Physics of the Top Quark and its Mass

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.. not just the heaviest SM particle

- Top quark: heaviest known particle
- Most sensitive to the mechanism of mass generation
- Peculiar role in the generation of flavor.
- Top might not be the SM-Top, but have a non-SM component.
- Top as calibration tool for new physics particles (SUSY and other exotics)
- Top production major background it new physics searches
- One of crucial motivations for New Physics

- Very special physics laboratory: $\Gamma_t \gg \Lambda_{QCD}$
  - Top treated a particle: $p_T$, spin, $\sigma_{tot}$, $\sigma$(single top), $\sigma$(tt+X),.. $\rightarrow q \gg \Gamma_t$
  - Quantum state sensitive low-E QCD and unstable particle effects: $m_t$, endpoint regions $\rightarrow q \sim \Gamma_t$
  - Multiscale problem: $p_T$, $m_t$, $\Gamma_t$, $\Lambda_{QCD}$, \ldots (depends on resolution of observable)
Top as a Lab: Rich Collection of Achievements

Just some selection pf recent results...

Outline

This talk is a based on a personal selection of new SM calculations that appeared from the last edition of SM@LHC.

- **Stable Tops**: NNLO+NNLL in QCD. NNLO QCD + complete-NLO, $t\bar{t}$ merged $i\bar{i}$ and $i\bar{t}$+jet at complete-NLO.
- **Decays and off-shell effects**: NLO QCD to lepton+jets channel.
- **$t$-channel single top** production and decay at NNLO QCD in NWA.
- $t\bar{t} + b$-jet NLO+PS in 4FS.
- $t\bar{t}W^\pm$ complete-NLO.
- $t\bar{t}t\bar{t}$ complete-NLO.  

From Davide Pagani (SM@LHC2018)

At QCD@LHC:

- Review $t\bar{t}H$  
  Talk by A. Kulesza

- $t\bar{t}b\bar{b}$ jet @ NLO  
  Talk by F. Buccioni

- $\sigma_{tot}(tt) @ >$ NNLO  
  Talk by J. Piclum

- off-shell $t\bar{t}\gamma$ @ NLO  
  Talk by T. Weber

- **on-shell tops**
- **off-shell tops**

Czakon, Ferroglia, Heymes, Pejcak, Scott, Wang, Yang 2018
Czakon, Heymes, Mitov, Pagani, Tsinikos, Zaro, 2018
Gütschow, Lindert, Schönherr, 2018
Jezo, Lindert, Oleari, Naron, Pozzorini, 2016
Denner, Pellen, 2016
Berger, Gao, Zhu, 2016, 2018
Bruchseifer, Caola, Melnikov, 2014
Jezo, Lindert, Moretti, Pozzorini, 2018

Frederix, Pagani, Zaro, 2017
Czakon, Ferroglia, Heymes, Pejcak, Scott, Wang, Yang 2018

- NNLO + NNLL'

Urge to identify “best” renormalization scale to reduce size of higher order corrections lifted.

→ collider differential cross sections are multi-scale problems

- Multi-scale problem:

In general there IS no “best” renormalization scale because no choice can eliminate all large logs
Top as a Lab: Rich Collection of Achievements

- Electroweak and QCD compete and interfere non-trivially
  → coherent treatment (resummation needed)
  e.g. parton showers

\[ p_{T, J_2} \]

\[ M_{\mu-b} \]

\[ M_{\text{top}_{\text{had}}} \]

\[ M_{\text{top}_{\text{lep}}} \]

- Breakdown of the picture of a top quark “particle”

From Davide Pagani (SM@LHC2018)

Denner, Pellen ’17
Top Mass Measurements

- Most precise measurements from direct reconstruction (uncertainties < 1 GeV)
- Other methods are less precise since top mass sensitivity smaller.
- Some of these are based on NLO (or NNLO) pQCD calculations in the pole mass scheme → “pole mass measurements”
**Top Mass Measurements**

