Role of the Z polarization in the $pp \to ZH$, $H \to b\bar{b}$ measurement.

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Based on
D. Gonçalves and JN (arXiv:1805.06385)
D. Gonçalves and JN (in preparation).

QCD@LHC, 27.08.2018, TU Dresden.

Introduction

Introduction: VH, $H \to b\bar{b}$ channels.

- $H \to b\bar{b}$ has the largest Higgs decay rate, $\sim 58\%$.
- The boosted VH production ($V = W, Z$) has the largest sensitivity to $H \to b\bar{b}$ (Butterworth et al 2008)
- Latest results on the VH, $H \to b\bar{b}$ are 4.9σ (ATLAS $\sim 80\mathrm{fb}^{-1}$), 3.8σ (CMS $\sim 36\mathrm{fb}^{-1}$).
- There are 3 channels, based on the number of charged leptons: $ZH \to \nu\nu b\bar{b}$, $WH \to \ell\nu b\bar{b}$, $ZH \to \ell\ell b\bar{b}$.

Channel	Significance	
	Exp.	Obs.
VBF+ggF	0.9	1.5
$t\bar{t}H$	1.9	1.9
VH	5.1	4.9
$H \to b\bar{b}$ Combination	5.5	5.4

Signal strength parameter	Signal strength	p_0		Significance	
		Exp.	Obs.	Exp.	Obs.
0-lepton	$1.04^{+0.34}_{-0.32}$	$9.5 \cdot 10^{-4}$	$5.1 \cdot 10^{-4}$	3.1	3.3
1-lepton	$1.09^{+0.46}_{-0.42}$	$8.7 \cdot 10^{-3}$	$4.9 \cdot 10^{-3}$	2.4	2.6
2-lepton	$1.38^{+0.46}_{-0.42}$	$4.0 \cdot 10^{-3}$	$3.3 \cdot 10^{-4}$	2.6	3.4
VH, $H \to b\bar{b}$ combination	$1.16^{+0.27}_{-0.25}$	$7.3 \cdot 10^{-6}$	$5.3 \cdot 10^{-7}$	4.3	4.9

(from ATLAS 2018)

Introduction: $Z(\ell^+\ell^-)H(b\bar{b})$ channel.

We focus only on 2-lepton channel $Z(\ell^+\ell^-)H(b\bar{b})$.

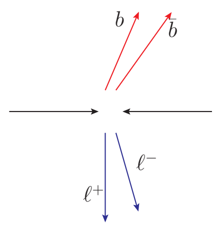
Process	0-lepton	1-lepton	2-lepton low-$p_{\mathrm{T}}(\mathrm{V})$	2-lepton high-$p_{\mathrm{T}}(\mathrm{V})$
Vbb	216.8	102.5	617.5	113.9
Vb	31.8	20.0	141.1	17.2
V+udscg	10.2	9.8	58.4	4.1
$\mathrm{t\bar{t}}$	34.7	98.0	157.7	3.2
Single top quark	11.8	44.6	2.3	0.0
VV(udscg)	0.5	1.5	6.6	0.5
VZ(bb)	9.9	6.9	22.9	3.8
Total background	315.7	283.3	1006.5	142.7
VH	38.3	33.5	33.7	22.1
Data	334	320	1030	179
S/B	0.12	0.12	0.033	0.15

(from CMS 2018)

- ▷ A higher signal sensitivity is gained in the high-$p_{\mathrm{T}}(Z)$ channel.
- ▷ $Z(\ell^+\ell^-)b\bar{b}$ (part of the $\mathcal{O}(\alpha_s^2)$ correction to the Drell-Yan Z production) is the dominant background.

Introduction: $Z(\ell^+\ell^-)H(b\bar{b})$ v.s. $Z(\ell^+\ell^-)b\bar{b}$ background.

$Z(\ell^+\ell^-)b\bar{b}$ is an irreducible background:



- ♠ 2-leptons come from $Z \to \ell^+\ell^-$ both in ZH signal and $Zb\bar{b}$ background.
- ♠ $Z \to \ell^+\ell^-$ angular distribution is uniquely determined by Z polarization.
- ♠ Z polarization is process-dependent, thus can be different between the signal and the background.

