Effects of magnetic field on plasma evolution in relativistic heavy-ion collisions

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Outline

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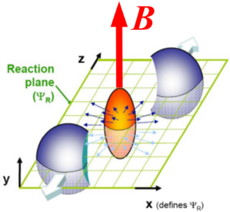
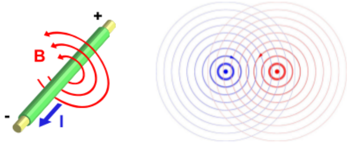
Introduction

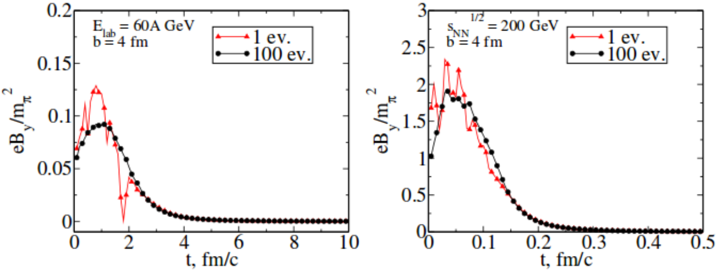
- Terrestrial heavy ion collision experiments like **RHIC** at BNL and **LHC** at CERN, produced new state of QCD matter which shows many features of hot **QGP**.
- The main goal of Heavy-ion collision experiments to explore QCD phase diagram, produce quark-gluon plasma and investigate its physical properties.
- An entire new line of explorations has been initiated in recent years by the very exciting possibility that in relativistic heavy-ion collision experiments extremely high magnetic fields are expected to arise.
- Two colliding nuclei generate **two electric currents in opposite directions**, and produce a **magnetic field perpendicular to the reaction plane**[1].





[1] QCD in strong magnetic field, talk given by M.N.Chernodub

- Very strong magnetic field is produced in HIC experiments.
  1. Earth's magnetic field: 10^{-5} Tesla.
  2. Magnetar's surface: 10^{10} Tesla.
  3. Heavy-ion collisions: 10^{15} Tesla (few m_π^2).
- Conversion of units: $m_\pi^2 \simeq 0.02GeV^2 \simeq 3 \times 10^{14}$ Tesla = 3×10^{18} Gauss.
- Estimates of magnetic field created in non-central collision within UrQMD model[2]:



[2] V.V.Skokov et al.arXiv: 0907.1396

- Much work has been done regarding effect of magnetic field in HIC, some of them are [2]
  1. Chiral Magnetic effect,
  2. Magnetic Catalysis and Inverse magnetic catalysis,
  3. Effect on HRG model ,
  4. Effect on the flow etc.
- Some of us had earlier showed the effect of magnetic field on the anisotropic flow[3] $\rightarrow$ enhancement in the elliptic flow coefficient v_2 by almost 30%.
- This happens due to the directional dependence of sound speed in the presence of magnetic field.
- It can be shown that in linear order flow velocity is directly proportional to sound speed square [3],

$$v_x = \frac{c_s^2 x}{\sigma_x^2} t, \qquad v_y = \frac{c_s^2 y}{\sigma_y^2} t$$

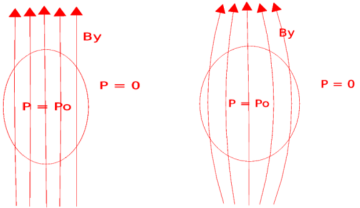
- Above estimates of the fluid velocity is for ideal hydrodynamical evolution without the magnetic field. In the presence of magnetic field situation becomes complex.

