N3LO QCD DIS Single-jet production and NNLO QCD e^+e^- event orientation with NNLOJET

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Outline

Genuine Subtraction Methods

N^3LO Single-Jet Production in NC DIS

NNLO QCD Event Orientations in e^+e^- annihilation

Genuine Subtraction Methods

Cross Sections at NNLO:

$$\sigma_{\mathrm{LO}} = \int\limits_{\mathrm{d}\Phi_m} \mathrm{d}\sigma_{\mathrm{B}},$$

$$\sigma_{\mathrm{NLO}} = \int\limits_{\mathrm{d}\Phi_{m+1}} \mathrm{d}\sigma_{\mathrm{NLO}}^{\mathrm{real(R)}} + \int\limits_{\mathrm{d}\Phi_m} \mathrm{d}\sigma_{\mathrm{NLO}}^{\mathrm{virtual(V)}} + \int\limits_{\mathrm{d}\Phi_m} \mathrm{d}\sigma_{\mathrm{NLO}}^{\mathrm{MF}},$$

$$\begin{aligned}\sigma_{\mathrm{NNLO}} &= \int\limits_{\mathrm{d}\Phi_{m+2}} \mathrm{d}\sigma_{\mathrm{NNLO}}^{\mathrm{RR}} + \int\limits_{\mathrm{d}\Phi_{m+1}} \mathrm{d}\sigma_{\mathrm{NNLO}}^{\mathrm{RV}} + \int\limits_{\mathrm{d}\Phi_m} \mathrm{d}\sigma_{\mathrm{NNLO}}^{\mathrm{VV}} \\ &+ \int\limits_{\mathrm{d}\Phi_m} \mathrm{d}\sigma_{\mathrm{NNLO}}^{\mathrm{MF,1}} + \int\limits_{\mathrm{d}\Phi_{m+1}} \mathrm{d}\sigma_{\mathrm{NNLO}}^{\mathrm{MF,2}}.\end{aligned}$$

Genuine Subtraction Methods

Counter Terms

$$\overbrace{\sigma_{\text{NNLO}}}^{\text{IR finite}} = \int_{d\Phi_{m+2}} \overbrace{\left[d\sigma^{\text{RR}}_{\text{NNLO}} - d\sigma^{\text{RR,S}}_{\text{NNLO}} \right]}^{\text{IR finite}}$$

$$+ \int_{d\Phi_{m+1}} \underbrace{\left[d\sigma^{\text{RV}}_{\text{NNLO}} - d\sigma^{\text{RV,T}}_{\text{NNLO}} \right]}_{\text{IR finite}}$$

$$+ \int_{d\Phi_{m}} \underbrace{\left[d\sigma^{\text{VV}}_{\text{NNLO}} - d\sigma^{\text{VV,U}}_{\text{NNLO}} \right]}_{\text{IR finite}} .$$

- Counter terms allow pointwise cancellation of IR singularities.

The Zero

$$\sigma_{\mathrm{NNLO}}^{MF} = \int_{\mathrm{d}\Phi_{m+2}} -\mathrm{d}\sigma_{\mathrm{NNLO}}^{\mathrm{RR,S}} + \int_{\mathrm{d}\Phi_{m+1}} -\mathrm{d}\sigma_{\mathrm{NNLO}}^{\mathrm{RV,T}} + \int_{\mathrm{d}\Phi_{m}} -\mathrm{d}\sigma_{\mathrm{NNLO}}^{\mathrm{VV,U}}$$

- Remaining terms in sum are massfactorization terms and result in PDF renormalization.

What Do Counter Terms Look Like?

Construction Principle

Construction of subtraction terms is based on

- The universal behaviour of QCD corrections in unresolved limits.
  $\rightarrow$ allows construction of counter terms according to factorization, i.e. (singular kernel)$\times$(correction at lower multiplicity).
- Factorization of phase space for suitable momentum mappings.
  $\rightarrow$ allows integration over phase space of singular kernel only
  $\rightarrow$ move subtraction terms across different phase space multiplicities.

NNLOJET

The method of antenna subtraction is implemented in the NNLOJET program, a semi-automated Monte Carlo for NNLO phenomenology.