In this work,

1. we show that Z polarization is very different between the ZH signal and the $Zb\bar{b}$ background.
2. we estimate the improvement on the signal sensitivty.

Z polarization in the ZH signal and the $Zb\bar{b}$ background.

Z polarization in ZH and $Zb\bar{b}$: General idea

The lepton direction is parametrized by two angles θ, ϕ in the Z rest frame as

$$\begin{aligned} Z:&\ \ \left(m_{\ell\ell}, 0, 0, 0\right), \\ \ell^-(\ell^+):&\ \frac{m_{\ell\ell}}{2}\left(1,\ \pm\sin\theta\cos\phi,\ \pm\sin\theta\sin\phi,\ \pm\cos\theta\right). \end{aligned}$$

In general, $Z \to \ell^+\ell^-$ angular $(\cos\theta, \phi)$ distribution can be described with 8 coefficients A_i ($i = 1$ to 8) as

$$\begin{aligned} \frac{1}{\sigma}\frac{d\sigma}{d\cos\theta d\phi} = &\ 1 + \cos^2\theta + A_1(1 - 3\cos^2\theta) + A_2\sin 2\theta\cos\phi + A_3\sin^2\theta\cos 2\phi \\ &+ A_4\cos\theta + A_5\sin\theta\cos\phi + A_6\sin\theta\sin\phi + A_7\sin 2\theta\sin\phi + A_8\sin^2\theta\sin 2\phi. \end{aligned}$$

Why 8? $\to$ because the degrees of freedom of polarization of a spin 1 particle is 8 in the most general case.

> Message: 8 coefficients $A_i (i = 1$ to $8)$ uniquely parametrize Z polarization, and determine $Z \to \ell^+\ell^- (\cos\theta, \phi)$ distribution.
> 8 coefficients $A_i (i = 1$ to $8)$ are process-dependent $\to$ evaluate in next slide.

As the coordinate system of the Z rest frame (i.e. the direction of the z axis), we choose the Collins-Soper frame (Collins, Soper 1977).

Z polarization in ZH and $Zb\bar{b}$: 8 coefficients A_i ($i = 1$ to 8)

We calculate the coefficients $A_i (i = 1 \text{ to } 8)$ at the LO and at the QCD NLO with MadGraph5aMC@NLO for 13 TeV LHC, imposing the signal selections such as $75 < m_{\ell\ell} < 105\ \mathrm{GeV}, \quad p_\mathrm{T}(Z) > 200\ \mathrm{GeV}$:

	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8
ZH (LO)	0.03(6)	0.2(1)	−80.0(1)	−0.08(8)	−0.01(8)	0.04(8)	0.1(1)	0.1(1)
ZH (NLO)	1.7(1)	0.0(3)	−75.0(3)	−0.1(2)	0.6(2)	−0.2(2)	−0.0(3)	0.1(3)
$Zb\bar{b}$ (LO)	47.0(1)	0.6(1)	44.7(1)	0.1(1)	0.2(1)	−0.1(1)	−0.1(1)	−0.0(1)

in unit of %. Shown in the parentheses is the statistical uncertainty for the last digit.

- Only A_1 and A_3 are siginificant.
- A_1 and A_3 (i.e. polarization) are very different between ZH signal and $Zb\bar{b}$ background.

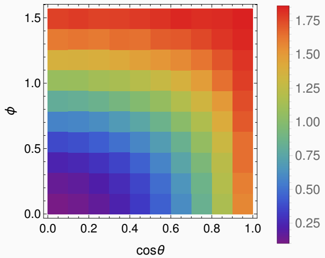
Effectively, $Z \to \ell^+\ell^-$ angular $(\cos\theta, \phi)$ distribution is determined by

$$\frac{1}{\sigma}\frac{d\sigma}{d\cos\theta d\phi} = 1 + \cos^2\theta + A_1(1 - 3\cos^2\theta) + A_3 \sin^2\theta \cos 2\phi.$$

θ and ϕ can be re-defined in the restricted ranges $0 \le \cos\theta \le 1$, $0 \le \phi \le \pi/2$. (originally, $-1 \le \cos\theta \le 1$, $0 \le \phi \le 2\pi$.)

