Comparing parton showers and NLL resummation

Daniel Reichelt
Work done in collaboration with Stefan Höche and Frank Siegert,
[arXiv:1711.03497]

GEORG-AUGUST-UNIVERSITÄT
GÖTTINGEN

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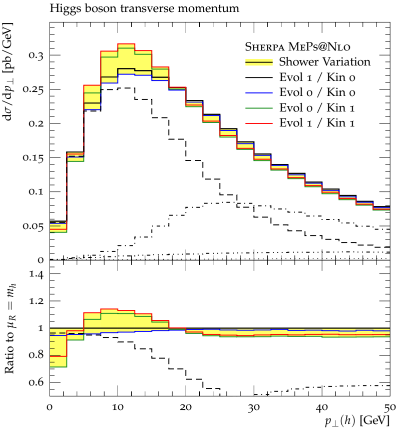
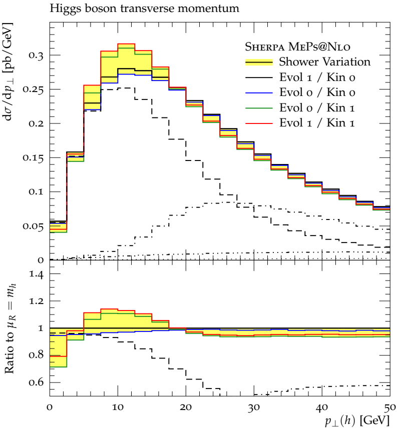


[Höche, Krauss, Schönherr 2014]



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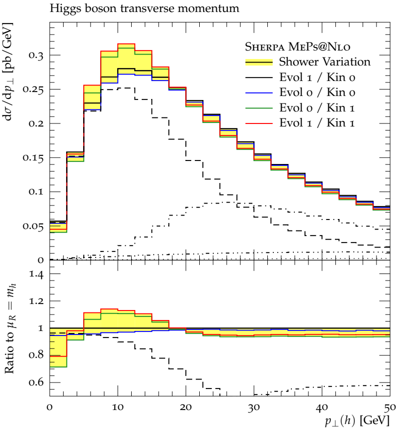
Large Resummation uncertainty:

- Motivates work on showers with better accuracy.



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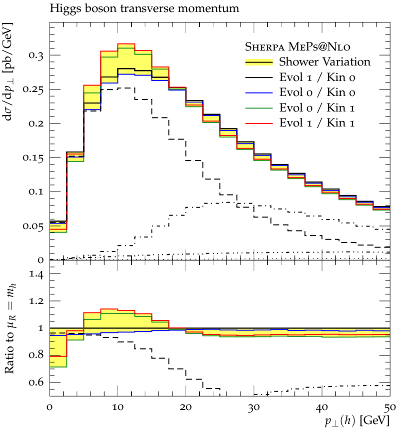

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  - Formally only lowest approximation.
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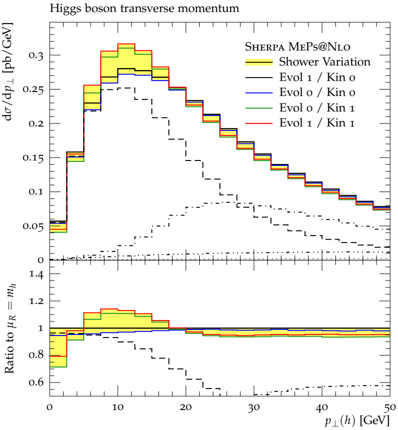
    - e.g. momentum conservation



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  - Parton showers "are better than formally expected":
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- No straightforward comparison possible.

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  - Turn on different contributions step by step
    ⇒ finally recover full parton shower.
  - Determine sizes of individual contributions.

