Magnetised hot nuclear matter.

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Noncentral Heavy Ion collisions

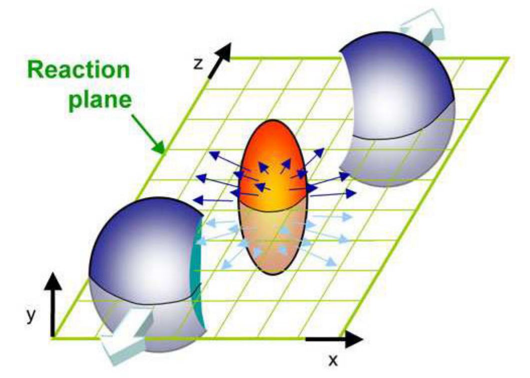


Figure: Pictorial representation of non central heavy ion collisions.

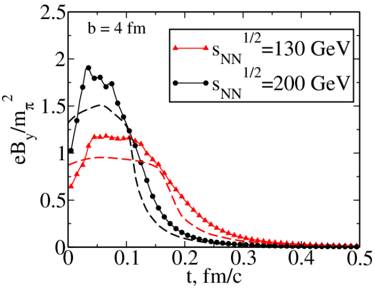
- A very strong magnetic field ($eB \approx m_\pi^2$ at RHIC) is generated in the direction perpendicular to the reaction plane, due to the relative motion of the ions themselves. ($m_\pi^2 = 1.96 \times 10^{-2}$ GeV$^2 \approx 10^{18}$ Gauss)

  - The very high initial magnitude of this magnetic field then decreases very fast, being inversely proportional to the square of time(?).

    [✎ L. McLerran and V. Skokov, 1305.0774]

    [✎ K. Tuchin, 1006.3051]

[✎ V. Skokov et al, IJMP, 2009]



- Many interesting phenomena and subsequent modification of the present theoretical tools.

Regimes of study and our work

- Strong magnetic field

  The initial magnitude of this magnetic field can be very high ($eB \approx 10m_\pi^2$ at LHC) at the time of the collision.

- Weak magnetic field

  As the field strength decreases very fast, by the time the gluons and quarks thermalise, the temperature becomes the largest of the energy scales.

- Arbitrary magnetic field.

- Works done so far, our work and the regime we worked in.
  [✎ A. Bandyopadhyay, CAI and M. G. Mustafa, PRD, 2016]

Notations

$$B = B\hat{z}$$

$$\begin{aligned} a^\mu &= a^\mu_{\shortparallel} + a^\mu_\perp \\ a^\mu_{\shortparallel} &= (a^0, 0, 0, a^3) \\ a^\mu_\perp &= (0, a^1, a^2, 0), \end{aligned}$$

$$\begin{aligned} g^{\mu\nu} &= g^{\mu\nu}_{\shortparallel} + g^{\mu\nu}_\perp \\ g^{\mu\nu}_{\shortparallel} &= \mathsf{diag}(1, 0, 0, -1) \\ g^{\mu\nu}_\perp &= \mathsf{diag}(0, -1, -1, 0), \end{aligned}$$

$$\begin{aligned} (a \cdot b) &= (a \cdot b)_{\shortparallel} - (a \cdot b)_\perp \\ (a \cdot b)_{\shortparallel} &= a^0 b^0 - a^3 b^3 \\ (a \cdot b)_\perp &= (a^1 b^1 + a^2 b^2), \end{aligned}$$

Fermions in a constant magnetic field

- The Lagrangian density of a fermion in a constant magnetic field ($B = B\hat{z}$) is given by,

$$\begin{aligned} \mathcal{L} &= \bar{\psi}\left(i\gamma^\mu D_\mu - m\right)\psi \\ D_\mu &= \partial_\mu - ieA_\mu^{ext} \end{aligned}$$

- The orbital energy levels get discretized, which are known as the Landau Levels (LL).

$$k_{\shortparallel}^2 - m_f^2 - 2nq_fB = k_0^2 - k_3^2 - m_f^2 - 2nq_fB = 0$$
$$\implies E_n = k_0 = \sqrt{k_3^2 + m_f^2 + 2nq_fB}.$$

$n = 0, 1, 2, \cdots$

- Each LL is degenerate.

