Two-Loop Five-Point Helicity Amplitudes in QCD via Integrand Reduction

Heribertus Bayu Hartanto

with S. Badger, C. Brønnum-Hansen and T. Peraro

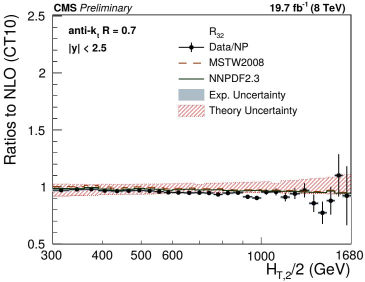
QCD@LHC2018, Dresden

August 28, 2018

NNLO frontier: 2 to 3 scattering

- $pp \to jjj$: $R_{3/2}$, $m_{jjj} \Rightarrow \alpha_s$ determination at multi-TeV range

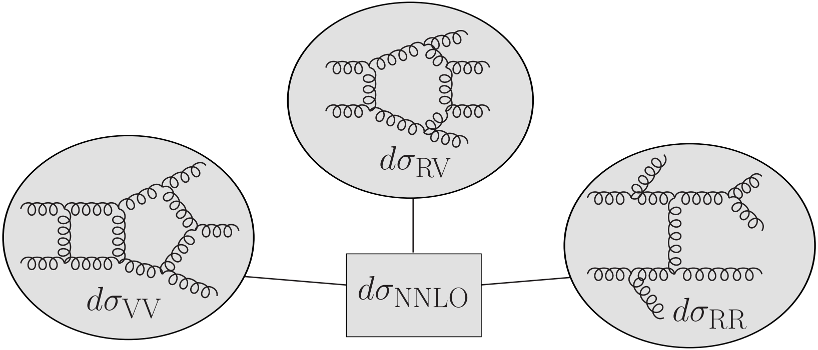
$R_{3/2} \sim \alpha_s \to$ cancellation of uncertainties



$$\alpha_s(M_Z) = 01150 \ \pm 0.0010(\text{exp}) \ \pm 0.0013(\text{PDF}) \ \pm 0.0015(\text{NP}) \ {}^{+0.0050}_{-0.0000}(\text{scale})$$

[CMS-PAS-SMP-16-008]

- $pp \to \gamma\gamma j$: background to Higgs p_T, signal/background interference effects
- $pp \to Hjj$: Higgs p_T, background to VBF (probes Higgs coupling)
- $pp \to Vjj$: Vector boson p_T, W^+/W^- ratios, multiplicity scaling
- $pp \to VVj$: background for new physics



q_T subtraction N-jettiness subtraction   projection to Born

antenna subtraction   CoLoRFulNNLO   STRIPPER

Nested Soft-Collinear Subtraction   geometric subtraction

$+$ ...

Integrand Reduction

[Ossola,Papadopoulos,Pittau,Mastrolia,Badger,Frellesvig,Zhang,Peraro,Mirabella, ...]

Colour ordered amplitude:

$$\mathcal{A}^{(2)}(\{p\}) = \int \left(\prod_i^2 \frac{d^d k_i}{i\pi^{d/2} e^{-\epsilon\gamma_E}} \right) \frac{\mathcal{N}(\{k\},\{p\})}{\mathcal{D}_1 \cdots \mathcal{D}_n}$$

Integrand Reduction: construct irreducible numerators $\Delta_i(\{k\},\{p\})$

$$\frac{\mathcal{N}(\{k\},\{p\})}{\mathcal{D}_1 \cdots \mathcal{D}_n} = \sum_{i \in T} \frac{\Delta_i(\{k\},\{p\})}{(\text{propagators})_i}$$

- $\mathcal{N}(\{k\},\{p\})$: process dependent numerator function
  - $\Rightarrow$ generalized unitarity cuts $\rightarrow$ product of tree amplitudes
    [Bern,Dixon,Dunbar,Kosower,1994;Britto,Cachazo,Feng,2004;Ellis,Giele,Kunszt,Melnikov,2007-2008; ...]
  - $\Rightarrow$ Feynman diagram input (`QGRAF`, `FeynArts`, ...)
- fit $\Delta_i(\{k\},\{p\})$ on the solutions of multiple cuts $\{\mathcal{D}_i = 0\}_{i \in T}$

Integrand Reduction

[Ossola,Papadopoulos,Pittau,Mastrolia,Badger,Frellesvig,Zhang,Peraro,Mirabella, ...]

