N3LO QCD DIS Single-jet production and NNLO QCD $e^+e^-$ event orientation with $\text{NNLOJET}$

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Outline

Genuine Subtraction Methods

$N^3LO$ Single-Jet Production in NC DIS

NNLO QCD Event Orientations in $e^+e^-$ annihilation
Genuine Subtraction Methods
Cross Sections at NNLO:

\[ \sigma_{\text{LO}} = \int \frac{d\sigma_B}{d\Phi_m}, \]

\[ \sigma_{\text{NLO}} = \int \frac{d\sigma_{\text{real}(R)}}{d\Phi_{m+1}} + \int \frac{d\sigma_{\text{virtual}(V)}}{d\Phi_m} + \int \frac{d\sigma_{\text{MF}}}{d\Phi_m}, \]

\[ \sigma_{\text{NNLO}} = \int \frac{d\sigma_{\text{RR}}}{d\Phi_{m+2}} + \int \frac{d\sigma_{\text{RV}}}{d\Phi_{m+1}} + \int \frac{d\sigma_{\text{VV}}}{d\Phi_m} \]

\[ + \int \frac{d\sigma_{\text{MF,1}}}{d\Phi_m} + \int \frac{d\sigma_{\text{MF,2}}}{d\Phi_{m+1}}. \]
Genuine Subtraction Methods

Counter Terms

\[ \sigma_{\text{NNLO}} = \int_{d\Phi_{m+2}} \left[ d\sigma_{\text{RR,NNLO}} - d\sigma_{\text{RR,S,NNLO}} \right] + \int_{d\Phi_{m+1}} \left[ d\sigma_{\text{RV,NNLO}} - d\sigma_{\text{RV,T,NNLO}} \right] + \int_{d\Phi_{m}} \left[ d\sigma_{\text{VV,NNLO}} - d\sigma_{\text{VV,U,NNLO}} \right]. \]

- Counter terms allow pointwise cancellation of IR singularities.
\[
\sigma^{MF}_{\text{NNLO}} = \int_{\Phi_{m+2}} -d\sigma^{\text{RR},S}_{\text{NNLO}} + \int_{\Phi_{m+1}} -d\sigma^{\text{RV},T}_{\text{NNLO}} + \int_{\Phi_{m}} -d\sigma^{\text{VV},U}_{\text{NNLO}}
\]

- Remaining terms in sum are massfactorization terms and result in PDF renormalization.
What Do Counter Terms Look Like?

Construction Principle

Construction of subtraction terms is based on

- The universal behaviour of QCD corrections in unresolved limits.
  → allows construction of counter terms according to factorization, i.e. \((\text{singular kernel}) \times (\text{correction at lower multiplicity})\).

- Factorization of phase space for suitable momentum mappings.
  → allows integration over phase space of singular kernel only
  → move subtraction terms across different phase space multiplicities.
The method of antenna subtraction is implemented in the NNLOjet program, a semi-automated Monte Carlo for NNLO phenomenology.

**Processes**

Many processes are already included at NNLO:

- $pp \rightarrow H + 0,1 \text{ jets}$ [arXiv:1408.5325],
- $pp \rightarrow Z(l^+l^-) + 0,1 \text{ jets}$ [arXiv:1607.01749],
- $pp \rightarrow W(l^+l^-) + 0,1 \text{ jets}$ [arXiv:1712.07543],
- NC & CC DIS single/dijets [arXiv:1606.03991],
- NC DIS single jet (N$^3$LO) [arXiv:1803:09973],
- $pp \rightarrow \text{dijets}$ [arXiv:1611.01460] (Joao tomorrow).
- $e^+e^- \rightarrow 3 \text{ jets}$ [arXiv:1709.01097],
- VBF at NNLO [arXiv:1802.02445]

**Reds** are focus of this talk.
$N^3$LO Single-Jet Production in NC DIS
Jet Production in NC DIS

Lepton-proton scattering in NC DIS:

- Process (a) is single-jet production
  - calculated to N3LO [arxiv:1803.09973]

- Processes (b) and (c) give dijet production
  - calculated to NNLO [arxiv:1703.05977 & arxiv:1606.03991]

- Process (d) is trijet production (only available to NLO).
The Projection-to-Born (P2B) method

The P2B method [Cacciari et al., ’15] is the simplest possible incarnation of an IR subtraction method. The requirements for the method’s applicability are:

1. Existence of a unique mapping from higher multiplicities to Born kinematics.

2. Process has been calculated inclusively to the desired order.

3. Differential results for the (+1)-jet process are available to one order lower.

- The weight of the IR finite (+1)-jet contribution is then projected to Born kinematics to give the required subtraction term for the (+1)-jet to the (+0)-jet transition.
Situation for DIS Single-Jet Production

- Born kinematics is completely fixed by values of $q$, the virtual vector boson's momentum, and Bjorken $x$. The momentum of the final-state jet is then given by (momentum conservation)

$$p_{1jet,B} = xP + q.$$ 

- Inclusive jet production in DIS is available to $N^3\text{LO}$ [Vermaseren et al.,'05]

- DIS dijet production known to NNLO

$\rightarrow$ All ingredients at hand to apply P2B to obtain single-jet production in DIS to $N^3\text{LO}$. 
At N^3LO the fully inclusive cross section contains:

\[
\frac{d\sigma_{X}^{N^3LO, \text{incl.}}}{d\mathcal{O}_B} = \int_{\Phi_{n+3}} d\sigma_{X}^{RRR} J(\mathcal{O}_B) + \int_{\Phi_{n+2}} d\sigma_{X}^{RRV} J(\mathcal{O}_B) \\
+ \int_{\Phi_{n+1}} d\sigma_{X}^{RVV} J(\mathcal{O}_B) + \int_{\Phi_{n}} d\sigma_{X}^{VVV} J(\mathcal{O}_B),
\]

where \( J(\mathcal{O}_B) \) is the jet function operating on Born kinematics.
Application of the P2B-Method

At $N^3$LO the fully inclusive cross section can be written as:

\[
\frac{d\sigma_{X}^{N^3\text{LO}}}{d\mathcal{O}} = \int_{\Phi_{n+3}} \left( d\sigma_X^{RRR} (J(\mathcal{O}_{n+3}) - J(\mathcal{O}_{n+3\rightarrow B})) 
- d\sigma_X^{S,a} (J(\mathcal{O}_{n+2}) - J(\mathcal{O}_{n+2\rightarrow B})) 
- d\sigma_X^{S,b} (J(\mathcal{O}_{n+1}) - J(\mathcal{O}_{n+1\rightarrow B})) \right)
+ \int_{\Phi_{n+2}} \left( d\sigma_X^{RRV} (J(\mathcal{O}_{n+2}) - J(\mathcal{O}_{n+2\rightarrow B})) 
- d\sigma_X^{T,a} (J(\mathcal{O}_{n+2}) - J(\mathcal{O}_{n+2\rightarrow B})) 
- d\sigma_X^{T,b} (J(\mathcal{O}_{n+1}) - J(\mathcal{O}_{n+1\rightarrow B})) \right)
+ \int_{\Phi_{n+1}} \left( d\sigma_X^{VV} (J(\mathcal{O}_{n+1}) - J(\mathcal{O}_{n+1\rightarrow B})) 
- d\sigma_X^{U} (J(\mathcal{O}_{n+1}) - J(\mathcal{O}_{n+1\rightarrow B})) \right)
+ \frac{d\sigma_{X}^{N^3\text{LO}, \text{incl.}}}{d\mathcal{O}_B} .
\]

- The red terms are exactly the contributions to the inclusive cross section apart from the three-loop correction, but with opposite sign. Green (NLO-like) and blue (NNLO-like) contributions cancel separately among each other.
- Each partonic multiplicity is individually IR finite.
Validation at NNLO (P2B vs. Antenna)

We calculated single-jet distributions measured by ZEUS [arXiv:hep-ex/0502029].
N$^3$LO results

\[ \sqrt{s} = 300 \text{ GeV} \]

\[ e^+ e^- \rightarrow e + \text{jet} + X \]

\[ \frac{d\sigma}{dE_{\text{jet}}} \text{ [pb/GeV]} \]

\[ \frac{d\sigma}{d\eta_{\text{jet}}} \text{ [pb]} \]

\[ \text{Ratio to NNLO} \]

\[ \eta_{\text{jet}} \]

\[ E_{\text{jet}} \text{ [GeV]} \]

\[ \text{NNLOJET} \]

\[ ZEUS \]

\[ \text{LO} \]

\[ \text{NLO} \]

\[ \text{NNLO} \]

\[ \text{N}^3\text{LO} \]

\[ 0 \]

\[ 0.2 \]

\[ 0.4 \]

\[ 0.6 \]

\[ 0.8 \]

\[ 1 \]

\[ 1.2 \]

\[ 1.4 \]

\[ 1.6 \]

