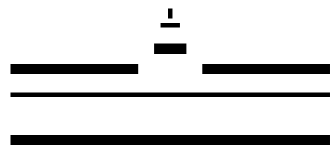


# ASSOCIATED PRODUCTION OF TOPS AND HIGGS: THEORETICAL PREDICTIONS

ANNA KULESZA  
UNIVERSITY OF MÜNSTER



WESTFÄLISCHE  
WILHELMS-UNIVERSITÄT  
MÜNSTER

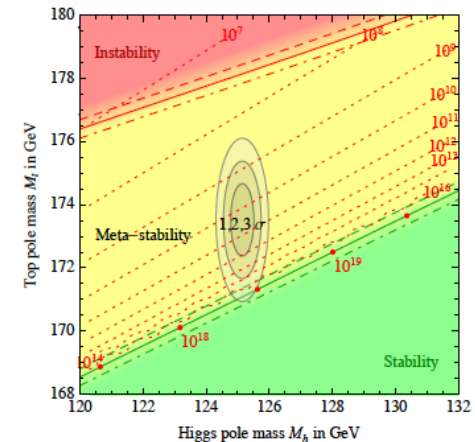
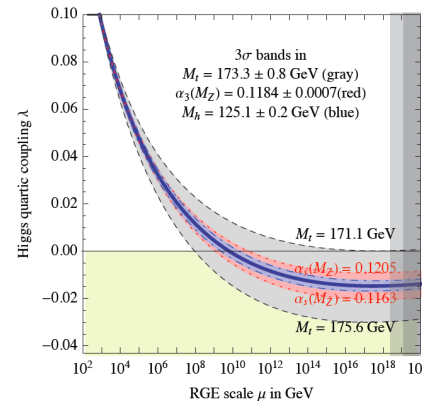
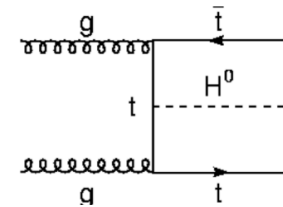
**DFG** Deutsche  
Forschungsgemeinschaft

QCD@LHC2018, TU DRESDEN, 27.08.2018

# ASSOCIATED HIGGS PRODUCTION WITH TOP QUARKS

- Direct probe of the strength of the top-Yukawa coupling without making any assumptions regarding its nature
- Crucial for understanding the Higgs sector and searches for deviations from the SM
- Far-reaching consequences -> stability of our Universe
- Fundamental Yukawa interactions in the SM not probed until very recently
- Small cross section  $\sigma \approx 500 \text{ fb}$  @13 TeV
  - need precise theory for signal and background → next talk
  - as well as advanced experimental techniques → previous talk