Colour ordered amplitude:

$$\mathcal{A}^{(2)}(\{p\}) = \int \left(\prod_i^2 \frac{d^d k_i}{i\pi^{d/2} e^{-\epsilon\gamma_E}} \right) \frac{\mathcal{N}(\{k\},\{p\})}{\mathcal{D}_1 \cdots \mathcal{D}_n}$$

Integrand Reduction: construct irreducible numerators $\Delta_i(\{k\},\{p\})$

$$\frac{\mathcal{N}(\{k\},\{p\})}{\mathcal{D}_1 \cdots \mathcal{D}_n} = \sum_{i\in T} \frac{\Delta_i(\{k\},\{p\})}{(\text{propagators})_i}$$

- $\mathcal{N}(\{k\},\{p\})$: process dependent numerator function
  - $\Rightarrow$ generalized unitarity cuts $\to$ product of tree amplitudes
    [Bern,Dixon,Dunbar,Kosower,1994;Britto,Cachazo,Feng,2004;Ellis,Giele,Kunszt,Melnikov,2007-2008; ...]
  - $\Rightarrow$ Feynman diagram input (`QGRAF`, `FeynArts`, ...)
- fit $\Delta_i(\{k\},\{p\})$ on the solutions of multiple cuts $\{\mathcal{D}_i = 0\}_{i\in T}$

how can we parameterise the irreducible numerator?

Integrand Parameterization

Expand loop momenta to physical and transverse components [van Neerven,Vermaseren,1983]

$$k_i^\mu = k_{\parallel,i}^\mu + k_{\perp,i}^\mu \qquad k_{\perp,i} = k_{\perp,i}^{[4]} + k_{\perp,i}^{[-2\epsilon]}$$

Spanning basis for each topology:

$$k_{\parallel,i}^\mu = \sum_{j=1}^{d_\parallel} a_{ij} v_j^\mu, \qquad k_{\perp,i}^{\mu,[4]} = \sum_{j=1}^{d_{\perp,[4]}} b_{ij} w_j^\mu, \qquad \text{with} \;\; v_i.w_j = 0$$

$$a_{ij}(k_i) \equiv a_{ij}(\{\mathcal{D}\},\{k.q\}), \qquad b_{ij}(k_i) \equiv b_{ij}(k_i.w_j), \qquad \mu_{ij} = -k_{\perp,i}^{[-2\epsilon]}.k_{\perp,j}^{[-2\epsilon]}$$

$k_i.q_j \rightarrow$ physical-space ISPs, $k_i.w_j \rightarrow$ spurious ISPs, $\mu_{ij} \rightarrow$ extra-dim ISPs

Relations between monomials in the ISPs:

$$\mu_{ij} = k_i.k_j - k_{\parallel,i}.k_{\parallel,j} - k_{\perp,i}^{[4]}.k_{\perp,j}^{[4]}$$

Any integrand:

$$\Delta_i(\text{ISP}) = \Delta_i(k_i.q_j, k_i.w_j, \mu_{ij})$$

Maximum rank of k in Δ is restricted by gauge theory power counting.

Constructing Integrand Basis

$$A^{(2),h} = \int [dk_1][dk_2] \sum_{i \in T} \frac{\Delta_i^h(k_i.q_j, k_i.w_j, \mu_{ij})}{(\text{propagators})_i}$$

$$= \int [dk_1][dk_2] \sum_{i \in T} \frac{\sum_{\vec{\alpha}} c^h_{i,\vec{\alpha}} \cdot (\mathbf{m}_i(k_j))^{\vec{\alpha}}}{(\text{propagators})_i}$$

$$\text{with} \qquad \mathbf{m}_i(k_j) = \{k_i.q_j, k_i.w_j, \mu_{ij}\}$$

- Write down an overcomplete set of monomials in $k_i.q_j, k_i.w_j, \mu_{ij}$ obeying power counting restrictions
- Order the monomials w.r.t. a set of criteria
  (e.g. prefer low rank over high rank, prefer μ_{ij} over $4d$)
- Map all monomials to $4d$ basis, $\mu_{ij} = f(k_i.q_j, k_i.w_j)$,
  then construct a linear system according to ordering of variables
- Solve the linear system for the independent monomials

