Resummation of Non-global Logarithms in Effective Field Theory

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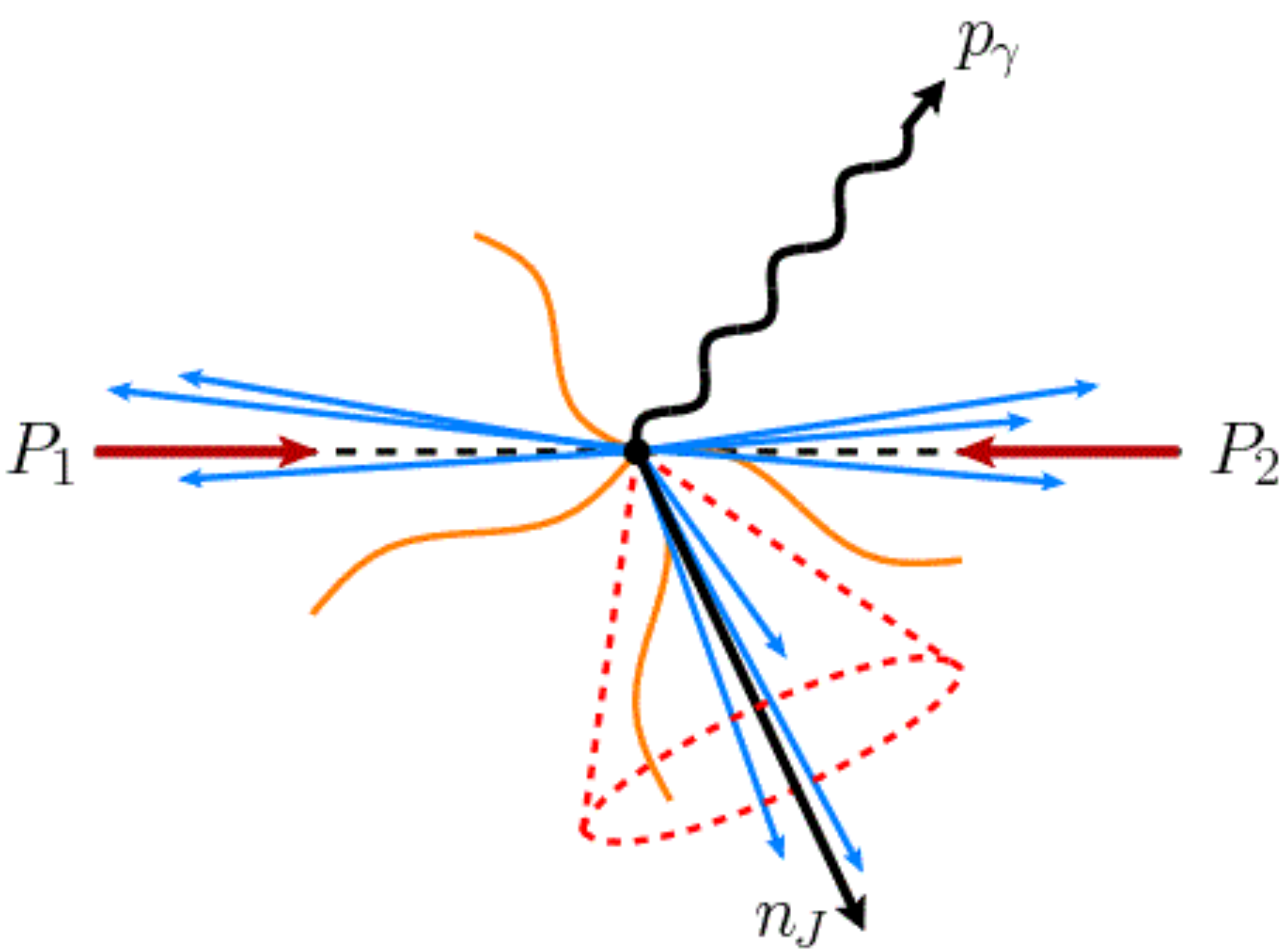
QCD@LHC 2018

Dresden 30.08.2018

A brief review

Non-global observable: Observables which are insensitive to emissions into certain regions of phase space



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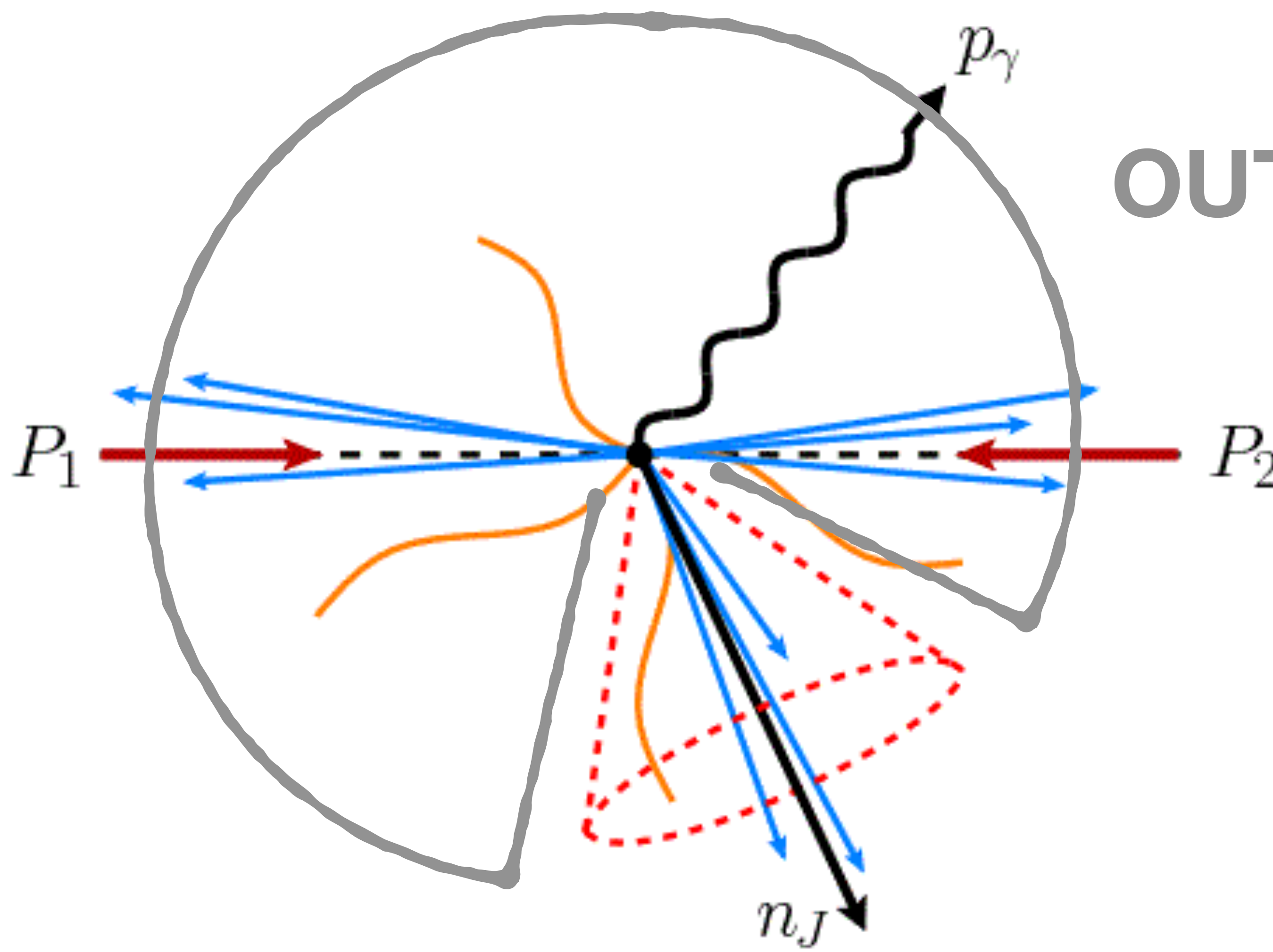

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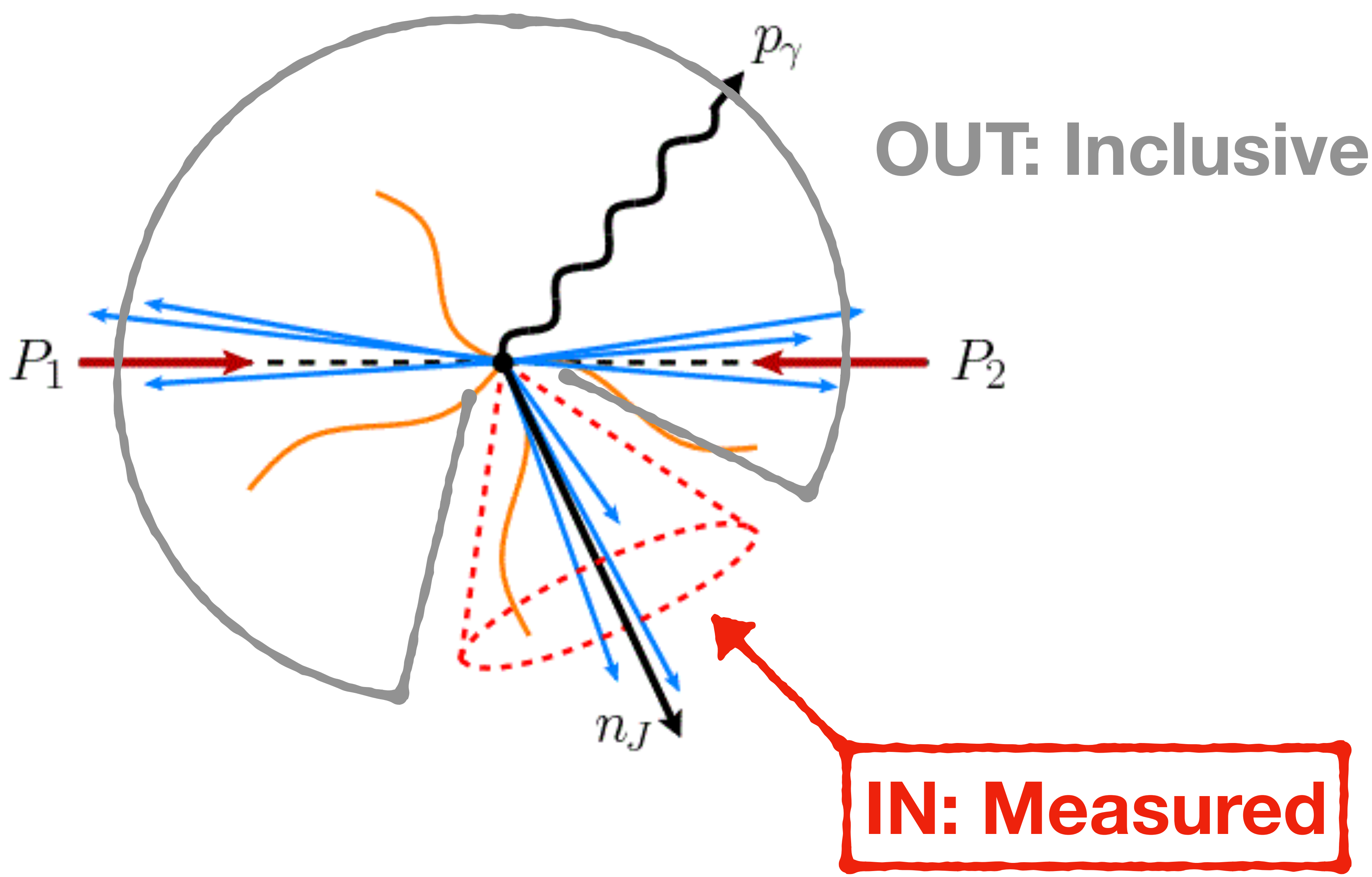
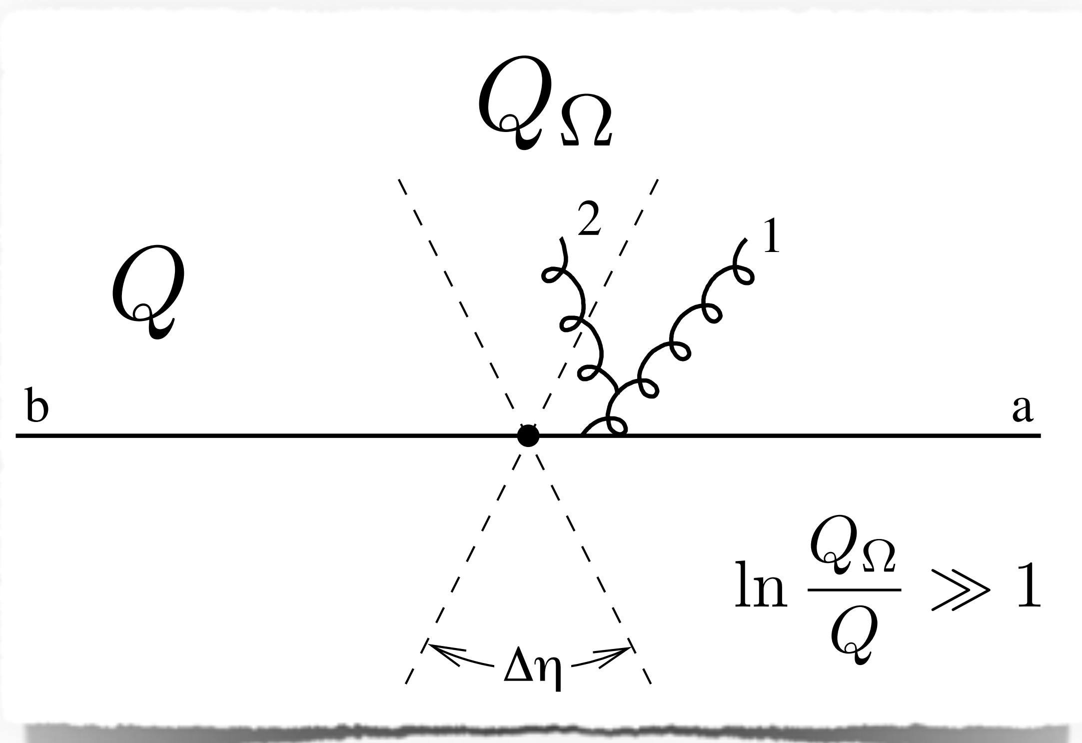
Non-global observable: Observables which are insensitive to emissions into certain regions of phase space