$$pp \rightarrow t\bar{t}H$$

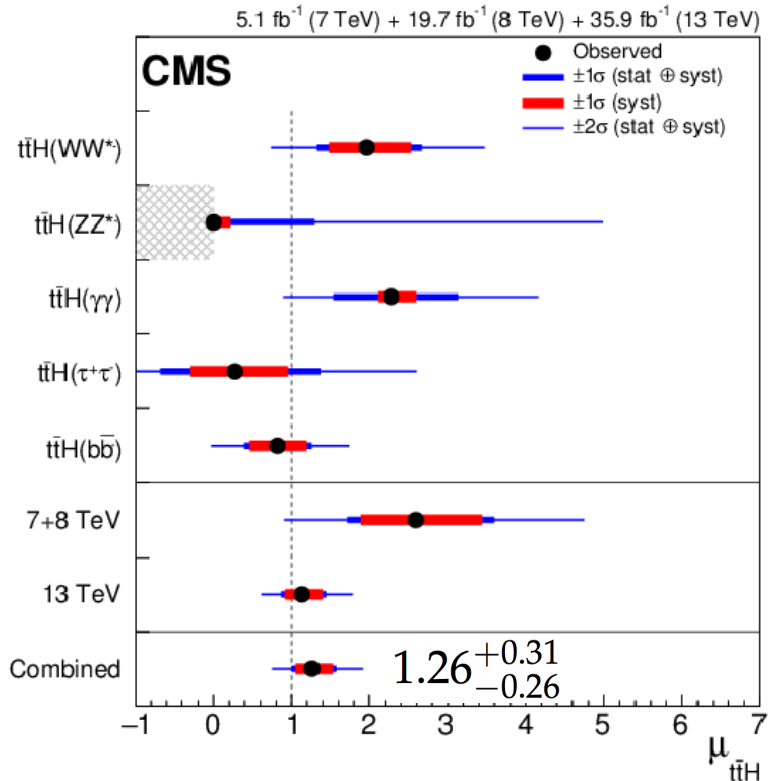


$$V = -\frac{m_H^2}{2} |H|^2 + \lambda |H^4|$$

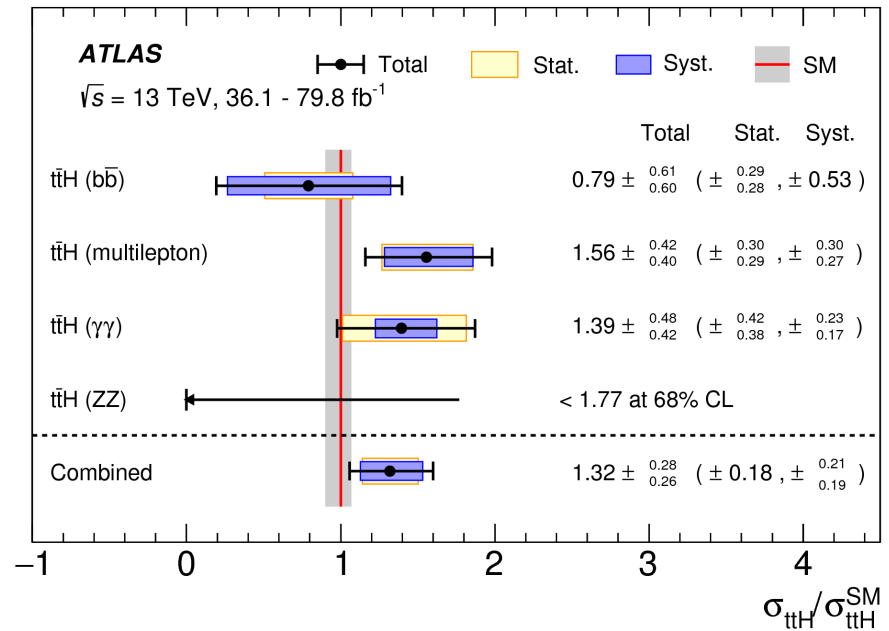
$$4\pi^2 \frac{d\lambda}{d \ln \mu^2} \cong -3y_t^4 + 6\lambda y_t^2 + 12\lambda^2 + \dots$$

[Buttazzo et al.'13]

# A NEW CHAPTER



CMS, Run1 + Run2: **5.2σ** (4.9σ exp.)



ATLAS, Run1 + Run2: **6.3σ** (5.1σ exp.)

# THEORY STATUS FOR TTBARH

## Fixed order perturbation theory

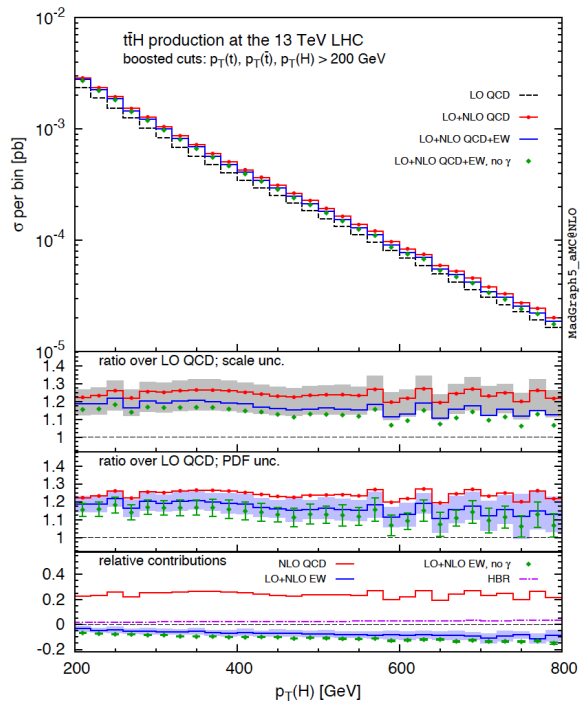
- **NLO QCD** available since (almost) 20 years [*Beenakker, Dittmaier, Krämer, Plumper, Spira, Zerwas '01-'02*][*Reina, Dawson'01*][*Reina, Dawson, Wackerath'02*]  
[*Dawson,Orr,Reina,Wackerath'03*] [*Dawson, Jackson, Orr, Reina, Wackerath'03*]
  - corrections to the total cross section of a few tens of percent (20-30%)
  - residual scale uncertainty of 10%
- **NLO QCD matched with parton showers**
  - POWHEG-Box [*Garzelli, Kardos,Papadopoulos,Trocsanyi'11*] [*Hartanto, Jäger,Reina, Wackerath'15*]
  - aMC@NLO [*Frederix, Frixione, Hirschi, Maltoni, Pittau, Torrielli'11*]
  - SHERPA+RECOLA (also for EW) [*Biedermann et al.'17*]

-

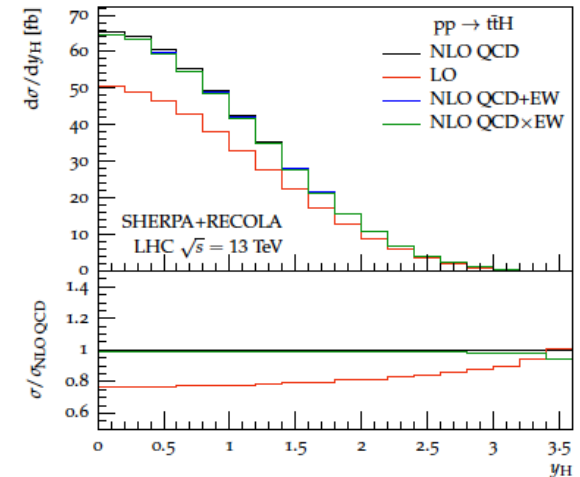
# TtH @NLO QCD AND EW

[Frixione, Hirschi, Pagani, Shao, Zaro'14-'15][Zhang, Ma, Zhang, Chen, Guo'14]

[Biedermann, Bräuer, Denner, Pellen, Schumann, Thompson'17]