[2] McLerran et. al. Nucl Phys A 803, 227, H. Taya PRD 92, 014038,

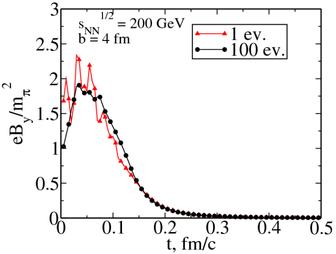
[3] R. K. Mohapatra, P. S. Saumia, and A. M. Srivastava, Mod. Phys. Lett.A 26, 2477 (2011)

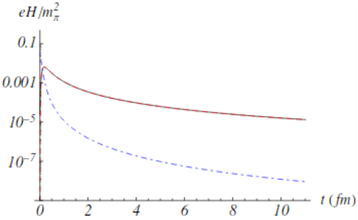
[3] J-Y Ollitrault, Eur. J. Phys. 29 (2008) 275-302.

- MHD allows different kinds of waves in the plasma. Different kinds of wave propagation can be analyzed by perturbing MHD equations $\rightarrow$ normal mode analysis.
  1. When the wave vector $\vec{k} \perp \vec{B}$, MHD equations gives magnetosonic wave of velocity,
  $$c_{\perp}^2 = c_s^2 + v_A^2, \qquad v_A \propto B_0$$
  2. When the wave vector $\vec{k}||\vec{B}$, MHD equations gives magnetosonic wave of velocity,
  $$c_{||}^2 = c_s^2$$
- Since the sound speed is larger in the direction perpendicular to the magnetic field anisotropic flow becomes larger for standard collision geometry.
- Naively, when magnetic field lines (frozen in the plasma) expand perpendicular to the direction of the magnetic field, it cost energy, and feel tension. Thus EoS becomes stiffer perpendicular to magnetic field which causes larger sound speed.



- Earlier it was thought that magnetic field rapidly decays after the collision. It is strong for a very short time, subsequently it rapidly decays.





- For conducting medium with $\sigma \sim 5.8MeV$, B is shown by the red curve (right fig.). Blue curve (right fig.) shows vacuum solution [4].
- It was pointed out by Tuchin that magnetic field does not decay quickly in a conducting medium.
- The magnetic field satisfies a diffusion equation with the diffusion constant equal to $1/(\sigma\mu)$, $\mu(\sim 1)$ is the magnetic permeability and $\sigma(\sim 0.04T)$ is the electrical conductivity.
- The time scale τ over which the magnetic field remains reasonably strong over length scale L is, $\tau \simeq L^2\sigma/4$. For $L = 6 - 10$ fm, $\tau \leq 1$ fm [5].

[4] K. Tuchin PRC 88(2013) 024911

[5] H.-T. Ding et al., Phys. Rev. D 83, 034504 (2011).

- Even if one takes the optimistic picture that due to non-zero conductivity of QGP, magnetic field does not decay rapidly, however the self-consistency of this picture can be questioned.
- Initially when the magnetic field was strong there was no plasma $\rightarrow$ only partonic DOF not in thermal equilibrium. So no plasma, no conductivity.
- If the parton system is assumed to have the QGP conductivity during its formation stages $\rightarrow$ then how magnetic field can penetrate the conducting plasma.
- Most of the earlier works have not taken into account evolution of the magnetic field coupled to the plasma [6].
- For simplicity we assume ideal RMHD with infinite conductivity, so magnetic field lines are frozen in the plasma.
- For a static conductor: $\vec{J} = \sigma \vec{E}$. For a conductor which is moving in a magnetic field $\vec{J} = \sigma(\vec{E} + \vec{v} \times \vec{B})$.
- Comoving electric field $\vec{E}' = \vec{E} + \vec{v} \times \vec{B}$ and $\vec{E}' = \eta \vec{J}$, where $\eta = 1/\sigma$.
- Ideal MHD limit corresponds to $\eta = 0$. In this limit the comoving electric field is zero.
- In ideal MHD, the magnetic flux through any co-moving closed circuit in the plasma remains constant. This important result is called the frozen flux condition. It means that the field lines can be thought of as being attached to the fluid (and vice versa).