Z polarization in ZH and $Zb\bar{b}$: $Z \to \ell^+\ell^-$ 2-dimensional $(\cos\theta, \phi)$ distribution.

Ratio of the normalized $Z \to \ell^+\ell^-$ angular $(\cos\theta, \phi)$ distribution for ZH signal to that for $Zb\bar{b}$ background:



- ▷ A large difference between the signal and the background is clearly visible in this restricted 2-dimensional $(\cos\theta, \phi)$ distribution!

Message: 2-dimensional $Z \to \ell^+\ell^-$ $(\cos\theta, \phi)$ distribution may be useful in distinguishing the ZH signal from the dominant-irreducible $Zb\bar{b}$ background.

Analysis at the hadron level

Analysis at the hadron level: Simulation setup.

- The ZH signal and backgrounds ($Zb\bar{b}$, $t\bar{t}$, ZZ) are simulated at the hadron level with Sherpa+OpenLoops. (Gleisberg et al 2009, Cascioli et al 2012, Denner et al 2017)
- The BDRS analysis is used for the $H \to b\bar{b}$ tagging; $p_{\mathrm{T}J} > 200$ GeV, $|\eta_J| < 2.5$, $|m_H^{\mathrm{BDRS}} - m_H| < 10$ GeV. (Butterwirth et al 2008)
- Two charged leptons (e or μ) with $p_{\mathrm{T}\ell} > 5$ GeV and $|\eta_\ell| < 2.5$ are requierd, which reconstruct a boosted Z boson: $75 < m_{\ell\ell} < 105$ GeV, and $q_{\mathrm{T}} \equiv p_{\mathrm{T}\ell\ell} > 200$ GeV.
- 70% b-tagging efficiency and 1% misstag rate are taken into account.
- 5% systematic uncertainties on the backgrounds are assumed.
- At the very end, we perform a two dimensional binned log-likelihood analysis based on the $Z \to \ell^+\ell^-$ angular $(\cos\theta, \phi)$ distribution, invoking the CL_s method (Read 2002).

Message: Our proposal uses only the lepton information, independent of how the $H \to b\bar{b}$ tagging performed is.

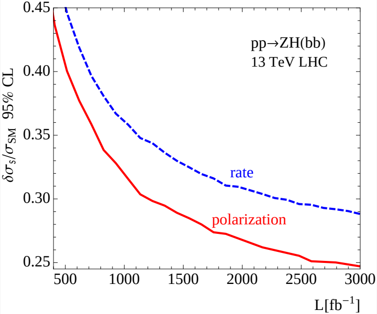
Analysis at the hadron level: Results.

x axis: Luminosity.

y axis: 95% CL upper bound on anomalous $Z(\ell\ell)H(b\bar{b})$ signal strength, $\frac{\delta\sigma}{\sigma_{\rm SM}} = \frac{\sigma - \sigma_{\rm SM}}{\sigma_{\rm SM}}$. (only the 2-lepton channel!)

Red curve: Takes into account the difference in Z polarization.

Blue curve: Does NOT take it into account.



- ▷ Enhancing the precision on the signal strength determination; from about 30% to 25% at $\mathrm{L} = 3\ \mathrm{ab}^{-1}$.

Message: Making use of the difference in Z polarization between the ZH signal and the background(s) seems to work well.

Summary

Summary.

We have studied the potential of exploiting the Z polarization to improve the sensitivity to the signal $pp \to Z(\ell\ell)H(b\bar{b})$.

- ♣ Process-dependent Z polarization determines $Z \to \ell^+\ell^-$ angular 2-dimensional $(\cos\theta, \phi)$ distribution.
- ♣ The ZH signal and the dominant-irreducible $Zb\bar{b}$ background, which is part of the $\mathcal{O}(\alpha_s^2)$ correction to the Drell-Yan Z production, predict very different states of Z polarization. These are found robust against the QCD NLO correction.
- ♣ This difference appears as the large difference in the $(\cos\theta, \phi)$ distribution.
- ♣ We have estimated the improvement by performing a 2-dimensional log-likelihood analysis based on the $(\cos\theta, \phi)$ distribution; improvement from about 30% to 25% in the precision on the signal strength determination at $\mathrm{L} = 3\ \mathrm{ab}^{-1}$.