- QCD corrections  $O(\alpha_s^3 \alpha)$  dominant
- K-factors in general not flat
- EW effects can be significant at large energies and  $p_T$ 's (Sudakov!), depending on the observable and cuts (more in the boosted region)
- QED effects small overall

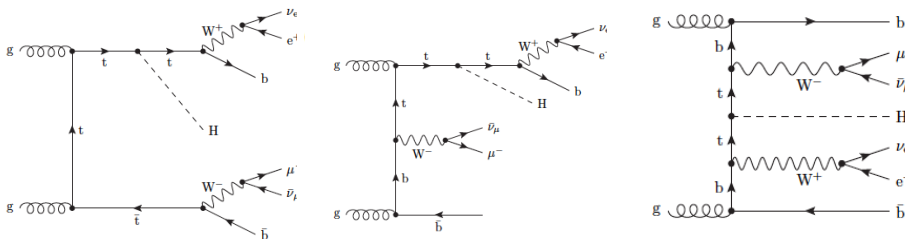


- multiplicative and additive combinations of QCD and EW corrections very close

# OFF-SHELL @NLO QCD

➤ Top decays into b quarks and leptons:  
 $pp \rightarrow ttH \rightarrow W^+W^-bbH \rightarrow e^+\nu_e \mu^-\nu_\mu bbH$

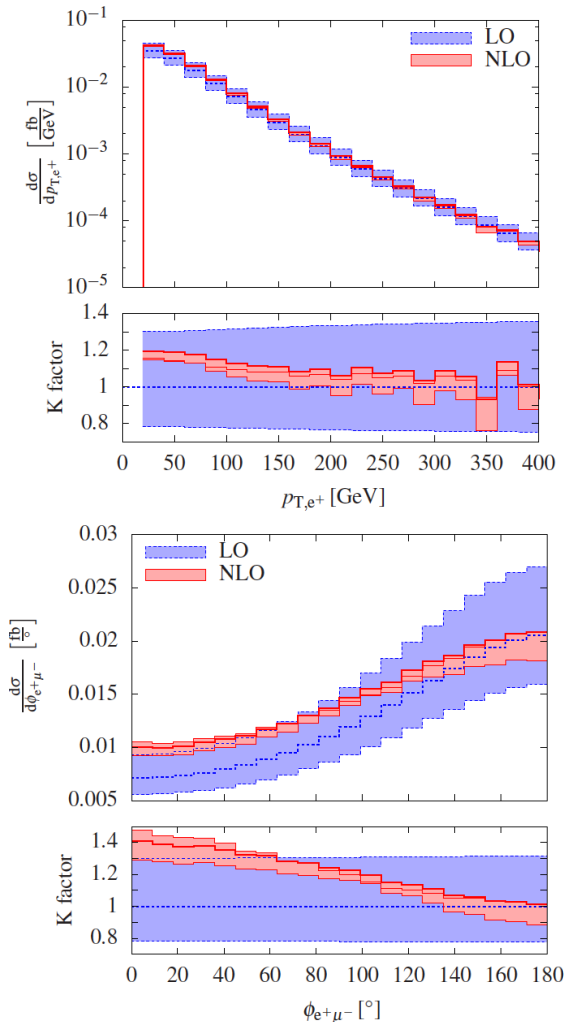
➤ NLO QCD [Denner, Feger'15] corrections to e.g.



-> RECOLA

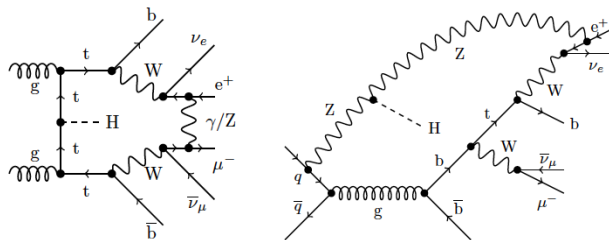
➤ All resonant, non-resonant, interference and off-shell effects included

➤ QCD NLO: non-resonant and off-shell effects below 1% for integrated cross sections



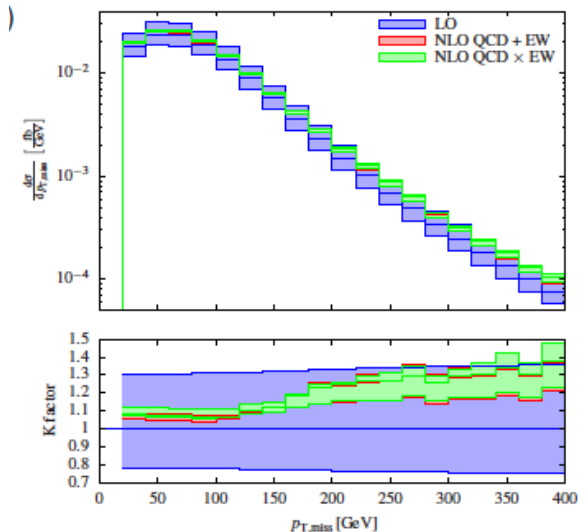
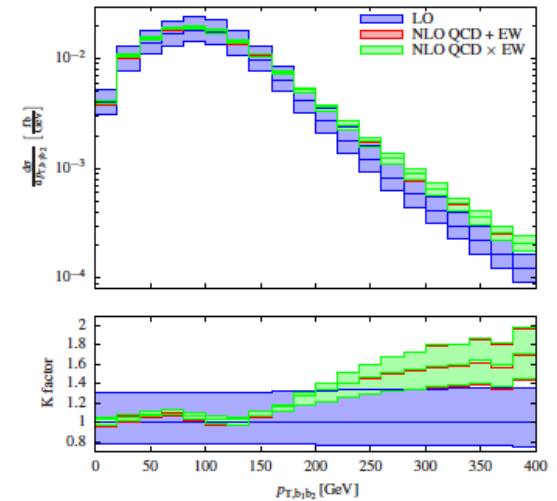
# NLO QCD AND EW WITH TOPS OFF-SHELL

