Inclusive jet and dijet production at the LHC

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• Theoretical framework: improved parton model formula

\[
\sigma(P_1, P_2) = \sum_{i,j} \int dx_1 dx_2 f_i(x_1, \mu_F^2) f_j(x_2, \mu_F^2) \hat{\sigma}_{ij}(p_1, p_2, \alpha_s(\mu^2), s/\mu^2, s/\mu_F^2)
\]

To apply:

• require large momentum transfer

• scale of the reaction \(Q^2 \gg \Lambda_{\text{hadronic}}\)

• quarks and gluons behave as free particles in the collision

• running coupling \(\alpha_s(Q^2)\) decreases at high-scales

• compute partonic cross section \(\sigma_{ij}\) in perturbation theory \(\rightarrow\) QCD Lagrangian
After hard scattering respect  *quark confinement*:

- individual *quarks* cannot be observed directly

  ![Diagram](image)

- force between *quarks* increases as they are separated

- I - soft and collinear radiation produces new *qq̄* pairs

- higher order corrections and parton shower matched to ME calculation simulate QCD radiation from the scattering scale $Q^2$ to the hadronization scale $\Lambda_{\text{hadronic}}$

- II - hadronization of *quarks* and *gluons* to form collimated bound states of baryons and mesons $\rightarrow$ final state jet

- properties of the final state jet follow closely the properties of the parton that initiated it local *parton-hadron duality*
Jet algorithms standardise the definition of jets and reduce the complexity of the final state to simpler calculable objects.

- find the smallest distance measure $d_{ij}$ between two particles and combine them if $d_{ij}$ smaller than the jet resolution size $R$

$$d_{ij} = \min(p_{t_i}^{2p}, p_{t_j}^{2p}) \frac{\Delta R_{ij}^2}{R^2}, \quad \Delta R_{ij}^2 = (y_i - y_j)^2 + (\phi_i - \phi_j)^2$$

$$d_{iB} = p_{t_i}^{2p},$$

- CMS and ATLAS have settled on anti-k$_T$ jets (p=-1)

$$\mathcal{J}_{n+1}(p_1, \ldots, p_i, p_j) \stackrel{p_i//p_j}{=} \mathcal{J}_n(p_1, \ldots, p_{ij})$$

$$\mathcal{J}_{n+1}(p_1, \ldots, p_n, p_i) \stackrel{p_i \rightarrow 0}{=} \mathcal{J}_n(p_1, \ldots, p_n)$$

- infrared-safe jet definition $\rightarrow$ allows inclusion of higher order perturbative corrections to the inclusive jet cross section
Inclusive jet cross section: theory status

Much progress in fixed-order calculations/resummed and parton-shower predictions

- **NLO QCD**
  - [Ellis, Kunszt, Soper '92] [Giele, Glover, Kosower '94] [Nagy 02]

- **NLO EW**
  - [Dittmaier, Huss, Speckner '13]
  - [Frederix, Frixione, Hirschi, Pagani, Shao, Zaro '17]

- **NLO QCD + PS (POWHEG)**
  - [Alioli, Hamilton, Nason, Oleari, Re '11]

- **NLO QCD + PS (MC@NLO)**
  - [Hoeche, Schonherr '12]

- **NLO QCD + Resummation (threshold+jet radius)**
  - [Dasgupta, Dreyer, Salam, Soyez '14] [Liu, Moch, Ringer '17]

- **NNLO QCD**
  - [Gehrmann-De Ridder, Gehrmann, Glover, JP '13] [Currie, Glover, JP '16]
  - [Currie, Gehrmann-De Ridder, Gehrmann, Glover, Huss, JP '17]
NLO QCD + PS (POWHEG)


- hardest emission generated first according to:

\[ d\sigma = \bar{B}(\Phi_B) \, d\Phi_B \left[ \Delta_R (p_{T \min}^\text{min}) + \frac{R(\Phi_R)}{\bar{B}(\Phi_B)} \Delta_R (k_T(\Phi_R)) \, d\Phi_{\text{rad}} \right] \]

