Precision extraction of the CKM element $|V_{cb}|$ and the SM predictions for $R(D^{(*)})$ from the decays $B \to D^{(*)}\ell\nu_\ell$ [1]

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[1] Based on JHEP 1712 (2017) 060 [S. Nandi, S. Patra, SJ]

Motivation

$$|V_{cb}| = \begin{cases} (38.71 \pm 0.75) \times 10^{-3} & \text{(exclusive)} \\ (42.00 \pm 0.65) \times 10^{-3} & \text{(inclusive)} \end{cases} \Bigg\} \approx 3\sigma \text{deviation}$$

$R(D^{(*)}) = \frac{Br(B \to D^{(*)}\tau\nu_\tau)}{Br(B \to D^{(*)}\ell\nu)}$ exceed the SM predictions by 4.17σ

- Additional form factors in $B \to D^{(*)}\tau\nu_\tau$ can't be extracted directly from available $B \to D^{(*)}\ell\nu$ data $\Longrightarrow$ Need to rely on theory inputs like HQET
- Ratio of form factors in $B \to D^{(*)}\ell\nu_\ell$ with QCDSR predictions differ from that predicted by lattice.

Form factor Ratio	Lattice Result (MILC)	QCDSR HQET prediction
$f_+(1)/f_0(1)$	1.3288 ± 0.0062	1.2859 ± 0.0374
$f_+(1.08)/f_0(1.08)$	1.2709 ± 0.0053	1.2380 ± 0.0296
$f_+(1.16)/f_0(1.16)$	1.2172 ± 0.0045	1.1935 ± 0.0231
$f_+(1)/f(1)$	0.2032 ± 0.0033	0.1820 ± 0.0053

- Is this due to the missing pieces in the HQET relations between the form factors (i.e. corrections at order α_s^2 and $\Lambda_{QCD}^2/m_{b,c}^2$)?

Motivation

- $R(D)$ updated using lattice input on the form factors in $B \to D\ell\nu_\ell$ beyond zero recoil (1606.0803)
- Different exclusive $|V_{cb}|$ with CLN and BGL parameterizations using recent Belle data for $B \to D^*\ell\nu$ decay (1703.06124)
- $R(D^*)$ and $|V_{cb}|$ updated from a combined fit to the $B \to D^{(*)}l\nu$ data using HQET framework (1703.05330)
- Our Work (1707.09977) : A combined analysis with the "complete" set of available data for the $B \to D^{(*)}l\nu$ decay modes together with up-to-date lattice and LCSR inputs, and extract $|V_{cb}|$ and predict $R(D^{(*)})$ in SM with both BGL and CLN parameterizations

QCD form factors

- The hadronic matrix element governing $B \to Dl\nu$ decay depends on two form factors,

$$\langle D(p')|\bar{c}\gamma^\mu b|\bar{B}(p)\rangle = f_+(q^2)(p+p')^\mu + f_-(q^2)(p-p')^\mu$$

that enter the differential decay rate in the combination

$$\frac{d\Gamma}{dq^2} = \frac{\eta_{EW}^2 G_F^2 |V_{cb}|^2 m_B \lambda^{\frac{1}{2}}}{192\pi^3}(1-\frac{m_l^2}{q^2})^2 \left[\frac{\lambda}{m_B^4}(1+\frac{m_l^2}{2q^2})f_+^2 + (1-r^2)^2\frac{3m_l^2}{2q^2}f_0^2\right]$$

where,

$$q^2 = (p-p')^2, \quad r = \frac{m_D}{m_B}, \quad \lambda = (q^2 - m_B^2 - m_D^2)^2 - 4m_B^2 m_D^2,$$

and,

$$f_0(q^2) = f_+(q^2) + \frac{q^2}{m_B^2 - m_D^2} f_-(q^2)$$

- At maximum recoil (i.e., $q^2 = 0$) : $f_+(0) = f_0(0)$
- f_0 contribution becomes irrelevant in the limit $m_l \to 0$

QCD form factors

- The matrix elements for $B \to D^* l\nu$ depend on four form factors,

$$\langle D^*(p',\epsilon)|\bar{c}\gamma^\mu b|\bar{B}(p)\rangle = ig\epsilon^{\mu\alpha\beta\gamma}\epsilon^*_\alpha p'_\beta p_\gamma,$$
$$\langle D^*(p',\epsilon)|\bar{c}\gamma^\mu\gamma_5 b|\bar{B}(p)\rangle = f\epsilon^{*\mu} + (\epsilon^*.p)[a_+(p+p')^\mu + a_-(p-p')^\mu]$$

that enter the differential rate in the combinations

$$\frac{d\Gamma}{dq^2} = \frac{\eta_{EW}^2 G_F^2 |V_{cb}|^2 \lambda^{\frac{1}{2}}}{192\pi^3 m_B^3}\left(1-\frac{m_l^2}{q^2}\right)^2 \left[\left(1+\frac{m_l^2}{2q^2}\right)\left(2q^2|f|^2 + |F_1|^2 + \frac{\lambda q^2}{2}|g|^2\right) + \frac{3\lambda m_l^2}{8q^2}|F_2|^2\right]$$

where,

$$F_1(q^2) = \frac{1}{2m_{D^*}}[\lambda a_+(q^2) - (q^2 - m_B^2 + m_{D^*}^2)f(q^2)],$$
$$F_2(q^2) = \frac{1}{m_{D^*}}[f(q^2) + (m_B^2 - m_{D^*}^2)a_+(q^2) + q^2 a_-(q^2)]$$

- F_2 contribution becomes irrelevant in the limit $m_l \to 0$

HQET form factors

- For $\bar{B} \to D$ decays,

$$\langle D(v')|\bar{c}\gamma^\mu b|\bar{B}(v)\rangle = \sqrt{m_B m_D}[h_+(w)(v+v')^\mu + h_-(w)(v-v')^\mu]$$