Non-Global Logs

- Non-global observables involve additional NGLs not captured by the usual resummation formula
- Exponentiating soft anomalous dimension only resum part of logs

$$\exp\left[-4\,C_F\Delta\eta\int_{\alpha(Q_\Omega)}^{\alpha(Q)}\frac{d\alpha}{\beta(\alpha)}\frac{\alpha}{2\pi}\right] = 1+4\frac{\alpha_s}{2\pi}C_F\Delta\eta\ln\frac{Q_\Omega}{Q}$$
$$+\left(\frac{\alpha_s}{2\pi}\right)^2\left(8\,C_F^2\Delta\eta^2-\frac{22}{3}\,C_FC_A\Delta\eta+\frac{8}{3}C_FT_Fn_f\Delta\eta\right)\ln^2\frac{Q_\Omega}{Q}$$



Non-global logs:

$$\left(\frac{\alpha_s}{2\pi}\right)^2 C_FC_A\left[-\frac{2\pi^2}{3}+4\,\mathrm{Li}_2\left(e^{-2\Delta\eta}\right)\right]\ln^2\frac{Q_\Omega}{Q}$$

(Dasgupta & Salam 2002)

LL resummation for non-global observables

- The leading logarithms arise from configuration in which the emitted gluons are strongly ordered

$$E_1 \gg E_2 \gg \cdots \gg E_m$$

- In the large-Nc limit, multi-gluon emission amplitudes become simple:

$$N_C^m g_s^{2m} \sum_{(1,\cdots,m)} \frac{p_a \cdot p_b}{(p_a \cdot p_1)(p_1 \cdot p_2)\cdots(p_m \cdot p_b)}$$

- Dasgupta-Salam shower

$$\mathcal{S}(\alpha_s L) \simeq \exp\left(-C_F C_A \frac{\pi^2}{3}\left(\frac{1+(at)^2}{1+(bt)^c}\right)t^2\right) \qquad a = 0.85\, C_A, \qquad b = 0.86\, C_A, \quad c = 1.33$$

(Dasgupta & Salam 2001)

- Banfi-Marchesini-Smye eqation

$$\partial_{\hat{L}} G_{kl}(\hat{L}) = \int \frac{d\Omega(n_j)}{4\pi} W_{kl}^j \left[\Theta_{\rm in}^{n\bar{n}}(j)\, G_{kj}(\hat{L})\, G_{jl}(\hat{L}) - G_{kl}(\hat{L})\right]$$

(Banfi, Marchesini & Smye 2002)

Some recent progress

- Dressed gluon expansion Larkoski, Moult & Neill '15 '16
- Color density matrix Caron-Huot '15
- Multi-Wilson-line theory in SCET Becher, Neubert, Rothen & DYS '15 '16
- Finite Nc results for hemisphere mass and inter-jet energy flow Hatta, Ueda '13, + Hagiwara '15
- Soft (Glauber) gluon evolution at amplitude level, finite Nc Martínez, Angelis, Forshaw, Plätzer & Seymour '18 **See Angelis's Talk**
- Reduced density matrix Neill & Vaidya '18

Soft radiations inside Non-global observables

- Non-global observables: soft radiation resolves the colors and directions of individual energetic partons.

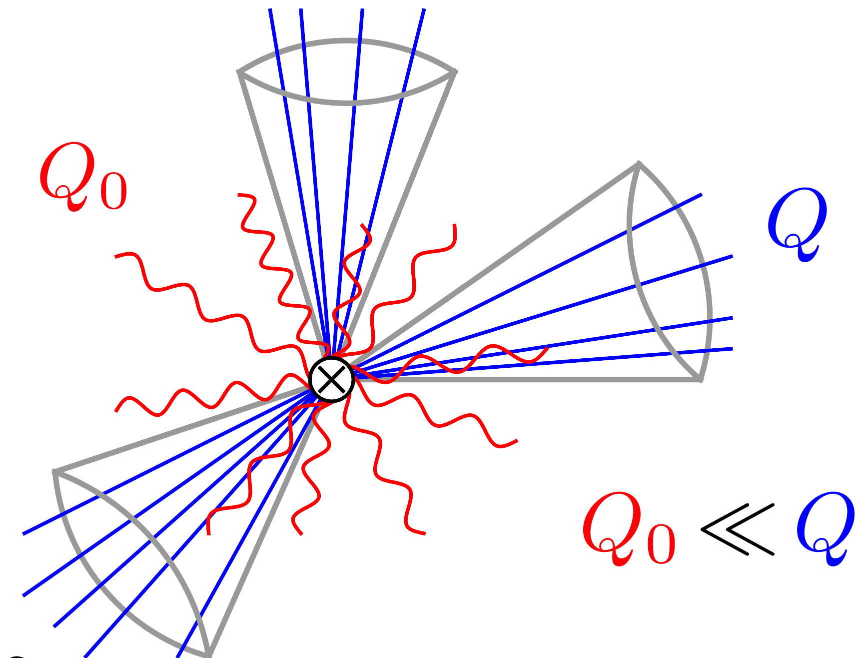
$$\sum_i Q_i \frac{p_i \cdot \epsilon(k)}{p_i \cdot k} \sim Q_{\text{tot}} \frac{n \cdot \epsilon}{n \cdot k}$$

- For a wide-angle jet, the energetic particles are not collinear.

- For a narrow-angle jets, we find that small-angle soft radiation plays an important role. Resolves directions of individual energetic partons!

  Becher, Neubert, Rothen & DYS '15; Chien, Hornig & Lee '15

Factorization for jet cross section



- For k jets process at lepton collider

$$d\sigma(Q,Q_0) = \sum_{m=k}^{\infty} \langle \boldsymbol{\mathcal{H}}_m(\{\underline{n}\},Q,\mu) \otimes \boldsymbol{\mathcal{S}}_m(\{\underline{n}\},Q_0,\mu) \rangle$$

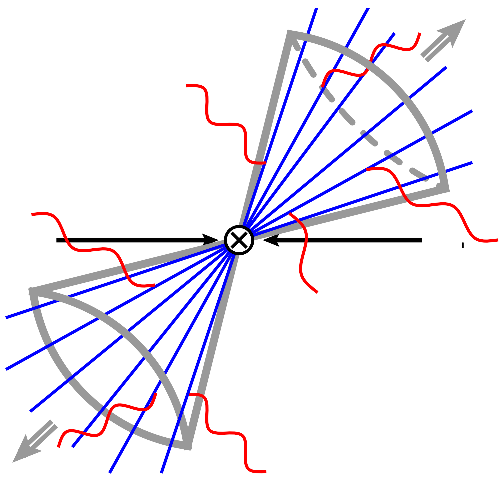
- Hard function: integrating over the energies of the hard particles, while keeping their direction fixed

$$\boldsymbol{\mathcal{H}}_m(\{\underline{n}\},Q,\mu) = \frac{1}{2Q^2} \sum_{\text{spins}} \prod_{i=1}^{m} \int \frac{dE_i\, E_i^{d-3}}{(2\pi)^{d-2}} \, |\mathcal{M}_m(\{\underline{p}\})\rangle\langle\mathcal{M}_m(\{\underline{p}\})| \, (2\pi)^d \, \delta\Big(Q - \sum_{i=1}^{m} E_i\Big) \, \delta^{(d-1)}(\vec{p}_{\text{tot}}) \, \Theta_{\text{in}}\big(\{\underline{p}\}\big)$$

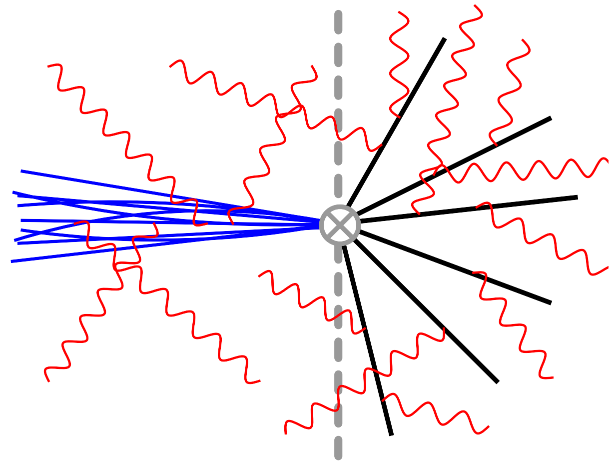
- Soft function

$$\boldsymbol{\mathcal{S}}_m(\{\underline{n}\},Q_0,\mu) = \sum\!\!\!\!\!\!\!\int_{X_s} \langle 0| \, \boldsymbol{S}_1^\dagger(n_1) \, \ldots \, \boldsymbol{S}_m^\dagger(n_m) \, |X_s\rangle\langle X_s| \, \boldsymbol{S}_1(n_1) \, \ldots \, \boldsymbol{S}_m(n_m) \, |0\rangle \, \theta(Q_0 - E_{\text{out}})$$

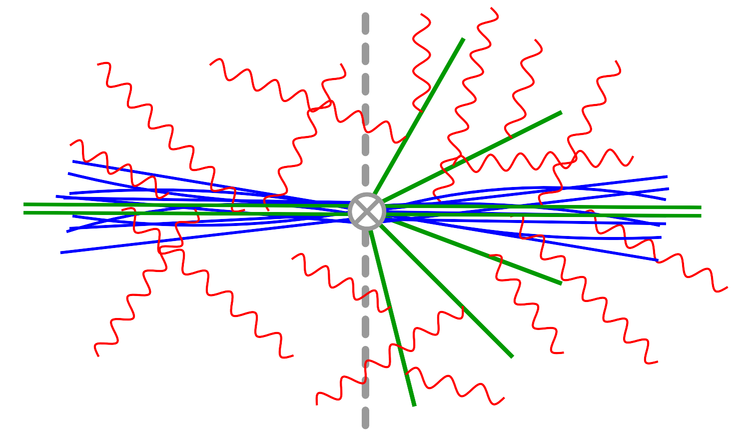
- $\otimes$ indicates integration over the direction of the energetic partons



