Double parton scattering: status of theory

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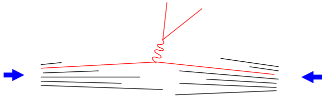
HELMHOLTZ RESEARCH FOR GRAND CHALLENGES

DESY

Hadron-hadron collisions

- standard description based on factorisation formulae

cross sect = parton distributions × parton-level cross sect



- factorisation formulae are for inclusive cross sections $pp \to Y + X$
  where $Y =$ produced by parton-level scattering, specified in detail
  $X =$ summed over, no details

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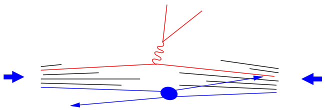
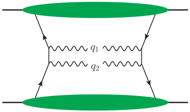


- factorisation formulae are for inclusive cross sections $pp \to Y + X$
  where $Y =$ produced by parton-level scattering, specified in detail
  $X =$ summed over, no details
- spectator interactions
  - cancel in inclusive cross sections thanks to unitarity
  - can be soft $\rightsquigarrow$ underlying event
  - or hard $\rightsquigarrow$ multiparton interactions
- here: double parton scattering with factorisation formula

  cross sect = double parton distributions × parton-level cross sections

Single vs. double parton scattering (SPS vs. DPS)

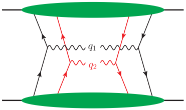
- example: prod'n of two gauge bosons, transverse momenta $\boldsymbol{q}_1$ and $\boldsymbol{q}_2$



single scattering:

$|\boldsymbol{q}_1|$ and $|\boldsymbol{q}_2| \sim$ hard scale Q

$|\boldsymbol{q}_1 + \boldsymbol{q}_2| \ll Q$



double scattering:

both $|\boldsymbol{q}_1|$ and $|\boldsymbol{q}_2| \ll Q$

- for transv. momenta $\sim \Lambda \ll Q$:

$$\frac{d\sigma_{\text{SPS}}}{d^2\boldsymbol{q}_1\, d^2\boldsymbol{q}_2} \sim \frac{d\sigma_{\text{DPS}}}{d^2\boldsymbol{q}_1\, d^2\boldsymbol{q}_2} \sim \frac{1}{Q^4\,\Lambda^2}$$

but single scattering populates larger phase space :

$$\sigma_{\text{SPS}} \sim \frac{1}{Q^2} \gg \sigma_{\text{DPS}} \sim \frac{\Lambda^2}{Q^4}$$

Single vs. double parton scattering (SPS vs. DPS)

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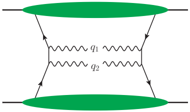
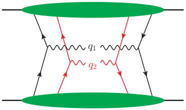




single scattering:

$|\boldsymbol{q}_1|$ and $|\boldsymbol{q}_2| \sim$ hard scale Q

$|\boldsymbol{q}_1 + \boldsymbol{q}_2| \ll Q$

double scattering:

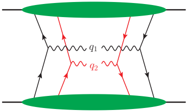
both $|\boldsymbol{q}_1|$ and $|\boldsymbol{q}_2| \ll Q$

- for small parton mom. fractions x
  double scattering enhanced by parton luminosity
- depending on process: enhancement or suppression
  from parton type (quarks vs. gluons), coupling constants, etc.

  example: same sign W pairs

  $$\left[\hat{\sigma}(u\bar{d} \to W^+)\right]^2 \propto \alpha_s^0 \quad \text{vs.} \quad \hat{\sigma}(qq \to qq + W^+W^+) \propto \alpha_s^2$$

DPS cross section: factorisation formula



$$\frac{d\sigma_{\text{DPS}}}{dx_1\, d\bar{x}_1\, dx_2\, d\bar{x}_2} = \frac{1}{C}\, \hat{\sigma}_1\, \hat{\sigma}_2 \int d^2\boldsymbol{y}\, F(x_1, x_2, \boldsymbol{y})\, F(\bar{x}_1, \bar{x}_2, \boldsymbol{y})$$

$$\begin{aligned} C &= \text{combinatorial factor} \\ \hat{\sigma}_i &= \text{parton-level cross sections} \\ F(x_1, x_2, \boldsymbol{y}) &= \text{double parton distribution (DPD)} \\ \boldsymbol{y} &= \text{transv. distance between partons} \end{aligned}$$