- with the born cross section replaced with the NLO differential cross section at fixed Born kinematics integrated over single emission → preserves NLO accuracy for inclusive quantities

\[ \bar{B}(\Phi_B) = B(\Phi_B) + \left[ V(\Phi_B) + \int d\Phi_{\text{rad}} R(\Phi_R) \right] \]

- probability of not having an emission harder than p_T → POWHEG Sudakov

\[ \Delta_R (p_T) = \exp \left[ - \int d\Phi_{\text{rad}} \frac{R(\Phi_R)}{\bar{B}(\Phi_B)} \theta (k_T(\Phi_R) - p_T) \right] \]

- interface with shower generator to develop the rest of the shower vetoing emissions harder than the first one
NLO QCD + PS (POWHEG)

- Comparison to cross section data $R = 0.4; 0.6 \text{ @ 8 TeV}$

- POWHEG prediction lower than fixed-order NLO QCDxNP → more in Bogdan’s talk

- Ratio to data less sensitive to the jet radius in POWHEG
NLO QCD + Resummation (threshold+jet radius)


Double differential single jet inclusive cross section:

\[
\frac{d^2 \sigma}{dp_T^2 dy} = \sum_{i_1 i_2} \int_0^1 \frac{V(1-w)}{dv} \int_{1-z}^{1} dz x_1^2 f_{i_1}(x_1) x_2^2 f_{i_2}(x_2) \frac{d^2 \hat{\sigma}_{i_1 i_2}}{dv dz}(v, z, p_T, R)
\]

- \( z = s_4 / s \) invariant mass recoiling against the jet
- in the small \( R \)-limit and threshold limit \( z \to 0 \) cross section factorizes within SCET

\[
\frac{d^2 \hat{\sigma}_{i_1 i_2}}{dv dz} = s \int ds_X ds_c ds_G \delta(zs - s_X - s_G - s_c) Tr[H_{i_1 i_2}(v, p_T, \mu_h, \mu) S_G(s_G, \mu_{sG}, \mu)]
\]
\[
\times J_X(s_X, \mu_X, \mu) \sum_m Tr[J_m(p_T R, \mu_J, \mu) \otimes \Omega_s c, m(s_c R, \mu_{sc}, \mu)]
\]

- perturbative contributions know at least to NLO
- joint resummation at NLL accuracy \( \alpha_s^n \left( \ln^k(z) / z \right) \); \( \alpha_s^n \ln^k(R) \)
- additive matching to fixed-order NLO QCD result

\[
\sigma_{NLO+NLL} = \sigma_{NLO} - \sigma_{NLO_{sing}} + \sigma_{NLL}
\]

- resummation scales \( \mu_i \) evolved through RGE equations to common hard scale \( \mu = \mu_h = p_T^{max} \)
NLO QCD + Resummation (threshold+jet radius)

Liu, Moch, Ringer [arXiv: 1808.04574]

- threshold logs lead to an enhancement of the cross section for large $p_T$ \( \mu = \mu_h = p_T^{max} \)
- resummation of small $R$-logs lead to a decrease in the cross section in the entire range $p_T$
NNLO QCD

Motivations:

- **Reduced scale dependence** of the prediction, i.e \( \mu_R \) with \( L_R = \log (\mu_R/\mu_0) \)

\[
\sigma(\mu_R, \alpha_s(\mu_R), L_R) = \left( \frac{\alpha_s(\mu_R)}{2\pi} \right)^2 \sigma_{ij}^0 + \left( \frac{\alpha_s(\mu_R)}{2\pi} \right)^3 \left\{ \sigma_{ij}^1 + 2\beta_0 L_R \sigma_{ij}^0 \right\} \\
+ \left( \frac{\alpha_s(\mu_R)}{2\pi} \right)^4 \left\{ \sigma_{ij}^2 + L_R \left( 3\beta_0 \sigma_{ij}^1 + 2\beta_1 \sigma_{ij}^0 \right) + L_R^2 3\beta_0^2 \sigma_{ij}^0 \right\} \\
+ \mathcal{O}(\alpha_s(\mu_R)^5)
\]