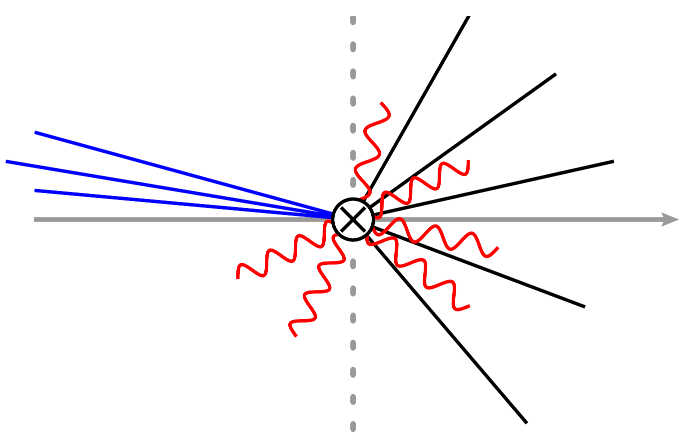
Jet cross section &
Gap fraction



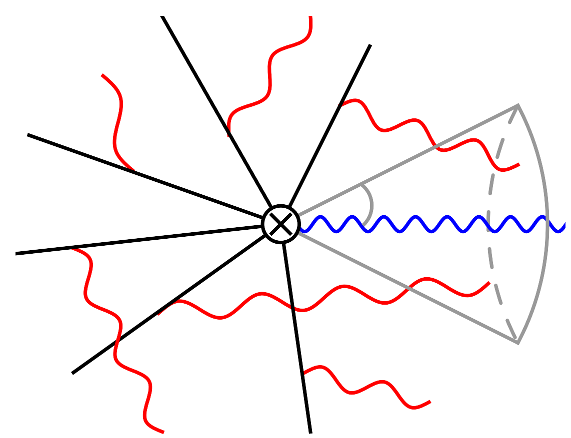
Light-jet mass



Hemisphere soft function



Narrow broadening



Photon isolation

Becher, Neubert, Rothen,
DYS ’15 ’16

Becher, Pecjak, DYS ’16

Becher, Rahn, DYS ’17

Balsiger, Becher, DYS, ‘18

RG evolution & Resummation

Then resummation cross section can be written as

$$d\sigma(Q,Q_0) = \sum_{l=k,\,m\geq l}^{\infty} \left\langle \boldsymbol{\mathcal{H}}_l(\{\underline{n}\},Q,\mu_h) \otimes \boldsymbol{U}_{lm}(\{\underline{n}\},\mu_s,\mu_h) \,\hat{\otimes}\, \boldsymbol{\mathcal{S}}_m(\{\underline{n}\},Q_0,\mu_s) \right\rangle$$

Wilson coefficients fulfill Renormalization Group equation

$$\frac{d}{d\ln\mu}\boldsymbol{\mathcal{H}}_m(\{\underline{n}\},Q,\mu) = -\sum_{l=k}^{m} \boldsymbol{\mathcal{H}}_l(\{\underline{n}\},Q,\mu)\,\boldsymbol{\Gamma}^H_{lm}(\{\underline{n}\},Q,\mu)$$

1. Compute $\boldsymbol{\mathcal{H}}_m$ at characteristic high scale $\mu_h \sim Q$

2. Evolve $\boldsymbol{\mathcal{H}}_m$ to the scale of low energy physics $\mu_s \sim Q_0$

3. Compute $\boldsymbol{\mathcal{S}}_m$ at $\mu_s \sim Q_0$

Resum large logarithms $\alpha_s^n \ln^m \frac{Q}{Q_0}$

LL resummation

- **LL resummation formula**

$$d\sigma_{\rm LL}(Q,Q_0) = \sum_{m=k}^{\infty} \big\langle \boldsymbol{\mathcal{H}}_k(\{\underline{n}\},Q,\mu_h) \otimes \boldsymbol{U}_{km}(\{\underline{n}\},\mu_s,\mu_h)\,\hat{\otimes}\,\mathbf{1} \big\rangle$$

One-loop anomalous dimension

$$\boldsymbol{\Gamma}^{(1)} = \begin{pmatrix} \boldsymbol{V}_k & \boldsymbol{R}_k & 0 & 0 & \dots \\ 0 & \boldsymbol{V}_{k+1} & \boldsymbol{R}_{k+1} & 0 & \dots \\ 0 & 0 & \boldsymbol{V}_{k+2} & \boldsymbol{R}_{k+2} & \dots \\ 0 & 0 & 0 & \boldsymbol{V}_{k+3} & \dots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

$$\begin{aligned} \boldsymbol{V}_m &= 2\sum_{(ij)} (\boldsymbol{T}_{i,L}\cdot\boldsymbol{T}_{j,L} + \boldsymbol{T}_{i,R}\cdot\boldsymbol{T}_{j,R}) \int \frac{d\Omega(n_l)}{4\pi}\, W_{ij}^l \\ &\qquad - 2\,i\pi \sum_{(ij)} (\boldsymbol{T}_{i,L}\cdot\boldsymbol{T}_{j,L} - \boldsymbol{T}_{i,R}\cdot\boldsymbol{T}_{j,R})\,\Pi_{ij}\,, \\ \boldsymbol{R}_m &= -4\sum_{(ij)} \boldsymbol{T}_{i,L}\cdot\boldsymbol{T}_{j,R}\, W_{ij}^{m+1}\,\Theta_{\rm in}(n_{m+1})\,. \end{aligned}$$

RG evolution = Parton Shower

$$d\sigma_{\rm LL}(Q,Q_0) = \big\langle \boldsymbol{\mathcal{H}}_k(t) + \int \frac{d\Omega_1}{4\pi} \boldsymbol{\mathcal{H}}_{k+1}(t) + \int \frac{d\Omega_1}{4\pi} \int \frac{d\Omega_2}{4\pi} \boldsymbol{\mathcal{H}}_{k+2}(t) + \dots \big\rangle$$

$$\begin{aligned} \boldsymbol{\mathcal{H}}_k(t) &= \boldsymbol{\mathcal{H}}_k(0)\, e^{t\boldsymbol{V}_k} \\ \boldsymbol{\mathcal{H}}_{k+1}(t) &= \int_0^t dt'\, \boldsymbol{\mathcal{H}}_k(t')\, \boldsymbol{R}_k\, e^{(t-t')\boldsymbol{V}_{k+1}} \\ \boldsymbol{\mathcal{H}}_{k+2}(t) &= \int_0^t dt'\, \boldsymbol{\mathcal{H}}_{k+1}(t')\, \boldsymbol{R}_{k+1}\, e^{(t-t')\boldsymbol{V}_{k+2}} \\ \boldsymbol{\mathcal{H}}_{k+3}(t) &= \dots \end{aligned}$$

$$t = \int_{\alpha(\mu)}^{\alpha(Q)} \frac{d\alpha}{\beta(\alpha)} \frac{\alpha}{4\pi}$$

We re-derive Dasgupta-Salam angular dipole shower!!!

RG evolution = Parton Shower

$$\boldsymbol{\mathcal{H}}_m(t) = \boldsymbol{\mathcal{H}}_m(t_0)\, e^{(t-t_0)\boldsymbol{V}_m} + \int_{t_0}^{t} dt'\, \boldsymbol{\mathcal{H}}_{m-1}(t')\, \boldsymbol{R}_{m-1}\, e^{(t-t')\boldsymbol{V}_m}$$

What is a shower?

A parton shower consists of three main features:

1. An ordering variable which defines the sequence according to which emissions are generated (such as k_t, angle, virtuality).
2. A branching probability $P(S_n.v)$ of finding a state S_n with n partons at scale v, which evolves as

$$\frac{dP(\mathcal{S}_n, v)}{d \ln 1/v} = -f(\mathcal{S}_n, v) P(\mathcal{S}_n, v)\,.$$

3. A kinematic mapping $\mathcal{M}$ from state $\mathcal{S}_n$ to $\mathcal{S}_{n+1}$

$$\mathcal{S}_{n+1} = \mathcal{M}(\mathcal{S}_n, v; i, j, \underbrace{z, \phi}_{\text{emission}})\,.$$

with an associated "splitting" weight function $d\mathcal{P}(\mathcal{S}_n, v; i, j, z, \phi)$, governing relative probabilities of new states.

Renormalization Scale or Observable

$\boldsymbol{V}_m$

$\boldsymbol{R}_m$

Frédéric Dreyer

Dreyer' talk on Tuesday

3/12

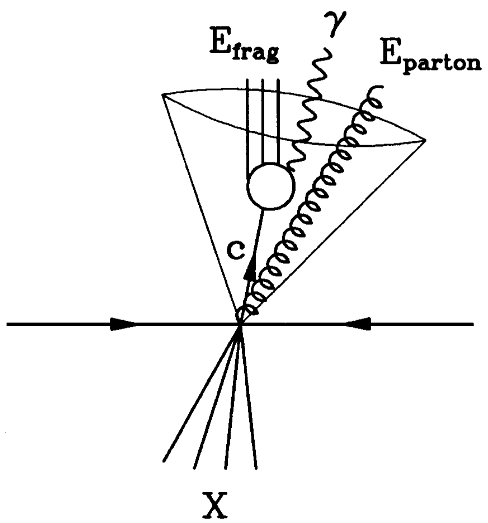
Automated resummation for Non-global observables

(Balsiger, Becher, DYS,'18)

$$d\sigma_{\rm LL}(Q,Q_0) = \sum_{m=k}^{\infty} \big\langle \boldsymbol{\mathcal{H}}_k(\{\underline{n}\},Q,\mu_h) \otimes \boldsymbol{U}_{km}(\{\underline{n}\},\mu_s,\mu_h)\,\hat{\otimes}\,\mathbf{1} \big\rangle$$

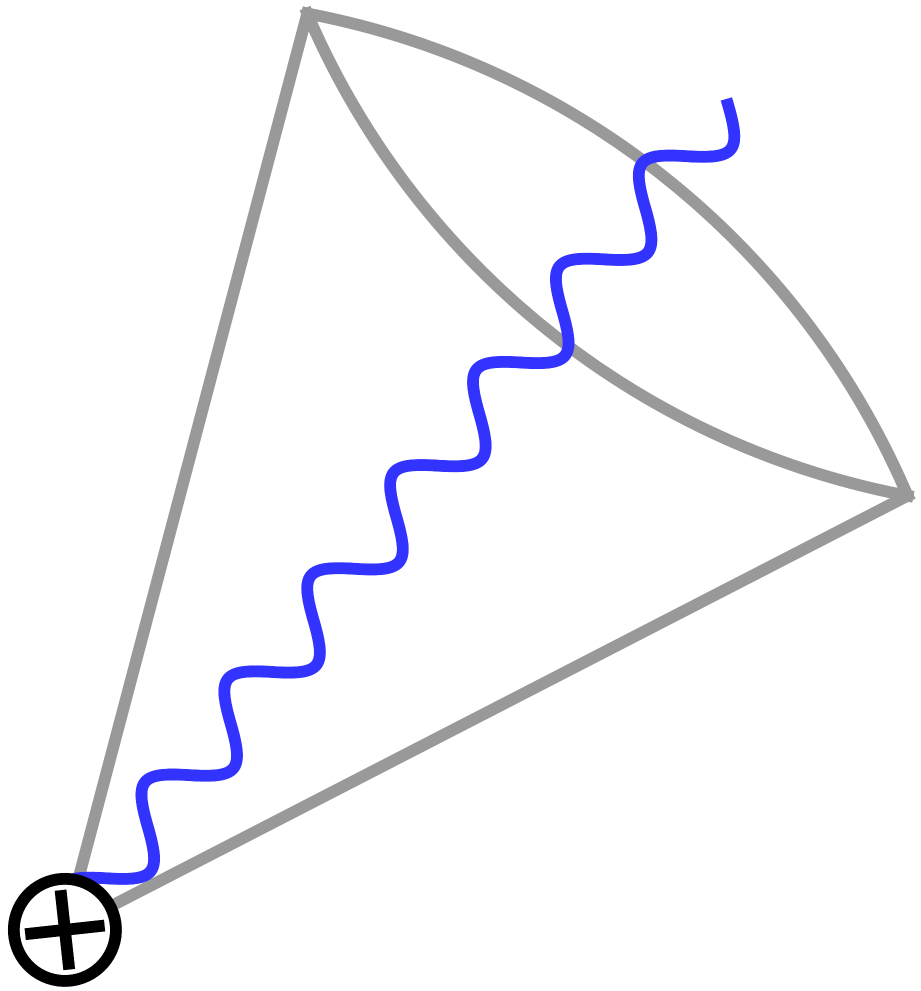
- Use Madgraph5_aMC@NLO generator
  - event file with directions and large-N_c color connections of hard partons
  - provides lowest multiplicity hard function for given process
- Run our shower on each event to generate additional partons and write result back into event file
- Analyze events, according to cuts on hard partons, obtain resummed cross section with hard cuts and veto scale