- For $\bar{B} \to D^*$ decays,

$$\begin{aligned}\langle D^*(v',\epsilon)|\bar{c}\gamma^\mu b|\bar{B}(v)\rangle =& i\sqrt{m_B m_D^*} h_V(w)\epsilon^{\mu\alpha\beta\gamma}\epsilon^*_\alpha v'_\beta v_\gamma,\\ \langle D^*(v',\epsilon)|\bar{c}\gamma^\mu\gamma_5 b|\bar{B}(v)\rangle =& \sqrt{m_B m_D^*}[h_{A_1}(w)(w+1)\epsilon^{*\mu} -\\ & (\epsilon^*.v)(h_{A_2}(w)v^\mu + h_{A_3}(w)v'^\mu)]\end{aligned}$$

where $w = v.v' = \frac{m_B^2 + m_{D^{(*)}}^2 - q^2}{2m_B m_{D^{(*)}}}$ $\left(w_{min} = 1, \quad w_{max} = \frac{m_B^2 + m_{D^{(*)}}^2}{2m_B m_{D^{(*)}}}\right)$

HQET form factors

- In the exact HQET limit, $h_i \propto \xi(w)$ with $\xi(1) = 1$
  where $\xi(w)$ = Isgur-Wise function
- Corrections to h_i's at order α_s and $\Lambda_{QCD}/m_{b,c}$ are known.
- The $\Lambda_{QCD}/m_{b,c}$ corrections in h_i's are expressed in terms of a few sub-leading Isgur-Wise functions (HQET parameters) :
  $\eta(1), \eta'(1), \chi_2(1), \chi_2'(1)$, and $\chi_3'(1)$
- QCDSR predictions for the sub-leading Isgur-Wise functons:
  $\eta(1) = 0.62 \pm 0.2, \quad \eta'(1) = 0 \pm 0.2, \quad \chi_2(1) = -0.06 \pm 0.02,$
  $\chi_2'(1) = 0 \pm 0.02, \quad \chi_3'(1) = 0.04 \pm 0.02$
- HQET parameters predicted in QCDSR have large errors
- In general, the ratios of form-factors are more sensitive to $\eta(1)$ than the other HQET parameters

Relating QCD and HQET form factors

$$f_+(w) = \frac{1+r_D}{2\sqrt{r_D}}\left[h_+(w) - \frac{1-r_D}{1+r_D}h_-(w)\right], \qquad f(w) = \sqrt{m_B m_D^*}(1+w)h_{A_1}(w),$$

$$f_0(w) = \frac{\sqrt{r_D}(1+w)}{1+r_D}\left[h_+(w) - \frac{1+r_D}{1-r_D}\frac{w-1}{w+1}h_-(w)\right], \qquad g(w) = \frac{h_V(w)}{\sqrt{m_B m_{D^*}}},$$

$$F_1(w) = (1+w)m_B\sqrt{m_B m_{D^*}}[(w-r_{D^*})h_{A_1}(w) - (w-1)(r_{D^*}h_{A_2}(w) + h_{A_3}(w))]$$

$$F_2(w) = \frac{1}{r_{D^*}}[(w+1)h_{A_1}(w) - (w-r_{D^*})h_{A_3}(w) - (1-wr_{D^*})h_{A_2}(w)]$$

where $r_{D^{(*)}} = m_{D^{(*)}}/m_B$

Defining some useful form factor ratios,

$$R_1(w) = \frac{h_v(w)}{h_{A_1}(w)} = (w+1)m_B m_D^* \frac{g(w)}{f(w)},$$

$$R_2(w) = \frac{h_{A_3}(w)}{h_{A_1}(w)} + r_{D^*}\frac{h_{A_2}(w)}{h_{A_1}(w)} = \frac{w-r}{w-1} - \frac{F_1(w)}{m_B(w-1)f(w)}$$

$$R_0(w) = \frac{(w+1)}{(1+r_{D^*})} - \frac{w-r_{D^*}}{1+r_{D^*}}\frac{h_{A_3}(w)}{h_{A_1}(w)} - \frac{1-wr_{D^*}}{1+r_{D^*}}\frac{h_{A_2}(w)}{h_{A_1}(w)} = \frac{(1+w)m_{D^*}}{1+r_{D^*}}\frac{F_2(w)}{f(w)}$$

Boyd-Grinstein-Lebed (BGL) Parametrization

- The form factors can be written as series in $z = \frac{\sqrt{w+1}-\sqrt{2}}{\sqrt{w+1}+\sqrt{2}}$ as,

$$F_i(z) = \frac{1}{P_i(z)\phi_i(z)} \sum_{n=0}^{N} a_n^{F_i} z^n \qquad (F_i = f_+, f_0, g, f, F_1 and F_2)$$

- The Blaschke factors eliminate the poles of $F_i's$ at $z = z_p$,

$$P_i(z) = \prod_{p_i} \frac{z - z_{p_i}}{1 - z z_{p_i}}$$

where $z_{p_i} = \frac{\sqrt{t_+ - m_{p_i}^2} - \sqrt{t_+ - t_-}}{\sqrt{t_+ - m_{p_i}^2} + \sqrt{t_+ - t_-}}$ and $t_\pm = (m_B \pm m_{D^{(*)}})^2$

- The product is extended to all the B_c resonances below the B-$D^{(*)}$ threshold $(m_B + m_{D^{(*)}})$ with appropriate quantum numbers.
- Unitarity constraint : $\sum_{n=0}^{N} (a_n^{F_i})^2 < 1$
- In our analysis, we take N=2