- Top decays into b quarks and leptons:  
 $pp \rightarrow ttH \rightarrow W^+W^-bbH \rightarrow e^+\nu_e \mu^-\bar{\nu}_\mu bbH$
- NLO EW corrections [Denner, Lang, Pellen, Uccirati'16] together with NLO QCD
- Tour de force calculations, virtual graphs involving nonagons



-> RECOLA

- Double pole approximation reliable at NLO



# BEYOND NLO QCD: RESUMMATION

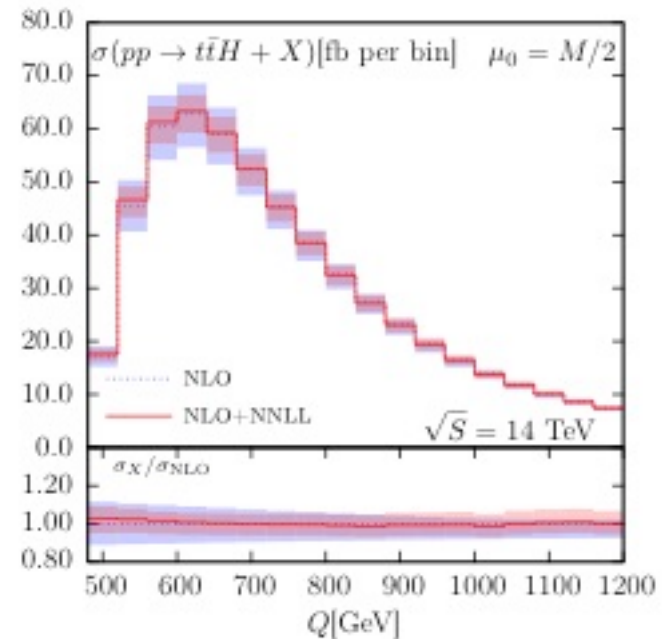
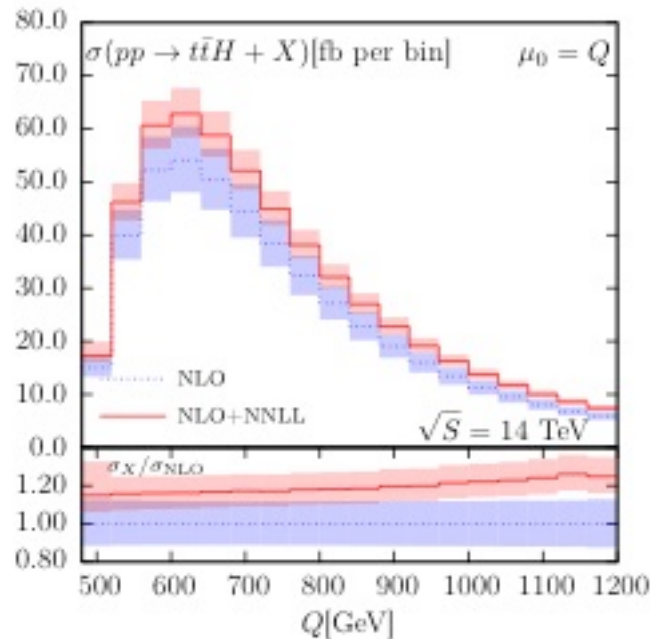
Resummation extends accuracy of the perturbative prediction beyond fixed-order by taking into account logarithmic contributions to all orders... **especially applicable when NNLO calculations out of reach**

- **NLL+NLO resummation in the absolute threshold limit**,  $\hat{s} \rightarrow M^2 = (m_3 + m_4 + m_5)^2$  obtained using direct QCD approach *[AK, Motyka, Stebel, Theeuwes'15]*
- **“Approximated” NNLO** based on the SCET approach to resummation **in the invariant mass limit**  $\hat{s} \rightarrow Q^2 = (p_3 + p_4 + p_5)^2$  *[Broggio, Ferroglia, Pecjak, Signer, Yang'15]*
- **NLL+NLO resummation in the invariant mass limit**, direct QCD *[AK, Motyka, Stebel, Theeuwes'16]*
- **NNLL+NLO resummation in the invariant mass limit**, hybrid SCET/direct QCD method *[Broggio, Ferroglia, Pecjak, Yang'16]*
- **NNLL+NLO resummation in the invariant mass limit**, direct QCD method *[AK, Motyka, Stebel, Theeuwes'17]*
- **NLL+NLO in the invariant mass limit for production with pseudoscalar Yukawa couplings** *[Broggio et al. '17]*