- follows from lowest order Feynman graphs
  proof of factorisation $\rightsquigarrow$ end of this talk
- can make $\hat{\sigma}_i$ differential in further variables (e.g. for jet pairs)
- can extend $\hat{\sigma}_i$ to higher orders in α_s (same as for SPS)
  get usual convolution integrals over x_i in $\hat{\sigma}_i$ and F

Paver, Treleani 1982, 1984; Mekhfi 1985, . . . , MD, Ostermeier, Schäfer 2012

Pocket formula

- results from simplest possible assumptions
- if two-parton density factorises as

$$F(x_1, x_2, \boldsymbol{y}) = f(x_1)\, f(x_2)\, G(\boldsymbol{y})$$

where $f(x_i) =$ usual PDF

- if assume same $G(\boldsymbol{y})$ for all parton types
then cross sect. formula turns into

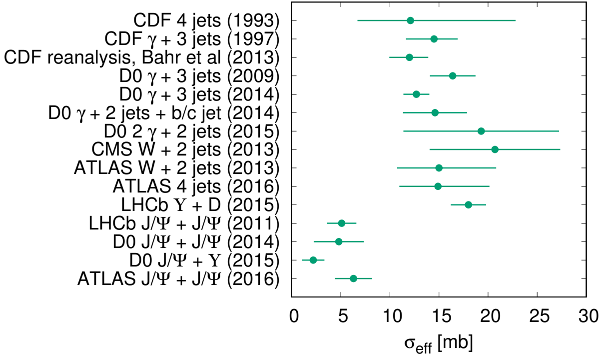
$$\frac{d\sigma_{\text{double}}}{dx_1\, d\bar{x}_1\;\, dx_2\, d\bar{x}_2} = \frac{1}{C}\, \frac{d\sigma_1}{dx_1\, d\bar{x}_1}\, \frac{d\sigma_2}{dx_2\, d\bar{x}_2}\, \frac{1}{\sigma_{\text{eff}}}$$

with $1/\sigma_{\text{eff}} = \int d^2\boldsymbol{y}\; G(\boldsymbol{y})^2$

$\rightsquigarrow$ scatters are completely independent

- underlies bulk of phenomenological estimates
- fails if any of the above assumptions is invalid
or if original cross sect. formula misses important contributions
cf. Calucci, Treleani 1999; Frankfurt, Strikman, Weiss 2003-04; Blok et al 2013

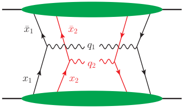
Experimental investigations (incomplete)



- other channels:
  - double open charm $C + C$ with $C = D^0, D^+, D_s^+, \Lambda_c^+$ LHCb 2012
  - $W + J/\Psi$, $Z + J/\Psi$ ATLAS 2014, 2015
  - $\Upsilon + \Upsilon$ (estimate $\sigma_{\text{eff}} \approx 2.2 \div 6.6\,\text{mb}$) CMS 2016
  - same-sign WW (LHC run 2) CMS 2017

Double parton scattering: ultraviolet problem

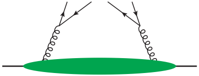
$$\frac{d\sigma_{\text{DPS}}}{dx_1\, d\bar{x}_1\, dx_2\, d\bar{x}_2} = \frac{1}{C}\, \hat{\sigma}_1 \hat{\sigma}_2 \int d^2\boldsymbol{y}\; F(x_1, x_2, \boldsymbol{y})\, F(\bar{x}_1, \bar{x}_2, \boldsymbol{y})$$



- for $\boldsymbol{y} \ll 1/\Lambda$ can compute

$$F(x_1, x_2, \boldsymbol{y}) \sim \frac{1}{\boldsymbol{y}^2} \text{ splitting fct} \otimes \text{usual PDF}$$

gives strong correlations between two partons



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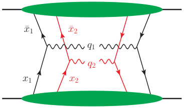
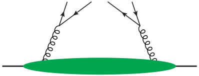