- Jets **modelled by extra partons** → perturbation theory can start **reconstructing** the shower

- Better matching of jet algorithm between **theory and experiment** and improved understanding of the jet shape
- At LO final state has no transverse momentum
- Better description of the transverse momentum of the final state due to double radiation off initial state
- NNLO provides the first serious estimate of the theory error → allows comparison with wealth of experimental jet data which have similar precision

**NNLO QCD**

![Graphs showing uncertainties](image)

dominant exp. systematic uncertainty: ±5-10%
NLO scale uncertainty ~10% NNLO needed
NNLO contributions

- Perturbative QCD expansion of the inclusive jet cross section at hadron colliders

\[ d\sigma = \sum_{i,j} \int \left[ d\sigma_{ij}^{\text{LO}} + \left( \frac{\alpha_s}{2\pi} \right) d\sigma_{ij}^{\text{NLO}} + \left( \frac{\alpha_s}{2\pi} \right)^2 d\sigma_{ij}^{\text{NNLO}} + \mathcal{O}(\alpha_s^3) \right] f_i(x_1)f_j(x_2)dx_1dx_2 \]

- NNLO gluonic contributions

\[ A_6^{(0)}(gg \to gggg) \quad A_5^{(1)}(gg \to ggg) \quad A_4^{(2)}(gg \to gg) \]

- tree-level 2→4 matrix elements
- one-loop 2→3 matrix elements
- two-loop 2→2 matrix elements
- NNLO DGLAP evolution
- NNLO PDF’s

[Berends, Giele '87] [Mangano, Parke, Xu '87] [Britto, Cachazo, Feng '06]
[Bern, Dixon, Kosower '93] [Kunszt, Signer, Trocsanyi '94]
[Anastasiou, Glover, Oleari, Tejeda-Yeomans '01] [Bern, De Freitas, Dixon '02]
[Moch, Vermaseren, Vogt '04]
[ABMP, CT, NNPDF, MMHT]
\[ d\hat{\sigma}_{NNLO} = \int d\Phi_4 \; d\hat{\sigma}_{NNLO}^{RR} + \int d\Phi_3 \; d\hat{\sigma}_{NNLO}^{RV} + \int d\Phi_2 \; d\hat{\sigma}_{NNLO}^{VV} \]

\[ d\hat{\sigma}_{NNLO}^{RR} = N \ d\Phi_4 (p_3, p_4, p_5, p_6; p_1, p_2) |M_{gg \to g g g g}^{(0)}|^2 J_2^{(4)} (p_3, p_4, p_5, p_6) \]

\[ d\hat{\sigma}_{NNLO}^{RV} = N \ d\Phi_3 (p_3, p_4, p_5; p_1, p_2) \]

\[ \left( M_{gg \to g g g}^{(0)*} M_{gg \to g g g}^{(1)} + M_{gg \to g g g}^{(0)} M_{gg \to g g g}^{(1)*} \right) J_2^{(3)} (p_3, p_4, p_5) \]

\[ d\hat{\sigma}_{NNLO}^{VV} = N \ d\Phi_2 (p_3, p_4; p_1, p_2) \]

\[ \left( M_{gg \to g g}^{(2)*} M_{gg \to g g}^{(0)} + M_{gg \to g g}^{(0)} M_{gg \to g g}^{(2)*} + |M_{gg \to g g}|^2 \right) J_2^{(2)} (p_3, p_4) \]

\begin{itemize}
  \item explicit infrared poles from loop integrations
  \item implicit poles in phase space regions corresponding to single and double unresolved gluon emission
  \item procedure to extract the infrared singularities and assemble all the parts
\end{itemize}
NNLO antenna subtraction

