Logarithmic accuracy of parton showers

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Parton showers and resummation

- Parton showers are ubiquitous in collider physics, however the theoretical accuracy of these tools is not always well understood.
- Resummations also provide all order calculations, allowing for higher logarithmic accuracy but with narrower scope of application.

	RESUMMATIONS	PARTON SHOWERS
Treatment of radiation	• Several simplifications: amplitudes, phase space, observable • All calculations derived in the on-shell/ singular limit (only logarithms)	• Radiation is described fully exclusively. Provide full set of final-state momenta • Full momentum conservation necessary (e.g. initial condition for hadronisation)
Observable dependence	• Tailored to the observable, e.g. global vs. non-global, specific approximations in each case	• A simple shower should be accurate for a broad family of observables at once
Logarithmic Accuracy	• Higher logarithmic orders achieved thanks to the above simplifications in the formulation	• Currently unknown. The goal of this talk is to initiate a formal study of this point

Accuracy of parton showers

- General purpose Monte Carlo event generators are used to simulate physics over broad range of scales: from hard matrix elements at TeV scale to GeV scale where quarks and gluons hadronise.
- Complicated tools with many subtleties, difficult to systematically quantify the accuracy that can be achieved with a given parton-shower algorithm.
- The accuracy of a shower is important in a number of contexts:
  - Quality of hadronisation models depend on the initial conditions provided by the shower.
  - Jet substructure and machine learning tools exploit very exclusive kinematic regions and leverage it to design better taggers.

What is a shower?

A parton shower consists of three main features:

1. An ordering variable which defines the sequence according to which emissions are generated (such as k_t, angle, virtuality).
2. A branching probability $P(S_n.v)$ of finding a state S_n with n partons at scale v, which evolves as

$$\frac{dP(\mathcal{S}_n, v)}{d\ln 1/v} = -f(\mathcal{S}_n, v)P(\mathcal{S}_n, v)\,.$$

3. A kinematic mapping $\mathcal{M}$ from state $\mathcal{S}_n$ to $\mathcal{S}_{n+1}$

$$\mathcal{S}_{n+1} = \mathcal{M}(\mathcal{S}_n, v; i, j, \underbrace{z, \phi}_{\text{emission}})\,.$$

with an associated "splitting" weight function $d\mathcal{P}(\mathcal{S}_n, v; i, j, z, \phi)$, governing relative probabilities of new states.

Case study: transverse-momentum ordered showers

As a case study, we will focus specifically on

- transverse-momentum ordered showers,
- with a dipole-local recoil

$$\tilde{p}_i + \tilde{p}_j \longrightarrow p_i + p_j + p_k$$

  where spectator j absorbs the longitudinal recoil.

which encompass several modern showers such as Pythia and Dire.

Pythia

- Evolution variable and branching:

$$v \equiv p_{\perp,\mathrm{evol}}$$

$$\rho^2_{\perp,\mathrm{evol}} = \frac{p^2_{\perp,\mathrm{evol}}}{(\tilde{p}_i + \tilde{p}_j)^2}\,, \qquad y = \frac{\rho^2_{\perp,\mathrm{evol}}}{z(1-z)}\,, \qquad \tilde{z} = \frac{(1-z)\left(z^2 - \rho^2_{\perp\mathrm{evol}}\right)}{z\,(1-z) - \rho^2_{\perp\mathrm{evol}}}$$

$$\rho_{\perp,\mathrm{evol}} \leq z \leq 1 - \rho_{\perp,\mathrm{evol}}$$

- $\mathrm{k_t}$ and rapidity of emission w.r.t. the emitter

$$\eta = \ln \frac{(1-\tilde{z})Q}{|k_\perp|}\,, \qquad |k^2_\perp| = \frac{\left(z^2 - \rho^2_{\perp\mathrm{evol}}\right)\left((1-z)^2 - \rho^2_{\perp\mathrm{evol}}\right)}{\left(z\,(1-z) - \rho^2_{\perp\mathrm{evol}}\right)^2}$$

Dire

- Evolution variable and branching:

$$v \equiv \sqrt{t}$$

$$\kappa^2 = \frac{t}{(\tilde{p}_i + \tilde{p}_j)^2}\,, \qquad y = \frac{\kappa^2}{1-z}\,, \qquad \tilde{z} = \frac{z-y}{1-y}$$

$$\frac{1}{2} - \sqrt{\frac{1}{4} - \kappa^2} \leq z \leq \frac{1}{2} + \sqrt{\frac{1}{4} - \kappa^2}$$

- $\mathrm{k_t}$ and rapidity of emission w.r.t. the emitter

$$\eta = \ln \frac{(1-\tilde{z})Q}{|k_\perp|}\,, \qquad |k^2_\perp| = (1-z)\,\frac{z(1-z) - \kappa^2}{(1-z-\kappa^2)^2}\,t$$