Resummation in isolation cross section



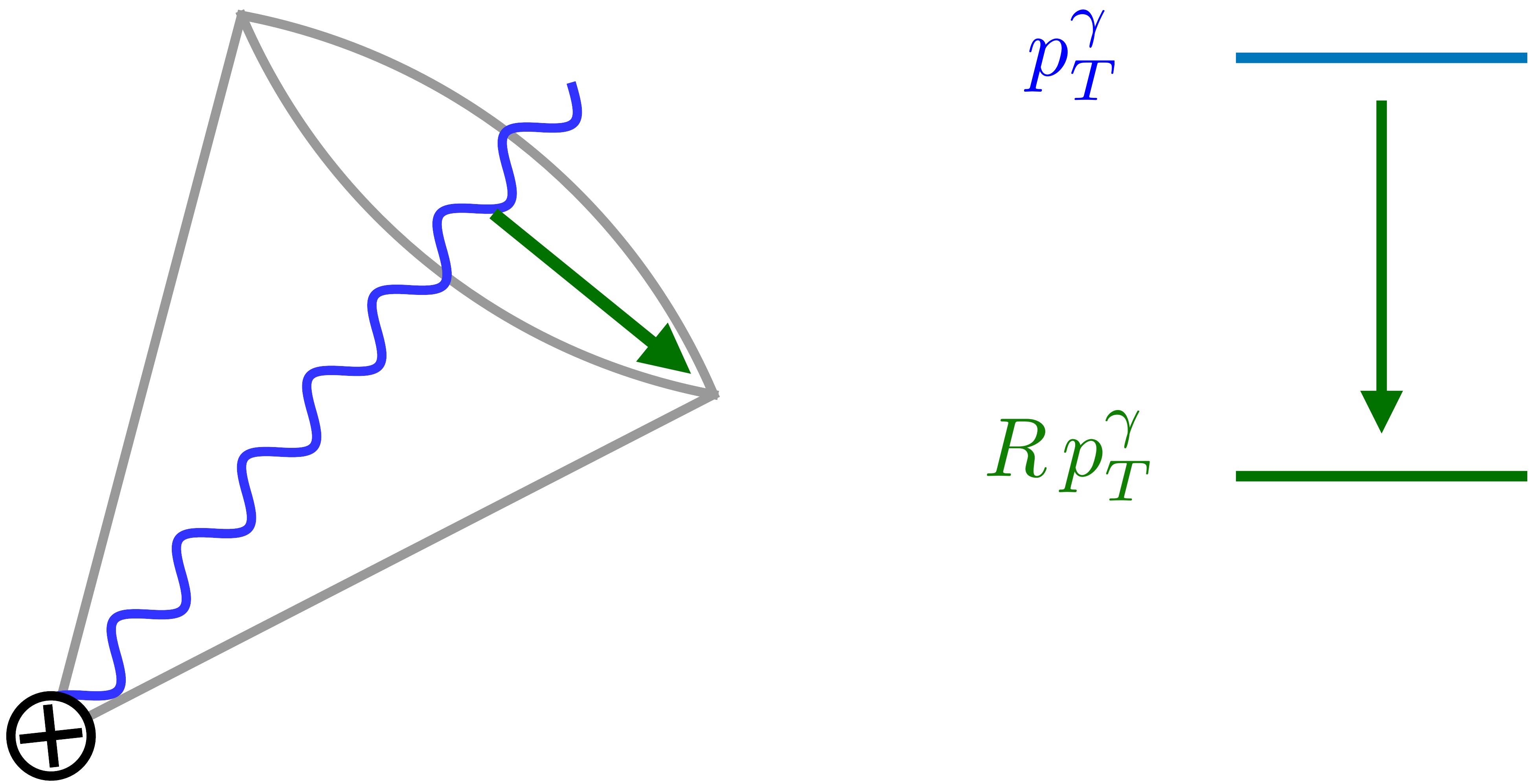
- Experiments use isolation to reduce photon from hard scattering from photons due to hadron decays.
- Experimentalists choose $\sum_{\text{had}\in\mathcal{C}(R)} E_T^{\text{had}} \leq \epsilon_\gamma E_\gamma^T$
    - E.g. ATLAS ’16 imposes $E_{\text{iso}}^T = 4.8\,\text{GeV} + 0.0042\,E_\gamma^T$ on hadronic energy inside cone.
- Smooth isolation Frixione ’98 $E_{\text{iso}}(r) = \epsilon_\gamma E_\gamma \left(\frac{1-\cos r}{1-\cos R}\right)^n$ $\lim_{r\to 0} E_{\text{iso}}(r) \to 0$
    - collinear safe; no fragmentation process
    - not applied in experiments now
- Soft-drop isolation Hall & Thaler ’18
    - democratic criteria; equivalent to smooth isolation at LO

New scales introduced by isolation



p_T^{γ}

New scales introduced by isolation



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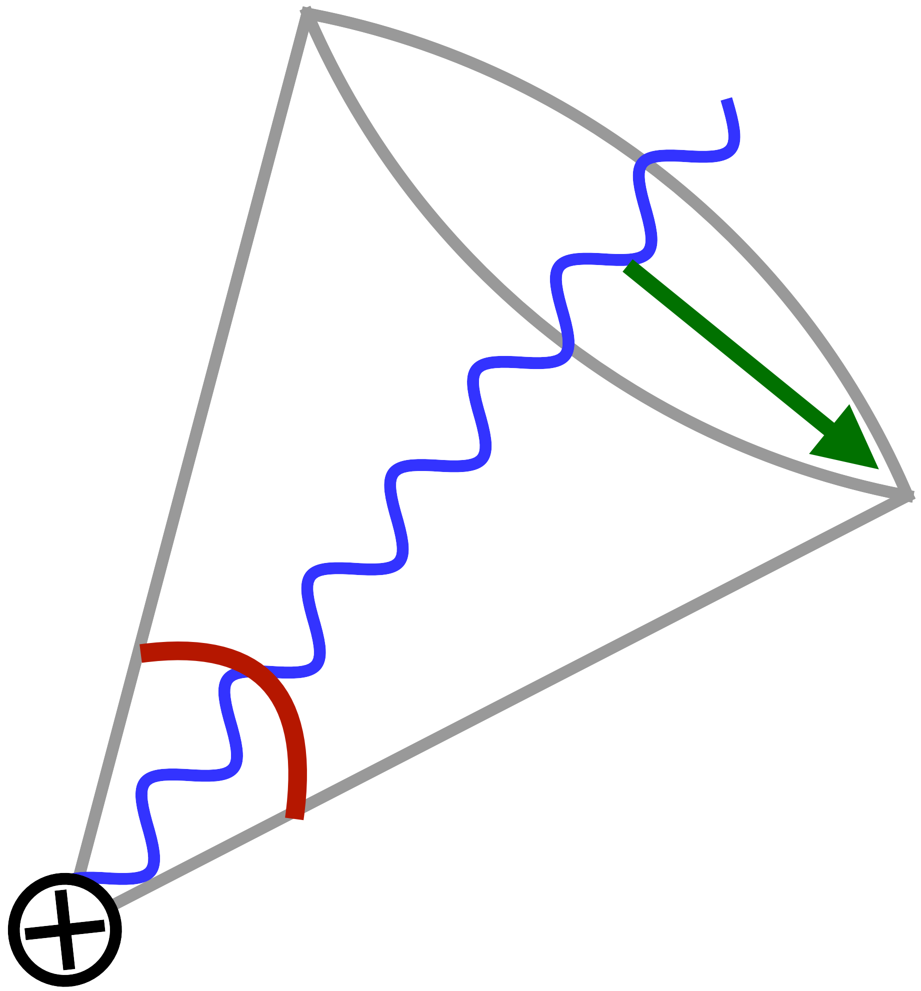

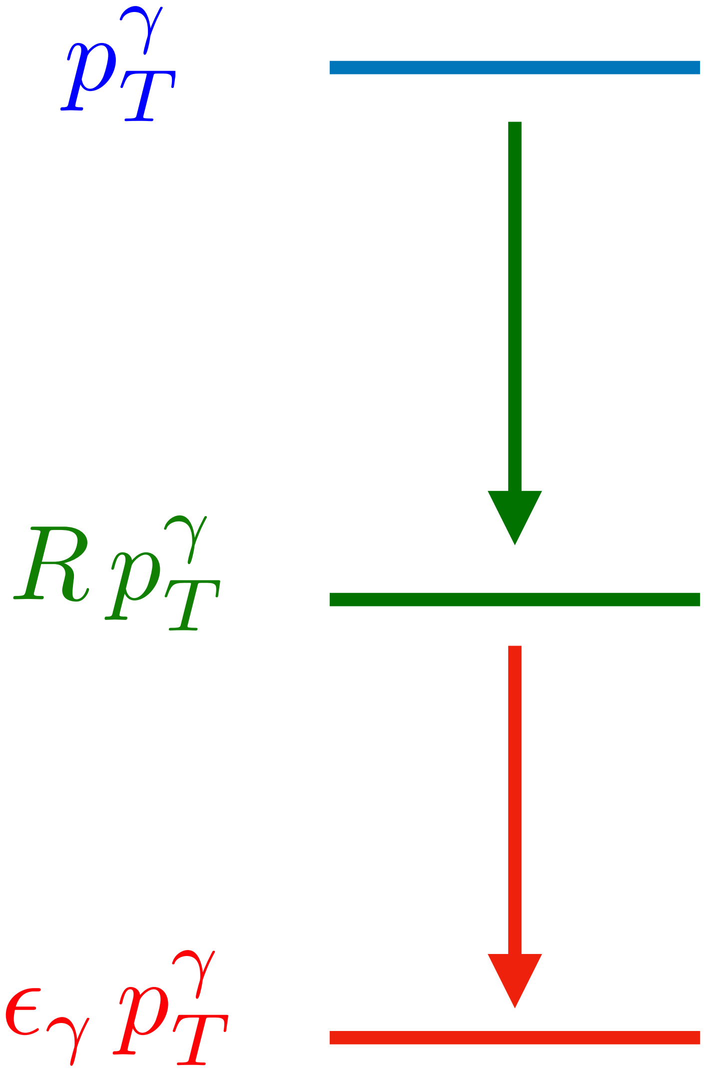

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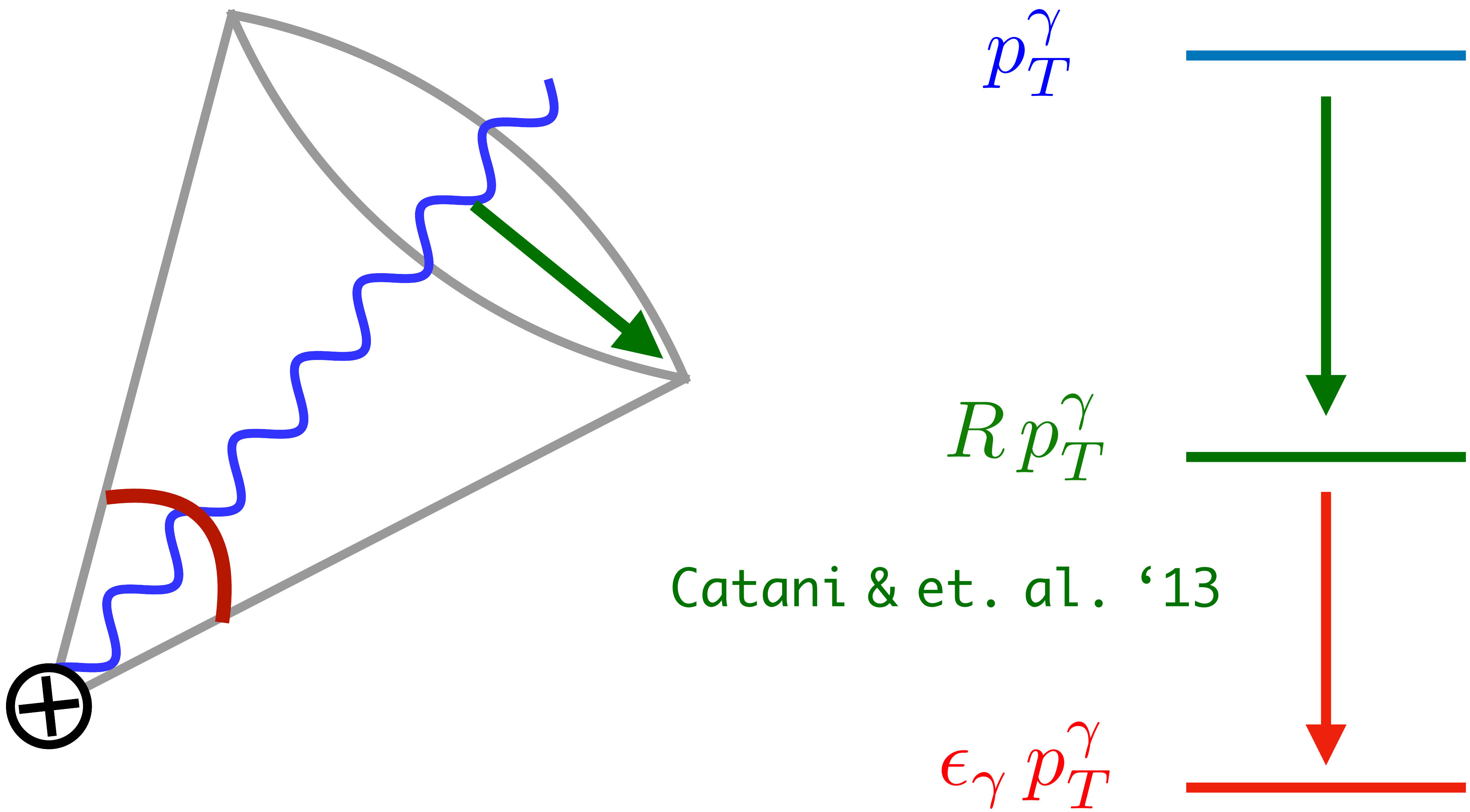

