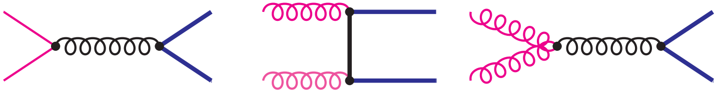
Top-Quark Pair Production Beyond NNLO

Jan Piclum

based on:

JP, C. Schwinn, JHEP 1803 (2018) 164 [arXiv:1801.05788]

Total Cross Section



$$\sigma_{t\bar{t}}(s) = \sum_{i,j} \int\limits_{4m_t^2}^{s} \mathrm{d}\hat{s}\, \mathcal{L}_{ij}(s, \hat{s}, \mu_f)\, \hat{\sigma}_{ij}(\hat{s}, \mu_f, \mu_r)$$

different ways to compute cross section:

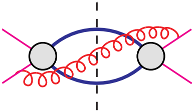
- fixed order $\rightarrow$ NNLO
- resum dominant contributions to all orders $\rightarrow$ NNLL
- use resummation to predict dominant terms of fixed-order result $\rightarrow$ approximate N^{3}LO

Dominant Terms

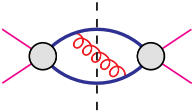
consider threshold limit: $\beta = \sqrt{1 - 4m_t^2/\hat{s}} \to 0$

Sterman 1987; Laenen et al. 1991; Catani et al.; Berger, Contopanagos; Kidonakis et al. 1996; Bonciani et al. 1998

Soft corrections:

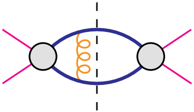


$\sim \alpha_s \ln^2 \beta$



$\sim \alpha_s \ln \beta$

Coulomb corrections:



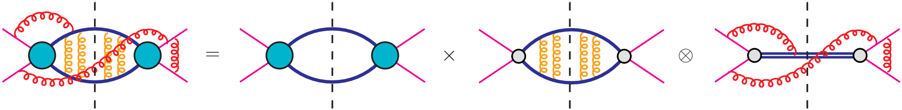
$\sim \alpha_s/\beta$

$\alpha_s/\beta \sim 1$, $\alpha_s \ln \beta \sim 1 \rightsquigarrow$ resum terms to all orders

Resummation in Momentum Space

Soft and Coulomb resummation: Beneke, Falgari, Schwinn 2009, 2010

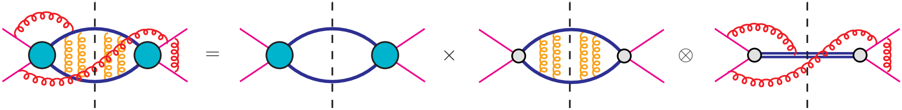
$$\hat{\sigma}_{ij} = \sum_R H_{ij}^R \int \mathrm{d}\omega \, J_R(m_t\beta^2 - \omega/2) \, W^R(\omega)$$



Resummation in Momentum Space

Soft and Coulomb resummation: Beneke, Falgari, Schwinn 2009, 2010

$$\hat{\sigma}_{ij} = \sum_R H_{ij}^R \int \mathrm{d}\omega\, J_R(m_t\beta^2 - \omega/2)\, W^R(\omega)$$

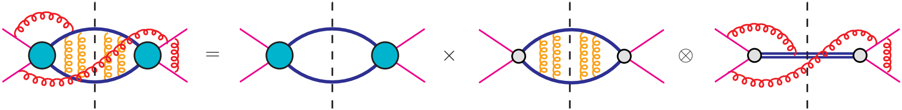


- factorisation formula is derived in EFT framework:
  - SCET for soft and collinear modes
  - pNRQCD for potential and soft modes
- factorisation of soft and Coulomb interaction is non-trivial
- soft function can be diagonalised by choice of colour basis

Resummation in Momentum Space

Soft and Coulomb resummation: Beneke, Falgari, Schwinn 2009, 2010

$$\hat{\sigma}_{ij} = \sum_R H_{ij}^R \int \mathrm{d}\omega \, J_R(m_t\beta^2 - \omega/2) \, W^R(\omega)$$