Processes

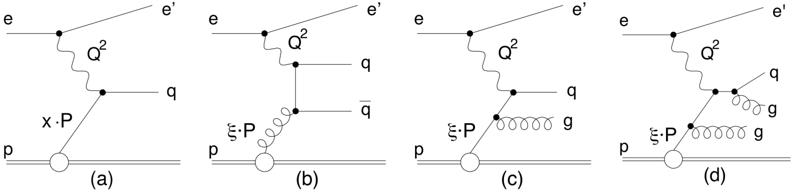
Many processes are already included at NNLO:

- pp$\rightarrow$H + 0,1 jets [arXiv:1408.5325],
- pp$\rightarrow$Z(l^+l^-) + 0,1 jets [arXiv:1607.01749],
- pp$\rightarrow$W(l^+l^-) + 0,1 jets [arXiv:1712.07543],
- NC & CC DIS single/dijets [arXiv:1606.03991],
- NC DIS single jet (N^3LO) [arXiv:1803:09973],
- pp$\rightarrow$dijets [arXiv:1611.01460] (Joao tomorrow).
- $e^+e^- \rightarrow$ 3 jets [arXiv:1709.01097],
- VBF at NNLO [arXiv:1802.02445]

Reds are focus of this talk.

N^3LO Single-Jet Production in NC DIS

Jet Production in NC DIS



Lepton-proton scattering in NC DIS:

- Process (a) is single-jet production
  $\rightarrow$ calculated to N3LO [arxiv:1803.09973]
- Processes (b) and (c) give dijet production
  $\rightarrow$ calculated to NNLO [arxiv:1703.05977 & arxiv:1606.03991]
- Process (d) is trijet production (only available to NLO).

The Projection-to-Born (P2B) method

The P2B method [Cacciari et al., '15] is the simplest possible incarnation of an IR subtraction method. The requirements for the method's applicability are:

1. Existence of a unique mapping from higher multiplicities to Born kinematics.
2. Process has been calculated inclusively to the desired order.
3. Differential results for the $(+1)$-jet process are available to one order lower.

- The weight of the IR finite $(+1)$-jet contribution is then projected to Born kinematics to give the required subtraction term for the $(+1)$-jet to the $(+0)$-jet transition.

Situation for DIS Single-Jet Production

- Born kinematics is completely fixed by values of q, the virtual vector boson's momentum, and Bjorken x. The momentum of the final-state jet is then given by (momentum conservation)

$$p_{1jet,B} = xP + q.$$

- Inclusive jet production in DIS is available to N^{3}LO [Vermaseren et al.,'05]
- DIS dijet production known to NNLO

$\rightarrow$ All ingredients at hand to apply P2B to obtain single-jet production in DIS to N^{3}LO.

N3LO Inclusive DIS Single-Jet Cross Section

At $\mathrm{N}^3\mathrm{LO}$ the fully inclusive cross section contains:

$$\frac{\mathrm{d}\sigma_X^{\mathrm{N^3LO,\,incl.}}}{\mathrm{d}\mathcal{O}_\mathrm{B}} = \int_{\Phi_{n+3}} \mathrm{d}\sigma_X^{RRR} J(\mathcal{O}_\mathrm{B}) + \int_{\Phi_{n+2}} \mathrm{d}\sigma_X^{RRV} J(\mathcal{O}_\mathrm{B}) + \int_{\Phi_{n+1}} \mathrm{d}\sigma_X^{RVV} J(\mathcal{O}_\mathrm{B}) + \int_{\Phi_{n}} \mathrm{d}\sigma_X^{VVV} J(\mathcal{O}_\mathrm{B}) \,,$$

where $J(\mathcal{O}_\mathrm{B})$ is the jet function operating on Born kinematics.