Resummation effects in isolation

- Tight isolation cut $\epsilon_\gamma \ll 1$
- Large logs $\ln \epsilon_\gamma$ from soft gluon radiation inside cone
- Fragmentation process are power suppressed
- At the NLO log term is $\alpha_s R^2 \ln \epsilon_\gamma$ Gordon & Vogelsang '94
- NLO results show no significant infrared sensitivity. Catani & et. al. '02

> destabilize the numerical convergence of the perturbative expansion. Nonetheless, owing to the presence of higher powers of $\ln \varepsilon_h$ at higher perturbative orders, the actual sensitivity of the cross section to very low values of ε_h is probably underestimated in the present NLO calculation.

- Non-global observables: more complicated logarithmic terms will appear beyond NLO

$$R^2 \times \alpha_s^n \ln^n \epsilon_\gamma \ln^{n-1} R$$

(Hatta, Iancu, Mueller, Triantafyllopoulos '17)

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> **definitely**

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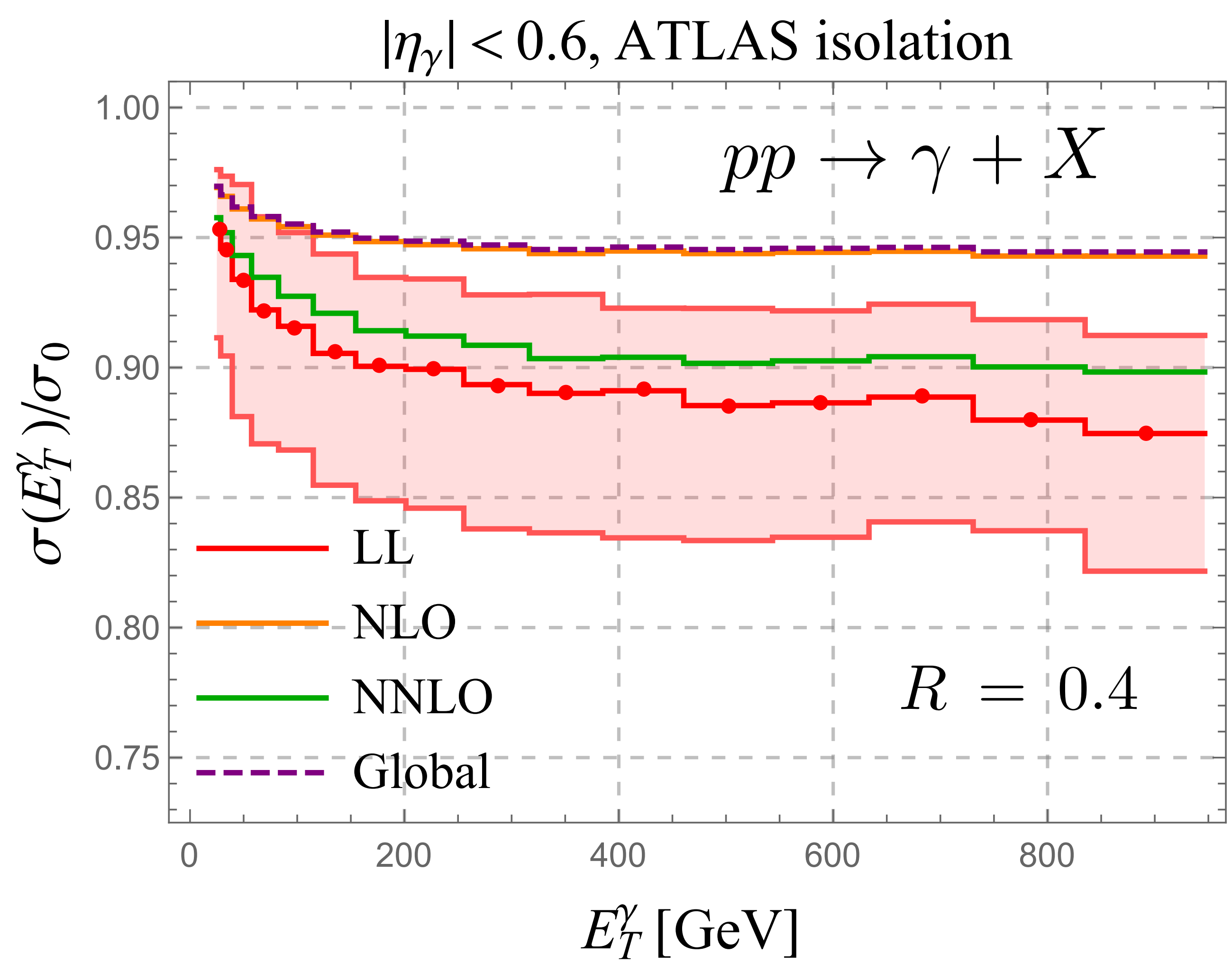
Resummation effects in γ isolation at the LHC



$$1 + \#\alpha_s R^2 \ln \epsilon_\gamma$$
$$+ \#\alpha_s^2 R^2 \ln R \ln^2 \epsilon_\gamma$$

- NLO: ~5% reduction, NNLO ~10%, resummed ~ 12%
- NGL dominates over global contribution: naive exponentiation (dashed)
- LL result suffers from large scale uncertainties

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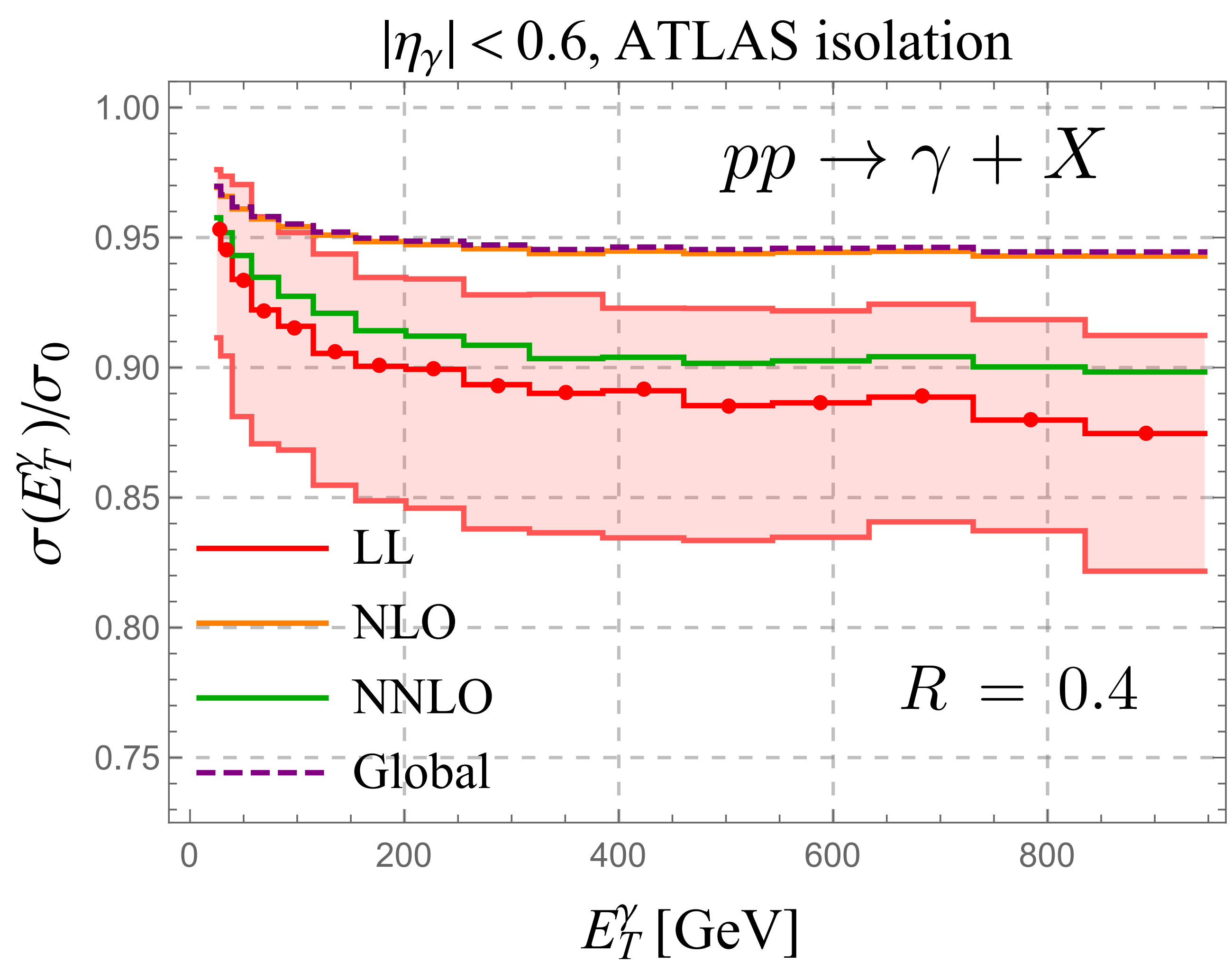

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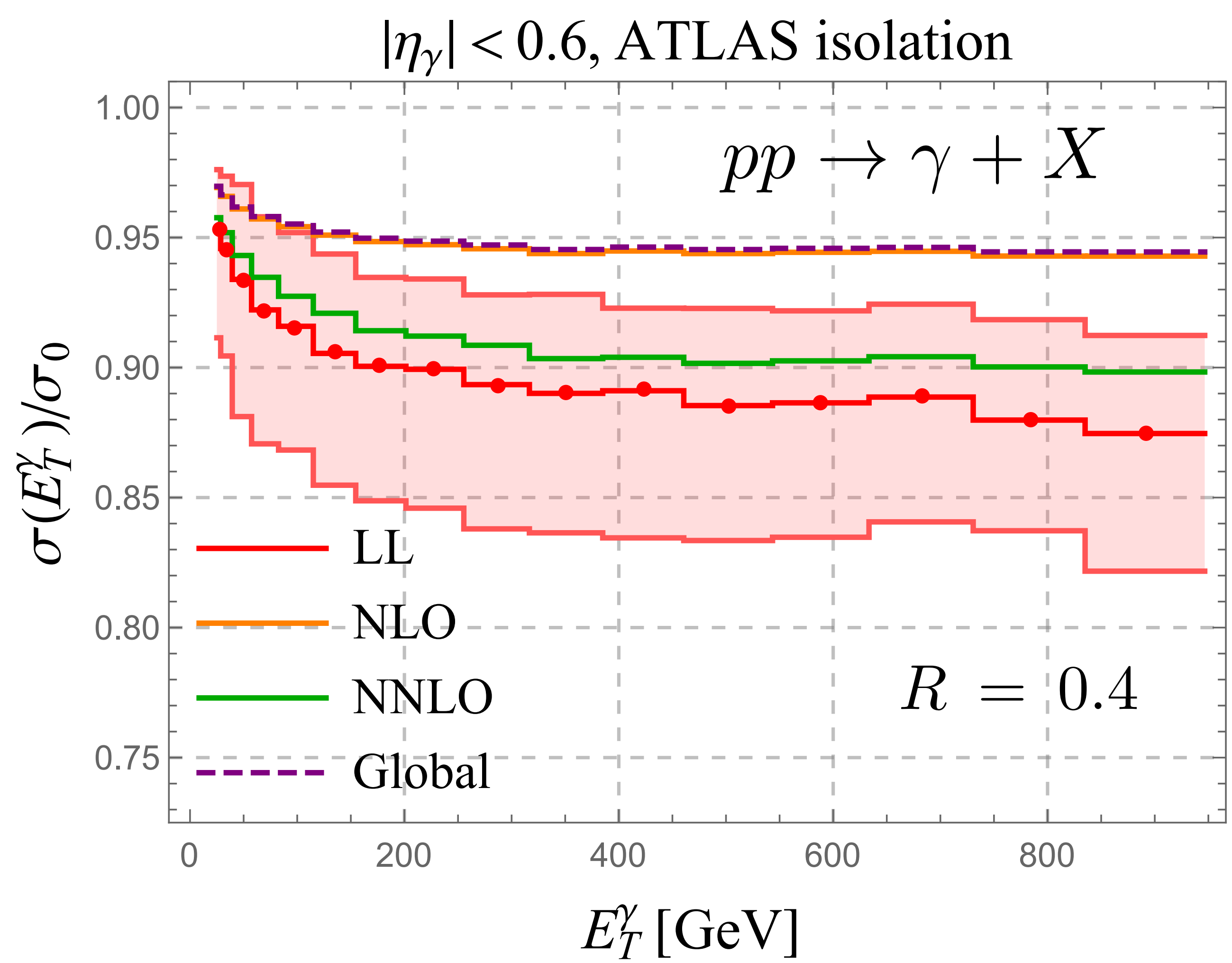