**Most precise method:**  Direct Reconstruction

- **Kinematic Fit**
  - Selected objects:
    - 4 untagged jets
    - 2 b-tagged jets
  - Constraints:
    - $2 x m_j = m_W$
    - $m_{top} = m_{j_{b1}} = m_{j_{b2}} = m_{antitop}$

- **High top mass sensitivity**
- **Precision of MC?**
- **Meaning of $m_t^{MC}$?**

$\Delta m_t \sim 200 \text{ MeV}$

$\begin{align*}
  m_t^{MC} & = 174.34 \pm 0.64 \quad (\text{Tevatron final, 2014}) \\
  m_t^{MC} & = 172.44 \pm 0.49 \quad (\text{CMS Run-1 final, 2015}) \\
  m_t^{MC} & = 172.84 \pm 0.70 \quad (\text{ATLAS Run-1 final, 2016})
\end{align*}$

- **Determination of the best-fit value of the Monte-Carlo top quark mass parameter**

QCD@LHC Workshop, Dresden, August 27-31, 2018
Top Mass Measurements

Why is there a controversy on the interpretation of $m_t^{MC}$?

- Picture of "top quark particle" does not apply
  - $m_t$ is a scheme-dependent parameter of a perturbative computation
    - In which scheme do MC event generators calculate?
  - Hadronization effects (affect all methods in a similar way, most relevant for most precise method)
  - Different / incoherent views about the numerical / conceptual importance of this issue

- Different views about the "precision" of MC event generators

- List of other sources of uncertainty that are independent of the $m_t^{MC}$ problem:
  - Jet, b-jet modelling
  - Fragmentation, hadronization model description
  - NLO matching
  - Color reconnection
  - Finite lifetime effects
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  - Color reconnection
  - Finite lifetime effects
Recent Work

Corcella, Franceschini, Kim, arXiv: 1712.05801 (m_t^{MC} from alternative methods)

- Dependence of m_t^{MC} determination from kinematic decay distributions on fragmentation parameters in Pythia 8 and Herwig 6
- Hadronization model parameters cannot be determined precise enough such that alternative methods (exclusive observables, m_B, E_B) can compete with direct mass measurements.
- **Endpoints not sensitive to hadronization model** variations (fragmentation)

Heinrich, Maier, Nisius, Schlenk, Schulze, Winter: 1709.08615 (alternative methods)

- Effects of off-shell top production compared to narrow width approximation (M_{j\ell})
- Effects related to off-shell effects as large as 0.5 – 1 GeV for m_t^{MC} determination

Ravasio, Jezo, Nason, Oleari, arXiv: 1801.03944 (m_t^{MC} from direct reconstruction)

- POWHEG study: NLO corrections in various approximations (production, decay, full off-shell) leads to small numerical differences (hva, \bar{t}t\bar{d}ec, \bar{b}\bar{b}4\ell)
- Numerical effects on the observed end point (e.g. peak position of reconstr. inv. Mass) MC dependent (Pythia compared to Herwig)
Content

• An approach to pin down $m_t^{MC}$ in a systematic way:
  → boosted top factorization
  → $e^+e^-$ 2-jettiness distribution

• Calibration of $m_t^{Pythia 8.2}$ using $e^+e^-$ 2-jettiness
  Butenschön, Dehnadi, Mateu, Preisser, Stewart,AHH; 2016

• Role of the shower cut in the coherent branching formalism and
  angular ordered parton showers
  Plätzer, Samitz ,AHH; 2018
  → Talk on Thursday

• Extension to pp collisions
  Mantry Pathak, Stewart,AHH; 2017

• Coherent branching mass

• Conclusions
An Approach to Pin Down $m_t^{MC}$

At the present stage it appears that a general answer to how to interpret $m_t^{MC}$ is not yet possible, so we can at least attempt to address the issue from the perspective of a particular observable.

**Need:**

Observable with **strong kinematic top mass dependence** that allows for a systematic theoretical description via

- Analytic QCD (e.g. QCD factorization, mass scheme under control)
- MC simulations

with the following properties:

- Hadron-level predictions
- NLO/NLL precision

The relation between analytic QCD and MC simulations can be studied in detail.