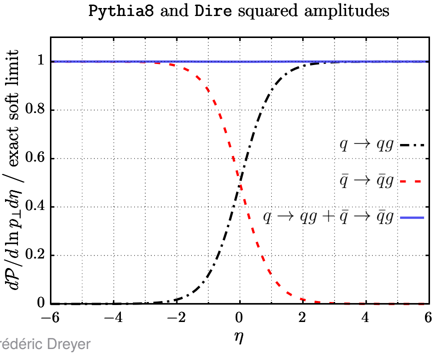
Singularity structure

- Expect a parton shower to reproduce the matrix element for any single-emission configuration with one or two singularities
- For multiple emissions, expect to control leading singularity of squared amplitude for any number of emissions.
- To reproduce the leading double logarithms, the soft and collinear limit for an emission should be

$$d\mathcal{P} = \frac{2C\alpha_s(p_\perp^2)}{\pi}\frac{dp_\perp}{p_\perp}d\eta$$

- For single-logarithmic accuracy, one also needs to reproduce emission pattern in the hard collinear and soft large-angle region.

Both Pythia and Dire divide the dipole at zero rapidity in the dipole's rest frame

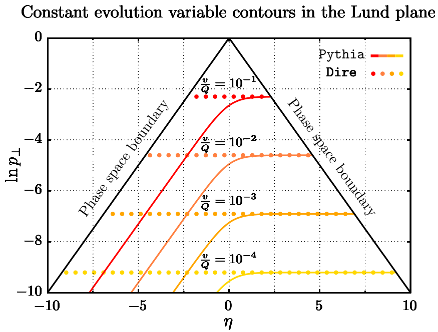


Pythia:

$$d\mathcal{P}_{q\to qg} = \frac{2\alpha_s(p^2_{\perp,\mathrm{evol}})C_F}{\pi}\frac{dp_\perp}{p_\perp}d\eta\left(\frac{e^{2\eta}}{1+e^{2\eta}}\right)$$

Dire:

$$d\mathcal{P}_{q\to qg} = \frac{2\alpha_s(t)C_F}{\pi}\frac{dp_\perp}{p_\perp}d\eta\left(\frac{e^{2\eta}}{1+e^{2\eta}}\right)$$



Pythia:

$$\eta = \frac{1}{2}\ln\left[\frac{(1-z)^2}{\rho^2_{\perp,\mathrm{evol}}} - 1\right], \quad |p^2_\perp| = p^2_{\perp,\mathrm{evol}}\left(1 - \frac{\rho^2_{\perp,\mathrm{evol}}}{(1-z)^2}\right)$$

Dire:

$$\eta = \frac{1}{2}\ln\left[\frac{(1-z)^2}{\kappa^2}\right], \qquad |p^2_\perp| = t$$

- Correct matrix element for single emission reproduced up to (Pythia) or including (Dire) running coupling effects.
- For Pythia, there is suppressed but non-zero probability to have arbitrarily small k_t for any finite value of the evolution variable v.

Two-emission case in the soft limit

Consider two soft-collinear emissions g_1, g_2 with $|\eta_1 - \eta_2| \gg 1$. The corresponding double emission probability is

$$dP_2 = \frac{C_F^2}{2!} \prod_{i=1,2} \left(\frac{2\alpha_s(p_{\perp,i}^2)}{\pi} \frac{dp_{\perp,i}}{p_{\perp,i}} d\eta_i \frac{d\phi_i}{2\pi} \right)$$

where $p_{\perp,i}$ and η_i are defined with respect to the q and $\bar{q}$ directions.

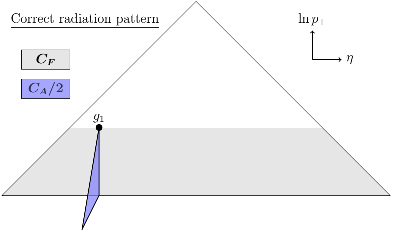
We want to answer the following question: do the parton showers reproduce this?

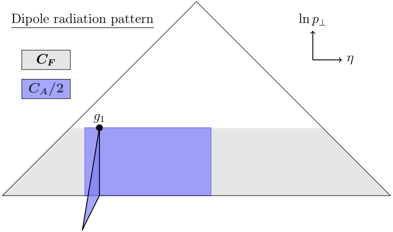
To generate two emissions:

- Start from $\bar{q}q$ and generate a first emission g_1 at value v_1 of ordering variable.
- With value v_2 of ordering variable, select one of the $\bar{q}g_1$ and g_1q dipole to emit g_2

Two-emission case: double strong ordering

- In the strongly ordered limit $v_1 \gg v_2$, kinematics of g_1 not affected by second emission.
- Splitting the $\bar{q}g_1$ dipole into two equal parts in its rest frame causes some part of the radiation assigned to the gluonic part to be in a phase space region where it is closer in angle to the $\bar{q}$ or q than it is to the gluon.
- This leads to a region with an incorrect colour factor $C_F C_A/2$ instead of C_F^2.