\[ d\hat{\sigma}_{\text{NNLO}} = \int d\Phi_4 \left( d\hat{\sigma}^{\text{RR}}_{\text{NNLO}} - d\hat{\sigma}^S_{\text{NNLO}} \right) \]
\[ + \int d\Phi_3 \left( d\hat{\sigma}^{\text{RV}}_{\text{NNLO}} - d\hat{\sigma}^T_{\text{NNLO}} \right) \]
\[ + \int d\Phi_2 \left( d\hat{\sigma}^{\text{VV}}_{\text{NNLO}} - d\hat{\sigma}^U_{\text{NNLO}} \right) \]

- matrix elements: universal factorization properties in IR limits
- phase space factorization

\[ d\Phi_{m+1}(p_1, \ldots, p_{m+1}; q) = d\Phi_m(p_1, \ldots, \tilde{p}_I, \tilde{p}_K, \ldots, p_{m+1}; q) \cdot d\Phi_{x_{ijk}}(p_i, p_j, p_k; \tilde{p}_I + \tilde{p}_K) \]
NNLO antenna subtraction

\[ d\hat{\sigma}_{\text{NNLO}} = \int d\Phi_4 \left( d\hat{\sigma}_{\text{NNLO}}^{RR} - d\hat{\sigma}_{\text{NNLO}}^S \right) \]

\[ + \int d\Phi_3 \left( d\hat{\sigma}_{\text{NNLO}}^{RV} - d\hat{\sigma}_{\text{NNLO}}^T \right) \]

\[ + \int d\Phi_2 \left( d\hat{\sigma}_{\text{NNLO}}^{VV} - d\hat{\sigma}_{\text{NNLO}}^U \right) \]

For inclusive jet and dijet production: \( pp \rightarrow \text{jet} + X ; \ pp \rightarrow 2\text{jet} + X \)

- NNLO corrections known \([\text{Currie, Gehrmann-De Ridder, Gehrmann, Glover, Huss, JP '17}]\)
- all channels \( \{ gg, qg, q\bar{q}, qq, q\bar{q}', q\bar{q}' \} \) at leading colour \( \alpha_s^2 N, \alpha_s^2 N N_F, \alpha_s^2 N_F^2 \)
- gg-channel with full colour
- sub-leading colour contributions: (suppressed by \( 1/N^2 \))
  - below two percent at NLO (all channels)
NNLOJET

**NNLO fully differential parton-level generator**

- Based on **antenna subtraction** for the **analytic cancellation** of **IR singularities** at **NNLO**

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**Infrastructure**

- Process management
- Phase space, histogram routines
- Validation and testing
- ApplFast interface in progress

**Processes implemented at NNLO**

- Z+(0,1) jet, W+(0,1) jet
- H+(0,1) jet
- DIS-2jet
- VBF H+2jet
- Inclusive jet production

Dijet inclusive production $\sqrt{s}=7$ TeV


Theory setup

- MMHT2014 nnlo
- anti-$k_T$ jet algorithm
- $p_{T1}>100$ GeV; $p_{T2}>50$ GeV;*
- $|y_{j1}|, |y_{j2}| < 3.0$
- $\mu_R=\mu_F=\{m_{jj}, <p_T>\}$
- vary scales by factors of 2 and 1/2

Comparison to data

- ATLAS 7 TeV; $L=4.5$ fb$^{-1}$
- $R=0.4$

*measurement requires observation of a dijet system in the final state; asymmetric $p_T$ cuts increase phase space available for real-gluon emission suppressing large logs in the QCD perturbative expansion of the observable
Dijet inclusive production: scale choice

\(pp \rightarrow 2\text{jets } + X:\)

• cross section measured differentially in:

\[ m_{jj}^2 = (p_{j1} + p_{j2})^2 \]

\[
y^* = \frac{1}{2} (y_{j1} - y_{j2})
\]