Thank you so much for your attention.

Appendix

- Multivariate analysis after basic event selections.
- Only $m(\ell^+\ell^-)$ as the information of the charged leptons:

Variable	0-lepton	1-lepton	2-lepton
p_{T}^{V}	$\equiv E_{\mathrm{T}}^{\mathrm{miss}}$	$\times$	$\times$
$E_{\mathrm{T}}^{\mathrm{miss}}$	$\times$	$\times$	$\times$
$p_{\mathrm{T}}^{b_1}$	$\times$	$\times$	$\times$
$p_{\mathrm{T}}^{b_2}$	$\times$	$\times$	$\times$
m_{bb}	$\times$	$\times$	$\times$
$\Delta R(\vec{b}_1, \vec{b}_2)$	$\times$	$\times$	$\times$
$\lvert\Delta\eta(\vec{b}_1, \vec{b}_2)\rvert$	$\times$		
$\Delta\phi(\vec{V}, \vec{bb})$	$\times$	$\times$	$\times$
$\lvert\Delta\eta(\vec{V}, \vec{bb})\rvert$			$\times$
m_{eff}	$\times$		
$\min[\Delta\phi(\vec{\ell}, \vec{b})]$		$\times$	
m_{T}^{W}		$\times$	
$m_{\ell\ell}$			$\times$
m_{top}		$\times$	
$\lvert\Delta Y(\vec{V}, \vec{bb})\rvert$		$\times$	
	Only in 3-jet events		
$p_{\mathrm{T}}^{\mathrm{jet}_3}$	$\times$	$\times$	$\times$
m_{bbj}	$\times$	$\times$	$\times$

(ATLAS 2017)

- Multivariate analysis after basic event selections.
- Only $m(\ell^+\ell^-)$ as the information of the charged leptons:

Variable	Description	Channels
$M(\mathrm{jj})$	dijet invariant mass	All
$p_\mathrm{T}(\mathrm{jj})$	dijet transverse momentum	All
$p_\mathrm{T}(\mathrm{j}_1)$, $p_\mathrm{T}(\mathrm{j}_2)$	transverse momentum of each jet	0- and 2-lepton
$\Delta R(\mathrm{jj})$	distance in η–ϕ between jets	2-lepton
$\Delta\eta(\mathrm{jj})$	difference in η between jets	0- and 2-lepton
$\Delta\phi(\mathrm{jj})$	azimuthal angle between jets	0-lepton
$p_\mathrm{T}(\mathrm{V})$	vector boson transverse momentum	All
$\Delta\phi(\mathrm{V},\mathrm{jj})$	azimuthal angle between vector boson and dijet directions	All
$p_\mathrm{T}(\mathrm{jj})/p_\mathrm{T}(\mathrm{V})$	p_T ratio between dijet and vector boson	2-lepton
$M(\ell\ell)$	reconstructed Z boson mass	2-lepton
$\mathrm{CMVA}_\mathrm{max}$	value of CMVA discriminant for the jet with highest CMVA value	0- and 2-lepton
$\mathrm{CMVA}_\mathrm{min}$	value of CMVA discriminant for the jet with second highest CMVA value	All
$\mathrm{CMVA}_\mathrm{add}$	value of CMVA for the additional jet with highest CMVA value	0-lepton
$p_\mathrm{T}^\mathrm{miss}$	missing transverse momentum	1- and 2-lepton
$\Delta\phi(\vec{p}_\mathrm{T}^\mathrm{miss},\mathrm{j})$	azimuthal angle between $\vec{p}_\mathrm{T}^\mathrm{miss}$ and closest jet ($p_\mathrm{T} > 30\,\mathrm{GeV}$)	0-lepton
$\Delta\phi(\vec{p}_\mathrm{T}^\mathrm{miss},\ell)$	azimuthal angle between $\vec{p}_\mathrm{T}^\mathrm{miss}$ and lepton	1-lepton
m_T	mass of lepton $\vec{p}_\mathrm{T} + \vec{p}_\mathrm{T}^\mathrm{miss}$	1-lepton
m_top	reconstructed top quark mass	1-lepton
N_aj	number of additional jets	1- and 2-lepton
$p_\mathrm{T}(\mathrm{add})$	transverse momentum of leading additional jet	0-lepton
SA5	number of soft-track jets with $p_\mathrm{T} > 5\,\mathrm{GeV}$	All

