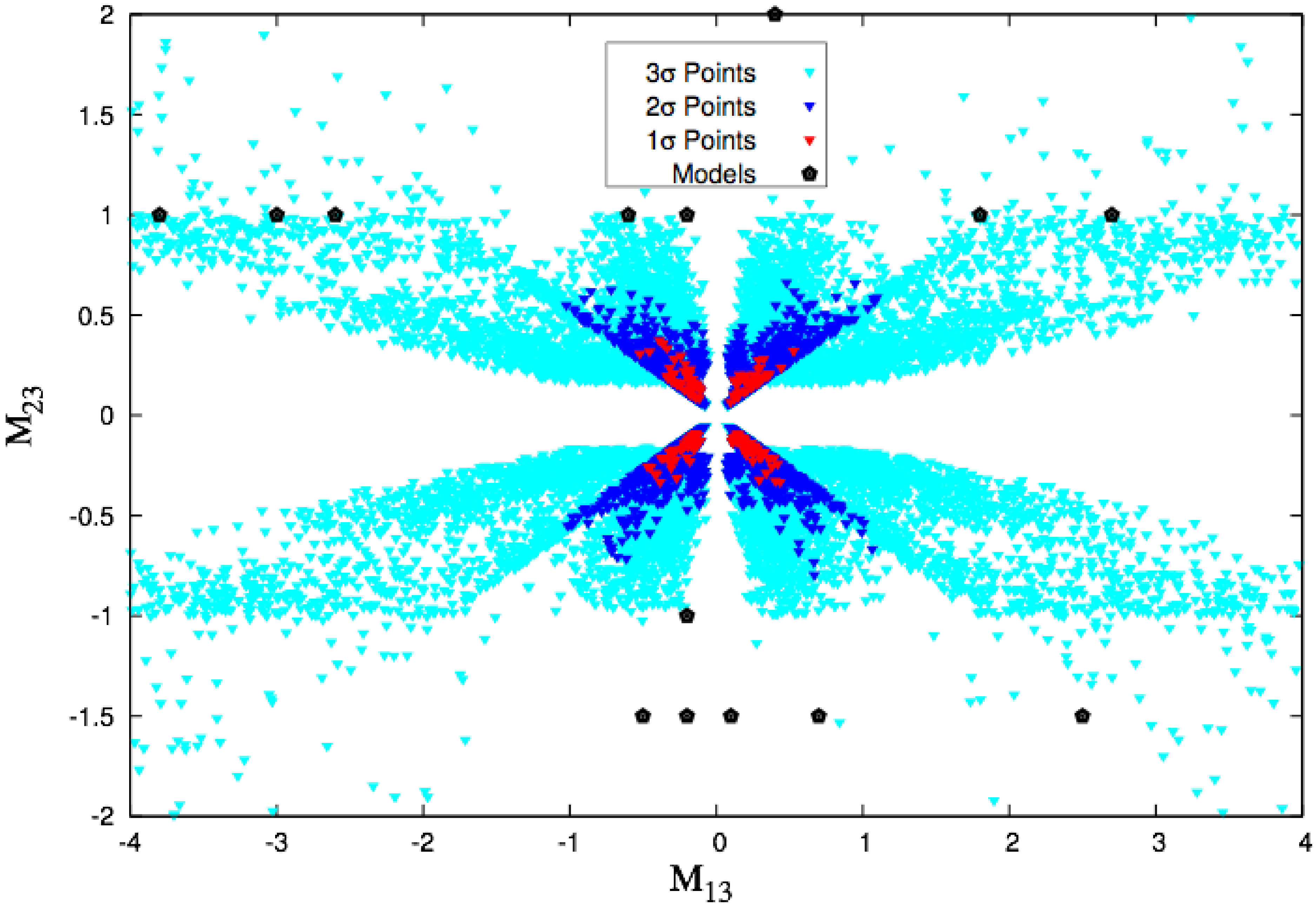
$$M_1 : M_2 : M_3$$

- **Low Scale Spectrum : Composition of LSP (possible DM) gets changed**

$M_1 : M_2 : M_3$ and Muon (g-2), Relic Density



Where are the “Models”?



Directions to be explored..

- Consider more generic structure of gaugino masses. Estimate Singlet-nonsinglet effects.
- Include the impact of possible intermediate scales as they possess different RGEs.
- Gaugino mass ratios are directly affected by these RGEs.
- Cheeck out other SUSY breaking scenarios, e.g. AMSB where gaugino mass ratios are proportional to their β-coefficients.

What about the X-gauginos

- **GUT symmetry is broken to the MSSM gauge group at the GUT scale itself — not necessary!**
- **Thus there are some broken generators.**
- **X-Gauginos : related to those broken generators.**
- **They also acquire masses in form:** $(A + \delta_X B)$

Radiative Correction to Vacuum Energy

- Vacuum Energy receives radiative correction from massive X-gauginos.

[Ellis et. al. Phys. Lett. B 155, Nucl. Phys. B 247]

- This contribution is very large: $\mathcal{O}(m_{3/2}^2 M_{Pl}^2)$
- To avoid such catastrophe, this contribution must vanish.
- This is possible if the X-gauginos are not affected by SUSY breaking, i.e.,

$$A/B = -\delta_X$$

Unique Gaugino Mass Ratio

$$\mathcal{M}_i \equiv M' \; [-\delta_X + \; \delta_j]$$

$\phi \in SU(5)$	δ_1	δ_2	δ_3	δ_X	$M_1 : M_2 : M_3$
24	1	3	-2	1/2	1/2 : 5/2 : -5/2
75	5	-3	-1	-2	7 : -1 : 1
200	10	2	1	3/2	17/2 : 1/2 : -1/2

Status of *flipped*-models

$$SO(10) \to SU(5) \otimes U(1)$$

$\phi \in SO(10)$	δ_5	δ_1	δ_X	$M_5 : M_1$
210	-1	4	-1	0 : 5
770	-1	-16	-1	0 : -15

$$E(6) \to SO(10) \otimes U(1)$$

$\phi \in E(6)$	δ_{10}	δ_1	δ_X	$M_{10} : M_1$
650	-6	30	-6	0 : 36
2430	4	108	4	0 : 104

What else….

Analyse all possible breaking patterns and check whether the scenarios are theoretically consistent or not

Non-Abelian vector Dark gauge Boson

Example Scenario

$$E(6) \rightarrow [SU(3)]^3 \rightarrow SU(3)_C \otimes SU(2)_L \otimes U(1)_L \otimes U(1)_R \otimes SU(2)_N$$
$$\rightarrow SM \otimes SU(2)_N \rightarrow SM$$

Additional global $U(1)_P$

Redefining lepton number $L = P + T_{3N}$

$R = (-1)^{3B+L+2J}$

Exotic fermions and SM Higgs play crucial roles in annihilation and coannihilation processes.