The relation between $m_t^{MC}$ and other field theory mass schemes can be understood and quantified by direct comparison.

Currently there are only very, very, … few such observables!
Factorization for Boosted Top Quarks

Observable: Thrust in e+e- (2-Jettiness $\tau_2$)

$$\tau = 1 - \max_n \sum_i |\vec{n} \cdot \vec{p}_i|$$

$$\tau \to 0 \quad \frac{M_1^2 + M_2^2}{Q^2}$$

Fat jet invariant mass distribution in the resonance region:

-Insensitive to top quark decay
-Can be extended to LHC (event shapes, fat jet masses)
-Can be extended to account for grooming effects (soft drop)
Boosted Top Factorization at Resonance

Jet functions:

\[ B_+ (\hat{s}, \Gamma_t, \mu) = \text{Im} \left[ \frac{-i}{12\pi m_J} \int d^4 x e^{ix.r} \langle 0 | T \left\{ \bar{h}_{\nu_+} (0) W_n (0) W_n^\dagger (x) h_{\nu_+} (x) \right\} | 0 \rangle \right] \]

- perturbative
- dependent on mass, width, color charge

Soft function:

\[ S_{\text{hemi}} (\ell^+, \ell^-, \mu) = \frac{1}{N_c} \sum_{X_{\ell}} \delta (\ell^+ - k_{s}^{+a}) \delta (\ell^- - k_{s}^{-b}) \langle 0 | \bar{Y}_n Y_n (0) | X_s \rangle \langle X_s | Y_n^\dagger Y_{\nu_n}^\dagger (0) | 0 \rangle \]

- non-perturbative
- Renormalization scale dependence perturbative
- dependent on color charge, kinematics

Independent of the mass and E_{cm}!
Boosted Top Factorization at Resonance

**full QCD:**

3 phase space regions:

- **n-collinear:** \((k_+, k_-, k_\perp) \sim (\Lambda, \frac{Q^2}{m^2} \Lambda, \frac{Q}{m} \Lambda)\)
- **\bar{n}-collinear:** \((k_+, k_-, k_\perp) \sim (\frac{Q^2}{m^2} \Lambda, \Lambda, \frac{Q}{m} \Lambda)\)
- **soft:** \((k_+, k_-, k_\perp) \sim (\Lambda, \Lambda, \Lambda)\)

\[
P_{t,\bar{t}} \approx m_t^2, \quad n^2 = 0, \quad \bar{n}^2 = 0
\]

\[
Y_n(x) = \bar{P} \exp \left(-ig \int_0^\infty ds \, n \cdot A_s(ns+x)\right)
\]

\[
\bar{Y}_n(x) = \bar{P} \exp \left(-ig \int_0^\infty ds \, \bar{n} \cdot \bar{A}_s(\bar{n}s+x)\right)
\]

\[
S_{\text{hemi}}(\ell^+, \ell^-, \mu) = \frac{1}{N_c} \langle 0 \mid (\bar{Y}_n)^{cd} (Y_n)^{ce} (0) \delta(\ell^- - (\hat{P}_a^+)^\dagger) \delta(\ell^- - \hat{P}_b^-) (Y_n)^{ef} (\bar{Y}_n)^{df} (0) \mid 0 \rangle
\]
Boosted Top Factorization at Resonance

**full QCD:**

\[ \Lambda \sim \Lambda_{QCD} \]

- n-collinear: \( (k_+, k_-, k_{\perp}) \sim (\Lambda, \frac{Q^2}{m^2} \Lambda, \frac{Q}{m} \Lambda) \)
- \( \bar{n} \)-collinear: \( (k_+, k_-, k_{\perp}) \sim (\frac{Q^2}{m^2} \Lambda, \Lambda, \frac{Q}{m} \Lambda) \)
- soft: \( (k_+, k_-, k_{\perp}) \sim (\Lambda, \Lambda, \Lambda) \)

\[
\frac{1}{(p_t + k)^2 - m_t^2} \sim (\frac{\sqrt{Q^2}}{m^2} \Lambda, k, k_{\perp}) \sim (\Lambda^2, m^2, m_{\perp})
\]