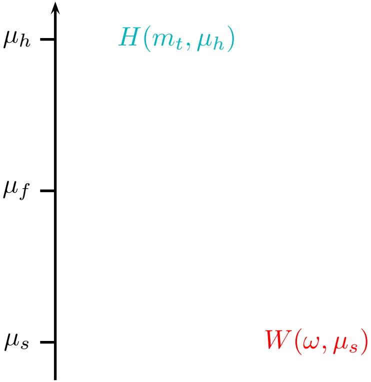
$$\hat{\sigma} \quad \propto \quad \hat{\sigma}^{(0)} \sum_k \left(\frac{\alpha_s}{\beta}\right)^k \exp[\underbrace{\ln\beta \, g_0(\alpha_s \ln\beta)}_{\text{LL}} + \underbrace{g_1(\alpha_s \ln\beta)}_{\text{NLL}} + \underbrace{\alpha_s g_2(\alpha_s \ln\beta)}_{\text{NNLL}} + \ldots]$$

$$\times \{1 \,(\text{LL, NLL})\,; \alpha_s, \beta \,(\text{NNLL})\,; \ldots\}$$

Resummation of Soft Logarithms

Becher, Neubert, Pecjak 2006; Becher, Neubert, Xu 2007;
Beneke, Falgari, Schwinn 2009;
Czakon, Mitov, Sterman 2009

- typical scales:
  $\mu_h \sim 2m_t$, $\mu_s \sim m_t\beta^2$
- hard and soft function obey RGEs
- solve RGEs in momentum space

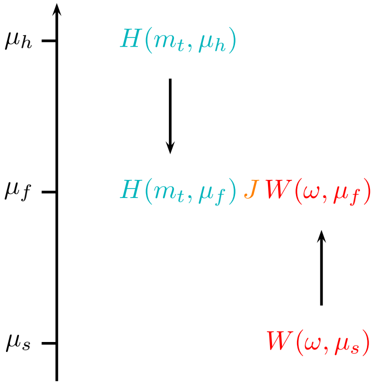


Resummation of Soft Logarithms

Becher, Neubert, Pecjak 2006; Becher, Neubert, Xu 2007;
Beneke, Falgari, Schwinn 2009;
Czakon, Mitov, Sterman 2009

- typical scales:
  $\mu_h \sim 2m_t$, $\mu_s \sim m_t\beta^2$
- hard and soft function obey RGEs
- solve RGEs in momentum space
- evolve H from μ_h to μ_f
- evolve W from μ_s to μ_f

$\Rightarrow$ resums logarithms of β

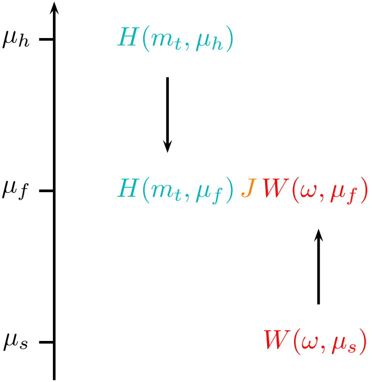


Resummation of Soft Logarithms

Becher, Neubert, Pecjak 2006; Becher, Neubert, Xu 2007;
Beneke, Falgari, Schwinn 2009;
Czakon, Mitov, Sterman 2009

- typical scales:
  $\mu_h \sim 2m_t$, $\mu_s \sim m_t\beta^2$
- hard and soft function obey RGEs
- solve RGEs in momentum space
- evolve H from μ_h to μ_f
- evolve W from μ_s to μ_f

$\Rightarrow$ resums logarithms of β



NNLL:

- 1-loop hard function — Czakon, Mitov 2008
- 1-loop soft function — Beneke, Falgari, Schwinn 2009
- 2-loop anomalous dimensions — Becher, Neubert 2009; Ferroglia et al. 2009; Beneke, Falgari, Schwinn 2009; Czakon, Mitov, Sterman 2009; Moch, Vermaseren, Vogt 2004
- 3-loop cusp anomalous dimension — Moch, Vermaseren, Vogt 2005