Fermion propagator in presence of magnetic field - I

- In presence of a constant magnetic field ($\vec{B} = B\hat{z}$),[✎ J. Schwinger, Phys. Rev, 1951]

$$S_m(x, x') = e^{\Phi(x,x')} \int \frac{d^4k}{(2\pi)^4} e^{-ik(x-x')} S_m(k),$$

  where $\Phi(x, x')$ is called the phase factor, which generally drops out in gauge invariant correlation functions.

- In momentum space the Schwinger propagator $S_m(k)$ can be written as an integral over proper time s, i.e.,

$$iS_m(k) = \int\limits_0^\infty ds \exp\left[is\left(k_{\shortparallel}^2 - m_f^2 - \frac{k_\perp^2}{q_f Bs}\tan(q_f Bs)\right)\right]$$
$$\times \left[\left(\not{k}_{\shortparallel} + m_f\right)\left(1 + \gamma_1\gamma_2 \tan(q_f Bs)\right) - \not{k}_\perp\left(1 + \tan^2(q_f Bs)\right)\right].$$

Fermion propagator in presence of magnetic field - II

- After performing the proper time integration, the fermion propagator can be represented as sum over discrete energy spectrum of the fermion

$$iS_m(k) = ie^{-\frac{k_\perp^2}{q_f B}} \sum_{n=0}^{\infty} \frac{(-1)^n D_n(q_f B, k)}{k_{\shortparallel}^2 - m_f^2 - 2nq_f B},$$

with Landau levels $n = 0,\, 1,\, 2, \cdots$ and

$$\begin{aligned} D_n(q_f B, k) \quad &= \quad (\not{k}_{\shortparallel} + m_f)\Big((1 - i\gamma^1\gamma^2)L_n\left(\frac{2k_\perp^2}{q_f B}\right) - (1 + i\gamma^1\gamma^2)L_{n-1}\left(\frac{2k_\perp^2}{q_f B}\right)\Big) \\ &\quad -4\not{k}_\perp L_{n-1}^1\left(\frac{2k_\perp^2}{q_f B}\right), \end{aligned}$$

where $L_n^\alpha(x)$ is the generalized Laguerre polynomial.

LLL Fermion propagator and Dimensional reduction

- For LLL approximation in the strong field limit the fermion propagator reduces to a simplified form as

$$iS_{ms}(k) = ie^{-k_\perp^2/q_f B} \frac{\not{k}_{\shortparallel} + m_f}{k_{\shortparallel}^2 - m_f^2}(1 - i\gamma_1\gamma_2),$$

- As $k_\perp^2 << q_f B$, an effective dimensional reduction from (3+1) to (1+1) takes place in the strong field limit.
- As a consequence the motion of the charged particle is restricted in the direction perpendicular to the magnetic field but can move along the field direction in LLL.
- This effective dimensional reduction also plays an important role in catalyzing the spontaneous chiral symmetry breaking.[✎ V. P. Gusynin et al, hep-ph/9509320]

Lowest Landau Level (LLL) approximation

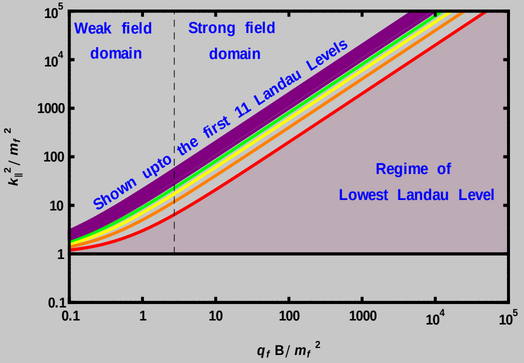


Figure: Thresholds corresponding to the first 11 Landau Levels are displayed as a function of $q_f B/m_f^2$. Also the regime of the LLL at strong magnetic field approximation is shown by the shaded area.