Application of the P2B-Method

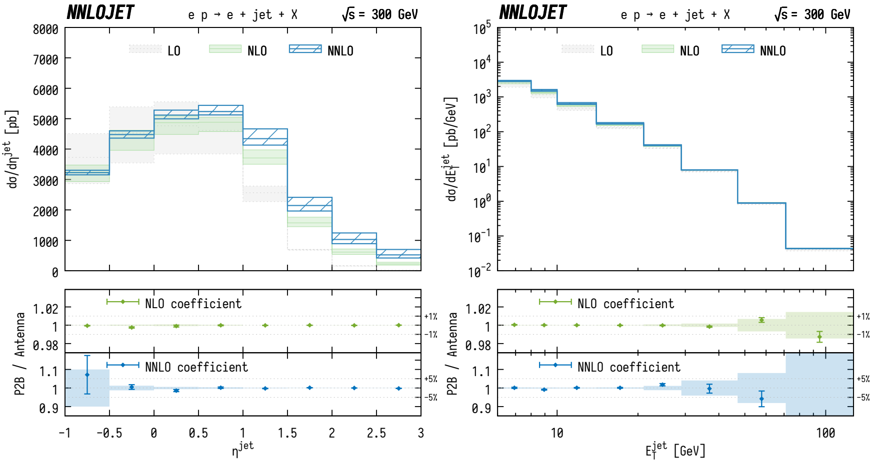
At N^3LO the fully inclusive cross section can be written as:

$$
\begin{aligned}
\frac{\mathrm{d}\sigma_X^{\mathrm{N^3LO}}}{\mathrm{d}\mathcal{O}} = & \int_{\Phi_{n+3}} \Big(\mathrm{d}\sigma_X^{RRR} \big(J(\mathcal{O}_{n+3}) - J(\mathcal{O}_{n+3\to\mathrm{B}}) \big) \\
& \qquad - \mathrm{d}\sigma_{X+j}^{S,a} \big(J(\mathcal{O}_{n+2}) - J(\mathcal{O}_{n+2\to\mathrm{B}}) \big) - \mathrm{d}\sigma_{X+j}^{S,b} \big(J(\mathcal{O}_{n+1}) - J(\mathcal{O}_{n+1\to\mathrm{B}}) \big) \Big) \\
& + \int_{\Phi_{n+2}} \Big(\mathrm{d}\sigma_X^{RRV} \big(J(\mathcal{O}_{n+2}) - J(\mathcal{O}_{n+2\to\mathrm{B}}) \big) \\
& \qquad - \mathrm{d}\sigma_{X+j}^{T,a} \big(J(\mathcal{O}_{n+2}) - J(\mathcal{O}_{n+2\to\mathrm{B}}) \big) - \mathrm{d}\sigma_{X+j}^{T,b} \big(J(\mathcal{O}_{n+1}) - J(\mathcal{O}_{n+1\to\mathrm{B}}) \big) \Big) \\
& + \int_{\Phi_{n+1}} \Big(\mathrm{d}\sigma_X^{RVV} \big(J(\mathcal{O}_{n+1}) - J(\mathcal{O}_{n+1\to\mathrm{B}}) \big) - \mathrm{d}\sigma_{X+j}^{U} \big(J(\mathcal{O}_{n+1}) - J(\mathcal{O}_{n+1\to\mathrm{B}}) \big) \Big) \\
& + \frac{\mathrm{d}\sigma_X^{\mathrm{N^3LO,\,incl.}}}{\mathrm{d}\mathcal{O}_\mathrm{B}} \,.
\end{aligned}
$$

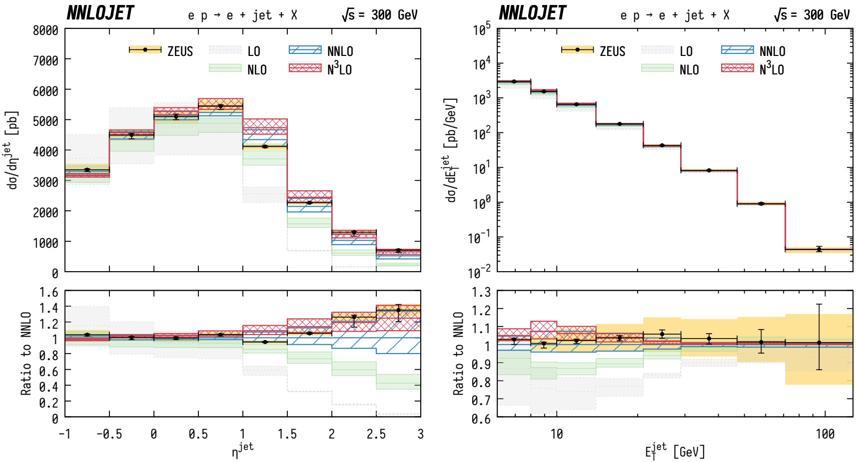
- The red terms are exactly the contributions to the inclusive cross section apart from the three-loop correction, but with opposite sign. Green(NLO-like) and blue(NNLO-like) contributions cancel separately among each other.
- Each partonic multiplicity is individually IR finite.