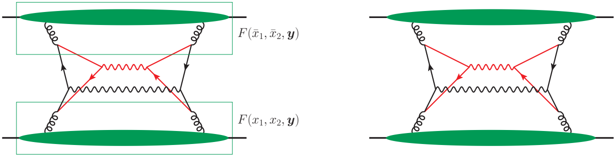
- for $\boldsymbol{y} \ll 1/\Lambda$ can compute

  $$F(x_1, x_2, \boldsymbol{y}) \sim \frac{1}{\boldsymbol{y}^2} \text{ splitting fct} \otimes \text{usual PDF}$$

  gives UV divergent cross section $\propto \int d^2\boldsymbol{y}/\boldsymbol{y}^4$
  in fact, formula not valid for $|\boldsymbol{y}| \sim 1/Q$



. . . and more problems



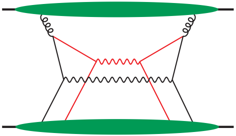
- double counting problem between double scattering with splitting (1v1) and single scattering at loop level (box graphs)

  MD, Ostermeier, Schäfer 2011; Gaunt, Stirling 2011; Gaunt 2012
  Blok, Dokshitzer, Frankfurt, Strikman 2011; Ryskin, Snigirev 2011, 2012
  already noted by Cacciari, Salam, Sapeta 2009

- also have graphs with splitting in one proton only: “2v1”

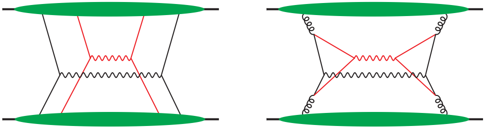
$$\sim \int d^2\boldsymbol{y}/\boldsymbol{y}^2 \times F_{\text{int}}(x_1, x_2, \boldsymbol{y})$$

B Blok et al 2011-13
J Gaunt 2012
B Blok, P Gunnellini 2015



A consistent solution

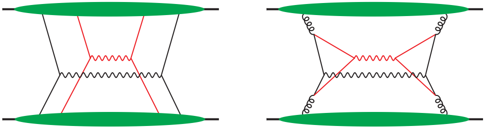
MD, J. Gaunt, K. Schönwald 2017



- regulate DPS: $\sigma_{\text{DPS}} \propto \int d^2\boldsymbol{y}\ \Phi^2(\nu y)\ F(x_1, x_2, \boldsymbol{y})\ F(\bar{x}_1, \bar{x}_2, \boldsymbol{y})$
  - $\Phi \to 0$ for $u \to 0$ and $\Phi \to 1$ for $u \to \infty$, e.g. $\Phi(u) = \theta(u-1)$
  - cutoff scale $\nu \sim Q$
  - $F(x_1, x_2, \boldsymbol{y})$ has both splitting and 'intrinsic' contributions
- keep definition of DPDs as operator matrix elements
  cutoff in $\boldsymbol{y}$ does not break symmetries that haven't already been broken

A consistent solution

MD, J. Gaunt, K. Schönwald 2017



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  - $F(x_1, x_2, \boldsymbol{y})$ has both splitting and 'intrinsic' contributions
- full cross section: $\sigma = \sigma_{\text{DPS}} - \sigma_{\text{sub}} + \sigma_{\text{SPS}}$
  - subtraction σ_{sub} to avoid double counting:
    $= \sigma_{\text{DPS}}$ with F computed for small $\boldsymbol{y}$ in fixed order perturb. theory
    much simpler computation than σ_{SPS} at given order

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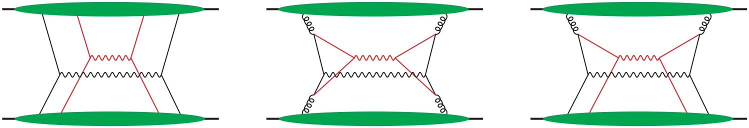

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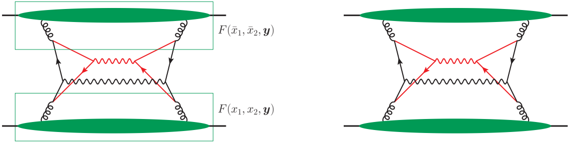
  - $F(x_1, x_2, \boldsymbol{y})$ has both splitting and 'intrinsic' contributions
- full cross section: $\sigma = \sigma_{\text{DPS}} - \sigma_{\text{sub (1v1 + 2v1)}} + \sigma_{\text{SPS}} + \sigma_{\text{tw2}\times\text{tw4}}$
  - subtraction σ_{sub} to avoid double counting:
    $= \sigma_{\text{DPS}}$ with F computed for small $\boldsymbol{y}$ in fixed order perturb. theory
    much simpler computation than σ_{SPS} at given order
  - analogous treatment for 2v1 term (cross talk with $\sigma_{\text{tw2}\times\text{tw4}}$)