Extracting $c_{i,\vec{\alpha}}$

$$\mathcal{A}^{(2),h} = \int [dk_1][dk_2] \sum_{i\in T} \frac{\Delta_i^h(k_i.q_j, k_i.w_j, \mu_{ij})}{(\text{propagators})_i} = \int [dk_1][dk_2] \sum_{i\in T} \frac{\sum_{\vec{\alpha}} c^h_{i,\vec{\alpha}} \cdot (\mathbf{m}_i(k_j))^{\vec{\alpha}}}{(\text{propagators})_i}$$

- Fit coefficients $c^h_{i,\vec{\alpha}}$ on multiple cuts

$\mathcal{N}\big|_{\text{8xcut}} = \Delta($  $)\big|_{\text{8xcut}}$ $\qquad$ $\mathcal{N}\big|_{\text{7xcut}} = \frac{\Delta(\text{ } }{}$  $)}{(k_1+k_2)^2}\Big|_{\text{7xcut}} + \Delta($  $)\big|_{\text{7xcut}}$

- Numerator inputs:

  $\Rightarrow$ product of trees (6d spinor helicity formalism [Cheung,O'Connell,2009]+ 6d trees)

  $\Rightarrow$ Feynman diagram input

- Momentum Twistor Variables [Hodges,2009]

  $\Rightarrow$ rational function of $3n-11$ variables (+1 overall dimension)

- Numerical evaluations over finite fields $\mathcal{Z}_p$ [von Manteuffel,Schabinger,2014;Peraro,2016]

  $\Rightarrow$ fast+exact, reconstruct $\mathcal{Q}$ from several $\mathcal{Z}_p$

- Analytic reconstruction from finite-field evaluations [Peraro,arXiv:1608.01902]

From Integrand to Integrated Form

Integrands contain spurious terms

$$\int [dk_1][dk_2] \frac{k_i.w_j}{\text{(propagators)}} = 0$$

Express integrals containing μ_{ij} as higher dimensional integrals
for example: [Bern,de Freitas,Dixon,2002]

$$\int [d^d k_1][d^d k_2] \frac{\mu_{11}}{D_1 \cdots D_n} = \epsilon \sum_{i \in \{2\} \cup \{12\}} \int [d^{(d+2)} k_1][d^{(d+2)} k_2] \frac{1}{D_1 \cdots D_i^2 \cdots D_n}$$

Evaluating integrals

- Proceed with IBP reduction + substitute Master Integrals
  many new ideas for efficient solutions of IBP systems [Kosower, Kajda, Gluza, Schabinger, von Manteuffel, Ita Larsen, Zhang, Böhm, Georgoudis, Schnemann, Abreu, Page, Febres-Cordero, Zeng]
- Direct numerical evaluations with sector decomposition method (`Fiesta` [Smirnov,etal], `SecDec` [Borowka,etal])

Status of 2-loop 5-point calculation

‘All plus‘ 5-gluon case

- Planar integrand, analytic D-dim integrand [Badger,Frellesvig,Zhang,arXiv:1310.1051]
- Full-colour integrand [Badger,Mogull,O'Connell,Ochirov,arXiv:1507.08797]
- Local integrands (also for 6-gluon) [Badger,Mogull,Peraro,arXiv:1712.03946]
- Planar compact analytic [Gehrmann,Henn,LoPresti,arXiv:1511.05409][Dunbar,Perkins,arXiv:1603.07514]

$2 \to 3$ **master integrals:**

- planar, massless [Gehrmann,Henn,Lo Presti,arXiv:1511.05409,1807.09812]
- planar, massless$+1$ external mass [Papadopoulos,Tommasini,Wever,arXiv:1511.09404]
- (some) non-planar [Chicherin,Henn,Mitev,arXiv:1712.09610]

Recent progress on amplitudes (planar sector):