Caprini-Lellouch-Neubert (CLN) Parametrization

- For $B \to D l \nu_l$ decay:

$$f_+(w) = f_+(1)[1 - 8\rho_D^2 + (51\rho_D^2 - 10)z^2 - (252\rho_D^2 - 84)z^3]$$
$$\frac{f_0(w)}{f_+(w)} = \frac{2r_D(1+w)}{(1+r_D)^2} 1.0036[1 - 0.0068w_1 + 0.0017w_1^2 - 0.0013w_1^3]$$

where $w_1 = w - 1$, $r_D = \frac{m_D}{m_B}$ and $z = \frac{\sqrt{w+1} - \sqrt{2}}{\sqrt{w+1} + \sqrt{2}}$

- For $B \to D^* l \nu_l$ decay:

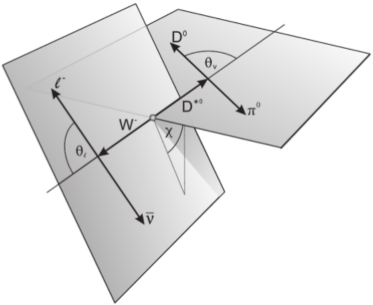
$$h_{A_1}(w) = h_{A_1}(1)(1 - 8\rho_{D^*}^2 z + (53\rho_{D^*}^2 - 15)z^2 - (231\rho_{D^*}^2 - 91)z^3),$$
$$R_1(w) = R_1(1) - 0.12(w-1) + 0.05(w-1)^2,$$
$$R_2(w) = R_2(1) + 0.11(w-1) - 0.06(w-1)^2,$$
$$R_0(w) = R_0(1) - 0.11(w-1) + 0.01(w-1)^2$$

- In the limit of vanishing lepton mass, f_0 and R_0 contributions becomes irrelevant.

Experimental Inputs

- For $B \to Dl\nu$ decay, we use the full dataset from Belle for $\Delta\Gamma/\Delta w$ including all the four subsamples $B^+ \to \bar{D}^0 e^+ \nu_e$, $B^+ \to \bar{D}^0 \mu^+ \nu_\mu$, $B^0 \to D^- e^+ \nu_e$ and $B^0 \to D^- \mu^+ \nu_\mu$ with 10 bins each in the range $1 \leq w \leq w_{max} (\simeq 1.6)$ alongwith their statistical and systematic uncertainties and the full systematic correlation matrix.

- For $B \to D^* l\nu$ decay, we use the unfolded binned differential decay rates by Belle for $\Delta\Gamma/\Delta x$ ($x = w, cos\theta_v, cos\theta_l$ and χ) with 10 bins each in the ranges $1 \leq w \leq 1.504$, $-1 \leq cos\theta_v, cos\theta_l \leq 1$ and $0 \leq \chi \leq 2\pi$ alongwith their uncertainties and the full correlation matrix.



Lattice and LCSR Inputs

- Defining, $\mathcal{G}(1)^2 = \frac{4r_D}{(1+r_D)^2} f_+(1)^2$
- Different values of $\mathcal{G}(1)$ used:

Source	$\mathcal{G}(1)$
Fermilab/MILC	1.0541(83)
HPQCD	1.035(40)
HQE(BPS Expansion)	1.04(2)

- Lattice QCD results of f_+ and f_0 for different values of w :

$f_+(w)$ & $f_0(w)$	Value from HPQCD	Correlation					
$f_+(1)$	1.178(46)	1.	0.994	0.975	0.507	0.515	0.522
$f_+(1.06)$	1.105(42)		1.	0.993	0.563	0.576	0.587
$f_+(1.12)$	1.037(39)			1.	0.617	0.634	0.649
$f_0(1)$	0.902(41)				1.	0.997	0.988
$f_0(1.06)$	0.870(39)					1.	0.997
$f_0(1.12)$	0.840(37)						1.
	Value from MILC						
$f_+(1)$	1.1994(95)	1.	0.967	0.881	0.829	0.853	0.803
$f_+(1.08)$	1.0941(104)		1.	0.952	0.824	0.899	0.886
$f_+(1.16)$	1.0047(123)			1.	0.789	0.890	0.953
$f_0(1)$	0.9026(72)				1.	0.965	0.868
$f_0(1.08)$	0.8609(77)					1.	0.952
$f_0(1.16)$	0.8254(94)						1.

Lattice and LCSR Inputs

- Zero-recoil (i.e. $w = 1$ or $z = 0$) value of $h_{A_1}(w)$ from unquenched Fermilab/MILC lattice data:

$$h_{A_1}(1) = 0.906 \pm 0.013$$

- Inputs from light cone sum rule (LCSR):

$$h_{A_1}(w_{max}) = 0.65(18), \quad R_1(w_{max}) = 1.32(4), \quad R_2(w_{max}) = 0.91(17)$$

where $w_{max} = \frac{m_B^2 + m_{D^{(*)}}^2}{2 m_B m_{D^{(*)}}}$

- Some important relations between relevant form factors in the BGL and CLN notations :

$$f(w) = \sqrt{m_B m_D^*}(1+w) h_{A_1}(w),$$

$$R_1(w) = (w+1) m_B m_D^* \frac{g(w)}{f(w)},$$

$$R_2(w) = \frac{w-r}{w-1} - \frac{F_1(w)}{m_B(w-1) f(w)}$$

CLN fit results

- Fit from $B \to Dl\nu$ data

Constraints	$\lvert V_{cb} \rvert$ ($\times 10^3$)	$\chi^2_{min}/d.o.f$	p-value (%)	$R(D)$
Using only $\mathcal{G}(1)$				
HPQCD+MILC	39.97(1.34)	**23.04/39**	**98.02**	**0.299(6)**
HPQCD+MILC+BPS	40.04(1.33)	**23.42/40**	**98.30**	**0.299(6)**
Belle	39.86(1.33)	4.57/8	80	0.298(6)
Using only $f_+(w)$				
HPQCD + MILC	**40.84(1.15)**	**31.22/43**	**90.91**	**0.305(3)**