[6] G. Inghirami et al., Eur. Phys. J. C76, 659 (2016)

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Relativistic Magnetohydrodynamics : Formalism

- The motion of an ideal relativistic magnetized fluid is described by [7],
  1. Mass conservation $\partial_\alpha(\rho u^\alpha) = 0$. We will be assuming zero baryon chemical potential situation so we will not use baryon number conservation equation.
  2. Conservation of energy momentum tensor: $\partial_\alpha \left[(\rho h + |b|^2) u^\alpha u^\beta - b^\alpha b^\beta + p\eta^{\alpha\beta} \right] = 0$
  3. Maxwell's equations: $\partial_\alpha (u^\alpha b^\beta - u^\beta b^\alpha) = 0$
- ρ is the rest mass density, u^α is the four velocity,
- b^α covariant magnetic field,
- h is specific enthalpy
- $p = p_g + |b|^2/2$ is the total pressure.
- $u^\alpha = \gamma(1, \vec{v})$
- $b^\alpha = \gamma(\vec{v}.\vec{B}, \frac{\vec{B}}{\gamma^2} + \vec{v}(\vec{v}.\vec{B}))$
- Normalizations: $u^\alpha u_\alpha = -1$, $u^\alpha b_\alpha = 0$, $|b|^2 = b^\alpha b_\alpha = \frac{|B|^2}{\gamma^2} + (\vec{v}.\vec{B})^2$,
- $\gamma = (1 - \vec{v}.\vec{v})^{-1/2}$

[7] A.Mignone et. al, Mon.Not. R. Astron.Soc. 000, 1 (2005)

- For computational purpose, the above equations can be conveniently put in the following form,

$$\frac{\partial U}{\partial t} + \sum_k \frac{\partial F^k(U)}{\partial x^k} = 0$$

- where,

$$U = (m_x, m_y, m_z, B_x, B_y, B_z, E)$$

-

$$F^x(U) = \begin{bmatrix} m_x v_x - B_x \frac{b_x}{\gamma} + p \\ m_x v_x - B_x \frac{b_y}{\gamma} \\ m_x v_x - B_x \frac{b_z}{\gamma} \\ 0 \\ B_y v_x - B_x v_y \\ B_z v_x - B_x v_z \\ m_x \end{bmatrix}$$

- $m_k = (\rho h \gamma^2 + B^2) v_k - (\vec{v}.\vec{B}) B_k$, $E = \rho h \gamma^2 - p_g + \frac{\vec{B}^2}{2} + \frac{v^2 B^2 - (\vec{v}.\vec{B})^2}{2}$

- Independent variables, $(\rho, \vec{v}, p_g, \vec{B})$, which have to be extracted from U.
- Set, $W = \rho h \gamma^2$, $S = \vec{m}.\vec{B}$, then

$$E = W - p_g + (1 - \frac{1}{2\gamma^2})|B^2| - \frac{S^2}{2W^2}$$

$$|m|^2 = (W + |B|^2)^2(1 - \frac{1}{\gamma^2}) - \frac{S^2}{W^2}(2W + |B|^2)$$

- In the beginning of each time step, $\vec{m}$, $\vec{B}$, S are known. γ in-terms of known quantities,
  $\gamma = \left(1 - \frac{S^2(2W+|B|^2)+|m|^2W^2}{(W+|B|^2)^2W^2}\right)^{-\frac{1}{2}}$, $p_g(W) = \frac{W}{4\gamma^2}$
- Unknown quantity W can be found out from,
  $f(W) = W - p_g + \left(1 - \frac{1}{2\gamma^2}\right)|B|^2 - \frac{S^2}{2W^2} - E = 0$
- Once W has been computed, one can get back γ and p_g. Velocities can be found by,

$$v_k = \frac{1}{W + |B|^2}\left(m_k + \frac{S}{W}B_k\right)$$

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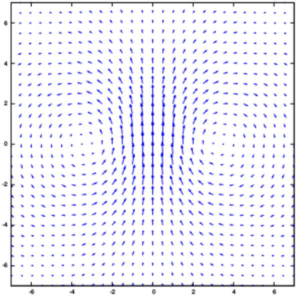
Simulation Details