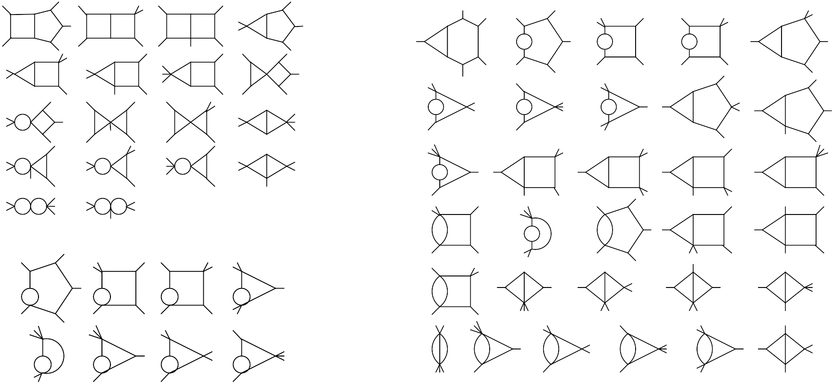
- *First look at two-loop five-gluon amplitudes in QCD* [Badger,Brønnum-Hansen,HBH,Peraro,arXiv:1712.02229]
- *Planar two-loop five-gluon amplitudes from numerical unitarity* [Abreu,Febres-Cordero,Ita,Page,Zeng,arXiv:1712.03946]
- *Efficient integrand reduction for particles with spin* [Boels,Jin,Luo,arXiv:1802.06761]
- *Two-loop five-point massless QCD amplitudes within the IBP approach* [Chawdry,Lim,Mitov,arXiv:1805.09182]

Leading colour 2-loop 5-gluon amplitudes

[Badger,Brønnum-Hansen,HBH,Peraro, Phys.Rev.Lett. 120 (2018) no.9, 092001]

$$\mathcal{A}^{(L)}(1,2,3,4,5) = n^L g_s^3 \sum_{\sigma\in S_5/Z_5} \mathrm{Tr}\left(T^{a_{\sigma(1)}}\cdots T^{a_{\sigma(5)}}\right) A^{(L)}\left(\sigma(1),\sigma(2),\sigma(3),\sigma(4),\sigma(5)\right)$$

$$n = m_\epsilon N_c \alpha_s/(4\pi), \quad m_\epsilon = i(4\pi)^\epsilon e^{-\epsilon\gamma_E}$$



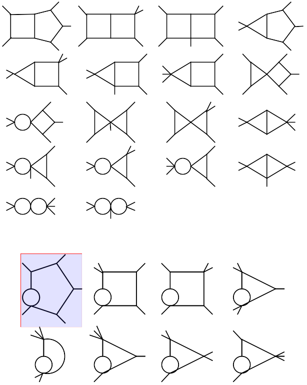
57 distinct topologies, 425 Δ (all permutations)

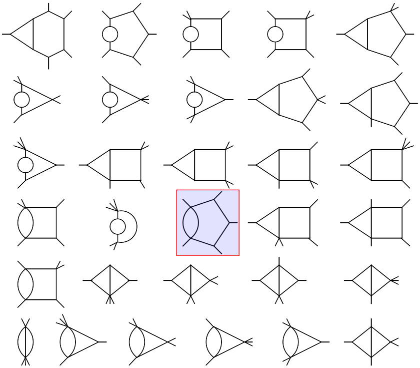
Leading colour 2-loop 5-gluon amplitudes

[Badger,Brønnum-Hansen,HBH,Peraro, Phys.Rev.Lett. 120 (2018) no.9, 092001]

$$\mathcal{A}^{(L)}(1,2,3,4,5) = n^L g_s^3 \sum_{\sigma \in S_5/Z_5} \mathrm{Tr}\left(T^{a_{\sigma(1)}} \cdots T^{a_{\sigma(5)}}\right) A^{(L)}\left(\sigma(1),\sigma(2),\sigma(3),\sigma(4),\sigma(5)\right)$$

$$n = m_\epsilon N_c \alpha_s/(4\pi), \quad m_\epsilon = i(4\pi)^\epsilon e^{-\epsilon\gamma_E}$$





57 distinct topologies, 425 Δ (all permutations)