Validation at NNLO (P2B vs. Antenna)

We calculated single-jet distributions measured by ZEUS [arXiv:hep-ex/0502029].



N^{3}LO results



Conclusions for Single-Jet Results

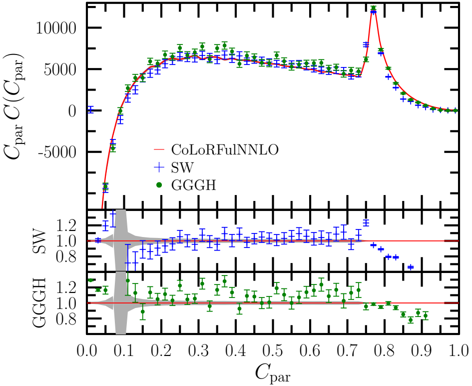
- N3LO PDFs not available.
- Experimental errors (jet energy uncertainty) still large.
- Calculation might gain importance in analysis of data from a future LHeC collider.
- Allows single-jet cross sections to be evaluated with fiducial cuts
  $\rightarrow$ no need to extrapolate experimental data
  $\rightarrow$ smaller errors.

NNLO QCD Event Orientations in e^+e^- annihilation

NNLO QCD Fixed-Order Predictions for $e^+e^- \to \gamma/Z \to 3$ Jets

Fixed-order predictions for canonical event shapes:

- Antenna subtraction [S. Weinzierl (2009), Gehrmann-DeRidder et.al (2007) EERAD3]
- CoLoRFulNNLO [Del Duca et.al(2016)]



E.g:

$$C_{par} = \frac{3}{2} \frac{\sum_{i,j} |\vec{p}_i||\vec{p}_j| \sin^2\theta_{ij}}{\left(\sum_i |\vec{p}_i|\right)^2}$$

[Del Duca et.al (2016)]

Is There Room for Improvement?

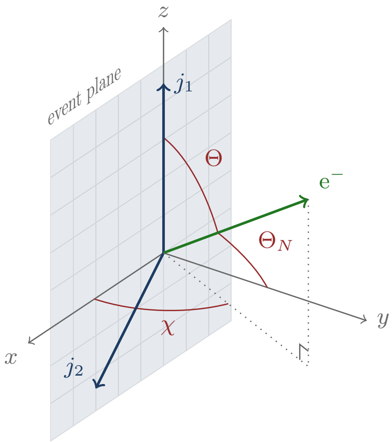
Previous calculations have been run for idealized lepton kinematics and for full 4π angular coverage:

$\rightarrow$ Lepton kinematics can be averaged out!

- Data has to be corrected for limited detector acceptance to match theoretical prediction
- SLD [hep-ex/9608016] found NLO effects to be small!

To be really precise, i.e. per-mille level, theoretical predictions should mirror experimental measurements:
$\rightarrow$ compare distributions in fiducial region
$\rightarrow$ use event orientations to get an indication for size of effects!

What Are Event Orientations?



For exclusive three-jet final states, event orientations are defined by:

- Θ, Θ_N, χ

Full lepton kinematic has to be considered in calculations!

Experimental Measurements

We compare orientation variables for exclusive three-jet final states measured by L3 experiment at the LEP collider with COM of M_Z. Jets are found using the JADE algorithm with parameter y_{cut}.

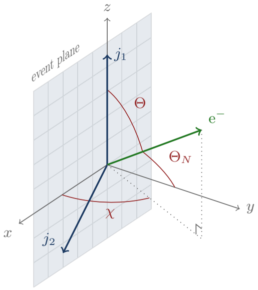
- L3 obtained two measurements:
  1. For $0.02 < y_{cut} < 0.05$
  2. For a fixed coarse jet resolution; $y_{cut} = 0.25$