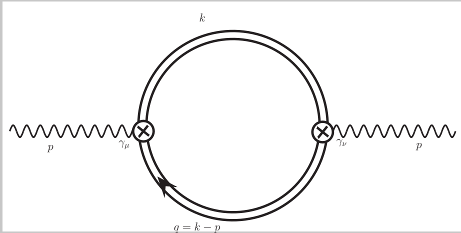
General expression

The electromagnetic spectral representation is extracted from the imaginary part of the two point correlation function $\Pi^{\mu}_{\mu}(p)$ as

$$\rho(p) \quad = \quad \frac{1}{\pi}\mathrm{Im}\ \chi(\mathrm{p}),$$

where $\chi(p) = \sum_f \ (1/q_f^2)\ \Pi^{\mu}_{\mu}$.

$$\Pi_{\mu\nu}(p)\Big|_{sfa} \qquad = -i\sum_f q_f^2 \int \frac{d^4k}{(2\pi)^4}\mathsf{Tr}_c\left[\gamma_\mu S_{ms}(k)\gamma_\nu S_{ms}(q)\right]$$



Polarization tensor in Vacuum

$$\Pi_{\mu\nu}(p)\Big|_{sfa} = -i\sum_f q_f^2 \int \frac{d^4k}{(2\pi)^4} \mathrm{Tr}_c\left[\gamma_\mu S_{ms}(k)\gamma_\nu S_{ms}(q)\right]$$

$$= \left(\frac{p_\mu^{\shortparallel} p_\nu^{\shortparallel}}{p_{\shortparallel}^2} - g_{\mu\nu}^{\shortparallel}\right)\Pi(p^2),$$

$$\Pi(p^2) = N_c \sum_f \frac{q_f^3 B}{8\pi^2 m_f^2}\, e^{-p_\perp^2/2q_fB} \times$$

$$\left[4m_f^2 + \frac{8m_f^4}{p_{\shortparallel}^2}\left(1-\frac{4m_f^2}{p_{\shortparallel}^2}\right)^{-1/2} \ln\frac{\left(1-\frac{4m_f^2}{p_{\shortparallel}^2}\right)^{1/2}+1}{\left(1-\frac{4m_f^2}{p_{\shortparallel}^2}\right)^{1/2}-1}\right].$$

- Due to the current conservation, the two point function is transverse.

$$p_{\shortparallel}^{\mu}\Pi_{\mu\nu}\Big|_{sfa} = 0$$

- The lowest threshold (LT) is $p_{\shortparallel}^2 = 4m_f^2$.
- $\Pi(p^2)$ is singular in presence of magnetic field at this threshold.
- This behavior is in contrast to that in absence of the magnetic field where the similar prefactor appears in the numerator.

Kinematic regions with respect to LT

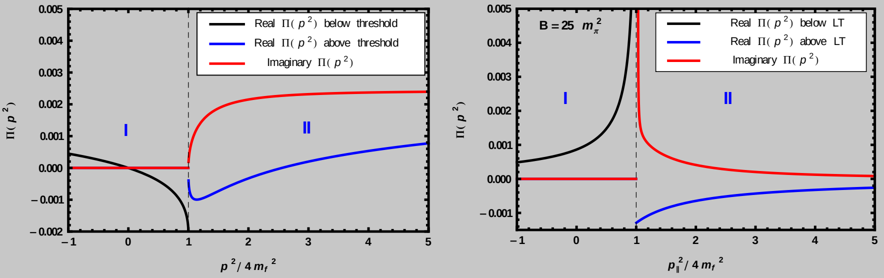


Figure: Plot of $\Pi(p^2)$ as a function of scaled photon momentum square in kinematic regions I $(p_{\shortparallel}^2 < 4m_f^2)$ and II $(p_{\shortparallel}^2 > 4m_f^2)$, both in absence of a magnetic field (left panel) and in presence of a strong magnetic field (right panel).

Electromagnetic spectral function in vacuum

- We now extract the vacuum spectral function in presence of strong magnetic field as

$$\rho\Big|_{sfa}^{\text{vacuum}} = N_c \sum_f \frac{q_f B m_f^2}{\pi^2 p_{\shortparallel}^2}\; e^{-p_\perp^2/2q_f B}\; \Theta\left(p_{\shortparallel}^2 - 4m_f^2\right)\left(1-\frac{4m_f^2}{p_{\shortparallel}^2}\right)^{-1/2} .$$

- The imaginary part is restricted by the LT, $p_{\shortparallel}^2 = 4m_f^2$. Beyond LT ($p_{\shortparallel}^2 > 4m_f^2$) there is nonzero continuous contribution to the electromagnetic spectral function.