(CMS 2018)

Signal regions	0-lepton		1-lepton		2-lepton			
	$p_T^V > 150\,GeV$, 2-b-tag		$p_T^V > 150\,GeV$, 2-b-tag		$75\,GeV < p_T^V < 150\,GeV$, 2-$b$-tag		$p_T^V > 150\,GeV$, 2-b-tag	
Sample	2-jet	3-jet	2-jet	3-jet	2-jet	≥3-jet	2-jet	≥3-jet
$Z + ll$	17 ± 11	27 ± 18	1.5 ± 1.0	3.4 ± 2.3	13.7 ± 8.7	49 ± 32	4.1 ± 2.8	30 ± 19
$Z + cl$	45 ± 18	76 ± 30	3.0 ± 1.2	6.9 ± 2.8	43 ± 17	170 ± 67	11.5 ± 4.6	88 ± 35
Z + HF	4770 ± 140	5940 ± 300	179.5 ± 9.1	348 ± 21	7400 ± 120	14160 ± 220	1421 ± 34	5370 ± 100
$W + ll$	20 ± 13	32 ± 22	31 ± 23	65 ± 48	< 1	< 1	< 1	< 1
$W + cl$	43 ± 20	83 ± 38	139 ± 67	250 ± 120	< 1	< 1	< 1	< 1
W + HF	1000 ± 87	1990 ± 200	2660 ± 270	5400 ± 670	1.8 ± 0.2	13.2 ± 1.5	1.4 ± 0.2	4.0 ± 0.5
Single top quark	368 ± 53	1410 ± 210	2080 ± 290	9400 ± 1400	188 ± 89	440 ± 200	23.1 ± 7.3	93 ± 26
$t\bar{t}$	1333 ± 82	9150 ± 400	6600 ± 320	50200 ± 1400	3170 ± 100	8880 ± 220	104 ± 6	839 ± 40
Diboson	254 ± 49	318 ± 90	178 ± 47	330 ± 110	152 ± 32	355 ± 68	52 ± 11	196 ± 35
Multi-jet e sub-ch.	–	–	100 ± 100	41 ± 35	–	–	–	–
Multi-jet μ sub-ch.	–	–	138 ± 92	260 ± 270	–	–	–	–
Total bkg.	7851 ± 90	19020 ± 140	12110 ± 120	66230 ± 270	10964 ± 99	24070 ± 150	1617 ± 31	6622 ± 78
Signal (fit)	128 ± 28	128 ± 29	131 ± 30	125 ± 30	51 ± 11	86 ± 22	27.7 ± 6.1	67 ± 17
Data	8003	19143	12242	66348	11014	24197	1626	6686

(ATLAS 2018)