• compare behaviour of the scales (normalised to data)

\[ \mu = m_{jj} ; \quad \mu = \langle p_T \rangle = \frac{1}{2} (p_{T1} + p_{T2}) \]

\text{small } |y^*|:

• both scales give reasonable predictions

\text{large } |y^*|:

• large negative NLO corrections, non-overlapping scale bands and residual NLO, NNLO scale uncertainties of \(~100\%, ~20\%\) with \(\mu = \langle p_T \rangle\)

• stable prediction with \(\mu = m_{jj}\)

\[ 0.0 < |y^*| < 0.5 \]

\[ 1.5 < |y^*| < 2.0 \]
Dijet inclusive production: $\mu = m_{jj}$

- Excellent convergence of the perturbative expansion; NNLO/NLO < 10% and flat

- Improved description of the dijet data at NNLO

- NNLO: scale choice issue resolved
Single jet inclusive production

Theory setup

- PDF4LHC15nnlo
- anti-$k_T$ jet algorithm
- $R=0.4$ ; $R=0.7$ ;
- $p_T>114$ GeV*
- $|y| < 4.7$*
- theory uncertainty: vary renormalization $\mu_R$ and factorization $\mu_F$ scales by factors $[1/2,2]$ around pre-defined central scale

Comparison to data

- CMS $\sqrt{s} = 13$ TeV ; $L=71$ pb$^{-1}$
- $R=0.4$ and $R=0.7$

*single jet inclusive observable obtained by summing over all jets that are observed in the event
Single jet inclusive production: scale choices \( \mu_R, \mu_F \)

- \( p_T \rightarrow \) transverse momentum of the individual jets
- \( p_{T1} \rightarrow \) transverse momentum of the leading jet
- \( H_T \rightarrow \) scalar sum of the transverse momenta of the reconstructed jets
- \( \hat{H}_T \rightarrow \) scalar sum of the transverse momenta of all partons
- \( \mu_R, \mu_F \) are arbitrary and unphysical parameters and are absent from the true result \( \rightarrow a \ priori \) each scale above is an equally valid scale choice

However, a suitable scale choice would

- minimize ratios of \( Q^2/\mu^2 \), i.e, faster perturbative convergence and smaller scale uncertainties
- avoid scales that introduce pathological behaviours in the prediction, i.e, \( \sigma < 0 \)
- avoid scales that are discontinuous on the phase space of the observable, i.e, no kinks in k-factors

\( \mu \sim p_T \)
\( \mu \sim p_{T1} \)
\( \mu \sim H_T \)
\( \mu \sim \hat{H}_T \)

\( \rightarrow \) recently derived NNLO predictions for inclusive jet production allow for the first time a robust study on scale setting, making use of the knowledge of three orders in the perturbative expansion of the observable
Individual jet contributions and jet fractions

- Single jet inclusive observable receives contributions from all jets in the event, at $O(\alpha_s^4)$

\[
\frac{d\sigma}{dp_T}(\mu = p_T) = \frac{d\sigma}{dp_{T1}}(\mu = p_{T1}) + \frac{d\sigma}{dp_{T2}}(\mu = p_{T2}) + \frac{d\sigma}{dp_{T3}}(\mu = p_{T3}) + \frac{d\sigma}{dp_{T4}}(\mu = p_{T4})
\]

**NLO:**
- leading jet dominates
- third jet negligible
- second jet sizeable at high $p_T$ negligible at low $p_T$

**NNLO:**
- leading and second jet fractions similar over the whole $p_T$ range
- significant increase in second jet $p_T$ contribution to the inclusive jet sample at NNLO with respect to NLO
Corrections to second jet distribution integrated over rapidity \( R=0.4 \)

- **NLO**: large and negative with huge uncertainty \( \rightarrow \) potentially large logs sensitive on IR effects; NNLO: large and positive