Leading Colour 2-loop 5-gluon: Euclidean Region

$$A^{(2)} = A^{(2),[0]} + (d_s - 2)A^{(2),[1]} + (d_s - 2)^2 A^{(2),[2]}$$

	ϵ^{-4}	ϵ^{-3}	ϵ^{-2}	ϵ^{-1}	ϵ^{0}
$\widehat{A}^{(2),[0]}_{--+++}$	12.5	27.7526	-23.7728	-168.1162	-175.2103
$P^{(2),[0]}_{--+++}$	12.5	27.7526	-23.7728	-168.1163	—
$\widehat{A}^{(2),[0]}_{-+-++}$	12.5	27.7526	2.5028	-35.8084	69.6695
$P^{(2),[0]}_{-+-++}$	12.5	27.7526	2.5028	-35.8086	—

	ϵ^{-4}	ϵ^{-3}	ϵ^{-2}	ϵ^{-1}	ϵ^{0}
$\widehat{A}^{(2),[1]}_{+++++}$	0	0	-2.5	-6.4324	-5.3107
$P^{(2),[1]}_{+++++}$	0	0	-2.5	-6.4324	—
$\widehat{A}^{(2),[1]}_{-++++}$	0	0	-2.5	-12.7492	-22.0981
$P^{(2),[1]}_{-++++}$	0	0	-2.5	-12.7492	—
$\widehat{A}^{(2),[1]}_{--+++}$	0	-0.625	-1.8175	-0.4869	3.1270
$P^{(2),[1]}_{--+++}$	0	-0.625	-1.8175	-0.4869	—
$\widehat{A}^{(2),[1]}_{-+-++}$	0	-0.625	-2.7759	-5.0018	0.1807
$P^{(2),[1]}_{-+-++}$	0	-0.625	-2.7759	-5.0018	—

	$\widehat{A}^{(2),[2]}_{+++++}$	$\widehat{A}^{(2),[2]}_{-++++}$	$\widehat{A}^{(2),[2]}_{--+++}$	$\widehat{A}^{(2),[2]}_{-+-++}$
ϵ^0	3.6255	-0.0664	0.2056	0.0269

$$s_{12} = -1,\, s_{23} = -\frac{37}{78},\, s_{34} = -\frac{2023381}{3194997},$$
$$s_{45} = -\frac{83}{102},\, s_{15} = -\frac{193672}{606645}$$

$$x_1 = -1, \quad x_2 = \frac{79}{90},\, x_3 = \frac{16}{61},\, x_4 = \frac{37}{78},\, x_5 = \frac{83}{102}$$

- $\Rightarrow$ reduce $\mathcal{O}(10^4)$ integrals to $\mathcal{O}(10^2)$ MIs using `FIRE` & `Reduze` (mostly analytic, some high-rank integrals numerical)
- $\Rightarrow$ Analytic MIs from [Gehrmann,Henn,Lo Presti] or alternatively [Papadopoulos,Tommasini,Wever] 6d/8d MIs to 4d MIs using `LiteRed` [Lee]
- $\Rightarrow$ IR poles [Catani,Becher,Neubert,Gardi,Magnea] with d_s dependence, extracted from FDH results [Gnendiger,Signer,Stockinger]
- $\Rightarrow$ verified by Abreu,Febres-Cordero,Ita,Page, Zeng [arxiv:1712.05721]
- $\Rightarrow$ by-passing original numerical evaluation with `Fiesta`/`SecDec`

Leading Colour 2-loop 5-gluon: Physical Region

MIs from Gehrmann,Henn,Lo Presti

$$A^{(2)} = A^{(2),[0]} + (d_s - 2)A^{(2),[1]} + (d_s - 2)^2 A^{(2),[2]}$$

$$s_{12} = \frac{113}{7}, \quad s_{23} = -\frac{152679950}{96934257}, \quad s_{34} = \frac{1023105842}{138882415}, \quad s_{45} = \frac{10392723}{3968069}, \quad s_{15} = -\frac{8362}{32585}$$

$$x_1 = \frac{113}{7}, \quad x_2 = -\frac{2}{9} - \frac{i}{19}, \quad x_3 = -\frac{1}{7} - \frac{i}{5}, \quad x_4 = \frac{1351150}{13847751}, \quad x_5 = -\frac{91971}{566867}$$