Outline

- Start from $q\bar{q}$ pair $\rightarrow$ look at observables vanishing in two jet limit.
- Consider additive observable, i.e. in presence of several soft gluons (In this talk $\Rightarrow$ Thrust $1 - T$, FC_1):

$$V(k_1, \ldots, k_n) = \sum_{i=1}^{n} V(k_i)$$

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$$\mathrm{R}_{NLL}(v) = 2 \int_{Q^2 v^{\frac{2}{a+b}}}^{Q^2} \frac{d\xi}{\xi} \left[\int_0^1 dz\, \frac{\alpha_s\left(\xi(1-z)^{\frac{2b}{a+b}}\right)}{2\pi} \frac{2\, C_F}{1-z} \Theta\left(\ln \frac{(1-z)^{\frac{2a}{a+b}}}{\xi/Q^2} \right) - \frac{\alpha_s(\xi)}{\pi}\, C_F B_q \right]$$

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  - Evaluate α_s in CMW scheme $\rightarrow$ cumulatively account for secondary emissions from gluons $\Rightarrow \mathcal{F}(v) = \lim_{\epsilon \to 0} \mathcal{F}_\epsilon(v)$,

$$\mathcal{F}_\epsilon(v) = e^{R'_{\mathrm{NLL}}(v) \ln \epsilon} \sum_{m=0}^{\infty} \frac{1}{m!} \left(\prod_{i=1}^{m} R'_{\mathrm{NLL}}(v) \int_\epsilon^1 \frac{d\zeta_i}{\zeta_i} \right) \Theta\left(1 - \sum_{j=1}^{m} \zeta_j \right)$$

- Parton Showers in a nutshell:
  - No-branching probability, e.g. from collinear factorization of matrix elements and unitarity: $\Pi(t', t) = e^{-R_{PS}(t,t')}$

$$\mathrm{R}_{PS}(v) = 2\int_{Q^2 v^{\frac{2}{a+b}}}^{Q^2} \frac{d\xi}{\xi} \int_{z_{\min}}^{z_{\max}} dz\, \frac{\alpha_s\left(\xi(1-z)^{\frac{2b}{a+b}}\right)}{2\pi}\, C_F \left[\frac{2}{1-z} - (1+z)\right] \Theta\left(\ln \frac{(1-z)^{\frac{2a}{a+b}}}{\xi/Q^2}\right).$$

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  - To be able to reproduce to analytic result:
    - ★ Evaluate α_s in CMW scheme $\rightarrow$ secondary gluon splittings are accounted for (for Observables considered here) $\rightarrow$ do not explicitly generate them
    - ★ Analyse Σ in parton shower:

$$\Sigma(v) = \exp\left[-R(Q, t_0)\right] \sum_{m=0}^{\infty} \frac{1}{m!} \left(\prod_{i=1}^{m} \int \frac{dt_i}{t_i} R'(t_i)\right) \Theta\left(v - \sum_{j=1}^{m} V(t_j)\right)$$

$$\Sigma(v) = \exp\left\{-\int_v \frac{d\xi}{\xi} R'_{>v}(\xi) - \int_{v_{\min}}^{v} \frac{d\xi}{\xi} R'_{<v}(\xi)\right\}$$
$$\times \sum_{m=0}^{\infty} \frac{1}{m!} \left(\prod_{i=1}^{m} \int_{v_{\min}} \frac{d\xi_i}{\xi_i} R'_{<v}(\xi_i)\right) \Theta\left(v - \sum_{j=1}^{m} V(\xi_j)\right)$$

$$\mathrm{R'}_{\lessgtr v}(\xi) = \frac{\alpha_s^{\lessgtr v,\mathrm{soft}}\left(\mu^2_{\lessgtr}\right)}{\pi} \int_{z^{\min}}^{z^{\max}_{\lessgtr v,\mathrm{soft}}} dz\, \frac{C_\mathrm{F}}{1-z} - \frac{\alpha_s^{\lessgtr v,\mathrm{coll}}\left(\mu^2_{\lessgtr v}\right)}{\pi} \int_{z^{\min}}^{z^{\max}_{\lessgtr v,\mathrm{coll}}} dz\, C_\mathrm{F} \frac{1+z}{2} \, .$$

	Resummation	Parton Shower
$z^{\max}_{>v,\mathrm{soft}}$	$1-(\xi/Q^2)^{\frac{a+b}{2a}}$	$1-(\xi/Q^2)^{\frac{a+b}{2a}}$
$\mu^2_{>v,\mathrm{soft}}$	$\xi(1-z)^{\frac{2b}{a+b}}$	$\xi(1-z)^{\frac{2b}{a+b}}$
$\alpha_s^{>v,\mathrm{soft}}$	2-loop CMW	2-loop CMW
$z^{\max}_{<v,\mathrm{soft}}$	$1-v^{\frac{1}{a}}$	$1-(\xi/Q^2)^{\frac{a+b}{2a}}$
$\mu^2_{<v,\mathrm{soft}}$	$Q^2 v^{\frac{2}{a+b}}(1-z)^{\frac{2b}{a+b}}$	$\xi(1-z)^{\frac{2b}{a+b}}$
$\alpha_s^{<v,\mathrm{soft}}$	1-loop	2-loop CMW