Polarization tensor in Medium

- To evaluate the spectral function in medium without any loss of information we can contract the indices μ and ν,

$$\Pi^{\mu}_{\mu}(p)\Big|_{sfa} = -i\sum_f q_f^2 \int \frac{d^4k}{(2\pi)^4}\mathrm{Tr}_c\left[\gamma_\mu S_{ms}(k)\gamma^\mu S_{ms}(q)\right]$$

$$= \quad -iN_c\sum_f \; e^{-p_\perp^2/2q_fB} \;\; \frac{q_f^3 B}{\pi}\int \frac{d^2k_{\shortparallel}}{(2\pi)^2}\frac{2m_f^2}{(k_{\shortparallel}^2-m_f^2)(q_{\shortparallel}^2-m_f^2)}.$$

- At finite temperature this can be written by replacing the p_0 integral by Matsubara sum as

$$\Pi^{\mu}_{\mu}(\omega,\mathbf{p})\Big|_{sfa} = -iN_c\sum_f \; e^{-p_\perp^2/2q_fB} \;\; \frac{2q_f^3 B m_f^2}{\pi}\left(iT\sum_{k_0}\right.$$

$$\left. \times\int \frac{dk_3}{2\pi}\frac{1}{(k_{\shortparallel}^2-m_f^2)(q_{\shortparallel}^2-m_f^2)}\right).$$

Mixed Representation technique

- In the mixed representation prescribed by Pisarski,

$$\frac{1}{k_{\shortparallel}^2 - m_f^2} \equiv \frac{1}{k_0^2 - E_k^2} = \int_0^\beta d\tau e^{k_0 \tau} \Delta_M(\tau, k),$$

and

$$\Delta_M(\tau, k) = \frac{1}{2E_k} \left[(1 - n_F(E_k))\, e^{-E_k \tau} - n_F(E_k) e^{E_k \tau} \right],$$

- Using these,

$$\Pi_\mu^\mu(\omega, \mathbf{p})\Big|_{sfa}$$

$$= N_c \sum_f e^{\frac{-p_\perp^2}{2q_f B}} \; \frac{2q_f^3 B m_f^2}{\pi} \int \frac{dk_3}{2\pi} \int_0^\beta d\tau \; e^{p_0 \tau} \; \Delta_M(\tau, k) \Delta_M(\tau, q).$$

Discontinuity

- One can now easily read off the discontinuity using

$$\text{Disc}\ \left[\frac{1}{\omega + \sum_i E_i}\right]_\omega = -\pi\delta(\omega + \sum_i E_i),$$

- The k_3 integral can now be performed using the following property of the delta function

$$\int_{-\infty}^{\infty} dp_3\ f(p_3)\ \delta[g(p_3)] = \sum_r \frac{f(p_{zr})}{g'(p_{zr})},$$

where the zeroes of the argument inside the delta function is called as p_{zr}.

Excluded processes

The most general form of the delta function would have been like $\delta(rE_k + lE_q - \omega)$ where r and l could have taken values ± 1.

- $r = +1$ and $l = +1$ represent pair creation process for $\omega > 0$.
- $r = -1$ and $l = -1$ correspond to a process with $\omega < 0$, which violates energy conservation as all the quasiparticles have positive energies and has been left out.
- $r = +1$ and $l = -1$ or vice versa represent the transitions between two different LL, n and n'. These processes are in principle allowed in presence of magnetic field.
- Because of the strong field limit the system is assumed to be confined within the LLL and those processes are excluded.