# INVARIANT MASS DISTRIBUTION

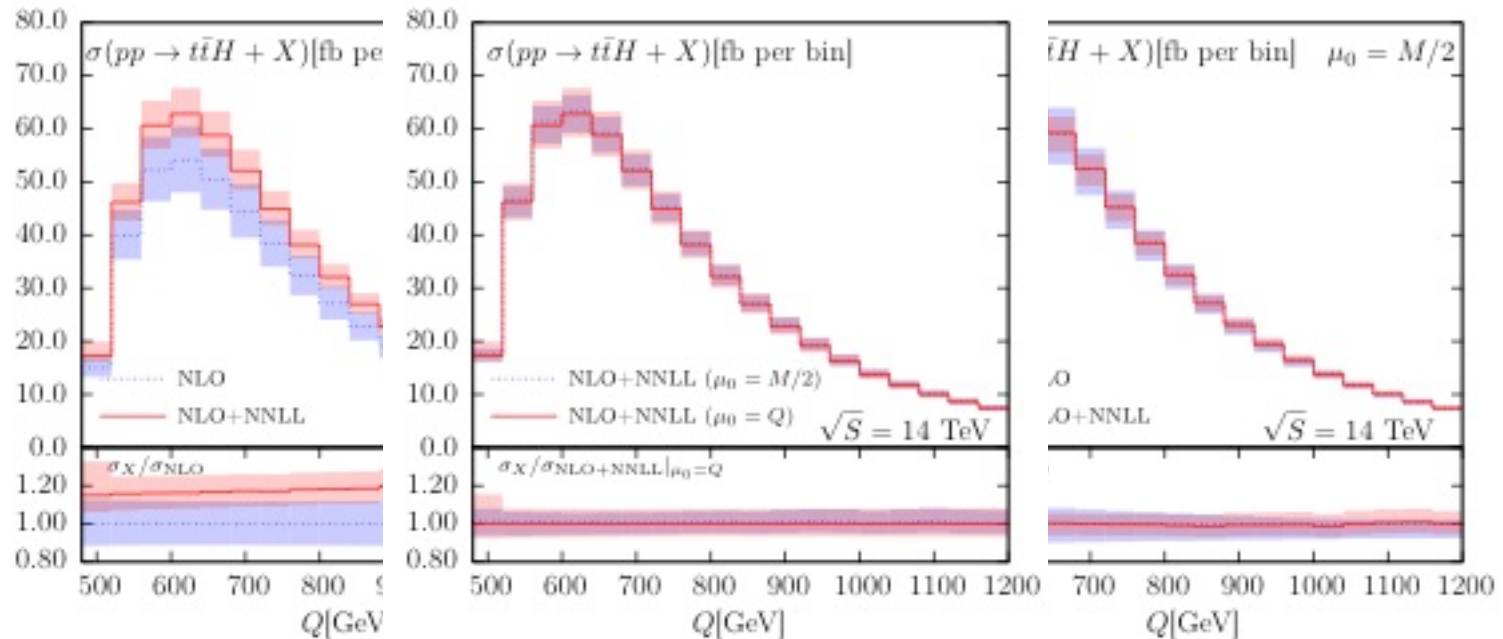
[AK, Motyka, Stebel, Theeuwes'17]



- NNLL+NLO distributions for two considered scale choices very close, NLO results differ visibly  
→  $K_{NNLL}$  factors also different
- NNLL+NLO error band slightly narrower than NLO (7-point method)

# INVARIANT MASS DISTRIBUTION

[AK, Motyka, Stebel, Theeuwes'17]



- NNLL+NLO distributions for the two scale choices very close, NLO results differ visibly  $\rightarrow K_{NNLL}$  factors also different
- NNLL+NLO error band slightly narrower than NLO (7-point method)

# TOTAL CROSS SECTION

[AK, Motyka, Stebel, Theeuwes'17]

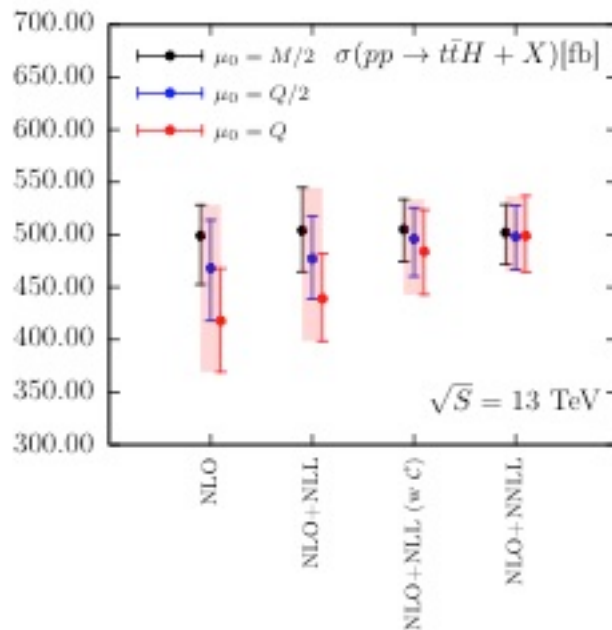
$\sqrt{S}$ [TeV]	$\mu_0$	NLO [fb]	NLO+NLL[fb]	NLO+NLL with $\mathcal{C}$ [fb]	NLO+NNLL[fb]
13	$Q$	418 <sup>+11.9%</sup> -11.7%	439 <sup>+9.8%</sup> -9.2%	484 <sup>+8.2%</sup> -8.5%	499 <sup>+7.6%</sup> -6.9%
	$Q/2$	468 <sup>+9.8%</sup> -10.7%	477 <sup>+8.6%</sup> -8.0%	496 <sup>+6.0%</sup> -7.2%	498 <sup>+6.0%</sup> -6.3%
	$M/2$	499 <sup>+5.9%</sup> -9.3%	504 <sup>+8.1%</sup> -7.8%	505 <sup>+5.7%</sup> -6.1%	502 <sup>+5.3%</sup> -6.0%