- Calculation of R(D) is straightforward
- Little increase in the central value of $|V_{cb}|$ with respect to Belle but the quality of our fit improves considerably
- Incorporating lattice inputs on $f_+(w)$, $|V_{cb}|$ increases by about 2% while the error reduces from 3.3% to 2.8%
- Predicted R(D) increases due to the use of $f_+(w)$ and there is a considerable reduction in the percentage error of the estimate

CLN fit results

- Fit to $B \to D^* l\nu$ data combined with constraint $h_{A_1}(1) = 0.906 \pm 0.013$

	Data+Lattice	Data+Lattice+LCSR
Parameters/ Observables	Best Fit ± Err. Values	Best Fit ± Err. Values
$\mathbf{\|V_{cb}\| \times 10^3}$	$\mathbf{38.23 \pm 1.46}$	$\mathbf{38.15 \pm 1.43}$
ρ^2_{D*}	1.17 ± 0.15	1.16 ± 0.14
$R_1(1)$	1.39 ± 0.09	1.37 ± 0.04
$R_2(1)$	0.91 ± 0.08	0.91 ± 0.07
$h_{A_1}(1)$	0.91 ± 0.01	0.91 ± 0.01
χ^2_{min}	34.14	34.62
dof	36	39
p-value	55.73%	69.10%

- Small decrease in the central value of $|V_{cb}|$ and a slight increase in uncertainty as compared to $B \to Dl\nu$ fit
- Error in extracted value of $R_1(1)$ reduces from 6.5% to 3% on incorporating LCSR constraints.
- Uncertainties in all other fit parameters reduces (though not considerably) with LCSR constraints

CLN fit results

- Combined fit to $B \to D^{(*)}\ell\nu_\ell$ data combined with constraint $h_{A_1}(1) = 0.906 \pm 0.013$

Parameters	Data+Lattice Best Fit ± Err. Values	Data+Lattice+LCSR Best Fit ± Err. Values
$\mathbf{\|V_{cb}\| \times 10^3}$	$\mathbf{39.82 \pm 0.90}$	$\mathbf{39.77 \pm 0.89}$
ρ_D^2	1.138 ± 0.023	1.138 ± 0.023
$\mathcal{G}(1)$	1.058 ± 0.007	1.058 ± 0.007
ρ_{D*}^2	1.269 ± 0.123	1.251 ± 0.113
$R_1(1)$	1.386 ± 0.087	1.371 ± 0.036
$R_2(1)$	0.880 ± 0.073	0.888 ± 0.065
$h_{A1}(1)$	0.900 ± 0.012	0.900 ± 0.012
χ^2_{min}	67.34	67.99
dof	79	82
p-value	82.21%	86.66%

- Combined fit shows considerable improvement over the $B \to D^* l\nu$ fit
- Extracted uncerainties of $|V_{cb}|$ reduces to $\approx 2\%$
- Central values of $\rho^2_{D^*}$ increases by approximately 8%
- Calculation of $R(D^*)$ depends on additional form factor ratio $R_0(w)$
- $R_0(1)$ can be estimated after the extraction of the five HQET parameters .

Extraction of HQET parameters

- Different cases for the fit of sub-leading Isgur-Wise functions:

Cases	Inputs for the fits
case-1	$R_1(1)$, $R_2(1)$, $f_+(w)/f_0(w)$ for w=1, 1.08, 1.16 (MILC) and w= 1.03, 1.06, 1.09, 1.12 (HPQCD)
case-2	case-1 with $R_1(w_{max})$, $R_2(w_{max})$ from LCSR.

- For completeness, we define some more cases by introducing additional parameters (Δs), which parametrize the unknown higher order corrections in the HQET form factor ratios
- In order to estimate the probable size of those missing corrections, we made the following replacements:

$$\frac{h_v}{h_{A_1}} \to \frac{h_v}{h_{A_1}}\Delta_v, \quad \frac{h_{A_3}}{h_{A_1}} \to \frac{h_{A_3}}{h_{A_1}}\Delta_{31}, \quad \frac{h_{A_2}}{h_{A_1}} \to \frac{h_{A_2}}{h_{A_1}}\Delta_{21}, \quad \frac{h_-}{h_+} \to \frac{h_-}{h_+}\Delta_{\mp}.$$

Form factor Ratio	Associated HQET form factor ratio(s)	Additional errors(s) involved
$f_+(w)/f_0(w)$	h_-/h_+	$\Delta_{\mp}$
$R_1(w) = func(g(w)/f(w))$	h_V/h_{A_1}	Δ_v
$R_2(w) = func(F_1(w)/f(w))$	$h_{A_2}/h_{A_1}, h_{A_3}/h_{A_1}$	Δ_{21}, Δ_{31}

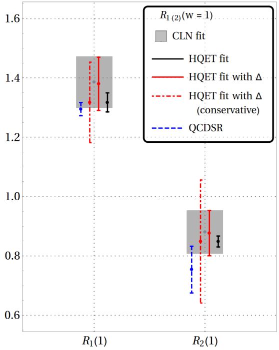
HQET fit results with CLN

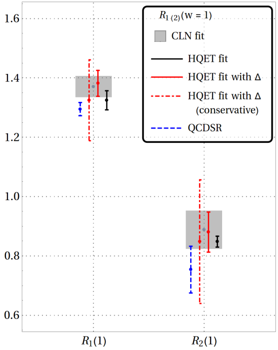
- Treating Δs as normally distributed nuisance parameters with each $\Delta = 1.0 \pm 0.2$:

Para-meters	case-1	case-1 with Δs	case-2	case-2 with Δs
$\eta(1)$	0.39(3)	0.40(5)	0.39(3)	0.40(5)
$\eta'(1)$	-0.002(100)	0.004(101)	-0.03(9)	0.001(101)
$\chi_2(1)$	-0.08(1)	-0.06(1)	-0.08(1)	-0.06(1)
$\chi_2{}'(1)$	-0.003(2)	-0.003(2)	-0.001(2)	-0.002(2)
$\chi_3{}'(1)$	0.04(2)	0.04(2)	0.05(2)	0.04(2)
Δ_v	-	1.05(6)	-	1.05(2)
Δ_{21}	-	1.00(20)	-	1.00(20)
Δ_{31}	-	1.04(8)	-	1.04(7)
$\Delta_{\mp}$		1.00(20)	-	1.00(20)
χ^2_{min}	4.34	3.36	9.20	3.62
dof	7	7	9	9
p-value	73.95%	85.00%	41.90%	93.46%

Conservative estimates for Δs

- Comparisions between different $R_1(1)$ and $R_2(1)$ obtained from different fits:





- Conservative estimate : $\Delta_v = 1 \pm 0.1$, $\Delta_{21} = 1 \pm 0.2$ and $\Delta_{31} = 1 \pm 0.2$

SM predictions for $R(D^{(*)})$ with CLN parameterization

- From fit to $B \to D^* l\nu$ data

	Data+Lattice	Data+Lattice+LCSR
Parameters/ Observables	Best Fit $\pm$ Err. Values	Best Fit $\pm$ Err. Values
$R_0(1)$	1.191 ± 0.017	1.195 ± 0.017
$R(D^*)$	0.255 ± 0.004	0.255 ± 0.004

- From combined fit to $B \to D^{(*)} l\nu$ data : In order to get a conservative estimate of $R(D^*)$, Δ_{31} and Δ_{21} are both taken as 1 ± 0.2

Parameters/ Observables	case-1	case-1 with Δ_{31} & Δ_{21} = 1.00(20)	case-2	case-2 with Δ_{31} & Δ_{21} = 1.00(20)
$R_0(1)$	1.192(17)	1.192(101)	1.196(17)	1.196(102)
$R(D)$ $R(D^*)$	**0.304(3)** **0.259(3)**	**0.304(3)** **0.259(6)**	**0.304(3)** **0.259(3)**	**0.304(3)** **0.259(6)**
Corr($R(D)$ -$R(D^*)$)	0.21	0.12	0.20	0.11

- The central values of $R(D^*)$ increase by approximately 2%
- Errors in $R(D^*)$ increase from 1.16% to 2.32% due to introduction of an additional error of about 20% in the HQET form factor ratios.

BGL fit results

- Fit to $B \to Dl\nu$ data :

Data+Lattice (HPQCD & MILC)			
Parameters/ Observables	Best Fit Values	$\pm$ Err	Err. from $\Delta\chi^2 = \pm 1$
$\mathbf{\|V_{cb}\| \times 10^3}$	**41.04**	**1.13**	$\left(\begin{smallmatrix}+1.12\\-1.13\end{smallmatrix}\right)$
$a_0^{f_+}$	0.0141	0.0001	(0.0001)
$a_1^{f_+}$	-0.0318	0.0028	(0.0028)
$a_2^{f_+}$	-0.0819	0.0199	(0.0199)
$a_1^{f_0}$	-0.1961	0.0136	(0.0136)
$a_2^{f_0}$	-0.2274	0.0942	(0.0942)
χ^2_{min}	33.37		
dof	46		
p-value	91.77%		
R(D)	0.302 $\pm$0.003		

- Lattice inputs for $f_+(w)$ and $f_0(w)$ from MILC and HPQCD collaborations make it possible to fit the $f_0(w)$ parameters together with $f_+(w)$
- The relation $f_+(w_{max}) = f_0(w_{max})$ is used to eliminate the parameter $a_0^{f_0}$ from the fit

BGL fit results

- Fit to $B \to D^* l\nu$ data :

Parameters	Data+Lattice		Data+Lattice+LCSR	
	Best Fit Values	Err. from $\Delta\chi^2 = 1$	Best Fit Values	Err. from $\Delta\chi^2 = 1$
$\mathbf{\lvert V_{cb}\rvert \times 10^3}$	**41.7**	$\begin{pmatrix} \mathbf{+2.0} \\ \mathbf{-2.1} \end{pmatrix}$	**40.6**	**(1.7)**
a_0^f	0.0109	(0.0002)	0.0109	(0.0002)
a_1^f	-0.0459	$\begin{pmatrix} +0.0527 \\ -0.0429 \end{pmatrix}$	-0.0518	$\begin{pmatrix} +0.0267 \\ -0.0131 \end{pmatrix}$
a_2^f	0.1513	$\begin{pmatrix} 0.8457 \\ -1.1508 \end{pmatrix}$	0.9942	$\begin{pmatrix} +0.0047 \\ -0.5019 \end{pmatrix}$
$a_1^{\mathcal{F}_1}$	-0.0092	$\begin{pmatrix} +0.0054 \\ -0.0050 \end{pmatrix}$	-0.0070	$\begin{pmatrix} +0.0048 \\ -0.0046 \end{pmatrix}$
$a_2^{\mathcal{F}_1}$	0.1150	$\begin{pmatrix} +0.0877 \\ -0.0921 \end{pmatrix}$	0.0932	$\begin{pmatrix} +0.0850 \\ -0.0883 \end{pmatrix}$
a_0^g	0.0111	$\begin{pmatrix} +0.0104 \\ -0.0075 \end{pmatrix}$	0.0257	$\begin{pmatrix} +0.0054 \\ -0.0034 \end{pmatrix}$
a_1^g	0.5786	$\begin{pmatrix} +0.3351 \\ -0.4007 \end{pmatrix}$	0.0836	$\begin{pmatrix} +0.0753 \\ -0.2157 \end{pmatrix}$
a_2^g	0.8155	$\begin{pmatrix} +0.1683 \\ -1.7701 \end{pmatrix}$	-0.9962	$\begin{pmatrix} +1.9958 \\ -0.0036 \end{pmatrix}$
χ^2_{min}	27.81		30.93	
dof	32		35	
p-value	67.87%		66.51%	