\[
(p_t^2 \approx m_t^2, \bar{n}^2 = 0)
\]

\[
W_n^{\dagger}(\infty, x) = P \exp \left( ig \int_0^\infty ds \bar{n} \cdot A_+(ns + x) \right)
\]

\[
h_{\nu+}(x)
\]

\[
W_n^{\dagger}(\infty, x) h_{\nu+}(x) \rightarrow \text{gauge independent}
\]

\[
B_+(\hat{s}, \Gamma_t, \mu) = \text{Im} \left[ \frac{-i}{12\pi m_J} \int d^4x \ e^{ir.x} \langle 0 | T \{ \bar{h}_{\nu+}(0) W_n(0) W_n^{\dagger}(x) h_{\nu+}(x) \} | 0 \rangle \right]
\]
2-Jettiness Distribution at NNLL/NLO

Hadron-level prediction:
\[
\frac{d\sigma}{d\tau_2} = f\left(m_t^{\text{MSR}}(R), \delta m^{\text{MSR}}, \alpha_s(M_Z), \Omega_1, \Omega_2, \mu_h, \mu_j, \mu_s, \mu_m, R, \Gamma_t\right)
\]

- Good convergence
- Reduction of scale uncertainty (NLL to NNLL)
- Control over whole distribution
- Any mass scheme possible

Q=700 GeV (p_T= 350 GeV) Q=1400 GeV (p_T= 700 GeV)

- Higher mass sensitivity for lower Q (p_T)
- Finite lifetime effects included
- Dependence on non-perturbative parameters
- Convergence: \(\Omega_{1,2,\ldots}\)

QCD@LHC Workshop, Dresden, August 27-31, 2018
Signal ttbar vs full ee $\rightarrow$ WWbb

MadGraph 5 study:

- Non-resonant contributions are irrelevant for $\tau_2$ distribution
  
  - PYTHIA (or similar MCs) will give a good description of the production process at LO
  - hemisphere invariant mass $\sim$ top invariant mass (no pollution from background)

  $\rightarrow$ Top width important effect
  $\rightarrow$ Non-trivial finite lifetime (interference, non-resonant) effects strongly suppressed.
Calibration of $m_t^{\text{Pythia 8.205}}$

Butenschön, Dehnadi, Mateu, Preisser, Stewart, AH; PRL 117 (2016) 153

- $\frac{d\sigma}{d\tau} = f(m_t^{\text{MSR}}, \alpha_s(m_Z), \Omega_1, \Omega_2, \ldots, \mu_H, \mu_J, \mu_S, \mu_M, R, \Gamma_t)$
  any scheme non-perturbative renorm. scales finite lifetime

- Generating PYTHIA Samples: (PYTHIA 8.205)
  at different energies: $Q = 600, 700, 800, \ldots, 1400$ GeV
  - masses: $m_t^{\text{MC}} = 170, 171, 172, 173, 174, 175$ GeV
  - width: $\Gamma_t = 1.4$ GeV
  - Statistics: $10^7$ events for each set of parameters

- Feed MC data into Fitting Procedure: all ingredients are there
  Fit parameters: $m_t^{\text{MSR}}, \alpha_s(m_Z), \Omega_1, \Omega_2, \ldots$
  - standard fit based on $\chi^2$ minimization
  - analysis with 500 sets of profiles ($\tau_2$ dependent renorm. scales) for the each MC sample
  - different Q-sets: 7 sets with energies between 600 - 1400 GeV
  - different n-sets: 3 choices for fitranges - (xx/yy)% of maximum peak height

- Tune 7 (Monash)
  Take $\alpha_s(M_Z)$ as input from world average.
  (Sensitivity to strong coupling very weak.)