\[ -1 \]

\[ -0.5 \]

\[ 0 \]

\[ 0.5 \]

\[ 1 \]

\[ 1.5 \]

\[ 2 \]

\[ 2.5 \]

\[ 3 \]

\[ 10 \]

\[ 100 \]
Conclusions for Single-Jet Results

- N3LO PDFs not available.
- Experimental errors (jet energy uncertainty) still large.
- Calculation might gain importance in analysis of data from a future LHeC collider.
- Allows single-jet cross sections to be evaluated with fiducial cuts
  → no need to extrapolate experimental data
  → smaller errors.
NNLO QCD Event Orientations in $e^+e^-$ annihilation
NNLO QCD Fixed-Order Predictions for $e^+e^- \rightarrow \gamma/Z \rightarrow 3$ Jets

Fixed-order predictions for canonical event shapes:

- Antenna subtraction [S. Weinzierl (2009), Gehrmann-DeRidder et.al (2007) EERAD3]
- CoLoRFulNNLO [Del Duca et.al(2016)]

E.g:

$$C_{par} = \frac{3}{2} \frac{\sum_{i,j} |\vec{p}_i| |\vec{p}_j| \sin^2 \theta_{ij}}{\left(\sum_i |\vec{p}_i|\right)^2}$$

[Del Duca et.al (2016)]
Is There Room for Improvement?

Previous calculations have been run for idealized lepton kinematics and for full $4\pi$ angular coverage:

→ Lepton kinematics can be averaged out!

- Data has to be corrected for limited detector acceptance to match theoretical prediction
- SLD [hep-ex/9608016] found NLO effects to be small!

To be really precise, i.e. per-mille level, theoretical predictions should mirror experimental measurements:
→ compare distributions in fiducial region
→ use event orientations to get an indication for size of effects!
What Are Event Orientations?

For exclusive three-jet final states, event orientations are defined by:

- $\Theta$, $\Theta_N$, $\chi$

Full lepton kinematic has to be considered in calculations!
Experimental Measurements

We compare orientation variables for exclusive three-jet final states measured by L3 experiment at the LEP collider with COM of $M_Z$. Jets are found using the JADE algorithm with parameter $y_{cut}$.

- L3 obtained two measurements:
  1. For $0.02 < y_{cut} < 0.05$
  2. For a fixed coarse jet resolution; $y_{cut} = 0.25$

All data:

- Corrections to $4\pi$ acceptance: only relevant in endpoint bins of event orientation distributions.
- Normalised to the three-jet cross section
  1. Distributions integrate to unity by construction.
  2. Leading order is independent of $\alpha_s$.

→ Look order-by-order for size of corrections.
Results for $\Theta_N$: Coarse vs Fine Jet Resolution
Results for $\Theta$: Coarse vs Fine Jet Resolution

**NNLOJET**

L3 data, JADE algorithm, $y_{cut} = 0.25$

$\mu_r^2 = M_Z^2$

**Ratio to LO**

$1/\sigma \left( \frac{d\sigma}{d(cos\theta)} \right)$

**NNLOJET**

L3 data, JADE algorithm, $0.02 < y_{cut} < 0.05$

$\mu_r^2 = M_Z^2$

**Ratio to LO**

$1/\sigma \left( \frac{d\sigma}{d(cos\theta)} \right)$
Results for $\chi$: Coarse vs Fine Jet Resolution

![Diagram showing event plane and jet vectors $j_1$, $j_2$, $\Theta$, $\Theta_N$, $\chi$, and $e^-$.]

Graphs showing:
- $1/\sigma (d\sigma/d\chi)$ for $\mu_r^2 = M_Z^2$
- Ratio to LO for $\chi$ [rad]

Graphs indicate:
- L3 data, JADE algorithm, $y_{cut} = 0.25$
- $0.98, 0.99, 1.00, 1.01, 1.02$
- $0, 0.2, 0.4, 0.6, 0.8, 1, 1.2, 1.4$
- $0.5, 0.6, 0.7, 0.8, 0.9, 1$
- L3 data, JADE algorithm, $0.02 < y_{cut} < 0.05$
- $0.98, 0.99, 1.00, 1.01, 1.02$
Conclusions

We find that event orientation variables

- are extremely robust under QCD corrections.
- and finer jet resolution has smaller corrections.

Our findings support the validity of applied acceptance corrections at LEP!

However, to obtain per-mille accuracy at a future linear collider comparison of data and theory in the fiducial region will be important.
Thank you for your attention!