Subtraction formalism at work



$$\sigma = \sigma_{\text{DPS}} - \sigma_{\text{sub}} + \sigma_{\text{SPS}}$$

- for $y \sim 1/Q$ have $\sigma_{\text{DPS}} \approx \sigma_{\text{sub}}$
  because pert. computation of F gives good approx. at considered order
  $\Rightarrow \sigma \approx \sigma_{\text{SPS}}$ dependence on $\Phi(\nu y)$ cancels between σ_{DPS} and σ_{sub}
- for $y \gg 1/Q$ have $\sigma_{\text{sub}} \approx \sigma_{\text{SPS}}$
  because DPS approximations work well in box graph
  $\Rightarrow \sigma \approx \sigma_{\text{DPS}}$ with regulator fct. $\Phi(\nu y) \approx 1$
- same argument for 2v1 term and $\sigma_{\text{tw}2\times\text{tw}4}$ (were neglected above)
- subtraction formalism works order by order in perturb. theory

J. Collins, Foundations of Perturbative QCD, Chapt. 10

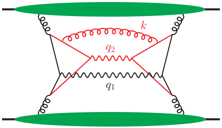
DGLAP evolution

- define DPDs as matrix elements of renormalised twist-two operators:

  $F(x_1, x_2, \boldsymbol{y}; \mu_1, \mu_2) \sim \langle p|\mathcal{O}_1(\mathbf{0}; \mu_1)\, \mathcal{O}_2(\boldsymbol{y}; \mu_2)|p\rangle \qquad f(x; \mu) \sim \langle p|\mathcal{O}(\mathbf{0}; \mu)|p\rangle$

  $\Rightarrow$ separate DGLAP evolution for partons 1 and 2:

$$\frac{\partial}{\partial \log \mu_i^2} F(x_i, \boldsymbol{y}; \mu_i) = P \otimes_{x_i} F \qquad \text{for } i = 1, 2$$



- DGLAP logarithm from strongly ordered region $|\boldsymbol{q}_1| \ll |\boldsymbol{k}| \sim |\boldsymbol{q}_2| \ll Q_2$ repeats itself at higher orders (ladder graphs)
- resummed by DPD evolution in σ_{DPS} if take $\nu \sim \mu_1 \sim Q_1$, $\mu_2 \sim Q_2$ and appropriate initial conditions (see next slide)
- can enhance DPS region over SPS region $|\boldsymbol{q}_1| \sim |\boldsymbol{q}_2| \sim Q_{1,2}$, which dominates by power counting

A model study

- take DPD model with $F = F_{\text{spl}} + F_{\text{int}}$

$$F_{\text{spl}}(x_1, x_2, \boldsymbol{y}; 1/y^*, 1/y^*) = F_{\text{perturb.}}(y^*)\, e^{-y^2\Lambda^2} \quad \text{with} \quad y^* = \frac{y}{\sqrt{1 + y^2/y_{\text{max}}^2}}$$

inspired by b^* of Collins, Soper, Sterman

$$F_{\text{int}}(x_1, x_2, \boldsymbol{y}; \mu_0, \mu_0) = f(x_1; \mu_0) f(x_2; \mu_0)\, \Lambda^2 e^{-y^2\Lambda^2}/\pi$$

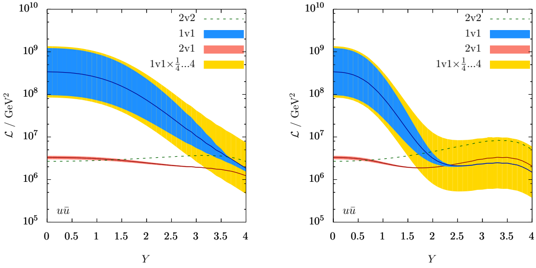
description simplified, actual model slightly refined