21 fit setups
Fit Result: Pythia 8.205 vs. Theory

$\Gamma_t=1.4 \text{ GeV}$, tune 7, $m_t^{\text{MC}} = 173 \text{ GeV}$

$\Omega_1 = 0.44 \text{ GeV}$, $m_t^{\text{MSR}(1\text{GeV})} = 172.81 \text{ GeV}$

• Good agreement of PYTHIA with NNLL/NLO theory predictions

• Perturbative uncertainties of theory predictions based on scale uncertainties (profiles)

• MC uncertainties:
  • Vertical: rescaled statistical error (PDF rescaling method) → independent on statistics
  • Horizontal: fit coverage from 21 fit setups (incompatibility uncertainty)
Fit Result: MSR vs. Pole Mass

500 profiles; $\alpha_s = .118$; $\Gamma_t = 1.4$ GeV; tune 7; $Q = 700, 1000, 1400$ GeV; peak(60/80)%

Input: $m_t^{MC} = 173$ GeV

fit to find $m_t^{MSR}(1$GeV$)$ or $m_t^{pole}$

- Good convergence & stability for MSR mass
- Mass $m_t^{MSR}(1$GeV$)$ mass definition closest to the MC top mass $m_t^{MC}$.
- Pole mass shows worse convergence.
- Pole mass not compatible with MC mass within errors
- 1100/700 MeV difference at NLL/NNLL
- $m_t^{pole} \neq M_t^{Pythia 8.2}$

Similar analyses from the 20 other Q-set and n-range setups.
Fit Result: $m_t^{MC}$ and $m_t^{MSR}(1\text{ GeV})$

- All investigated MC top mass values show consistent picture
- MC top quark mass is indeed closely related to MSR mass within uncertainties: $m_t^{MC} \approx m_t^{MSR}(1\text{GeV})$

<table>
<thead>
<tr>
<th>mass</th>
<th>order</th>
<th>central</th>
<th>perturb. incompatibility</th>
<th>total</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_t^{MSR}$</td>
<td>NLL</td>
<td>172.80</td>
<td>0.26</td>
<td>0.14</td>
</tr>
<tr>
<td>$m_t^{MSR,1\text{GeV}}$</td>
<td>N$^2$LL</td>
<td>172.82</td>
<td>0.19</td>
<td>0.11</td>
</tr>
<tr>
<td>$m_t^{pole}$</td>
<td>NLL</td>
<td>172.10</td>
<td>0.34</td>
<td>0.16</td>
</tr>
<tr>
<td>$m_t^{pole}$</td>
<td>N$^2$LL</td>
<td>172.43</td>
<td>0.18</td>
<td>0.22</td>
</tr>
</tbody>
</table>

$m_t^{MSR}(1\text{GeV}) = 172.82 \pm 0.22 \text{ GeV}$
Fit Result: Tune Dependence

500 profiles; $\Gamma_t = 1.4, -1$ GeV; tune 1, 3, 7;
diff. Q-sets; peak(60/80)%

$m_t^{\text{PYTHIA}} = 173$ GeV

- tune dependence:
  $m^{\text{MSR}[\text{tune}]} - m^{\text{MSR}[7]}

- clear sensitivity to tune
- $m^{\text{MC}}$ will depend on tune
- tune dependence is not a calibration uncertainty:
  (different tune $\Rightarrow$ different MC $\Rightarrow m^{\text{MC}}_t$)
Fit Result: Top Width Dependence

Top width dependence

$$\Delta m = m_t^{\text{MSR}[\Gamma_t]} - m_t^{\text{MSR}[\Gamma_t=1.4]}$$

- Clear sensitivity to top width value.
- Pythia resonance peak position does not depend on value of $\Gamma_t$
- Theory resonance peak position increases with $\Gamma_t$
- Conclusion: **Pythia does not describe the top width dependence in a way compatible with theory.**

The $m_t^{\text{MC}}$ calibration results obtained with Pythia$^{8.205}$ contain a systematic shift related to Pythia's incorrect description of finite-lifetime effects.

- Three colors: tunes 1, 3, 7
- Error bars: standard deviation of best mass value distribution in 500 profile function fits
Pythia

QCD Factorization

MC generators themselves need to be scrutinized and understood thoroughly when/before addressing the interpretation of the MC top quark mass.

Central Questions

• Does the MC contain sufficient higher-order information to specify a quark mass scheme?