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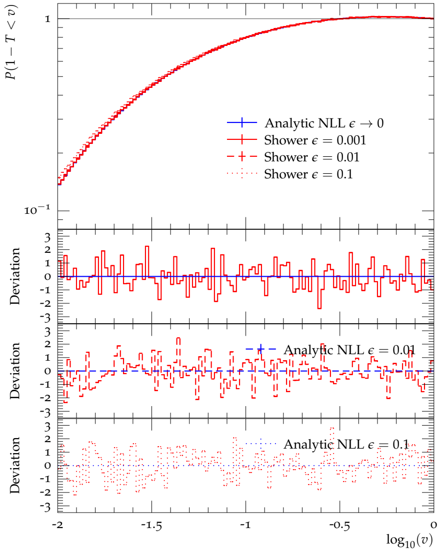
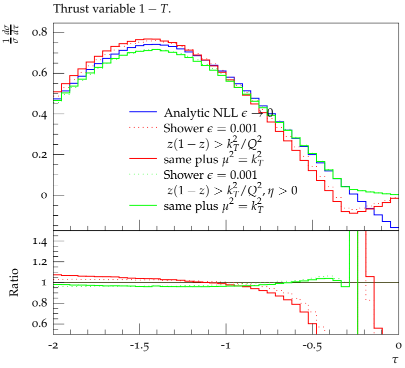
$z^{\max}_{>v,\mathrm{coll}}$	1	$1-(\xi/Q^2)^{\frac{a+b}{2a}}$
$\mu^2_{>v,\mathrm{coll}}$	ξ	$\xi(1-z)^{\frac{2b}{a+b}}$
$\alpha_s^{>v,\mathrm{coll}}$	1-loop	2-loop CMW
$z^{\max}_{<v,\mathrm{coll}}$	0	$1-(\xi/Q^2)^{\frac{a+b}{2a}}$
$\mu^2_{<v,\mathrm{coll}}$	n.a.	$\xi(1-z)^{\frac{2b}{a+b}}$
$\alpha_s^{<v,\mathrm{coll}}$	n.a.	2-loop CMW

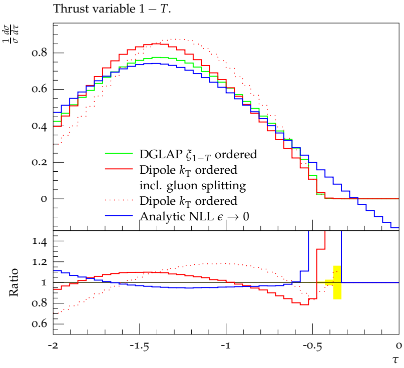
- Validation: Making all choices resummation-like in the parton shower reproduces resummation.



Outline



(a)



(b)

- choose $z_{min/max}$ as in PS → momentum conservation
- do phase space sectorization as in PS → z_{max}^{coll}
- additionally, we are now free to choose $\mu^2 = k_T^2$ everywhere
- compare to dipole showers

Outline

1. General Setup
2. Results
3. An application

- Understanding hadronization corrections for analytic calculations
- Motivation:
  - Applications of soft drop groomed observables in phenomenology [Larkoski, Marzani, Soyez, Thaler 2014]
  - e.g. soft-drop thrust [Baron, Marzani, Theeuwes]
  - Usual findings: greatly reduces dependence on non-perturbative physics modelling
  - However: usually relying on MC parton level/hadron level comparison $\rightarrow$ the parton level input in analytic calculations can be very different from the shower
  - Naive analytic models/parametrizations of hadronization not working for soft drop groomed observables

- Soft Drop in $e^+e^- \to$ jets:
  - recluster jet/hemisphere into two jets (usually using C/A)
  - check if

$$\frac{\min[E_i, E_j]}{E_i + E_j} > z_{\text{cut}} \, (1 - \cos\Theta_{ij})^{\beta/2}$$

  - if not, disregard softer jet, repeat
- here: $z_{\text{cut}} = 0.1$, $\beta = 0$.