Resummation of Coulomb Corrections

Resummation of Coulomb corrections from non-relativistic Greens function

Fadin, Khoze 1987; Peskin, Strassler 1990; ...

$$\left[-\frac{\vec{\nabla}^2}{m_t} + D_R \frac{\alpha_s}{r}\right] G^{(0)}_{C,R}(\vec{r}, \vec{r}\,', E) = \delta(\vec{r} - \vec{r}\,')$$

$$J_R(E) = 2\,\mathrm{Im}\, G^{(0)}_{C,R}(0, 0; E)\,, \quad E = \sqrt{\hat{s}} - 2m_t$$

- includes bound states below threshold ($E < 0$)
- depends on Coulomb scale: $\mu_C \sim m_t \beta$

Resummation of Coulomb Corrections

Resummation of Coulomb corrections from non-relativistic Greens function

Fadin, Khoze 1987; Peskin, Strassler 1990; ...

$$\left[-\frac{\vec{\nabla}^2}{m_t} + D_R \frac{\alpha_s}{r}\right] G^{(0)}_{C,R}(\vec{r}, \vec{r}\,', E) = \delta(\vec{r} - \vec{r}\,')$$

$$J_R(E) = 2\,\mathrm{Im}\, G^{(0)}_{C,R}(0, 0; E)\,, \quad E = \sqrt{\hat{s}} - 2m_t$$

- includes bound states below threshold ($E < 0$)
- depends on Coulomb scale: $\mu_C \sim m_t\beta$

corrections at NNLL:

- NLO Coulomb potential $\rightsquigarrow \mathcal{O}(\alpha_s^2/\beta)$
- NNLO non-Coulomb potential and kinetic energy $\rightsquigarrow \mathcal{O}(\alpha_s^2 \ln\beta)$

Approximate N^{3}LO cross section

- expand N^{3}LL in α_s to obtain dominant terms at N^{3}LO
- N^{3}LO$_{\text{app}}$ provides an approximation of full N^{3}LO:

$$\begin{aligned}\hat{\sigma}^{(3)}_{\text{app}} &= \hat{\sigma}^{(0)}\alpha_s^3\Big[a_6\ln^6\beta+\cdots+a_3\ln^3\beta+a_2\ln^2\beta+a_1\ln\beta \\ &\quad+\frac{1}{\beta}\left(b_4\ln^4\beta+\cdots+b_1\ln\beta+b_0\right) \\ &\quad+\frac{1}{\beta^2}\left(d_2\ln^2\beta+d_1\ln\beta+d_0\right)+C^{(3)}\Big]\end{aligned}$$

- NNLL provides $a_{6,\ldots,3}$, $b_{4,\ldots,1}$, and $d_{2,1,0}$

previous results:

- NNLO$_{\text{app}}$ — Moch, Uwer 2008; Beneke et al. 2009; Kidonakis 2010; Ahrens et al. 2011
- N^{3}LO$_{\text{app}}$ for double differential cross section — Kidonakis 2014
- N^{3}LO$_{\text{app}}$ for gg channel in Mellin space approach — Muselli et al. 2015

N^3LO_{app} Ingredients

- 2-loop hard function — Bärnreuther, Czakon, Fiedler 2013
- 2-loop soft function — Becher, Neubert, Xu 2008; Czakon, Fiedler 2013; Belitsky 1998
- various 3-loop anomalous dimensions
  Moch, Vermaseren, Vogt 2004; Mitov, Moch 2006; Ahrens et al. 2008; Becher, Neubert 2009
- P-wave contribution for gg channels — Falgari, Schwinn, Wever 2012
- LL next-to-eikonal correction from DY and Higgs production
  Krämer, Laenen, Spira 1996; Laenen et al. 2010; Del Duca et al. 2017
- N^3LO Coulomb function: generalise results from $e^+e^- \to t\bar{t}$
  Beneke, Kiyo, Maier, Marquard, Penin, JP, Rauh, Schuller, Steinhauser
- distributional correction from Coulomb bound state — Beneke, Ruiz-Femenía 2016