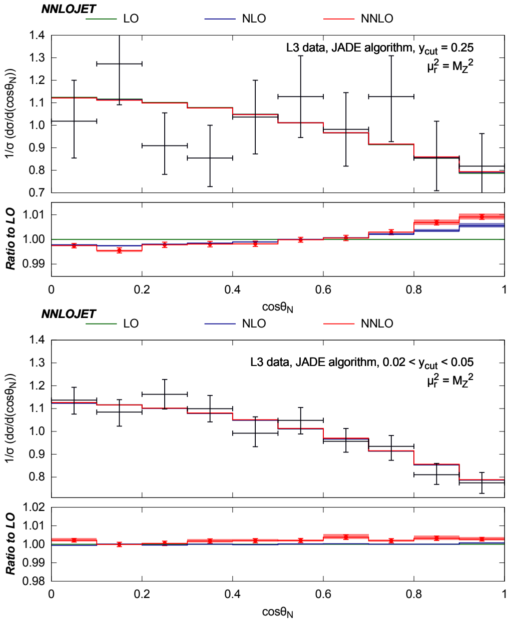
All data:

- Corrections to 4π acceptance: only relevant in endpoint bins of event orientation distributions.
- normalised to the three-jet cross section
  1. distributions integrate to unity by construction.
  2. leading order is independent of α_s.

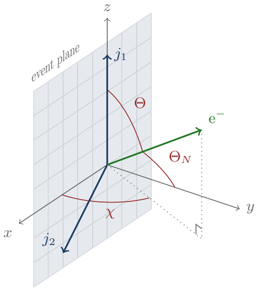
→Look order-by-order for size of corrections.

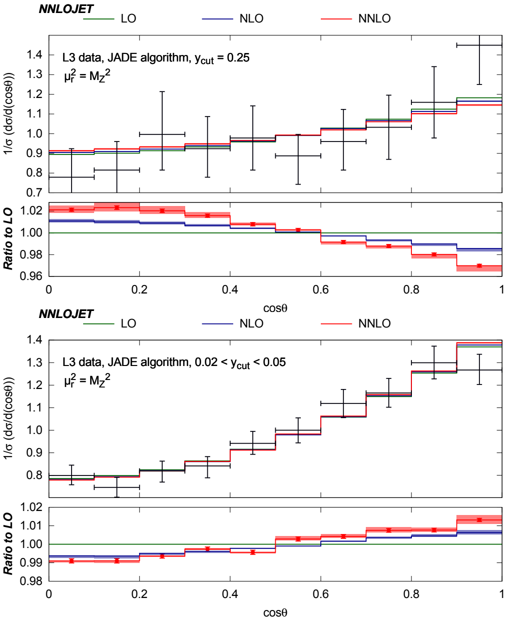
Results for Θ_N: Coarse vs Fine Jet Resolution



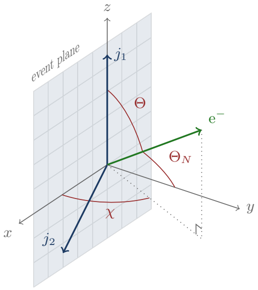


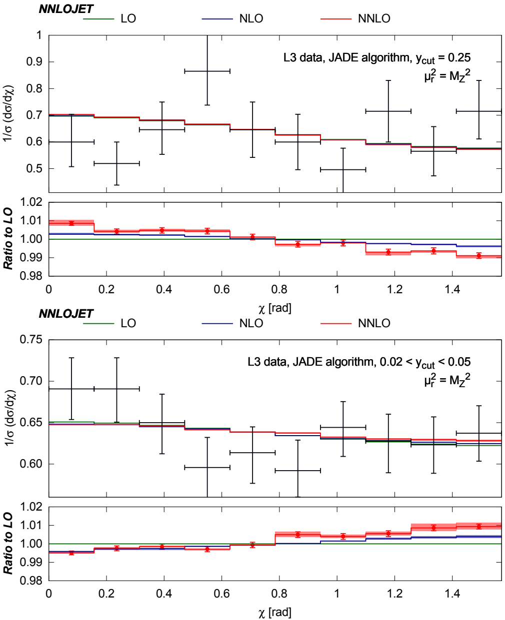
Results for Θ: Coarse vs Fine Jet Resolution





Results for χ: Coarse vs Fine Jet Resolution





Conclusions

We find that event orientation variables

- are extremely robust under QCD corrections.
- and finer jet resolution has smaller corrections.

Our findings support the validity of applied acceptance corrections at LEP!

However, to obtain per-mille accuracy at a future linear collider comparison of data and theory in the fiducial region will be important.

Thank you for your attention!