- $F_{\text{perturb.}}(y)$ ensures correct perturbative behaviour at small y
  DGLAP logarithms built up between splitting scale $\sim 1/y^*$ and $\sim Q$
- in subtraction term σ_{sub} term take instead
  $F_{\text{spl}}(x_1, x_2, \boldsymbol{y}; Q, Q) = F_{\text{perturb.}}(y)$
  hard scattering at fixed order, no DPD evolution here
- following plots: show double parton luminosity
  $$\mathcal{L} = \int d^2\boldsymbol{y}\; \Phi^2(\nu y)\, F(x_1, x_2, \boldsymbol{y})\, F(\bar{x}_1, \bar{x}_2, \boldsymbol{y})$$
  with separate contributions from 1v1, 2v1, 2v2

DPS parton luminosities for illustration, model parameters not tuned

- plot $\mathcal{L}$ vs. rapidity Y of q_1 with q_2 central (left) or at $-Y$ (right) with $\mu_{1,2} = Q_{1,2} = M_W$ at $\sqrt{s} = 14\,\mathrm{TeV}$
- blue band: vary ν from $0.5\,M_W \ldots 2\,M_W$
  yellow band: naive scale variation for $\sigma_{\text{1v1}} \propto \nu^2$ from $\int_{1/\nu^2} dy^2\, (1/y^2)^2$

$u\bar{u}$ on $\bar{u}u$



- if ν variation large then need $-\,\sigma_{\text{sub (1v1)}} + \sigma_{\text{SPS}}$

  $\rightsquigarrow$ use 1v1 as estimate for importance of SPS box graph contributions

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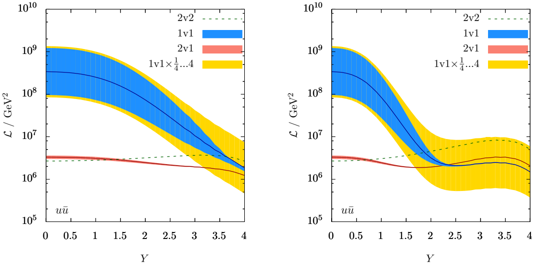

- large rapidity separation $\rightsquigarrow$ x_1 and x_2 asymmetric
  $\rightsquigarrow$ region $y \gg 1/\nu$ in 1v1 enhanced by DPD evolution
  $\rightsquigarrow$ dominates $\sigma_{1\text{v}1}$ $\rightsquigarrow$ weak sensitivity on ν

DPS parton luminosities for illustration, model parameters not tuned

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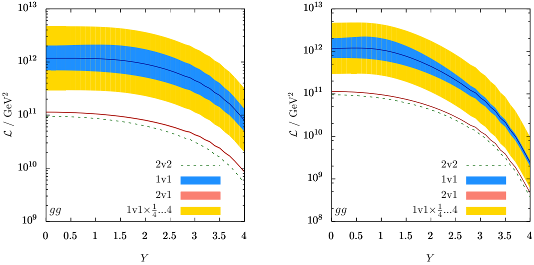
gg on gg



- gluons: prominent evolution effects at all Y

DPS parton luminosities for illustration, model parameters not tuned

- plot $\mathcal{L}$ vs. rapidity Y of q_1 with q_2 central (left) or at $-Y$ (right) with $\mu_{1,2} = Q_{1,2} = M_W$ at $\sqrt{s} = 14\,\mathrm{TeV}$
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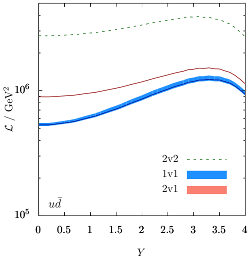
$u\bar{d}$ on $\bar{d}u$

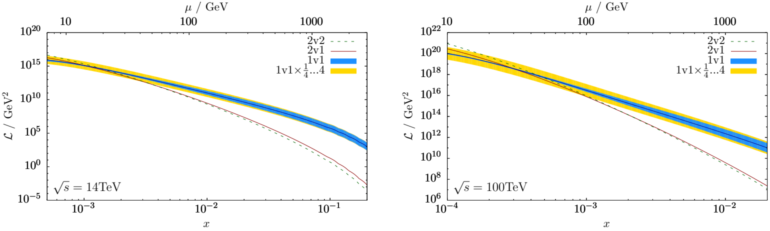


- $u\bar{d}$ induced by splitting at $\mathcal{O}(\alpha_s^2)$, e.g. by $u \to ug \to ud\bar{d}$
  parton combination relevant for W^+W^+ production