• Do we have information on how higher-order corrections may look like?
  → “MC scheme”

• Does the MC provide a description that is in full accordance to the SM (QCD+electroweak)?
  • Log summation
  • Properties of hadronization model
  • Finite Lifetime effects
  • …

• Which role do MC specific implementations play?
  • Shower cutoff $Q_0$
  • Kinematic reconstruction
  • …
Cutoff in Angular Ordered Parton Showers

→ Coherent branching: (basis of the Herwig parton shower)

2-Jettiness $\tau_2$ distribution (for $e^+e^-$ and boosted tops) can be analytically computed in QCD factorization (SCET) and coherent branching (CB).

Issues that have been addressed for $\tau_2$: → More details in my talk on Thursday
(Boosted top quarks, Narrow width approximation !!)

$Q_0=0$:

- Equivalence of CB and SCET at NLL order for $Q_0=0$ (massive quark case new!)
- Generator mass $m_t$ is the pole mass $m_t^{pole}$

$Q_0>0$:

- Pole of the top quark propagator = $m_t^{CB}(Q_0) \neq m_t^{pole}$
- Equivalence of CB and SCET at NLL enforces that shower cut $Q_0$ acts like change in the mass scheme $m_t^{pole} \rightarrow m_t^{CB}(Q_0)$ and a modification of hadronization effects

Plätzer, Samitz, AHH; arXiv:1807.06617

Catani, Webber, Marchesini, 1991
$Q_0$ Dependence: Herwig vs analytic QCD

Peak position of $M_T = \frac{Q^2 r_2}{2m_t}$ \( (Q = E_{cm}) \)

- Depends on value of $Q_0$
- Different $Q$ behavior of $Q_0$ effects concerning hadronization and the top mass

- Herwig simulations in full agreement with analytic calculations
- $M_{bl}$ and $M_{bW}$ distributions compatible
- Change of $Q_0$:
  Physical predictions unchanged when hadronization model tune is retuned (tune is $Q_0$ dependent) and generator model is interpreted as the CB mass:

$$m_t^{CB}(Q_0) = m_t^{pole} - \frac{2}{3} Q_0 \alpha_s(Q_0) + O(\alpha_s(Q_0)^2)$$

Plätzer, Samitz, AHH; arXiv:1807.06617
CB Mass $m_t^{CB}(Q_0)$: Universality

Plätzer, Samitz, AHH; arXiv:1807.06617

Result obtained for

• 2-Jettiness $\tau_2$ distribution  →  Valid for all hemisphere mass based event shape
type observables free of non-global logs (jet masses).
   No factorization predictions for direct reconstruction.

• Boosted top quarks  →  Coherent branching formalism only valid in quasi-
collinear limit.
   No parton shower formalism exists for low $p_T$ tops!

• NWA  →  Coherent branching formalism only valid in for stable
top quarks (NWA).
   No formalism exists for unstable tops!

Observable-indpendence of $m_t^{MC} = m_t^{CB}(Q_0)$ for any observable not proven at this time.

But – if universality applies – the answer is

$$m_t^{MC} = m_t^{CB}(Q_0) = m_t^{pole} - \frac{2}{3} Q_0 \alpha_s(Q_0) + \mathcal{O}(\alpha_s^2)$$
Extension to pp Collisions

- jet observable ★ ★ Jet Mass in Jet of radius $R$
- suitable top mass for jets ★
- initial state radiation ★
- final state radiation ★
- underlying event
- color reconnection ★
- beam remnant ★ Jet veto
- parton distributions ★ multiple channels
- sum large logs $Q \gg m_t \gg \Gamma_t$ ★
- Pile-up

Note: no star here

Better: factorization for pp

No star here
Extension to pp Collisions

\[ p_T \gg m_t \gg \Gamma_t > \Lambda_{QCD} \]