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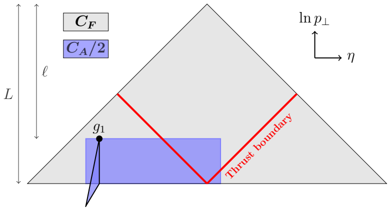
Difference between dipole and correct pattern for thrust:

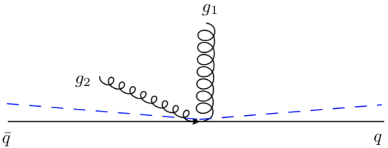
$$\delta\Sigma(L) = -\frac{1}{64}\left(\frac{2\alpha_s C_F}{\pi}\right) L^4 \left(\frac{C_A}{2C_F} - 1\right)$$

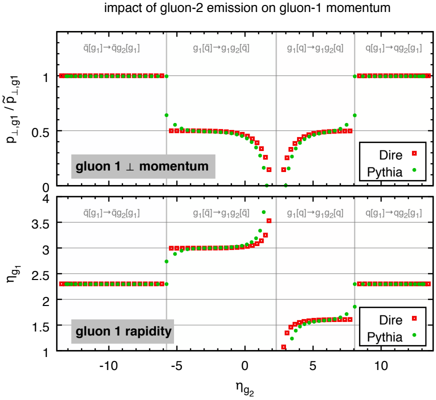


- Consider now the case where the ordering variables are of the same order $v_2 \sim v_1/2$.
- First emission is affected by recoil after second emission

When g_2 is at relatively central rapidities, g_1 acquires a transverse recoil, leading to $p_{\perp,g_1}/\tilde{p}_{\perp,g_1} = 1/2$.

$$g_1[\bar{q}] \to g_1 g_2[\bar{q}] : \boldsymbol{p}_{\perp,g_1} = \tilde{\boldsymbol{p}}_{\perp,g_1} - \boldsymbol{p}_{\perp,g_2}\,,$$
$$\eta_{g_1} = \tilde{\eta}_{g_1} - \ln \frac{|\boldsymbol{p}_{\perp,g_1}|}{|\tilde{\boldsymbol{p}}_{\perp,g_1}|}$$

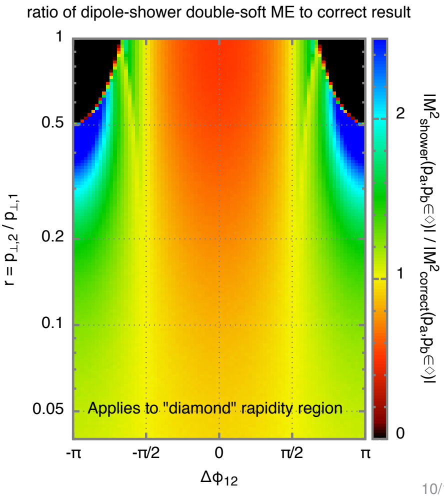




Two-emission case: single strong ordering

- Consider now the case where the ordering variables are of the same order $v_2 \sim v_1/2$.
- First emission is affected by recoil after second emission

- Effective matrix element does not reproduce correct soft limit at α_s^2
- Leads to incorrect NLL terms in many observables.



Logarithmic analysis at second order

Shortcomings of showers impact the logarithmic accuracy for a wide range of observables.

We focus on event shape variables and study probability that the event shape has a value smaller than e^{-L}

$$\Sigma(L) = \exp\left[L g_1(\alpha_s L) + g_2(\alpha_s L) + \alpha_s g_3(\alpha_s L) + \cdots\right] + \mathcal{O}\alpha_s e^{-L}\,,$$

In leading-N_C limit, impact of recoil can be written

$$\delta\Sigma(L) = \bar\alpha^2 \int_0^1 \frac{dv_1}{v_1} \int_{\ln v_1}^{\ln 1/v_1} d\eta_1 \int_0^{v_1} \frac{dv_2}{v_2} \int_{\frac{1}{2}(\eta_1+\ln v_1)}^{\frac{1}{2}(\eta_1+\ln 1/v_1)} d\eta_2 \int_0^{2\pi} \frac{d\phi_{12}}{2\pi} \times \times \left[\Theta\big(e^{-L} - V(p_1^{\text{shower}}, p_2)\big) - \Theta\left(e^{-L} - V(p_1^{\text{correct}}, p_2)\right)\right],$$

Observable	$\text{NLL}_{\ln\Sigma}$ discrepancy
$1-T$	$0.116^{+0.004}_{-0.004}\,\bar\alpha^3 L^3$
vector p_t sum	$-0.349^{+0.003}_{-0.003}\,\bar\alpha^3 L^3$
B_T	$-0.0167335\,\bar\alpha^2 L^2$
y_3^{cam}	$-0.18277\,\bar\alpha^2 L^2$
FC_1	$-0.066934\,\bar\alpha^2 L^2$

CONCLUSIONS

Conclusions

- Introduced formalism needed to study multi-scale accuracy of parton showers.
- Focusing on transverse-momentum ordered shower with dipole-local recoil, studied singular limits up to second order.
- We identified issues that affect the LL accuracy at subleading N_c and the NLL accuracy at leading N_c for a wide range of event-shape observables.
- These issues will impact the prospects for precision studies relying on parton showers.

Established a basis to understand the logarithmic properties of parton shower algorithms.