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From LL to NLL: Sub-leading NGLs

- In order to resume sub-leading NGLs, one needs
  - One-loop soft function $\mathcal{S}_m^{(1)}$
  - One-loop hard function $\mathcal{H}_k^{(1)}$ and tree level hard function $\mathcal{H}_{k+1}^{(1)}$
  - Two-loop anomalous dimensions: $\Gamma^{(2)} = \begin{pmatrix} \boldsymbol{v}_2 & \boldsymbol{r}_2 & \boldsymbol{d}_2 & 0 & \dots \\ 0 & \boldsymbol{v}_3 & \boldsymbol{r}_3 & \boldsymbol{d}_2 & \dots \\ 0 & 0 & \boldsymbol{v}_4 & \boldsymbol{r}_4 & \dots \\ 0 & 0 & 0 & \boldsymbol{v}_5 & \dots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$

    See Caron-Huot '15 + Herranen '16
  - Monte-Carlo implementation of all ingredients

From LL to NLL: Sub-leading NGLs

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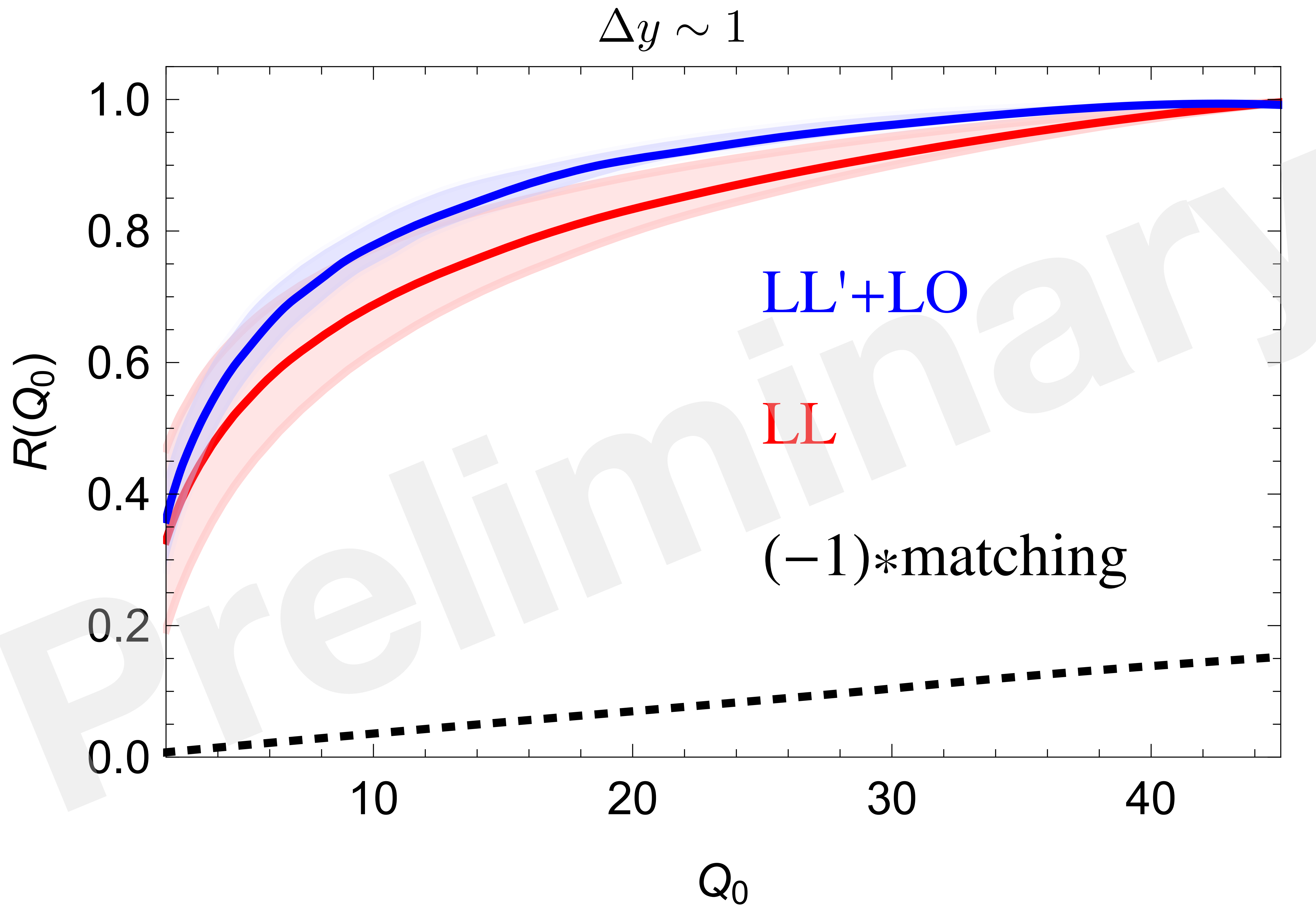
    See Caron-Huot ’15 + Herranen ’16
  - Monte-Carlo implementation of all ingredients

Inter-jet energy flow at e+e- collider

(Balsiger, Becher & DYS, work on progress)



Summary

We extend factorization and resummation in SCET to non-global observables

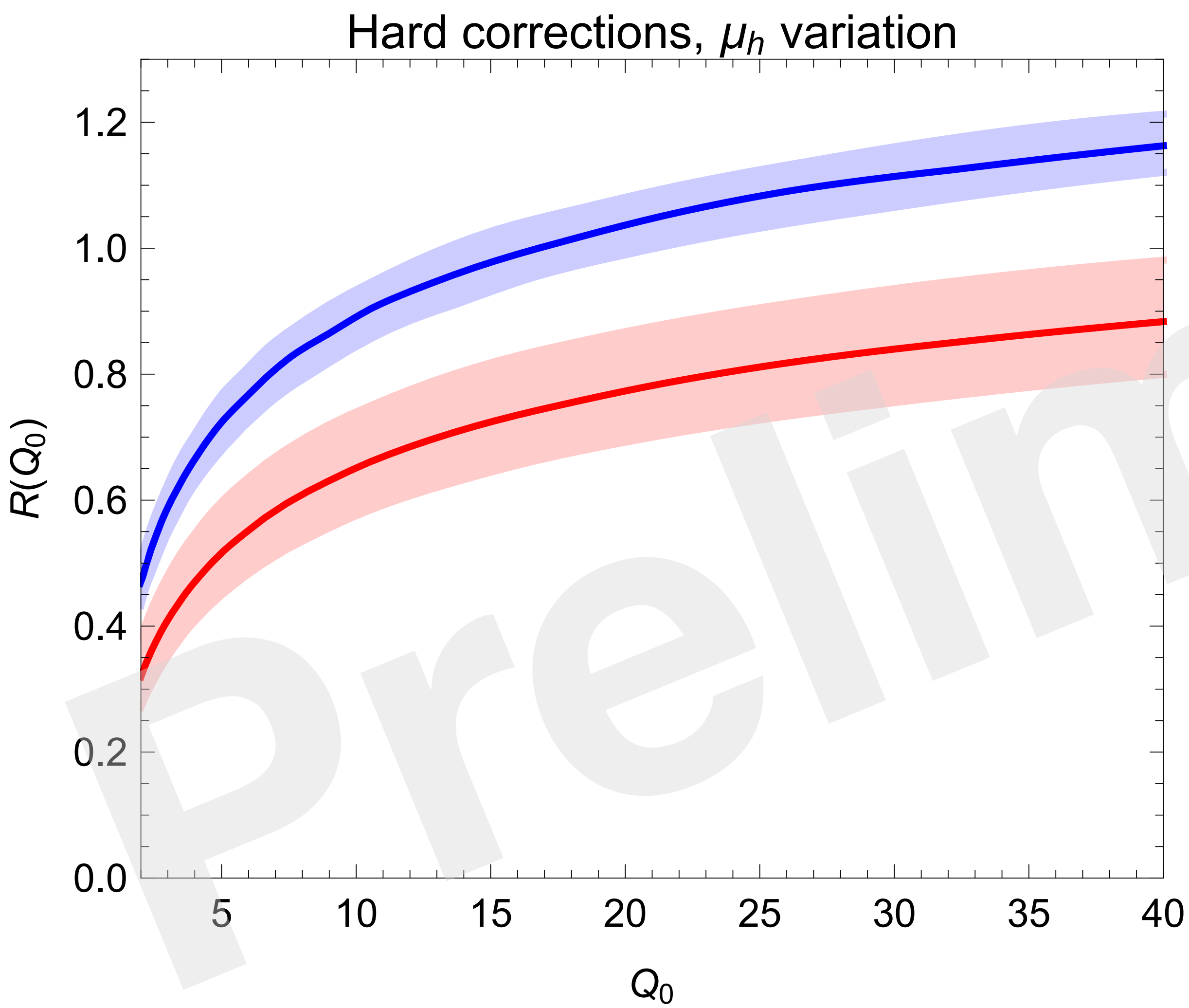
- have analyzed a variety of such observables
- multi-Wilson line operators are key ingredient