- **Boosted Tops** \( p_T \gg m_t \) retain top decay products
- **Fat Jets** \( R \gg \frac{m_t}{p_T} \)
- **Sensitivity** \( \hat{s} \sim \Gamma_t \) for measurement of jet-mass \( m_J \)
  \[ \hat{s} = \frac{m_J^2 - m_t^2}{m_t} \]
- **Grooming** \( z_{cut}, \beta \)
- **Jet Veto** \( T_{cut} \) or \( p_{Tcut} \)

(Perturbative and Nonperturbative effects give \( \Gamma > \Gamma_t \))
Extension to pp Collisions

Extension to pp (in principle) straightforward: (e.g. N-jettiness & X-Cone jets)

\[ \frac{d^2 \sigma}{dM_{j_1}^2 dM_{j_2}^2 dT_{\text{cut}}} = \text{tr} \left[ \hat{H}_{Qm} \hat{S}(T_{\text{cut}}, R, \ldots) \otimes F \right] \otimes J_B \otimes J_B \otimes \mathcal{I} \otimes f f \]

Issue is that UE / MPI is significant:

Same jet functions as e^+e^-

1 GeV shift

input mass in Pythia \( m_t = 173.1 \text{ GeV} \)
Extension to pp Collisions

Extension to pp (in principle) straightforward: (e.g. N-jettiness & X-Cone jets)

\[
\frac{d^2 \sigma}{dM_{j_1}^2 dM_{j_2}^2 dT_{\text{cut}}} = \text{tr} \left[ \hat{H}_{Qm} \hat{S}(T_{\text{cut}}, R, \ldots) \otimes F \right] \otimes J_B \otimes J_B \otimes \mathcal{I} \otimes f f
\]

Issue is that UE / MPI is significant:

Same jet functions as e^+e^-

BUT control of Underlying Event is model dependent.

Same model used for Hadronization can describe UE by (primarily) tuning one parameter \( \Omega \).

\[
\Omega = \int dk \: k F(k)
\]

Stewart, Tackmann, Waalewijn, 2015
Extension to pp Collisions

- Grooms soft radiation from the jet

\[
\frac{\min(p_{T_i}, p_{T_j})}{p_{T_i} + p_{T_j}} > z_{\text{cut}} \left( \frac{\Delta R_{ij}}{R_0} \right)^\beta
\]

\[
z > z_{\text{cut}} \theta^\beta
\]

two grooming parameters

- Allows for factorization calculations

Frye, Larkowski, Schwartz, Yan, 2016

Mode separation: additional soft-collinear modes
Extension to pp Collisions

AH, Mantry, Pathak, Stewart; arXive:1708.02586

Can only apply a “light soft drop” for tops:

\[
\frac{\Gamma_t}{m} \left( \frac{Q}{2m} \right)^\beta \gg z_{cut} \gg \frac{2m\Gamma_t}{Q^2}
\]

Ensure soft drop does not touch \( J_B \)

Ensure soft drop removes global soft radiation from measurement

Factorization with Soft Drop on one jet:

\[
\frac{d^2\sigma}{dM^2_JdT_{cut}} = \text{tr} \left[ \hat{H}_{Qm} \hat{S}(T_{cut}^\beta, Q_{z_{cut}}, \beta, \ldots) \otimes F \right] \otimes J_B \otimes \mathcal{I} \otimes f f \\
\times \left\{ \int d\ell dk J_B \left( \frac{\ell - Q}{m}, \Gamma_t, \delta m \right) S_C \left[ \left( \ell - k \left( \frac{k}{Q_{cut}} \right)^{1+\beta} \right) Q_{cut}^{1+\beta}, \beta, \mu \right] F_C(k) \right\}
\]

(“high-\( p_T \) factorization”)
Extension to pp Collisions

AH, Mantry, Pathak, Stewart; arXive:1708.02586

**z\text{cut} dependence**

predict transition for “light Soft Drop” ✓

most contamination is removed

![Graph showing the dependence of \( z_{\text{cut}} \) on \( M_J \) for different values of \( p_T \) and \( R = 1 \). The graph illustrates the effect of varying \( z_{\text{cut}} \) on the invariant mass distribution, with the red arrow indicating a predicted transition for “light Soft Drop.”]
Extension to pp Collisions