- We have performed 3+1 dimensional code and use lattice of size $200 \times 200 \times 200$ with lattice spacing of 0.1 fm.
- For evolution we use leapfrog algorithm of 2nd order accuracy.
- We perform low energy collisions with $\sqrt{s}$ = 20 GeV and with Cu nuclei.
- Because of computational limitations we have taken radius of copper nucleus as 4.0 fm with skin 0.4 fm.
- Glauber like initial conditions are used for the initial energy density profile where a nucleus-nucleus collision is viewed as a sequence of independent binary nucleon-nucleon collisions.
- The parameters are tuned to an initial central temperature of 160 - 180 MeV assuming energy density of ideal gas of quarks and gluons for the two flavor case with zero chemical potential.

- We have taken EOS of ideal relativistic gas $p_g = \rho/3$ and zero chemical potential for simplicity.
- Initial fluid velocity in the transverse plane taken to be zero. Longitudinal velocity profile with $\propto z$ suitable maximum velocity is taken.
- We have done our calculation in central rapidity region.
- Magnetic field produced by two oppositely moving, uniform charged spheres with appropriate Lorentz γ factor is taken as the initial magnetic field profile at time τ_0 after the collision.

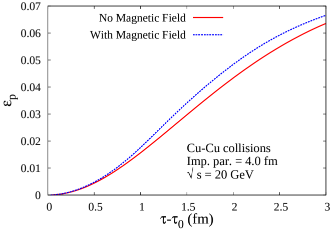
$$\vec{B'} = \gamma(\vec{B} - \vec{\beta} \times \vec{E}) - \frac{\gamma^2}{\gamma + 1}\vec{\beta}(\vec{\beta}.\vec{B})$$

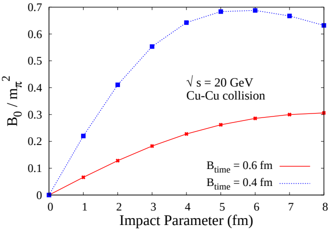


Outline

Magnetic field dependence of elliptic flow

- Conventional momentum anisotropy defined as: $\epsilon_p = \frac{T^{xx} - T^{yy}}{T^{xx} + T^{yy}}$

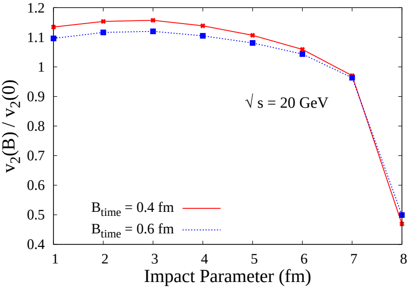


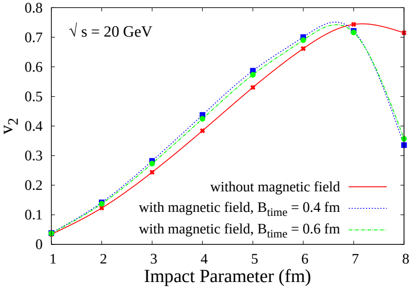


- As expected, ϵ_p increases gradually with time.

$$\epsilon_p = \frac{v_x^2 - v_y^2}{v_x^2 + v_y^2 + \frac{1}{2\gamma^2}}$$

- In the presence of magnetic field ϵ_p increases more rapidly, clearly showing enhancement of momentum anisotropy due to magnetic field.
- Although here we have represented ϵ_p as momentum anisotropy, but for the rest of the results we will use $v_2 = \frac{T^{0x} - T^{0y}}{T^{0x} + T^{0y}}$ as the definition of elliptic flow.