N^3LO_{app} Uncertainties

missing pieces for colour-octet channels:

- 3-loop massive soft anomalous dimension $\to a_1$
- 2-loop $1/m$ potential and (ultra-)soft correction $\to a_{2,1}$
- 3-loop constants $\to C^{(3)}$

for colour-singlet channel only $C^{(3)}$ is missing

approximation uncertainty estimated from:

- lower orders of massive soft anomalous dimension
- colour-singlet results for $1/m$ potential and ultrasoft correction
- scale dependence of 3-loop constants
- comparing expansion in $\beta = \sqrt{1 - \frac{4m_t^2}{\hat{s}}}$ and $v = \sqrt{\frac{\sqrt{\hat{s}}}{m_t} - 2}$

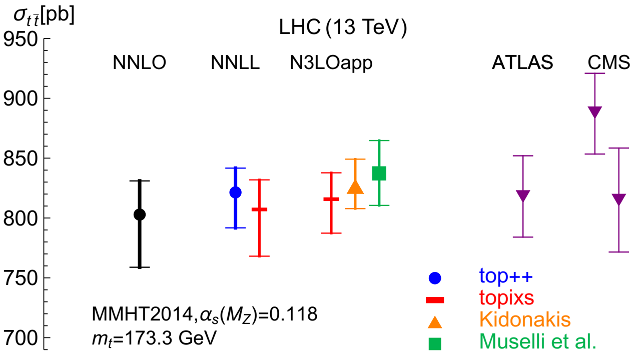
Results for LHC

$\sigma_{t\bar{t}}$ [pb]	LHC @ 7 TeV	LHC @ 8 TeV	LHC @ 13 TeV
NNLO	$171.8\,^{+\ 7.0\ +8.6}_{-11.0\ -8.2}$	$245.2\,^{+\ 9.6\ +11.4}_{-15.3\ -11.0}$	$802.9\,^{+28.1\ +29.7}_{-45.0\ -29.2}$
NNLL+NNLO	$173.3\,^{+6.5\ +8.7}_{-7.8\ -8.3}$	$247.1\,^{+\ 8.9\ +11.5}_{-11.5\ -11.1}$	$807.1\,^{+24.7\ +29.9}_{-39.0\ -29.4}$
N^3LO_{app}	$175.6\,^{+4.9\ +8.7}_{-6.0\ -8.6}$	$250.2\,^{+7.0\ +11.5}_{-8.6\ -11.4}$	$815.7\,^{+22.0\ +29.9}_{-28.4\ -30.4}$
ATLAS	173.3 ± 10.1	241.4 ± 8.5	818 ± 36
CMS			888^{+33}_{-34}

obtained with TOPIXS 3.0 using m_t=173.3 GeV, MMHT 2014 NNLO, $\alpha_s(M_Z) = 0.118 \pm 0.002$
uses NNLO result by Bärnreuther, Czakon, Mitov, Fiedler
experiment: LHC combinations ATLAS-CONF-2012-134, ATLAS-CONF-2014-054, ATLAS 1606.02699, CMS 1701.06228 using m_t=172.5 GeV

- theory uncertainty decreases at higher orders
- N^3LO_{app} theory uncertainty $\sim \pm$ 3%
- PDF+α_s uncertainty $\sim \pm$ 4–5%

Comparison with Other Results



⇒ good agreement between different methods

Summary

- constructed approximate N^3LO cross section for top-pair production
- based on combined soft and Coulomb resummation
- includes all known results for gg and $q\bar{q}$ channels
- reduces the theory uncertainty to $\sim \pm$ 3%
- results are implemented in version 3 of the public program `TOPIXS`

http://users.ph.tum.de/t31software/topixs/

Comparison of Different PDF Sets