	ϵ^{-4}	ϵ^{-3}	ϵ^{-2}	ϵ^{-1}	ϵ^{0}
$\widehat{A}^{(2),[0]}_{--+++}$	12.5	-9.17716 + 47.12389 i	-107.40046 - 25.96698 i	17.24014 - 221.41370 i	388.44694 - 167.45494 i
$P^{(2),[0]}_{--+++}$	12.5	-9.17716 + 47.12389 i	-107.40046 - 25.96698 i	17.24013 - 221.41373 i	—
$\widehat{A}^{(2),[0]}_{-+-++}$	12.5	-9.17716 + 47.12389 i	-111.02853 - 12.85282 i	-39.80016 - 216.36601 i	342.75366 - 309.25531 i
$P^{(2),[0]}_{-+-++}$	12.5	-9.17716 + 47.12389 i	-111.02853 - 12.85282 i	-39.80018 - 216.36604 i	—

	ϵ^{-4}	ϵ^{-3}	ϵ^{-2}	ϵ^{-1}	ϵ^{0}
$\widehat{A}^{(2),[1]}_{+++++}$	0	0	-2.5	0.60532 - 12.48936 i	35.03354 + 9.27449 i
$P^{(2),[1]}_{+++++}$	0	0	-2.5	0.60532 - 12.48936 i	—
$\widehat{A}^{(2),[1]}_{-++++}$	0	0	-2.5	-7.59409 - 2.99885 i	-0.44360 - 20.85875 i
$P^{(2),[1]}_{-++++}$	0	0	-2.5	-7.59408 - 2.99885 i	—
$\widehat{A}^{(2),[1]}_{--+++}$	0	-0.625	-0.65676 - 0.42849 i	-1.02853 + 0.30760 i	-0.55509 - 6.22641 i
$P^{(2),[1]}_{--+++}$	0	-0.625	-0.65676 - 0.42849 i	-1.02853 + 0.30760 i	—
$\widehat{A}^{(2),[1]}_{-+-++}$	0	-0.625	-0.45984 - 0.97559 i	1.44962 + 0.53917 i	-0.62978 + 2.07080 i
$P^{(2),[1]}_{-+-++}$	0	-0.625	-0.45984 - 0.97559 i	1.44962 + 0.53917 i	—

	$\widehat{A}^{(2),[2]}_{+++++}$	$\widehat{A}^{(2),[2]}_{-++++}$	$\widehat{A}^{(2),[2]}_{--+++}$	$\widehat{A}^{(2),[2]}_{-+-++}$
ϵ^{0}	0.60217 - 0.01985 i	-0.10910 - 0.01807 i	-0.06306 - 0.01305 i	-0.03481 - 0.00699 i

Leading colour 2-loop $qggg\bar{q}$: Euclidean Region

$$\mathcal{A}^{(L)}(1_q, 2_g, 3_g, 4_g, 5_{\bar{q}}) = n^L g_s^3 \sum_{\sigma \in S_3} \left(T^{a_{\sigma(2)}} T^{a_{\sigma(3)}} T^{a_{\sigma(4)}}\right)_{i_1}^{\bar{i}_5} A^{(L)}(1_q, \sigma(2)_g, \sigma(3)_g, \sigma(4)_g, 5_{\bar{q}})$$

$$n = m_\epsilon N_c \alpha_s/(4\pi), \quad m_\epsilon = i(4\pi)^\epsilon e^{-\epsilon\gamma_E}$$

$$s_{12} = -1, \quad s_{23} = -\frac{37}{78}, \quad s_{34} = -\frac{2023381}{3194997}, \quad s_{45} = -\frac{83}{102}, \quad s_{15} = -\frac{193672}{606645}$$

$$x_1 = -1, \quad x_2 = \frac{79}{90}, \quad x_3 = \frac{16}{61}, \quad x_4 = \frac{37}{78}, \quad x_5 = \frac{83}{102}$$