- The lattice input $h_{A_1}(1) = 0.906 \pm 0.013$ to the fit decides the value of a_0^f through the relation $f = \sqrt{m_B m_D^*}(1+w)h_{A_1}$
- The relation $F_1(1) = (m_B - m_{D^*})f(1)$ is used to eliminate the parameter $a_0^{\mathcal{F}_1}$ from the fit

BGL combined fit to $B \to D^{(*)} l\nu$ data

Parameters	Data+Lattice Best Fit Values	Data+Lattice Err. from $\Delta\chi^2 = 1$	Data+Lattice+LCSR Best Fit Values	Data+Lattice+LCSR Err. from $\Delta\chi^2 = 1$
$\mathbf{\lvert V_{cb}\rvert \times 10^3}$	**41.2**	**(1.0)**	**40.9**	**(0.9)**
a_0^f	0.0109	(0.0002)	0.0109	(0.0001)
a_1^f	-0.0366	$\binom{+0.0409}{-0.0422}$	-0.0534	$\binom{+0.0194}{-0.0112}$
a_2^f	-0.0340	$\binom{+1.0312}{-0.9652}$	0.9936	$\binom{+0.0049}{-0.4022}$
$a_1^{\mathcal{F}_1}$	-0.0084	$\binom{+0.0045}{-0.0044}$	-0.0074	$\binom{+0.0043}{-0.0042}$
$a_2^{\mathcal{F}_1}$	0.1054	$\binom{+0.0846}{-0.0855}$	0.0983	$\binom{+0.0821}{-0.0830}$
a_0^g	0.0112	$\binom{+0.0108}{-0.0075}$	0.0256	$\binom{+0.0052}{-0.0033}$
a_1^g	0.5882	$\binom{+0.3320}{-0.4233}$	0.0800	$\binom{+0.0722}{-0.2131}$
a_2^g	0.8038	$\binom{+0.1783}{-1.7582}$	-0.9925	$\binom{+1.9887}{-0.0038}$
$a_0^{f_+}$	0.0141	(0.0001)	0.0141	(0.0001)
$a_1^{f_+}$	-0.0320	(0.0027)	-0.0317	(0.0027)
$a_2^{f_+}$	-0.0816	(0.0199)	-0.0822	(0.0198)
$a_1^{f_0}$	-0.1967	(0.0134)	-0.1956	(0.0134)
$a_2^{f_0}$	-0.2291	(0.0941)	-0.2259	(0.0940)
χ^2_{min}	61.26		64.35	
dof	79		82	
p-value	93.04%		88.35%	

- Uncertainties of extracted $|V_{cb}| \approx 2\% \implies$ Most precise estimate obtained so far from a combined analysis
- Central values of $|V_{cb}|$ increases by 3.5%(3%) without (with) LCSR as compared to CLN combined fit

Additional form-factor F_2

- Calculation of $R(D^*)$ in BGL parameterization depends on additional form-factor F_2
- In order to extract the expansion coefficients of F_2 (i.e., $a_n^{\mathcal{F}_2}$ for n=0,1,2), we use:

$$F_2(w) = \left(\frac{F_2(w)}{F_i(w)}\right)_{HQET} F_i(w), \quad i \neq 2.$$

- Here, $F_i(w)$'s can be anyone of $f_+(w)$, $f_0(w)$, $F_1(w)$ and $f(w)$ and are known either from BGL fits or from lattice data.
- The only unknowns on the R.H.S. of the equation are the 5 HQET parameters : $\eta(1), \eta'(1), \chi_2(1), \chi_2'(1),$ and $\chi_3'(1)$.
- Need to fit these Isgur-Wise functions in order to extract the coefficients of F_2 and hence, predict $R(D^*)$
- $a_1^{\mathcal{F}_2}$ is eliminated using the QCD relation : $F_2(q^2 = 0) = \frac{2F_1(q^2=0)}{m_B^2 - m_{D^*}^2}$

Relating BGL form-factors to HQET form-factors

$$\frac{F_1(w)}{f(w)} = m_B(w-1)\left(\frac{w-r_{D^*}}{w-1} - \frac{h_{A_2}}{h_{A_1}} r_{D^*} - \frac{h_{A_3}}{h_{A_1}}\right),$$

$$\frac{F_2(w)}{f(w)} = \frac{1}{m_B r_{D^*}}\left(1 - \frac{h_{A_2}}{h_{A_1}}\frac{1-r_{D^*}w}{1+w} - \frac{h_{A_3}}{h_{A_1}}\frac{w-r_{D^*}}{1+w}\right)$$

$$\frac{F_2(w)}{F_1(w)} = \frac{F_2(w)}{f(w)}\frac{f(w)}{F_1(w)}$$

$$\frac{F_2(w)}{f_+(w)} = 2\frac{\left(1+w-\frac{h_{A_2}}{h_{A_1}}(1-r_{D^*}w)+\frac{h_{A_3}}{h_{A_1}}(r_{D^*}-w)\right)}{\frac{\sqrt{r_{D^*}}}{\sqrt{r_D}}\frac{h_+}{h_{A_1}}\left(\frac{h_-}{h_+}(r_D-1)+(1+r_D)\right)},$$

$$\frac{F_2(w)}{f_0(w)} = \frac{\left(1-\frac{h_{A_2}}{h_{A_1}}(1-r_{D^*}w)+\frac{h_{A_3}}{h_{A_1}}(r_{D^*}-w)\right)}{\frac{\sqrt{r_{D^*}}\sqrt{r_D}}{r_D+1}\frac{h_+}{h_{A_1}}\left(\frac{h_-}{h_+}\frac{(r_D+1)(w-1)}{(r_D-1)(w+1)}+1\right)}.$$

Apart from h_{A_2}/h_{A_1}, h_{A_3}/h_{A_1} and h_-/h_+, these ratios are sensitive to h_+/h_{A_1}.