EM Spectral function in medium

- The spectral function in strong field approximation is finally obtained as

$$
\begin{aligned}
\rho\Big|_{sfa} &= \frac{1}{\pi}\mathcal{I}m\ \chi(p)\Big|_{sfa} \\
= \quad & N_c\sum_f \frac{q_f B m_f^2}{\pi^2 p_{\shortparallel}^2}\ e^{-p_\perp^2/2q_fB}\ \Theta\left(p_{\shortparallel}^2 - 4m_f^2\right)\left(1-\frac{4m_f^2}{p_{\shortparallel}^2}\right)^{-1/2} \\
& \times\Big[1-n_F(p_+^s)-n_F(p_-^s)\Big]
\end{aligned}
$$

where

$$
p_\pm^s = \frac{\omega}{2} \pm \frac{p_3}{2}\sqrt{\left(1-\frac{4m_f^2}{p_{\shortparallel}^2}\right)}.
$$

EM Spectral function

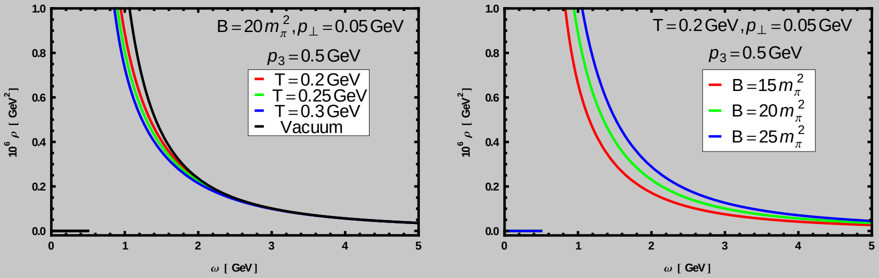


Figure: Variation of the spectral function with photon energy for different values of T at fixed B, $p_\perp$ and p_3 (*Left Panel*) and for different values of magnetic field at fixed T, $p_\perp$ and p_3 (*Right Panel*).The value of the magnetic field is chosen in terms of the pion mass m_π.

EM Spectral function - Summary

- The electromagnetic spectral function vanishes in the massless limit of quarks because of the dimensional reduction. Physically, this is quite natural as the process of creating an l, $\bar{l}$ pair moving in opposite directions involves chirality breaking. [✎ A. V. Smilga, PRD 45, 1378]
- The threshold, $p_{\shortparallel}^2 = 4m_f^2$ is independent of the magnetic field strength. It is also independent of T as $q_f B >> T^2$ in the strong field approximation.
- Like vacuum case here also the spectral function vanishes below the threshold because the polarization tensor is purely real below the threshold.

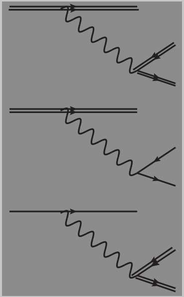
EM Spectral function - Summary

- When $p_{\shortparallel}^2 = 4m_f^2$, the spectral strength diverges because of the factor $\left(1 - 4m_f^2/p_{\shortparallel}^2\right)^{-1/2}$ that appears due to the dimensional reduction.
- The fermions are virtually paired up in LLL providing a strongly correlated system in presence of strong magnetic field and there could be generation of dynamical mass for fermions through fermion-antifermion condensates.
- The spectral strength falls off with increase of ω as there is nothing beyond the LLL in strong field approximation. To improve the high energy behavior of the spectral function one requires weak field approximation ($T^2 \; >> \; q_f B$).

Dilepton production

The dilepton production from a magnetized hot and dense matter can generally be dealt with three different scenarios.

- Both quarks and leptons move in a magnetized medium.
- Only the quarks move in a magnetized medium but not the final lepton pairs.
- Only the final lepton pairs move in the magnetic field.



Dilepton Rate - Plots

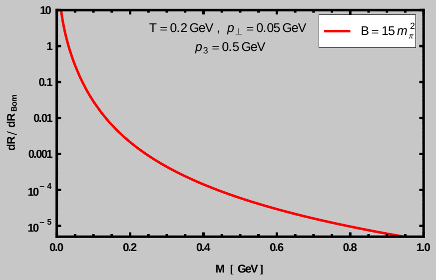


Figure: Plot of ratio of the Dilepton rate in the strong magnetic field approximation to the Born rate (perturbative leading order) for nonzero external three momentum of photon.