	ϵ^{-4}	ϵ^{-3}	ϵ^{-2}	ϵ^{-1}	ϵ^0
$\widehat{A}^{(2)}_{++++-}$	0	0	-4	-13.5322768031	6.048656403
$P^{(2)}_{++++-}$	0	0	-4	-13.5322768028	—
$\widehat{A}^{(2)}_{+++--}$	8	7.9682909085	-52.3927085027	-140.1563714534	47.5687220127
$P^{(2)}_{+++--}$	8	7.9682909085	-52.3927085034	-140.1563714829	—
$\widehat{A}^{(2)}_{++-+-}$	8	7.9682909085	-32.2213536407	-47.9234973502	145.9720111187
$P^{(2)}_{++-+-}$	8	7.9682909085	-32.2213536403	-47.9234973889	—
$\widehat{A}^{(2)}_{+-++-}$	8	7.9682909084	-40.8851109385	-87.0299398048	101.2329971544
$P^{(2)}_{+-++-}$	8	7.9682909085	-40.8851109386	-87.0299398374	—

Fully numerical, IR poles checked in HV scheme, preliminary!

Leading colour 2-loop $q\bar{q}gQ\bar{Q}$: Euclidean Region

$$\mathcal{A}^{(L)}(1_q, 2_{\bar{q}}, 3_g, 4_Q, 5_{\bar{Q}}) = n^L g_s^3 \left[(T^{a_3})_{i_4}^{\;\bar{i}_2} \delta_{i_1}^{\;\bar{i}_5} A^{(L)}(1_q, 2_{\bar{q}}, 3_g, 4_Q, 5_{\bar{Q}}) + (1 \leftrightarrow 4, 2 \leftrightarrow 5) \right]$$

$$n = m_\epsilon N_c \alpha_s/(4\pi), \quad m_\epsilon = i(4\pi)^\epsilon e^{-\epsilon\gamma_E}$$

$$s_{12} = -1, \quad s_{23} = -\frac{37}{78}, \quad s_{34} = -\frac{2023381}{3194997}, \quad s_{45} = -\frac{83}{102}, \quad s_{15} = -\frac{193672}{606645}$$

$$x_1 = -1, \quad x_2 = \frac{79}{90}, \quad x_3 = \frac{16}{61}, \quad x_4 = \frac{37}{78}, \quad x_5 = \frac{83}{102}$$

	ϵ^{-4}	ϵ^{-3}	ϵ^{-2}	ϵ^{-1}	ϵ^{0}
$\widehat{A}^{(2)}_{+-++-}$	4.5	2.2831548748	-32.0984879203	-41.3935016796	149.3305056672
$P^{(2)}_{+-++-}$	4.5	2.2831548747	-32.0984879200	-41.3935017002	—
$\widehat{A}^{(2)}_{+--+-}$	4.5	2.2831548747	-4.6165799244	-6.3236903564	-32.0327897453
$P^{(2)}_{+--+-}$	4.5	2.2831548747	-4.6165799294	-6.3236903481	—
$\widehat{A}^{(2)}_{+-+-+}$	4.5	2.2831548747	-38.294786128	-43.5232972386	-56.7196838792
$P^{(2)}_{+-+-+}$	4.5	2.2831548747	-38.294786128	-43.5232972694	—
$\widehat{A}^{(2)}_{+---+}$	4.5	2.2831548747	-26.7131604905	-69.7580529817	22.2365337061
$P^{(2)}_{+---+}$	4.5	2.2831548747	-26.7131604895	-69.758052969	—

Fully numerical, IR poles checked in HV scheme, preliminary!

Can We Improve Integrand Basis?

So far ...

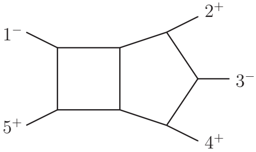
- Set up integrand reduction framework for massless 2-loop 5-point amplitudes
- Results for $ggggg$, $qggg\bar{q}$ and $q\bar{q}Q\bar{Q}g$ partonic channels

Issues ...