DPS parton luminosities for illustration, model parameters not tuned

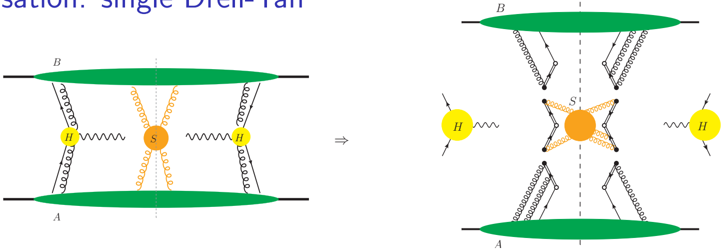
- plot $\mathcal{L}$ vs. $x = x_1 = x_2 = \bar{x}_1 = \bar{x}_2$ at fixed $\sqrt{s}$
  $\mu_{1,2} = Q_{1,2} = x\sqrt{s}$

gg on gg



- DPS region enhanced for small x by evolution

Factorisation: single Drell-Yan

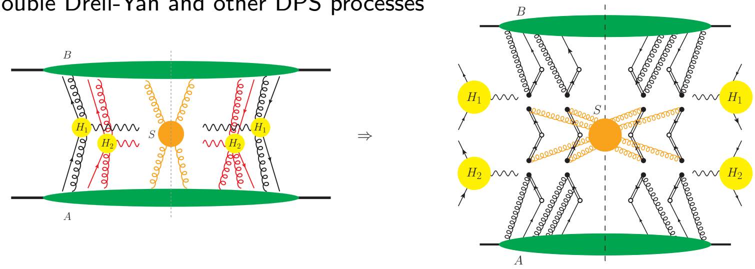


- fast-moving longitudinal gluons coupling to hard scattering
  - include in Wilson lines in parton density
- soft gluon exchange between left- and right-moving partons
  - include in soft factors = vevs of Wilson lines
    needs: eikonal approximation, Ward identities, Glauber cancellation
  - essential for establishing factorisation
  - permits resummation of Sudakov logarithms
    when p_T of gauge bosons is measured

Collins, Soper, Sterman 1980s (CSS); Collins 2011

Double parton scattering

- can generalise previous treatment from single to double Drell-Yan and other DPS processes



- basic steps can be repeated:
  - collinear gluons ⇝ Wilson lines in DPDs
  - soft gluons ⇝ soft factor, Glauber gluons cancel

MD, D Ostermeier, A Schäfer 2011; MD, J Gaunt, P Plößl, A Schäfer 2015

Double parton scattering

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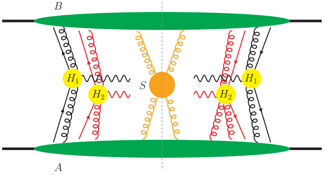

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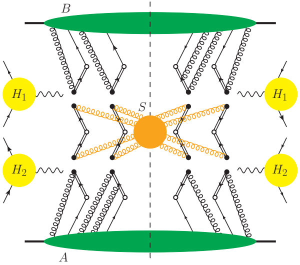

- colour structure ≫ complicated
  - matrices in colour space, final result reasonably simple
  - have DPS equivalent of CSS resummation formula for production of two gauge bosons (or other colourless particles)

A Vladimirov 2016–17; M Buffing, T Kasemets, MD 2017

Summary

- double parton scattering important
  in specific kinematics/for specific processes
- significant progress towards a systematic formulation of
  DPS factorisation in QCD
- solution for UV problem of DPS $\leftrightarrow$ double counting with SPS
  - simple UV regulator for DPS using distance y between partons
  - simple subtraction term to avoid double counting
  - can use DPS to estimate size of box graph corrections in SPS

  naturally includes DGLAP logarithms in DPS
  evolution $\rightsquigarrow$ DPS with $y \gg 1/Q$ can dominate for small x_1 and/or x_2
- factorisation complicated by colour structure but tractable
  have full generalisation of CSS formula to double Drell-Yan