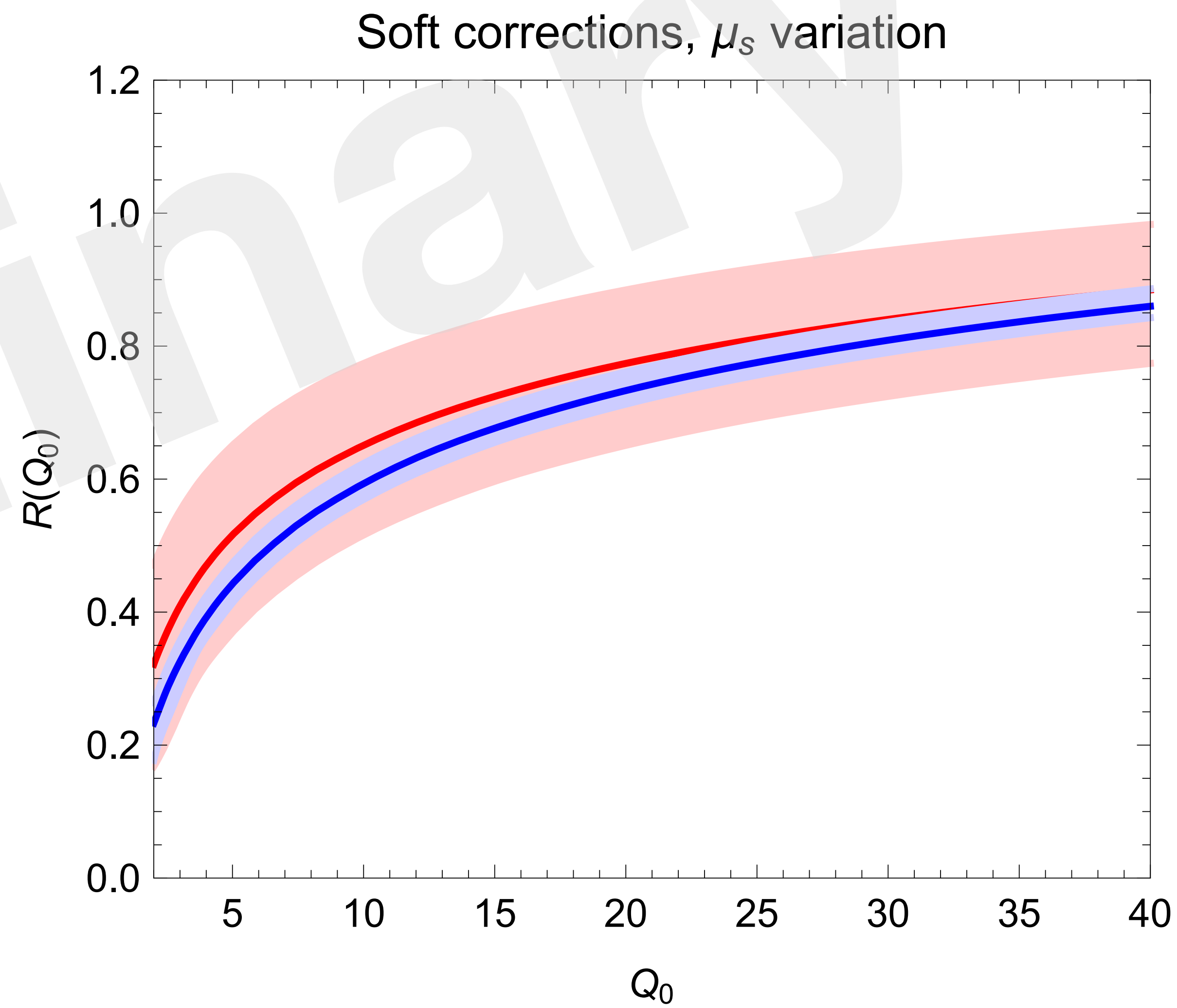
We obtain a parton shower from effective field theory

- not restricted to leading logarithms or large Nc
- not a General Purpose Parton Shower, but helpful to understand how to extend showers to higher accuracy
- flexible implementation of LL shower using LHE event files
- include one-loop matching coefficients: first important step in sub-leading NGLs resummation

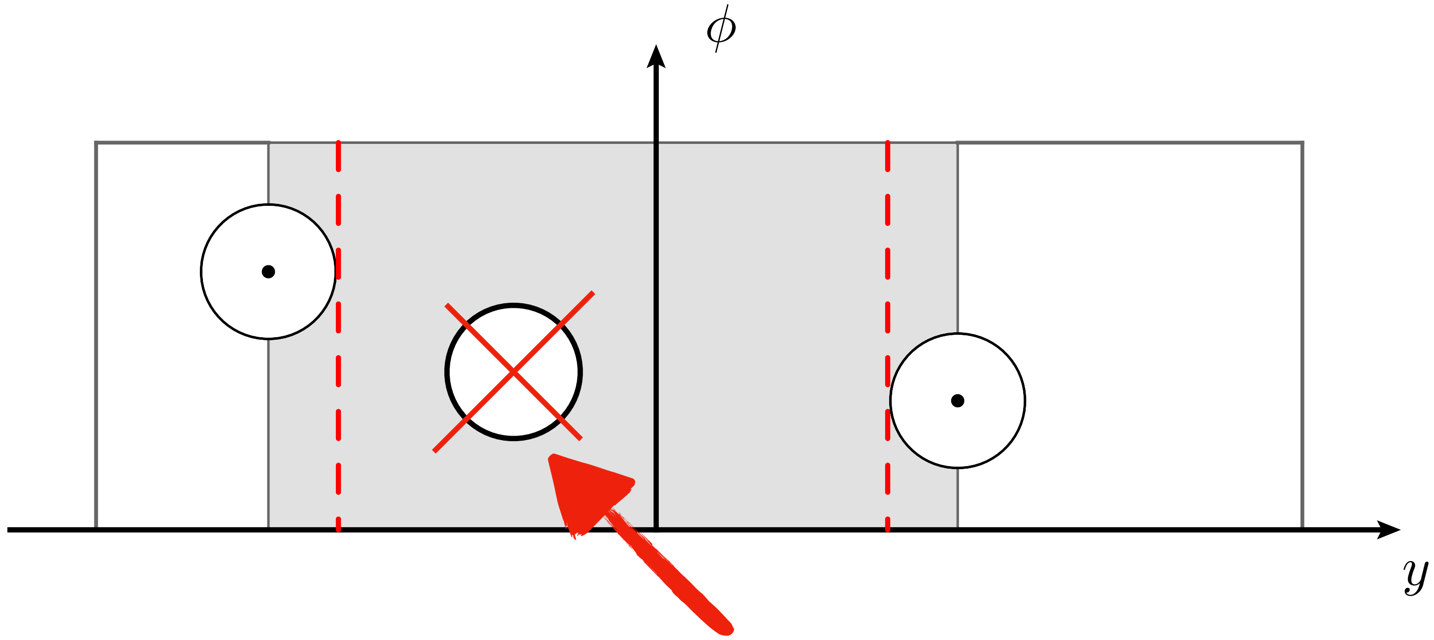
Thank you

Numerical results





Gap fraction at the LHC



$$R(\overline{p}_T, Q_0) = \frac{\sigma_{2-\text{jet}}(\overline{p}_T, Q_0)}{\sigma_{2-\text{jet}}(\overline{p}_T, Q_0 = \overline{p}_T)}$$

$$\overline{p}_T = \frac{1}{2}\left(p_{T,\,\text{jet1}} + p_{T,\,\text{jet2}}\right)$$

veto events with $p_{T,\,\text{jet3}} > Q_0$

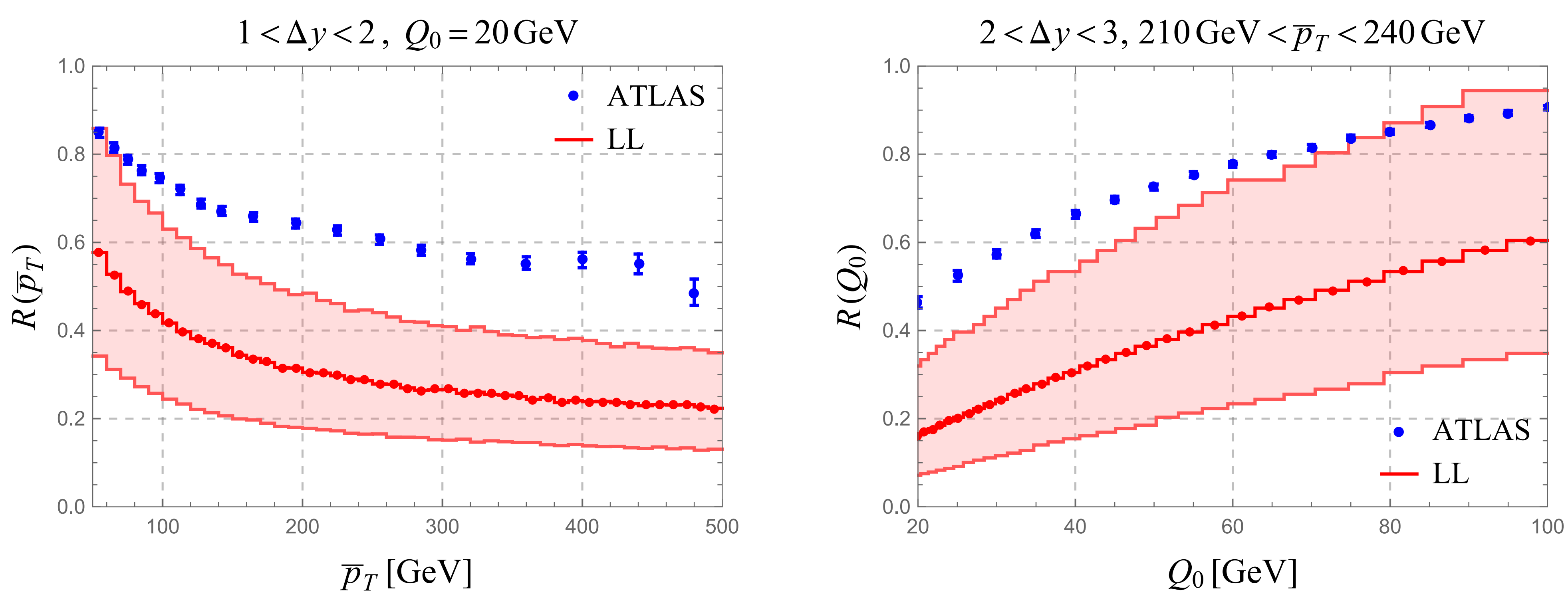
- Moderate non-perturbative corrections
- Need soft gluon resummation at small Q_0
- ATLAS ’11 ’14 observe MC predictions are not always consistent with data
- GL with full color, NGLs estimated by a K-factor
- Numerical solution of BMS equation but with approximated veto region

Resummation

- Factorization formula to all order is unknown due to glauber gluons
- We work in LL and large Nc limit

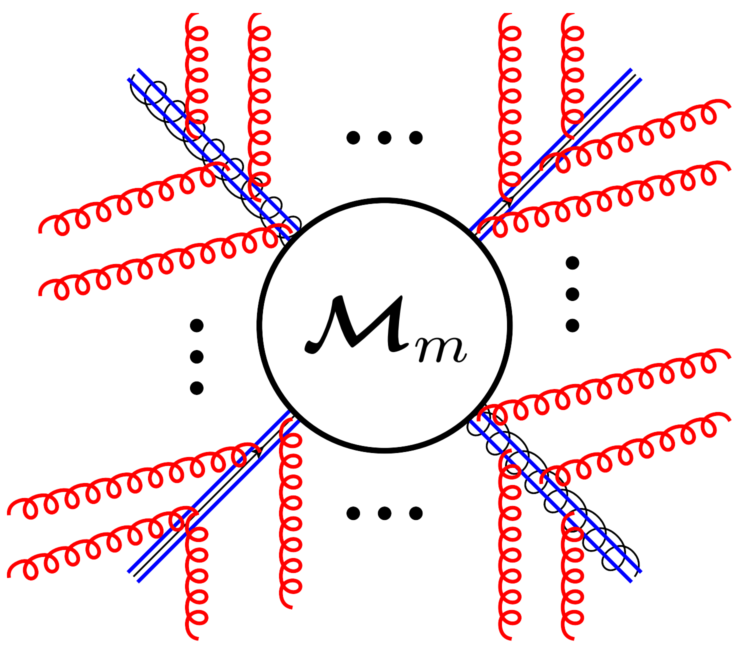
$$\frac{d\sigma(Q_0)}{d\Delta y\, d\overline{p}_T} = \sum_{a,b=q,\bar{q},g} \int dx_1 dx_2 f_a(x_1,\mu_f) f_b(x_2,\mu_f) H_2^{ab}(\hat{s},\Delta y,\overline{p}_T,\mu_h)\langle U_{2m}(\mu_s,\mu_h)\hat{\otimes}1\rangle$$

- scale setting $\mu_f = \mu_h = \overline{p}_T$ and $\mu_s = Q_0$
- focus on central jets, no collinear logs



Factorization

- The operator for the emission from an amplitude with m hard partons



hard scattering amplitude with m particles (vector in color space)

$$\boldsymbol{S}_1(n_1)\,\boldsymbol{S}_2(n_2)\,\ldots\,\boldsymbol{S}_m(n_m)\,|\mathcal{M}_m(\{\underline{p}\})\rangle$$

soft Wilson lines along the directions of the energetic particles (color matrices)

$$\boldsymbol{S}_i(n_i) = \mathbf{P}\exp\left(ig_s\int_0^\infty ds\, n_i\cdot A_s^a(sn_i)\,\boldsymbol{T}_i^a\right)$$

Resummation effects in isolation

- Small cone radius $R \ll 1$

Catani & et. al. '02

Isolation radius	Direct contribution		Fragmentation contribution		Total
R	Born	NLO	Born	NLO	NLO
1.0	1764.6	3318.4	265.0	446.7	3765.1
0.7	1764.6	3603.0	265.0	495.0	4098.0
0.4	1764.6	3968.9	265.0	555.6	4524.5
0.1	1764.6	4758.2	265.0	678.9	5431.1
Without isolation	1764.6	3341.1	1724.3	1876.8	5217.9