$\mu_0$	$K_{\text{NLO+NNLL}}$
$Q$	1.19
$Q/2$	1.06
$M/2$	1.01

# TOTAL CROSS SECTION

[AK, Motyka, Stebel, Theeuwes'17]

$\sqrt{S}$ [TeV]	$\mu_0$	NLO [fb]	NLO+NLL[fb]	NLO+NLL with $\mathcal{C}$ [fb]	NLO+NNLL[fb]
13	$Q$	$418^{+11.9\%}_{-11.7\%}$	$439^{+9.8\%}_{-9.2\%}$	$484^{+8.2\%}_{-8.5\%}$	$499^{+7.6\%}_{-6.9\%}$
	$Q/2$	$468^{+9.8\%}_{-10.7\%}$	$477^{+8.6\%}_{-8.0\%}$	$496^{+6.0\%}_{-7.2\%}$	$498^{+6.0\%}_{-6.3\%}$
	$M/2$	$499^{+5.9\%}_{-9.3\%}$	$504^{+8.1\%}_{-7.8\%}$	$505^{+5.7\%}_{-6.1\%}$	$502^{+5.3\%}_{-6.0\%}$



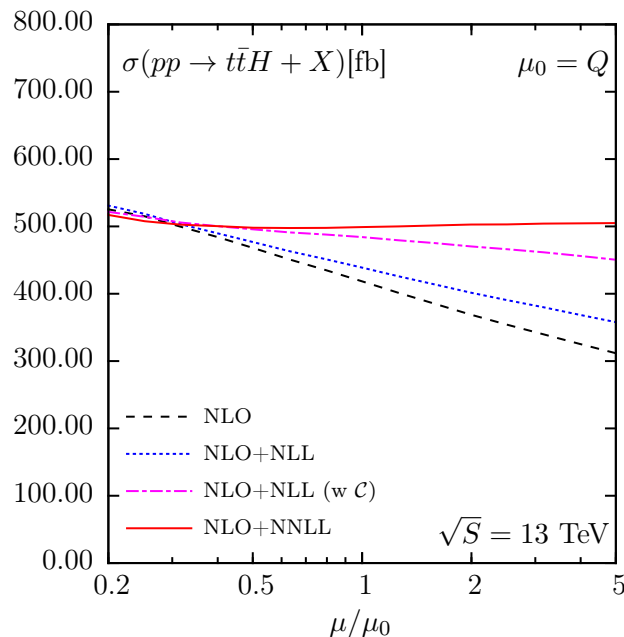
- Compared to NLO, remarkable stability of NLO+NNLL
- Stability improves with increasing accuracy of resummation
- Reduction of the theory scale error
- “Best” NNLL+NLO prediction in agreement with NLO at  $\mu_0 = M/2$

$$\sigma_{\text{NLO+NNLL}} = 500^{+7.5\%+3.0\%}_{-7.1\%-3.0\%} \text{ fb}$$

# SCALE DEPENDENCE OF THE TOTAL CROSS SECTION

[AK, Motyka, Stebel, Theeuwes'17]

$$\mu_F = \mu_R = \mu$$



- For  $\mu_0 = Q$ , decrease in scale dependence with increasing accuracy. NLO+NNLL scale dependence in the  $\mu_0/2 - 2\mu_0$  range of order 1%.
- For  $\mu_0 = M/2$ , mostly similar behaviour
- Apparent cancellations between  $\mu_F$  and  $\mu_R$  scale dependence  $\rightarrow$  7-point used for estimation of scale variation error

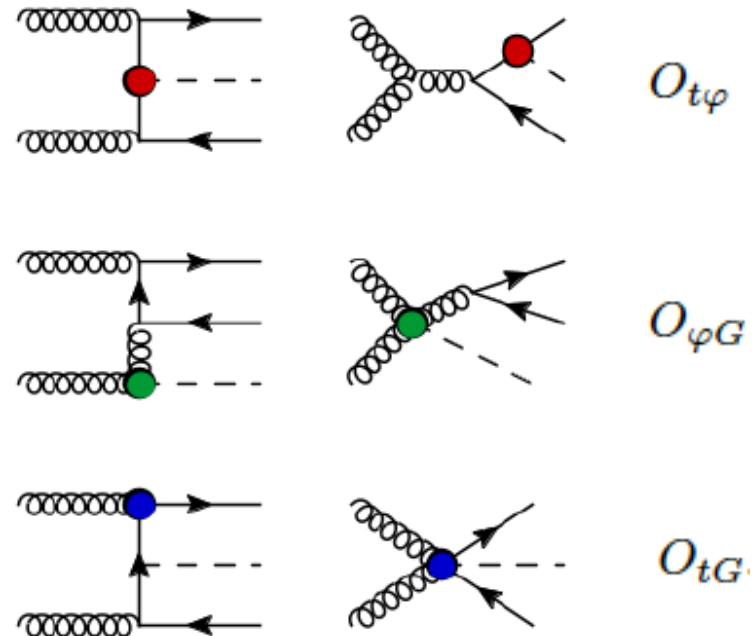
# NLO QCD TO TTH IN THE EFT (I)