- analytic form of the integrands are very large
- large cancellation between topologies
- $\mathcal{O}(\epsilon)$ amplitudes are not manifest ($d_s = 2$, +++++, −++++)
- high-rank integrals
  $\rightarrow$ difficult to solve IBP systems analytically
  $\rightarrow$ very large analytic expressions

Improving Integrand Basis (1)

Pentagon-box topology



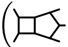
Old Basis

54 non-zero integrand coeffs
28 non-zero coeffs at $\mathcal{O}(\epsilon^0)$
worst case:
$(k_2 \cdot p_2)^4 \mu_{11}$ (rank 4, 1 dot)

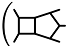
New Basis

49 non-zero integrand coeffs
3 non-zero coeffs at $\mathcal{O}(\epsilon^0)$
$k_1 \cdot n_1 \; k_2 \cdot n_2$ (rank 2)
μ_{11} (1 dot)

$$\mathrm{ISP} = \{k_1 \cdot p_1, k_2 \cdot p_2, k_2 \cdot p_3, \mu_{11}, \mu_{12}, \mu_{22}\}$$

I () $[(k_2 \cdot p_2)^4 \mu_{11}] = \frac{\#}{\epsilon} + \#$

but

I () $[\langle 1|k_2|5]^4 \mu_{11}] = \mathcal{O}(\epsilon)$

$$\langle 1|k_2|5] = A + B\ k_2 \cdot p_2 + C\ k_2 \cdot p_3 + \mathrm{props}$$

New ISP basis

$$\{k_1 \cdot n_1, k_1 \cdot n_1^*, k_2 \cdot n_2, k_2 \cdot n_2^*, \mu_{11}, \mu_{12}, \mu_{22}\}$$

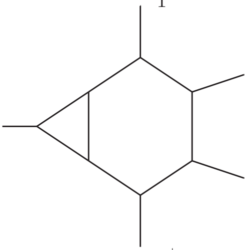
$$k_2 \cdot n_2 = \langle 1|k_2|5]$$
$$k_1 \cdot n_1 = \mathrm{tr}_+ (2\,(k_1 - p_2)\,(k_1 - p_{23})\,4)$$

$$\mathrm{tr}_+ (ijkl) = [i|j|k|l|i\rangle$$

[Arkani-Hamed,Bourjaily,Cachazo,Trnka,2012]
[Bourjaily,Hermann,Trnka,2017] [Badger,Peraro,Mogull,2016]

Improving Integrand Basis (2)

Hexagon-triangle topology



Old Basis

16 non-zero integrand coeffs

16 non-zero coeffs at $\mathcal{O}(\epsilon^0)$

worst case:

$(k_2 \cdot p_1)^3 \mu_{11}$ (rank 3, 1 dot)

New Basis

10 non-zero integrand coeffs

0 non-zero coeffs at $\mathcal{O}(\epsilon^0)$

$$\mathrm{ISP} = \{k_2 \cdot p_1, k_2 \cdot p_2, k_2 \cdot p_3, \mu_{11}, \mu_{12}, \mu_{22}\}$$

New ISP basis

$$\{k_2 \cdot n_1, k_2 \cdot n_1^*, k_2 \cdot p_1, \mu_{11}, \mu_{12}, \mu_{22}\}$$

$$k_2 \cdot n_1 = \langle 1|k_2|5]$$

Summary:

- D-dimensional integrand reduction method at two loops + generalized unitarity cuts
- Analytic reconstructions of integrand coefficients from finite-field evaluations
- Benchmark results for massless 2-loop 5-point amplitudes
  $\Rightarrow ggggg$, $qggg\bar{q}$, $q\bar{q}gQ\bar{Q}$

Summary:

- D-dimensional integrand reduction method at two loops + generalized unitarity cuts
- Analytic reconstructions of integrand coefficients from finite-field evaluations
- Benchmark results for massless 2-loop 5-point amplitudes
  $\Rightarrow ggggg$, $qggg\bar{q}$, $q\bar{q}gQ\bar{Q}$

Outlook:

- Improve integrand representation (local basis?)
- Non-planar?, Off-shell leg?

Summary:

- D-dimensional integrand reduction method at two loops + generalized unitarity cuts
- Analytic reconstructions of integrand coefficients from finite-field evaluations
- Benchmark results for massless 2-loop 5-point amplitudes
  $\Rightarrow ggggg$, $qggg\bar{q}$, $q\bar{q}gQ\bar{Q}$

Outlook:

- Improve integrand representation (local basis?)
- Non-planar?, Off-shell leg?

THANK YOU!!!