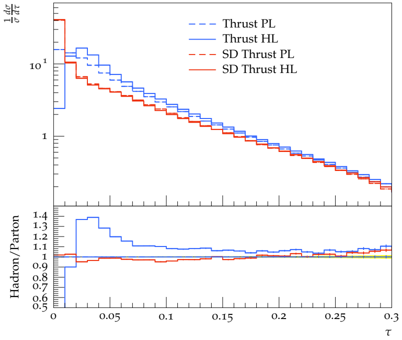
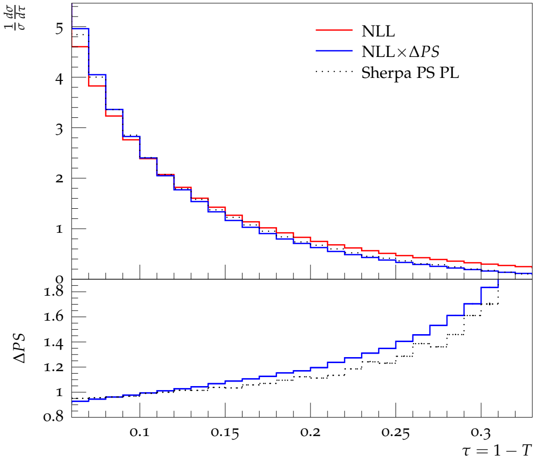


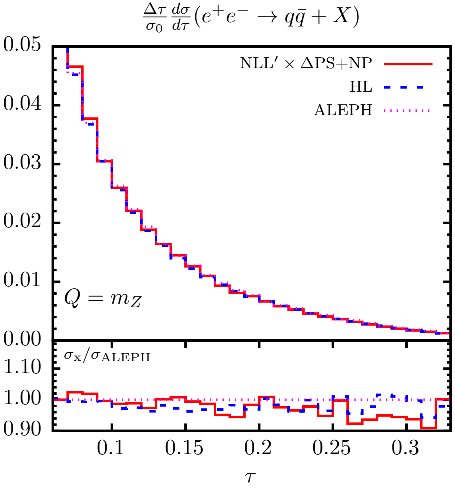
Figure: Sherpa with (HL) and without (PL) hadronization effects taken into account.

- Analytic hadronization model:
  - Cumulative distribution Σ convoluted with function F parametrizing non-perturbative effects.
  - e.g. $F(k) = 4k/\Omega^2 \exp(-2k/\Omega)$

- In phenomenological relevant region: momentum conservation gives most relevant contribution
- use this to extract perturbative $\Delta\mathrm{PS}$

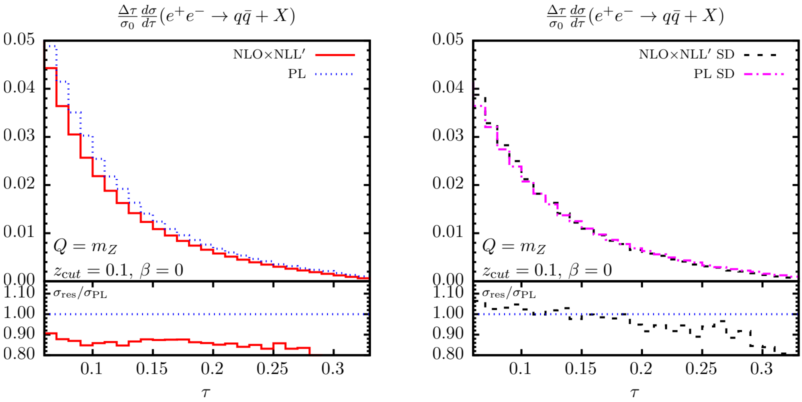


- If we understand the perturbative difference we can use the hadronization models interchangeably



[Plot by Vincent Theeuwes]

- Outlook:
  - Realistic calculation: (at least) matching to NLO
  - Need to understand $\Delta\mathrm{PS}$ at this level, or establish it is small



(a) Thrust (b) SD Thrust

[Plots by Vincent Theeuwes]

- Summary:
    - Constructed parton shower *exactly* emulating NLL resummation.
    - Used to determine *numerical* size of individual contributions.
    - Interpretation: inherent uncertainty in resummation and parton shower.
    - Use this to understand how to consistently deduce hadronization corrections from MC for soft drop groomed observables.