Extraction of HQET parameters

Cases	Inputs for the HQET fits	Normalizations used for F_2 extraction
case-3	$\frac{F_1(w)}{f(w)}$ for w=1.03, 1.06, 1.09 and $\frac{f_+(w)}{f_0(w)}$ for w=1, 1.03, 1.06, 1.09 from BGL fit results	$\frac{F_2(w)}{f(w)}$ and $\frac{F_2(w)}{F_1(w)}$
case-4	case-3 with $R_1(w_{max})$ and $R_2(w_{max})$ from LCSR	case-3
case-5	$\frac{f_+(w)}{f_0(w)}$ for w=1, 1.08, 1.16 (MILC) and w=1.03, 1.06, 1.09, 1.12 (HPQCD)	$\frac{F_2(w)}{f_+(w)}$ and $\frac{F_2(w)}{f_-(w)}$
case-6	case-5 with $R_1(w_{max})$ and $R_2(w_{max})$ from LCSR	case-5

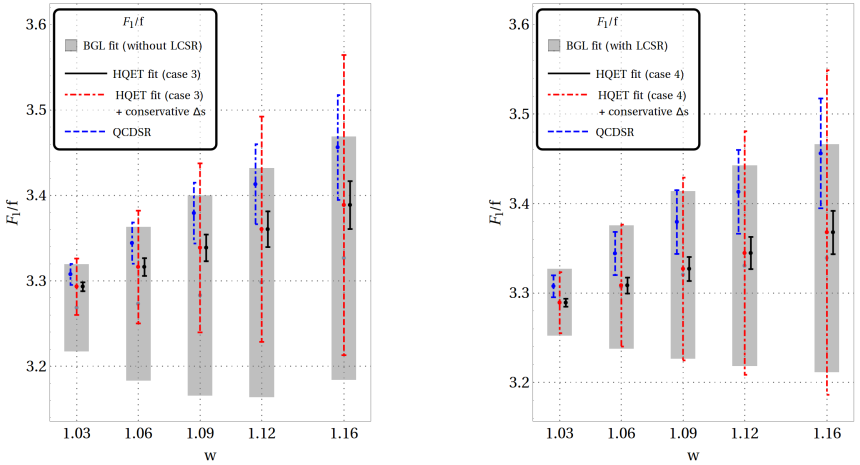
Form factor ratios used	Associated HQET form factor ratio(s)	Additional errors(s) involved
$\frac{F_1(w)}{f(w)}$, $R_2(w) = func\left(\frac{F_1(w)}{f(w)}\right)$, $\frac{F_2(w)}{f(w)}$, $\frac{F_2(w)}{F_1(w)}$	$\frac{h_{A_2}}{h_{A_1}}$, $\frac{h_{A_3}}{h_{A_1}}$	Δ_{21}, Δ_{31}
$\frac{f_+(w)}{f_0(w)}$	$\frac{h_-}{h_+}$	$\Delta_{\mp}$
$R_1(w) = func\left(\frac{g(w)}{f(w)}\right)$	h_V/h_{A_1}	Δ_v
$\frac{F_2(w)}{f_+(w)}$, $\frac{F_2(w)}{f_-(w)}$	$\frac{h_{A_2}}{h_{A_1}}$, $\frac{h_{A_3}}{h_{A_1}}$, $\frac{h_-}{h_+}$, $\frac{h_+}{h_{A_1}}$	$\Delta_{21}, \Delta_{31}, \Delta_{\mp}, \Delta$

HQET fit results with BGL

Treating the Δs as normally distributed nuisance parameters with each $\Delta = 1.0 \pm 0.2$

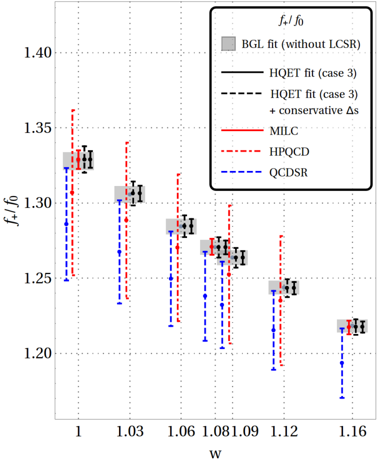
Para- meters	case-3	case-3 with Δs	case-4	case-4 with Δs	case-5	case-5 with Δs	case-6	case-6 with Δs
$\eta(1)$	0.39(3)	0.39(5)	0.38(3)	0.40(5)	0.39(4)	0.40(5)	0.40(3)	0.40(5)
$\eta'(1)$	0.10(7)	0.12(5)	0.08(7)	0.14(7)	0.01(12)	0.004(101)	-0.02(10)	0.003(101)
$\chi_2(1)$	-0.07(6)	-0.05(6)	-0.11(5)	-0.08(6)	-0.06(6)	-0.06(6)	-0.06(6)	-0.06(6)
$\chi_2'(1)$	0.007(60)	-0.02(4)	0.006(59)	-0.004(30)	-0.003(60)	-0.003(60)	-0.002(59)	-0.003(60)
$\chi_3'(1)$	0.06(5)	0.06(4)	0.06(5)	0.04(4)	0.04(6)	0.04(6)	0.05(6)	0.04(6)
Δ_v	-	-	-	1.06(3)	-	-	-	1.06(3)
$\Delta_\mp$	-	0.98(20)	-	1.00(20)	-	1.00(20)	-	1.00(20)
Δ_{21}	-	1.05(20)	-	1.02(20)	-	-	-	1.00(20)
Δ_{31}	-	1.03(10)	-	1.07(7)	-	-	-	1.01(13)
χ^2_{min}	1.71	1.73	7.63	1.88	3.84	3.26	7.05	3.36
dof	5	5	7	7	5	5	7	7
p-value	88.77%	88.54%	36.62%	96.60%	57.25%	66.02%	42.36%	84.97%