Selection	0-lepton	1-lepton		2-lepton
		e sub-channel	μ sub-channel	
Trigger	$E_{\mathrm{T}}^{\mathrm{miss}}$	Single lepton	$E_{\mathrm{T}}^{\mathrm{miss}}$	Single lepton
Leptons	0 loose leptons with $p_{\mathrm{T}} > 7$ GeV	1 tight electron $p_{\mathrm{T}} > 27$ GeV	1 medium muon $p_{\mathrm{T}} > 25$ GeV	2 loose leptons with $p_{\mathrm{T}} > 7$ GeV ≥ 1 lepton with $p_{\mathrm{T}} > 27$ GeV
$E_{\mathrm{T}}^{\mathrm{miss}}$	> 150 GeV	> 30 GeV	–	–
$m_{\ell\ell}$	–	–		81 GeV $< m_{\ell\ell} <$ 101 GeV
Jets	Exactly 2 or 3 jets			Exactly 2 or ≥ 3 jets
Jet p_{T}	> 20 GeV			
b-jets	Exactly 2 b-tagged jets			
Leading b-tagged jet p_{T}	> 45 GeV			
H_{T}	> 120 (2 jets), >150 GeV (3 jets)	–		–
$\min[\Delta\phi(\vec{E}_{\mathrm{T}}^{\mathrm{miss}}, \vec{\mathrm{jets}})]$	$> 20^{\circ}$ (2 jets), $> 30^{\circ}$ (3 jets)	–		–
$\Delta\phi(\vec{E}_{\mathrm{T}}^{\mathrm{miss}}, \vec{bb})$	$> 120^{\circ}$	–		–
$\Delta\phi(\vec{b}_1, \vec{b}_2)$	$< 140^{\circ}$	–		–
$\Delta\phi(\vec{E}_{\mathrm{T}}^{\mathrm{miss}}, \vec{E}_{\mathrm{T,trk}}^{\mathrm{miss}})$	$< 90^{\circ}$	–		–
p_{T}^{V} regions	> 150 GeV			(75, 150] GeV, > 150 GeV
Signal regions	✓	$m_{bb} \geq 75$ GeV or $m_{\mathrm{top}} \leq 225$ GeV		Same-flavour leptons Opposite-sign charge ($\mu\mu$ sub-channel)
Control regions	–	$m_{bb} < 75$ GeV and $m_{\mathrm{top}} > 225$ GeV		Different-flavour leptons

(ATLAS 2017)

Variable	0-lepton	1-lepton	2-lepton
$p_{\mathrm{T}}(\mathrm{V})$	>170	>100	$[50, 150], >150$
$M(\ell\ell)$	—	—	$[75, 105]$
p_{T}^{ℓ}	—	$(>25, >30)$	>20
$p_{\mathrm{T}}(\mathrm{j}_1)$	>60	>25	>20
$p_{\mathrm{T}}(\mathrm{j}_2)$	>35	>25	>20
$p_{\mathrm{T}}(\mathrm{jj})$	>120	>100	—
$M(\mathrm{jj})$	$[60, 160]$	$[90, 150]$	$[90, 150]$
$\Delta\phi(\mathrm{V}, \mathrm{jj})$	>2.0	>2.5	>2.5
$\mathrm{CMVA}_{\mathrm{max}}$	$>\mathrm{CMVA}_{\mathrm{T}}$	$>\mathrm{CMVA}_{\mathrm{T}}$	$>\mathrm{CMVA}_{\mathrm{L}}$
$\mathrm{CMVA}_{\mathrm{min}}$	$>\mathrm{CMVA}_{\mathrm{L}}$	$>\mathrm{CMVA}_{\mathrm{L}}$	$>\mathrm{CMVA}_{\mathrm{L}}$
N_{aj}	<2	<2	—
$N_{\mathrm{a}\ell}$	$=0$	$=0$	—
$p_{\mathrm{T}}^{\mathrm{miss}}$	>170	—	—
$\Delta\phi(\vec{p}_{\mathrm{T}}^{\mathrm{miss}}, \mathrm{j})$	>0.5	—	—
$\Delta\phi(\vec{p}_{\mathrm{T}}^{\mathrm{miss}}, \vec{p}_{\mathrm{T}}^{\mathrm{miss}}(\mathrm{trk}))$	<0.5	—	—
$\Delta\phi(\vec{p}_{\mathrm{T}}^{\mathrm{miss}}, \ell)$	—	<2.0	—
Lepton isolation	—	<0.06	$(<0.25, <0.15)$
Event BDT	>-0.8	>0.3	>-0.8

(CMS 2018)