Dilepton Rate - Observations

- The LLL dynamics in strong field approximation enhances the dilepton rate as compared to the Born rate for a very low invariant mass (≤ 100 MeV).
- At high mass it falls off very fast similar to that of the spectral function since there is no higher LL in strong field approximation.
- The enhancement found in the strong field approximation in the rate will contribute to the dilepton spectra at low invariant mass.

Debye screening

- In the static limit the Debye screening mass is obtained as

$$
\begin{aligned}
m_D^2 &= \Pi_{00}(|\vec{p}| = 0, \omega \to 0) \\
&= N_c \sum_f \frac{q_f^3 B}{\pi T} \int_0^\infty \frac{dk_3}{2\pi}\, n_F(E_k)\,[1 - n_F(E_k)]
\end{aligned}
$$

- Massive case $(m_f \neq 0) \Longrightarrow$ numerically evaluated.

- On the other hand, for the massless case $(m_f = 0)$ a simple analytical expression is obtained as

$$
\Pi_{00}\Big|_{|\vec{p}|, m_f = 0, \omega \to 0}^{sfa} = N_c \sum_f \frac{q_f^3 B}{4\pi^2}.
$$

Debye Screening - Plots

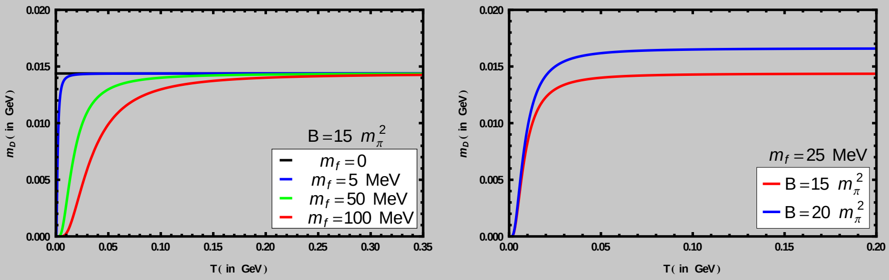
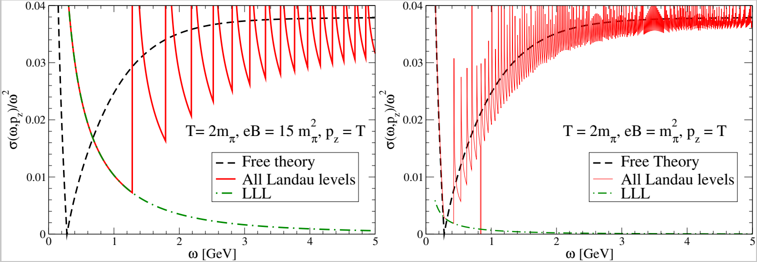


Figure: *Left panel:* Variation of the Debye screening mass with temperature for different quark masses massive at a fixed value of B. *Right panel:* Comparison of the temperature Variation of the Debye screening mass for two values of B $(= 15m_\pi^2$ and $20m_\pi^2)$.

Debye screening - Summary

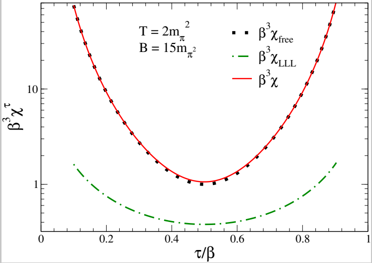
- A hot magnetized medium is associated with three scales: the dynamical mass m_f, temperature T and the magnetic field B.
- When the quark mass, $m_f = 0$, Debye screening is independent of T because the only scale in the system is the magnetic field ($q_f B \;>>\; T^2$), and the thermal scale gets canceled out exactly as found analytically.
- For massive quarks, the three scales became very distinct and as $T < m_f$, the quasiquark mass brings the Debye screening down. Eventually the screening mass vanishes completely when $T = 0$.
- As $T > m_f$, the screening becomes independent of other two scales ($m_f^2 \leq T^2 \leq q_f B$).

Spectral function in pseudoscalar channel



- Spectral function in the pseudoscalar channel for different values of field strength and temporal pseudoscalar correlator.

  [✎ P. Chakraborty, 1711.04404]



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Thank You