[Maltoni, Vrynidou, Zhang'16]

- Looking for possible deviations from the SM
  - model-independent theoretical framework of effective field theories
  - SMEFT: Standard Model with higher dimensional operators

$$\mathcal{L}_{\text{EFT}} = \mathcal{L}_{\text{SM}} + \sum_i \frac{C_i}{\Lambda^2} O_i + \mathcal{O}(\Lambda^{-4}) + h.c.,$$

Tree level contributions from



same operators also probed in  $H$ ,  $Hj$  and  $HH$  production  $\leftrightarrow$  cross-talk

# NLO QCD TO TTH IN THE EFT (II)

[Maltoni, Vrynidou, Zhang'16]

MG5\_aMC

MMHT2014 LO/NLO PDFs

$$\mu_0^{R,F} = m_t + m_H/2$$

$$\mu_0^{EFT} = m_t$$

$$\sigma = \sigma_{SM} + \sum_i \frac{1\text{TeV}^2}{\Lambda^2} C_i \sigma_i + \sum_{i \leq j} \frac{1\text{TeV}^4}{\Lambda^4} C_i C_j \sigma_{ij}.$$

Total cross sections:

13 TeV	$\sigma$ LO	$\sigma$ NLO	K
$\sigma_{SM}$	$0.464^{+0.161+0.000+0.005}_{-0.111-0.000-0.004}$	$0.507^{+0.030+0.000+0.007}_{-0.048-0.000-0.008}$	1.09
$\sigma_{t\phi}$	$-0.055^{+0.013+0.002+0.000}_{-0.019-0.003-0.001}$	$-0.062^{+0.006+0.001+0.001}_{-0.004-0.001-0.001}$	1.13
$\sigma_{\phi G}$	$0.627^{+0.225+0.081+0.007}_{-0.153-0.067-0.005}$	$0.872^{+0.131+0.037+0.013}_{-0.123-0.035-0.016}$	1.39
$\sigma_{tG}$	$0.470^{+0.167+0.000+0.005}_{-0.114-0.002-0.004}$	$0.503^{+0.025+0.001+0.007}_{-0.046-0.003-0.008}$	1.07
$\sigma_{t\phi,t\phi}$	$0.0016^{+0.0005+0.0002+0.0000}_{-0.0004-0.0001-0.0000}$	$0.0019^{+0.0001+0.0001+0.0000}_{-0.0002-0.0000-0.0000}$	1.17
$\sigma_{\phi G,\phi G}$	$0.646^{+0.274+0.141+0.018}_{-0.178-0.107-0.010}$	$1.021^{+0.204+0.096+0.024}_{-0.178-0.085-0.029}$	1.58
$\sigma_{tG,tG}$	$0.645^{+0.276+0.011+0.020}_{-0.178-0.015-0.010}$	$0.674^{+0.036+0.004+0.016}_{-0.067-0.007-0.019}$	1.04
$\sigma_{t\phi,\phi G}$	$-0.037^{+0.009+0.006+0.000}_{-0.013-0.007-0.000}$	$-0.053^{+0.008+0.003+0.001}_{-0.008-0.004-0.001}$	1.42
$\sigma_{t\phi,tG}$	$-0.028^{+0.007+0.001+0.000}_{-0.010-0.001-0.000}$	$-0.031^{+0.003+0.000+0.000}_{-0.002-0.000-0.000}$	1.10
$\sigma_{\phi G,tG}$	$0.627^{+0.252+0.053+0.014}_{-0.166-0.047-0.008}$	$0.859^{+0.127+0.021+0.017}_{-0.126-0.020-0.022}$	1.37

➔ K-factors depend on EFT operators, corrections up to 60%

➔ Renormalization and factorization scale variation dominant uncertainty, substantially reduced @NLO

$$\mu_0^{R,F}/2 < \mu_F = \mu_R < 2 \mu_0^{R,F}$$

$$\mu_0^{EFT}/2 < \mu^{EFT} < 2 \mu_0^{EFT}$$

pdf uncertainty

# NLO QCD TO TTH IN THE EFT (II)

[Maltoni, Vrynidou, Zhang'16]

$$\sigma = \sigma_{SM} + \sum_i \frac{1\text{TeV}^2}{\Lambda^2} C_i \sigma_i + \sum_{i \leq j} \frac{1\text{TeV}^4}{\Lambda^4} C_i C_j \sigma_{ij}.$$