The angles θ ($0 \le \theta \le \pi/2$) and ϕ ($0 \le \phi \le \pi/2$) defined in the Collins-Soper frame can be obtained from

$$|\cos\theta| = \frac{2\left|q^0 p_\ell^3 - q^3 p_\ell^0\right|}{Q\sqrt{Q^2 + |\vec{q}_{\mathrm{T}}|^2}}\,,$$

$$|\cos\phi| = \frac{2}{\sin\theta}\,\frac{\left|Q^2 \vec{p}_{\mathrm{T}\ell}\cdot\vec{q}_{\mathrm{T}} - |\vec{q}_{\mathrm{T}}|^2 p_\ell\cdot q\right|}{Q^2|\vec{q}_{\mathrm{T}}|\sqrt{Q^2 + |\vec{q}_{\mathrm{T}}|^2}}$$

where $q^\mu = (q^0, \vec{q}_{\mathrm{T}}, q^3)$ and $p_\ell^\mu = (p_\ell^0, \vec{p}_{\mathrm{T}\ell}, p_\ell^3)$ are four-momenta of the Z boson and one of the leptons, respectively, in the laboratory frame. We stress that p_ℓ^μ can be the momentum of either ℓ^- or ℓ^+ (*i.e.* either gives the same θ and ϕ values). This is simply because interchanging ℓ^- and ℓ^+ corresponds to $\theta \to \pi - \theta$ *and* $\phi \to \phi + \pi$ (i.e. $\cos\theta \to -\cos\theta$ and $\cos\phi \to -\cos\phi$).

In terms of the scattering amplitudes $\mathcal{M}^{\lambda}_{\lambda_1\lambda_2}$, where $\lambda_{1,2}$ denote the helicity of the initial gluons, the functions f_i can be written as

$$f_1 = \overline{\sum_{\lambda_1,\lambda_2}} \frac{1}{2}(|\mathcal{M}^{+}_{\lambda_1\lambda_2}|^2 + |\mathcal{M}^{-}_{\lambda_1\lambda_2}|^2 + |\mathcal{M}^{0}_{\lambda_1\lambda_2}|^2)$$

$$f_2 = \overline{\sum_{\lambda_1,\lambda_2}} \frac{1}{2}|\mathcal{M}^{0}_{\lambda_1\lambda_2}|^2$$

$$f_3 = \overline{\sum_{\lambda_1,\lambda_2}} \frac{1}{\sqrt{2}} Re[\mathcal{M}^{0}_{\lambda_1\lambda_2}(\mathcal{M}^{+}_{\lambda_1\lambda_2})^* - \mathcal{M}^{-}_{\lambda_1\lambda_2}(\mathcal{M}^{0}_{\lambda_1\lambda_2})^*]$$

$$f_4 = \overline{\sum_{\lambda_1,\lambda_2}} Re[\mathcal{M}^{-}_{\lambda_1\lambda_2}(\mathcal{M}^{+}_{\lambda_1\lambda_2})^*]$$

$$f_5 = \overline{\sum_{\lambda_1,\lambda_2}} (|\mathcal{M}^{+}_{\lambda_1\lambda_2}|^2 - |\mathcal{M}^{-}_{\lambda_1\lambda_2}|^2)$$

$$f_6 = \overline{\sum_{\lambda_1,\lambda_2}} \sqrt{2} Re[\mathcal{M}^{0}_{\lambda_1\lambda_2}(\mathcal{M}^{+}_{\lambda_1\lambda_2})^* + \mathcal{M}^{-}_{\lambda_1\lambda_2}(\mathcal{M}^{0}_{\lambda_1\lambda_2})^*]$$

$$f_7 = \overline{\sum_{\lambda_1,\lambda_2}} \sqrt{2} Im[\mathcal{M}^{0}_{\lambda_1\lambda_2}(\mathcal{M}^{+}_{\lambda_1\lambda_2})^* + \mathcal{M}^{-}_{\lambda_1\lambda_2}(\mathcal{M}^{0}_{\lambda_1\lambda_2})^*]$$