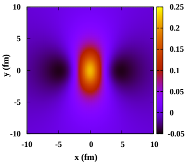


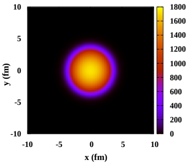
- The effect of magnetic field on elliptic flow is shown in the above figures.
- According to our used definition of v_2:

$$v_2 = \frac{T^{x0} - T^{y0}}{T^{x0} + T^{y0}}$$

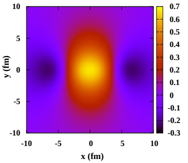
- Calculated magnetic field is monotonically increasing function of the impact parameter almost for the entire range considered here.
- We see clear enhancement in v_2 due to magnetic field. In the left figure $v_2(B)/V_2(0)$ has maximum value at the impact parameter of about 3 fm.
- Interestingly, for large impact parameter (near about 6.5 fm) there is no effect of magnetic field on v_2 and for larger impact parameters, magnetic field actually leads to suppression of v_2.

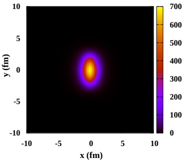
- Non-trivial behavior of magnetic field dependence of v_2 and it appears to originate from the differences in the profiles of magnetic field vs. the energy density profile.
- For low impact parameter (1fm) magnetic field is well inside the plasma region, hence the argument of anisotropic sound speed holds true.





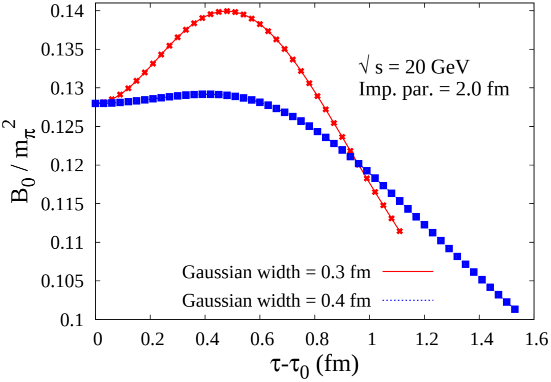
- For large impact parameter (7 fm) extension of magnetic field is outside the plasma region. Lenz's law opposes the expansion of the conducting plasma in the direction perpendicular to the magnetic field.





Enhancement of magnetic field due to fluctuation

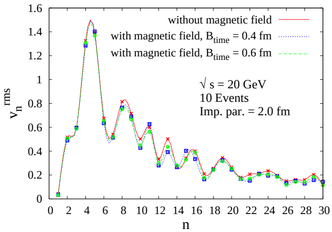
- At an average level one expects that magnetic field decreases as plasma evolves.
- However plasma has strong initial state fluctuations in the energy density.
- As fluctuations evolve, the dynamics of magnetic flux lines become very complex.
- It is clearly possible that in some region plasma expansion dilutes the magnetic flux, while due to energy density inhomogeneities, the neighboring region may get concentration of magnetic flux, thereby locally increasing the magnetic field.



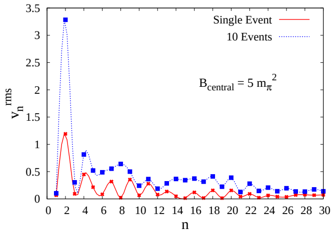
- sharper fluctuations can lead to larger increase the local magnetic field.

Effects of magnetic field on the power spectrum of flow fluctuations

- v_n denotes the n_{th} Fourier coefficient of the momentum anisotropy in $\delta p/p$.
- We calculate v_n^{rms} value rather than average v_n. In case of event average, average value of v_n will be zero due to random orientations of different events.



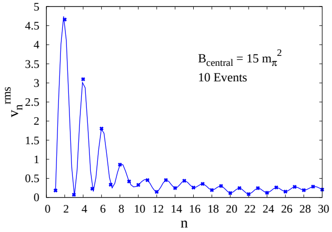
Effect of magnetic field (0.1 m_π^2 and 0.4 m_π^2) is small



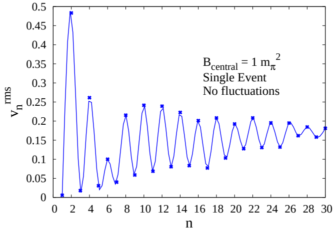
Even-odd power difference is seen in first few flow coefficients

- For power spectrum calculation Bjorken expansion is not taken into account for numerical stability.
- Left plot the power spectrum calculated after the time evolution of about 2 fm. For the right plot due to the strong magnetic field the simulation could be carried out only for relatively short time of 0.6 fm.