- NLO results are unphysical as $R = 0.1$!!!
- Log(R) spoils perturbative convergence
- This log comes from mismatching between inside and outside radiation
- LL resummation has been studied by Catani & et. al. '13
  - Higher-order effects are moderate for $R \sim 0.4$

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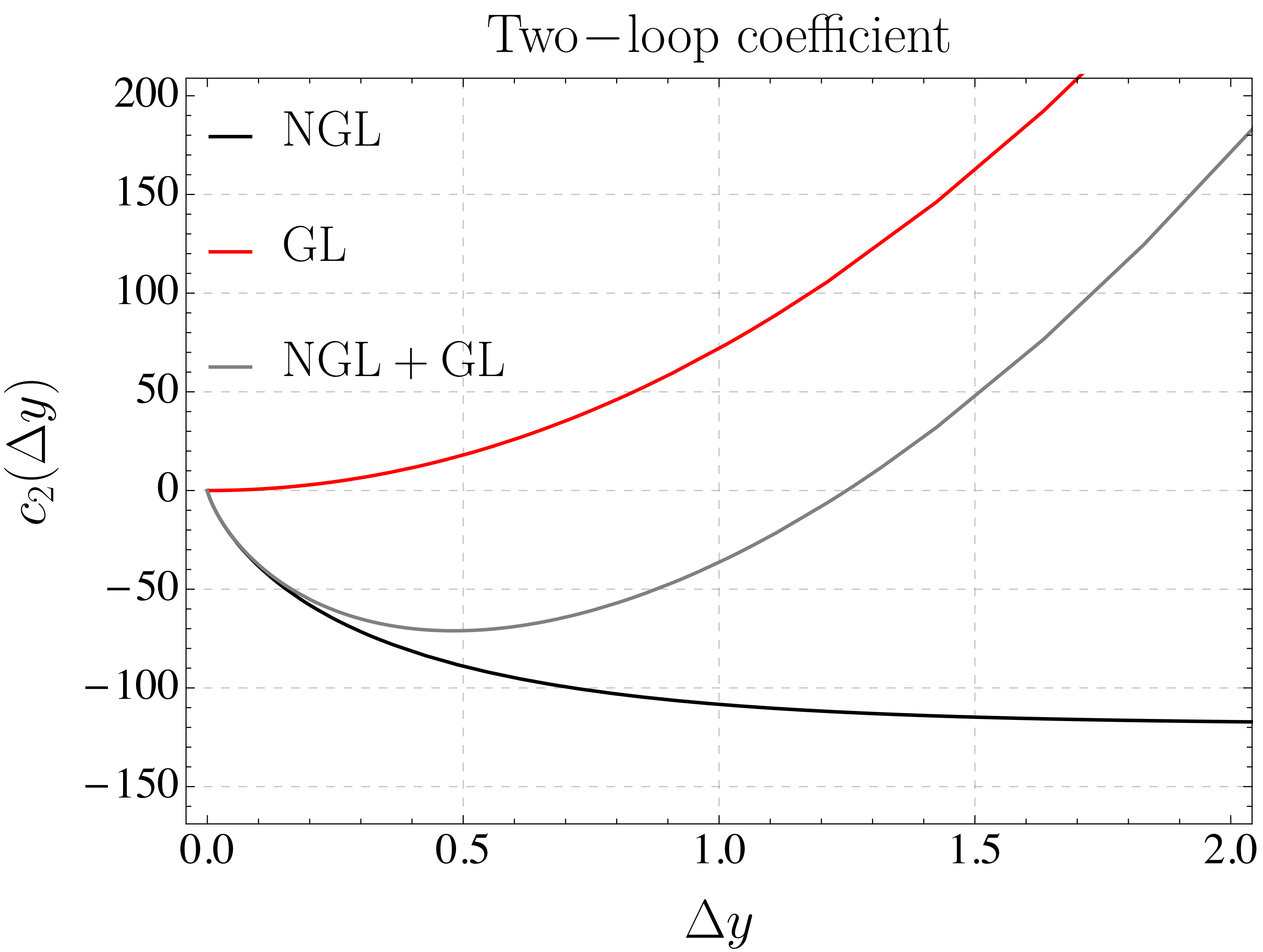
Global vs Non-global Logs

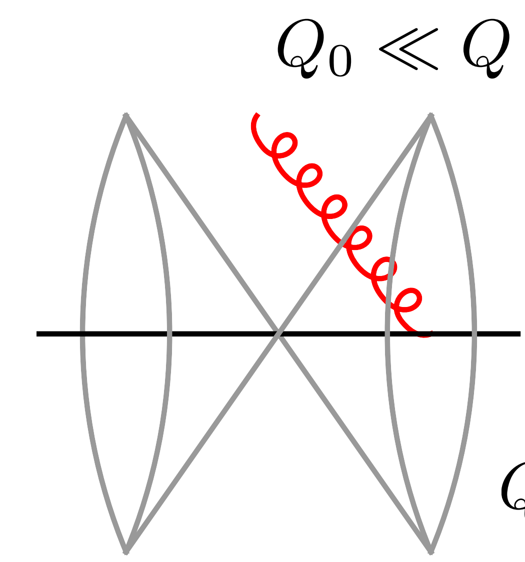


Collinear limit and NGLs

(Hatta, Iancu, Mueller, Triantafyllopoulos '17)

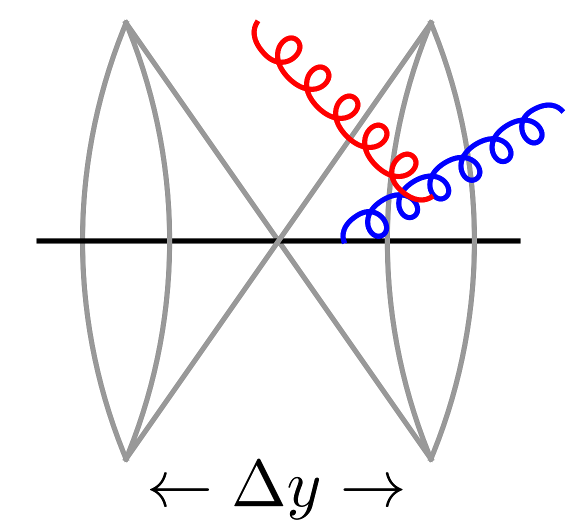
- E.g. Inter-jet energy flow @ e^+e^- colliders
  - Soft radiations from two Wilson lines (global)

$$\frac{\sigma_{\rm GL}^{\rm LL}}{\sigma_0} = \exp\left[-8\, C_F \Delta y\, t\right] \qquad\qquad t = \int_{\alpha(Q_0)}^{\alpha(Q)} \frac{d\alpha}{\beta(\alpha)} \frac{\alpha}{4\pi}$$



  - Leading NGLs at two-loops

$$\frac{\sigma_{\rm NGL}^{\rm LL}}{\sigma_0} = 4\, C_F C_A \left[-\frac{2\pi^2}{3} + 4\,{\rm Li}_2\left(e^{-2\Delta y}\right)\right] t^2$$



  - Narrow gap limit: $\Delta y \to 0$

$$\frac{\sigma_{\rm NGL}^{\rm LL}}{\sigma_0} = 4\, C_F C_A \Big[8\, \Delta y\big(\ln(2\Delta y) - 1\big) - 4\, \Delta y^2 + \ldots\Big] t^2$$

  - Collinear enhancement from boundary region

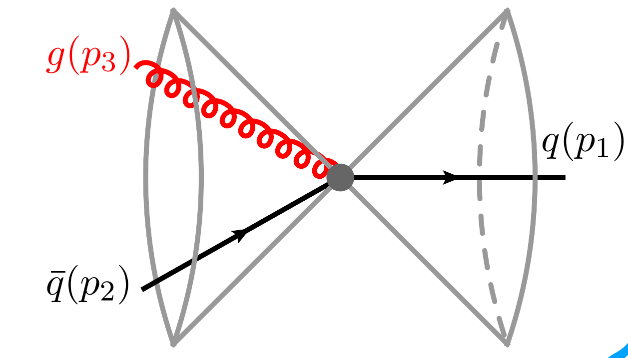
- One-loop hard function

$$\mathcal{H}_2^{(1)} = \frac{N_c}{2}\left[-8\ln^2\frac{\mu_h}{Q} - 12\ln\frac{\mu_h}{Q} - 16 + \frac{7}{3}\pi^2\right]$$

$$\begin{aligned}\mathcal{H}_{3,\mathrm{I}}^{(1)} = \frac{N_c}{2}\Bigg\{ &\left[4\ln^2\frac{\mu_h}{Q} - 4\ln\frac{\mu_h}{Q}\ln\left(\frac{\delta^2}{1+\delta^2}\right) - \frac{\pi^2}{6} + \ln^2\left(\frac{\delta^2}{1+\delta^2}\right)\right]\delta(u)\delta(v) \\ &+ \left[-\ln\frac{\mu}{Q}F(0,v) + \frac{F(0,v)}{2}\ln\left(\frac{\delta^2}{1+\delta^2}\right)\right]\delta(u)\left(\frac{1}{v}\right)_+ + \frac{F(0,v)}{2}\delta(u)\left(\frac{\ln v}{v}\right)_+ \\ &+ \left[-2\ln\frac{\mu}{Q}F(u,0) + \frac{2u^2}{(1+u)^3} + F(u,0)\ln\left(\frac{\delta^2}{1+\delta^2}\right) - 2F(u,0)\ln(1+u)\right]\delta(v)\left(\frac{1}{u}\right)_+ \\ &+ 2F(u,0)\delta(v)\left(\frac{\ln u}{u}\right)_+ + F(u,v)\left(\frac{1}{u}\right)_+\left(\frac{1}{v}\right)_+ \Bigg\}\end{aligned}$$

- MC implementation:
  - Phase space slicing method: large log cancellation
  - Numerical extrapolation method: point-by-point shower
  - Two independent methods consistent
  - Subtraction method? Can hard function be provided by MG5_aMC@NLO?

$$\sin^2\frac{\theta_2}{2} = \frac{\delta^2 u^2 v}{1+\delta^2}, \qquad \sin^2\frac{\theta_3}{2} = \frac{\delta^2 v}{1+\delta^2}.$$



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Ingredients at LL′ level

$$d\sigma^{\mathrm{LL}'}(Q,Q_0) = \sum_{m=2}^{\infty} \big\langle \boldsymbol{\mathcal{H}}_2^{(0)}(\{\underline{n}\},Q,\mu_h) \otimes \boldsymbol{U}_{2m}(\{\underline{n}\},\mu_s,\mu_h)\,\hat{\otimes}\,\mathbf{1} \big\rangle$$
$$+ \alpha_s(\mu_h) \sum_{m=2}^{\infty} \big\langle (\boldsymbol{\mathcal{H}}_2^{(1)}(\{\underline{n}\},Q,\mu_h)) \otimes \boldsymbol{U}_{2m}(\{\underline{n}\},\mu_s,\mu_h)\,\hat{\otimes}\,\mathbf{1} \big\rangle$$
$$+ \alpha_s(\mu_h) \sum_{m=3}^{\infty} \big\langle (\boldsymbol{\mathcal{H}}_3^{(1)}(\{\underline{n}\},Q,\mu_h)) \otimes \boldsymbol{U}_{3m}(\{\underline{n}\},\mu_s,\mu_h)\,\hat{\otimes}\,\mathbf{1} \big\rangle$$
$$+ \alpha_s(\mu_s) \sum_{m=2}^{\infty} \big\langle (\boldsymbol{\mathcal{H}}_2^{(0)}(\{\underline{n}\},Q,\mu_h)) \otimes \boldsymbol{U}_{2m}(\{\underline{n}\},\mu_s,\mu_h)\,\hat{\otimes}\,\boldsymbol{\mathcal{S}}_m^{(1)}(\{\underline{n}\},Q_0,\mu_s)) \big\rangle$$

- One-loop soft function can be implemented by re-weighting the events when they radiate outside jet cone (inside gap)

$$S_m^{(1)} = \frac{N_c}{2} \sum_{i,i+1} \int dy\, \frac{d\phi}{2\pi} \left[-4\ln\frac{\mu_s}{Q_0} + 4\ln\frac{2\,|\sin\phi|}{f(\phi,y)} \right] \Theta_{\mathrm{out}}(y,\phi)$$