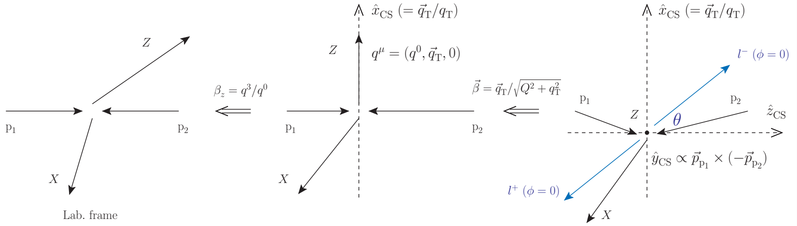
$$f_8 = \overline{\sum_{\lambda_1,\lambda_2}} \frac{1}{\sqrt{2}} Im[\mathcal{M}^{0}_{\lambda_1\lambda_2}(\mathcal{M}^{+}_{\lambda_1\lambda_2})^* - \mathcal{M}^{-}_{\lambda_1\lambda_2}(\mathcal{M}^{0}_{\lambda_1\lambda_2})^*]$$

$$f_9 = \overline{\sum_{\lambda_1,\lambda_2}} Im[\mathcal{M}^{-}_{\lambda_1\lambda_2}(\mathcal{M}^{+}_{\lambda_1\lambda_2})^*].$$

Lepton p_{T} in terms of the Collins-Soper angles

Lepton p_{T} in terms of the CS angles: general formula

Lepton p_{T} in Lab. frame has a simple formula:



$$\text{In the Collins Soper frame}: \left(p^*_{\ell^-(\ell^+)}\right)^\mu = \frac{m_{\ell\ell}}{2}\left(1,\ \pm\sin\theta\cos\phi,\ \pm\sin\theta\sin\phi,\ \pm\cos\theta\right).$$

$\Downarrow$ boost to the lab. frame

$$\text{In the Lab. frame}: \vec{p}_{\mathrm{T}\ell^-(\ell^+)} = \frac{1}{2}\left(q_{\mathrm{T}} \pm \sqrt{m_{\ell\ell}^2 + q_{\mathrm{T}}^2}\sin\theta\cos\phi,\quad \pm m_{\ell\ell}\sin\theta\sin\phi\right).$$

$\Downarrow$ calculate the absolute values

$$p_{\mathrm{T}\ell^-(\ell^+)} = \frac{1}{2}\sqrt{q_{\mathrm{T}}^2 + m_{\ell\ell}^2\sin^2\theta + q_{\mathrm{T}}^2\sin^2\theta\cos^2\phi \pm 2q_{\mathrm{T}}\sqrt{m_{\ell\ell}^2 + q_{\mathrm{T}}^2}\sin\theta\cos\phi}.$$

That's all!

Lepton p_{T} in terms of the CS angles: difference between ZH and $Zb\bar{b}$

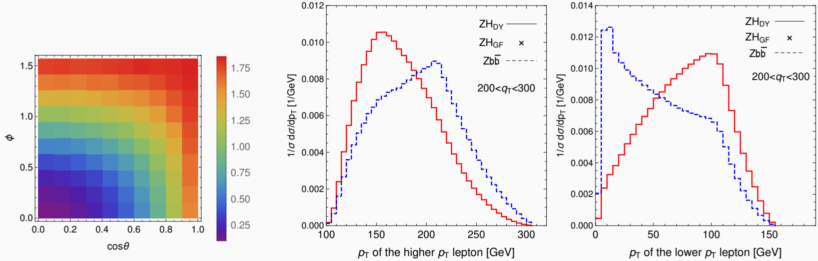
$$p_{\mathrm{T}\ell 1(2)} = \frac{1}{2}\sqrt{q_{\mathrm{T}}^2 + Q^2\sin^2\theta + q_{\mathrm{T}}^2\sin^2\theta\cos^2\phi \pm 2q_{\mathrm{T}}\sqrt{Q^2+q_{\mathrm{T}}^2}\sin\theta|\cos\phi|}.$$

Signal predicts more events at $\phi \sim \pi/2$: p_{T} of the 2 leptons are equivalent,

$$p_{\mathrm{T}\ell 1} = p_{\mathrm{T}\ell 2} = \frac{1}{2}\sqrt{q_{\mathrm{T}}^2 + Q^2\sin^2\theta}.$$

Background predicts more events at $\phi \sim 0$: 1 lepton is very hard, another is very soft,

$$p_{\mathrm{T}\ell 1(2)} = \frac{1}{2}\left|q_{\mathrm{T}} \pm \sqrt{Q^2+q_{\mathrm{T}}^2}\sin\theta\right|.$$



Message: The difference in Z polarization largely appears in lepton p_{T}.