- In case of large magnetic field due to numerical stability we have taken a simpler profile for magnetic field where the profile in the (x-z) plane is chosen to be proportional to the energy density profile in the (x-z) plane at $y = 0$.
- Peak value of the magnetic field is chosen by hand and the magnetic field is taken to be constant along the y axis, which is consistent with the Gauss's law.

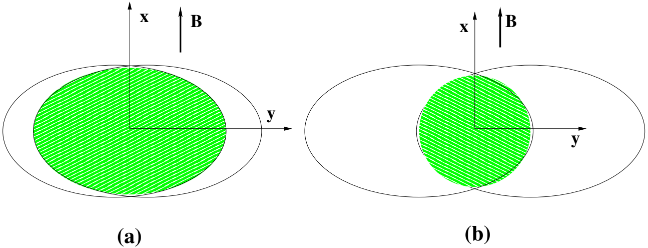


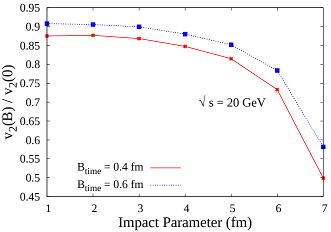
Strong difference in the power of even and odd values of flow coefficients

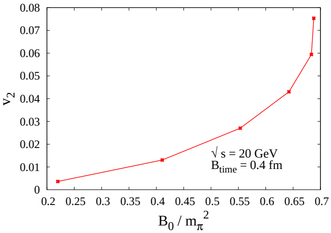


Strong difference in the power of even and odd values of flow coefficients due to magnetic field only

Anomalous elliptic flow for deformed nucleus





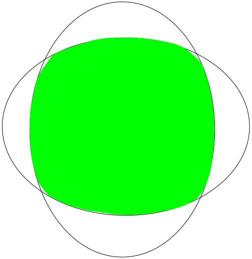


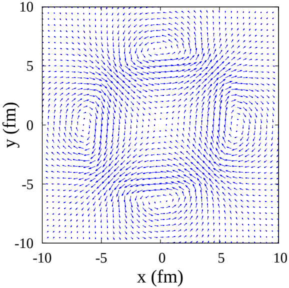
Suppression of v_2 for the case when the QGP region is elliptical in shape but the magnetic field points along the semi-minor axis, x-axis in this case.

v_2 for the case when the QGP region is roughly isotropic in shape but still non-zero magnetic field is present leading to non-zero v_2, even though no elliptic flow is expected.

Quadrupole magnetic field from deformed nucleus

- A very interesting possibility arises when considering collision of deformed nuclei (Uranium) in crossed configuration.





- Crossed configuration of collision of deformed nuclei. Note that the overlap region will be reasonably isotropic, with possibly strong v_4 component.
- Now there are four spectator parts whose motion should lead to quadrupolar magnetic field configuration.
- Quadrupolar field will tend to focus plasma motion along the longitudinal direction, thereby affecting Bjorken longitudinal expansion.
- This should lead to suppression of transverse flow at non-zero rapidity.

Conclusions

- Magnetic field can change elliptic flow with dependence on the impact parameter of the collisions. It can be very important in the study of viscosity of QGP and provides signal of presence of magnetic field.
- In fact the suppression of elliptic flow can be a good signal of presence of magnetic field.
- We found that magnetic field can get enhanced in the presence of fluctuations.
- Effect of magnetic field on the flow fluctuations is very interesting, specifically even odd difference in flow fluctuation for very large magnetic field.
- Deformed nuclei can give very interesting possibility in the magnitude and profile of magnetic field like qudrupolar field configuration which can give beam focussing.