- **Stabilization of the predictions at NNLO** (in line with the LO) \( \rightarrow \) functional form of the scale matters
Differential corrections for leading and subleading jet

- large cancellations between corrections to first (blue) and second jet (red) at NLO (solid), NNLO (dashed)

- smaller perturbative coefficients for $\mu = 2p_T$ ; $\mu = \hat{H}_T$ for leading and subleading jet
$\mu = p_T$ versus $\mu = p_{T1}$: Similarities and Differences

$p_T = p_{T1}$

- for leading-order kinematics ($2 \rightarrow 2$ kinematics)
- for high $p_T$-jets (back to back and balanced in $p_T$)

$p_T \neq p_{T1}$

- events with three or more hard jets (small rate as seen before)
- events with jets outside the fiducial cuts (worse for small $R$)
Decomposition of events contributing to a single bin in $p_{T2}$

Large cancellations/imbalance between positive real emission and large negative virtual correction

- worse for $\mu = p_{T1}$ that changes event-by-event in the distribution; $\mu = p_T$ remains constant
Excellent convergence of the perturbative expansion and overlapping scale uncertainty bands observed for $\mu = 2p_T$ and $\mu = \hat{H}_T$
Define scale choice criteria for single jet inclusive cross section

Studied the IR sensitivity of the different ingredients and introduced an extended set of criteria to help identify the most appropriate scale choice for the perturbative description of single jet inclusive production.

- **(a) perturbative convergence**: size of the corrections to the inclusive cross section reduces at each successive order

- **(b) scale uncertainty as theory estimate**: overlapping scale uncertainty bands between the last two orders, i.e., between NLO and NNLO

- **(c) perturbative convergence of the individual jet spectra**: perturbative convergence of the corrections to the individual $p_T^1$ and $p_T^2$ distributions

- **(d) stability of the second jet distribution**: require the predictions and associated scale uncertainty to provide physical, positive cross sections

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<th>scale</th>
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<th>(b)</th>
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<td>$p_T^{1,1}$</td>
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<td>$2p_T^{1,1}$</td>
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<td>$\hat{H}_T/2$</td>
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<td>$\hat{H}_T$</td>
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(a) $R = 0.7$

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<th>criterion</th>
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<tr>
<td>$\mu = 2p_T$ and $\mu = \hat{H}_T$ as scales that satisfy all the criteria above for both cone sizes $R=0.4$ and $R=0.7$ ⇒ $\mu = p_T^{1,1}$ strongly disfavoured</td>
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<td>(b) $R = 0.4$</td>
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Table 2: Summary of scales vs. criteria for (a) $R=0.7$ and (b) $R=0.4$ cone sizes.
Comparison with LHC CMS data $R=0.7$

- small positive NNLO corrections improve the agreement with CMS data with respect to NLO

- **significant reduction in scale uncertainty** from NLO to NNLO → roughly more than a factor of 2 in a wide range of $p_T$ and rapidity
Comparison with LHC CMS data $R=0.4$

- improved agreement with data at NNLO with respect to NLO

- both scale choices are stable and provide reasonable predictions for $R=0.7$ and $R=0.4$
Summary/Outlook

Summary:

- **Significant theoretical progress** in the description of inclusive jet and dijet production at the LHC
- Theoretical developments **driven** by the increase in precision of the experimental measurements
- New theoretical calculations available that can be used to understand the impact of the effects of higher order corrections in the description of jet data at hadron colliders

Outlook

- Perform further **quantitative comparisons** between data and theory (different energies, covering wide jet $p_T$ and rapidity and jet cone sizes)
- Use new data to understand effects in tuning of hadronization and underlying event parameters and respective uncertainties
- Extend existing phenomenological predictions to triple-differential measurements and angular observables and jet shapes
- Study sensitivity of jet-based observables to $\alpha_s$ and PDF extractions and assess ultimate reach in precision in a combined fit