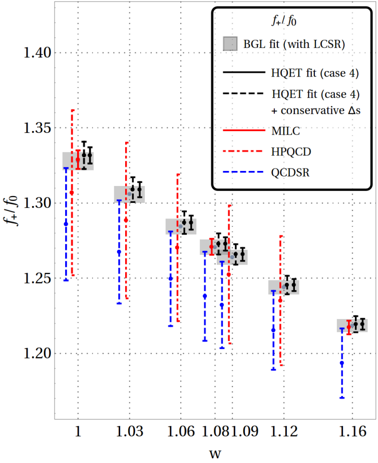
Different fit results of $F_1(w)/f(w)$



Conservative estimate : $\Delta_{21} = 1 \pm 0.2$ and $\Delta_{31} = 1 \pm 0.2$

Different fit results of $f_+(w)/f_0(w)$





Conservative estimate : $\Delta_v = 1 \pm 0.1$

SM prediction of $R(D^*)$ with BGL

The probable size of the additional error(Δ) in h_+/h_{A_1} in the ratio $F_2(w)/f_{+/0}(w)$ is obtained using :

$$\left(\frac{f_0(1)}{f(1)}\right)_{lattice} \approx \left(\frac{f_0(1)}{f(1)}\right)_{HQET}$$

$$\Downarrow$$

$$\Delta = 1 \pm 0.1$$

In order to get a conservative estimate of $R(D^*)$, Δ_{31}, Δ_{21} and $\Delta_{\mp}$ are all taken as 1 ± 0.2

Parameters/ Observables	case-3	case-3 with Δs	case-4	case-4 with Δs	case-5	case-5 with Δs	case-6	case-6 with Δs
$a_0^{\mathcal{F}_2}$	0.053(1)	0.053(4)	0.053(1)	0.053(5)	0.058(1)	0.058(8)	0.058(1)	0.058(8)
$a_2^{\mathcal{F}_2}$	0.21(6)	0.21(8)	-0.14(3)	-0.17(10)	-0.48(1)	-0.42(2)	-0.39(1)	-0.33(1)
$R(D)$	0.302(3)	0.302(3)	0.302(3)	0.302(3)	0.302(3)	0.302(3)	0.302(3)	0.302(3)
$R(D^*)$	**0.255(5)**	**0.255(5)**	**0.257(5)**	**0.257(5)**	**0.258(5)**	**0.258(7)**	**0.260(5)**	**0.260(7)**
Corr($R(D)$ -$R(D^*)$)	0.12	0.11	0.12	0.10	0.14	0.10	0.13	0.09

Present Scenario of V_{cb}

- From experimental collaborations :

Belle($B \to Dl\nu$), 1510.03657,	$\lvert V_{cb}\rvert_{CLN} = (39.86 \pm 1.33) \times 10^{-3}$
Belle($B \to Dl\nu$), 1510.03657,	$\lvert V_{cb}\rvert_{BGL} = (40.83 \pm 1.13) \times 10^{-3}$
Belle($B \to D^* l\nu$), 1702.01521,	$\lvert V_{cb}\rvert_{CLN} = (38.2 \pm 1.5) \times 10^{-3}$

- Fits only to Belle $B \to D^* l\nu$ data :

Bigi, Gambino, Schacht, 1703.06124,	$\lvert V_{cb}\rvert_{CLN} = (38.2 \pm 1.5) \times 10^{-3}$
Bigi, Gambino, Schacht, 1703.06124,	$\lvert V_{cb}\rvert_{BGL} = (41.7^{+2.0}_{-2.1}) \times 10^{-3}$
Grinstein Kobach, 1703.08170,	$\lvert V_{cb}\rvert_{BGL} = (41.9^{+2.0}_{-1.9}) \times 10^{-3}$

- Fits to combined Belle $B \to D^{(*)} l\nu$ data:

Jaiswal, Nandi, Patra, 1707.09977,	$\lvert V_{cb}\rvert_{BGL} = (40.90 \pm 0.94) \times 10^{-3}$
Jaiswal, Nandi, Patra, 1707.09977,	$\lvert V_{cb}\rvert_{CLN} = (39.77 \pm 0.89) \times 10^{-3}$

SM Predictions for $R(D^{(*)})$: Present Scenario

Reference (Scenario)	$R(D)$	$R(D^*)$	Correlation
Data [HFAG]	0.407 ± 0.046	0.304 ± 0.015	−20%
Lattice [FLAG]	0.300 ± 0.008	—	—
Fajfer et al. '12	—	0.252 ± 0.003	—
Bernlochner *et al.* '17 ($L_{w\geq1}$)	0.298 ± 0.003	0.261 ± 0.004	19%
Bernlochner *et al.* '17 ($L_{w\geq1}$+SR)	0.299 ± 0.003	0.257 ± 0.003	44%
Bigi, Gambino '16	0.299 ± 0.003	—	—
Bigi, Gambino, Schacht '17	—	0.260 ± 0.008	—
Jaiswal, Nandi, Patra '17 (case-6)	0.302 ± 0.003	0.260 ± 0.007	9%
Jaiswal, Nandi, Patra '17 (case-4)	0.302 ± 0.003	0.257 ± 0.005	10%

Conclusion

- No clear resolution of the $|V_{cb}|$ puzzle; this long lasting discrepancy between the inclusive and exclusive determinations of $|V_{cb}|$ have to be thoroughly reconsidered

- Shift in the SM prediction of $R(D^{(*)})$:

Observable(s)	Existing Discrepency between SM and expt.	Discrepancy between SM and expt. using our results
$R(D)$	2.30σ	2.29σ
$R(D^*)$	3.45σ	3.01σ
$[R(D), R(D^*)]$	4.17σ	3.84σ

- Updated results allow us to determine the new physics effects in $B \to D^{(*)}\tau\nu$ rates with improved precision

- Predictions can be systematically improved with more data from Belle II and LHCb


Thank you
for
listening!