AH, Mantry, Pathak, Stewart; arXive:1708.02586

Hadronization + MPI: MC mass and MSR mass compatible

Result very encouraging:
Realistic full hadron level jet mass distributions for boosted top quarks.
“Light grooming” restriction may be lifted. → separation of ultra-collinear modes
Relation of $m_t^{CB}(Q_0)$ to other Masses

Herwig 7: $Q_0 = 1.25\ \text{GeV} \quad \rightarrow \quad m_t^{\text{Herwig}} = m_t^{CB}(1.25\ \text{GeV})$

**MSR Mass**

$$m_t^{MC} = m_t^{CB}(Q_0) = m_t^{\text{pole}} - \frac{2}{3} Q_0 \alpha_s(Q_0) + \mathcal{O}(\alpha_s^2) = m_t^{\text{pole}} - 0.67 Q_0 \alpha_s(Q_0) + \mathcal{O}(\alpha_s^2)$$

$$m_t^{MSR}(Q_0) = m_t^{\text{pole}} - \frac{4}{3\pi} Q_0 \alpha_s(Q_0) + \mathcal{O}(\alpha_s^2) = m_t^{\text{pole}} - 0.42 Q_0 \alpha_s(Q_0) + \mathcal{O}(\alpha_s^2)$$

$$\rightarrow m_t^{MSR}(Q_0) - m_t^{CB}(Q_0) = 0.24 Q_0 \alpha_s(Q_0) + \mathcal{O}(\alpha_s^2)$$

$$m_t^{MSR}(Q_0) - m_t^{CB}(Q_0) = (0.190 \pm 0.070)\ \text{GeV}$$

\[ \bar{\alpha}_S^{MS}(M_Z) = 0.118 \]

- CB and MSR masses do not suffer from the $\mathcal{O}(\Lambda_{QCD})$ renormalon $\rightarrow$ good convergence
- Uncertainty estimated from difference between $\alpha_s$ in $\overline{\text{MS}}$ and MC schemes
- MSR mass can be related to $\overline{m}_t(\overline{m}_t)$ with uncertainty of 15 MeV.

- Precision sufficient for all possible applications at the LHC
- Two-loop corrections needed for ILC top quark physics
Relation of $m_t^{CB}(Q_0)$ to other Masses

Herwig 7: $Q_0 = 1.25$ GeV $\rightarrow$ $m_t^{Herwig} = m_t^{CB}(1.25$ GeV$)$

Pole Mass

$$m_t^{MSR}(Q_0) - m_t^{CB}(Q_0) = (0.190 \pm 0.070) \text{ GeV}$$

$$m_t^{pole} - m_t^{MSR}(Q_0) = (0.350 \pm 0.250) \text{ GeV}$$

$\rightarrow$ $m_t^{pole} - m_t^{CB}(Q_0) = (0.540 \pm 0.260) \text{ GeV}$

- Pole mass suffers from the $O(\Lambda_{QCD})$ renormalon $\rightarrow$ irreducible ambiguity 250 MeV
- Pole mass around 0.5 GeV larger than Herwig top generator mass.
  Shift as large as current experimental uncertainty from direct methods.

Lepenik, Preisser, AHH  2017
$\pm 110$ MeV: Beneke, Marquard, Nason, Steinhauser  2017
Conclusions / Outlook

- QCD factorization ($e^+e^- \tau_2$ boosted) – MC (angular ordered parton shower) studies
  - We know $m_t^{MC}$ at NLO for angular ordered parton showers (Herwig)
    Current limitation: NWA, boosted tops
  - Numerical calibration tool (NNLL/NLO) available for any MC

- Finite lifetime effects well known in QCD factorization
  → MC improvements in reach

- QCD factorization for top decay differential observables for boosted top quarks:
  → not available, difficult, but appear possible

- Comparable studies for direct reconstruction (low $p_T$ unboosted top quarks):
  QCD factorization / analytic QCD currently out of reach → serious challenge

- Top mass problem requires scrutiny of MC event generators and is a vehicle to
  better understand the systematics of MCs and improve their theoretical prediction