MG5\_aMC

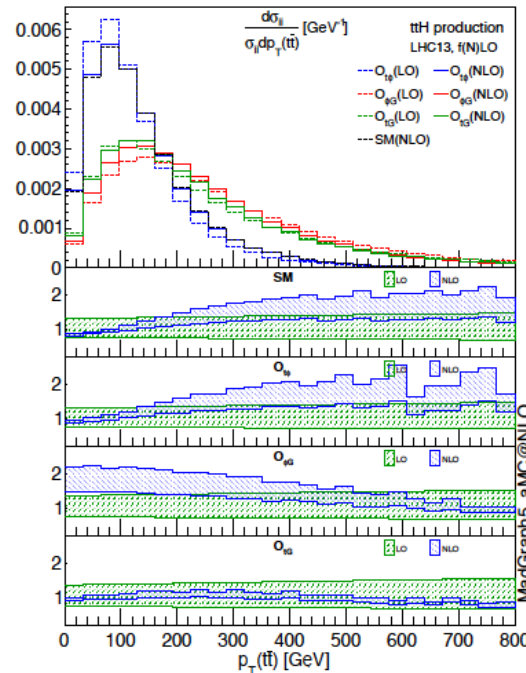
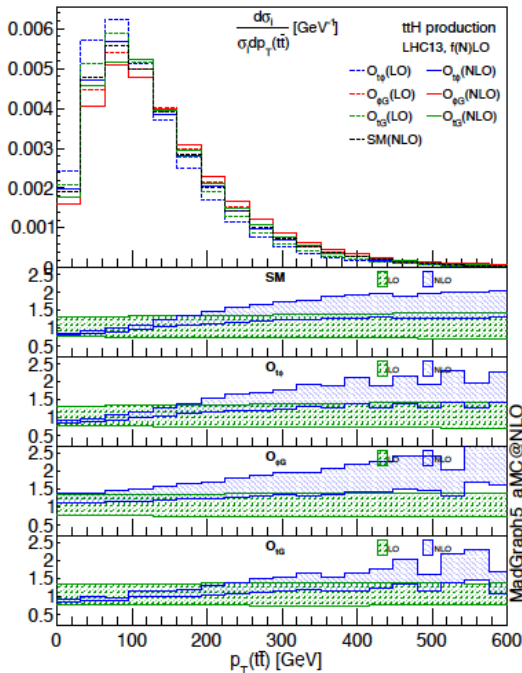
MMHT2014 LO/NLO PDFs

$$\mu_0^{R,F} = m_t + m_H/2$$

$$\mu_0^{EFT} = m_t$$



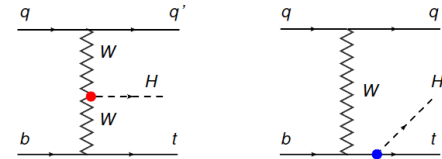
K-factors in differential distributions not flat and dependent on EFT operators





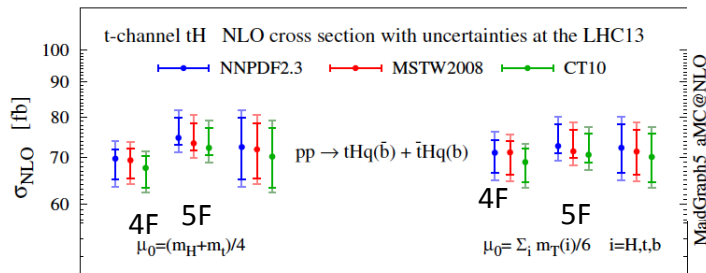
# HIGGS + SINGLE TOP

➤ Destructive interference between t-channel diagrams involving top Yukawa coupling and Higgs coupling to gauge bosons → small cross section → sensitive to the relative size and phase of the couplings

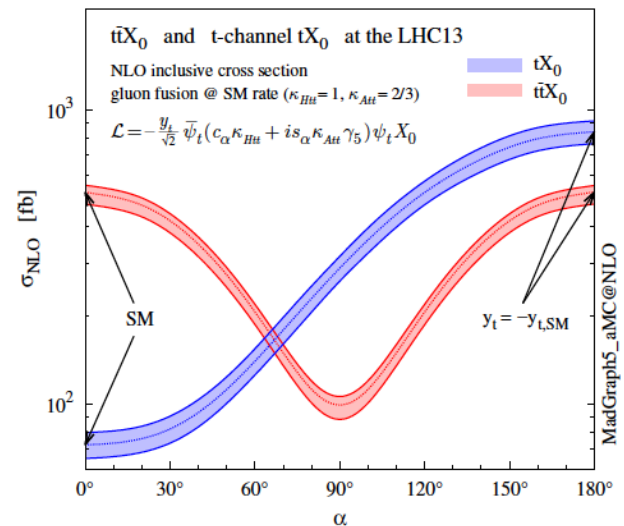


➤ NLO cross section calculated in 4FS and 5FS schemes, good agreement

➤ tH provides complementary information to ttH on CP-violating Yukawa coupling



➤ s-channel contribution small (~3 fb @NLO) compared to t-channel (~72 fb @NLO)



[Demartin, Maltoni, Mawatari, Zaro'15]

# SUMMARY

- ttH production @ the LHC is one of the most promising windows onto new physics → precise theoretical prediction are essential → a lot of recent progress!
- Fixed order predictions combine now NLO QCD and NLO EW corrections, also including top decays
- Resummed calculations reach NNLL+NLO accuracy
  - Remarkable stability of the NNLL+NLO differential and total cross sections w.r.t. scale variation; improving stability with growing accuracy
  - Reduction (albeit small using the 7-point method) of the theory error due to scale variation
- Using EFT, information can be combined from different Higgs production processes, helping to resolve effects of various operators

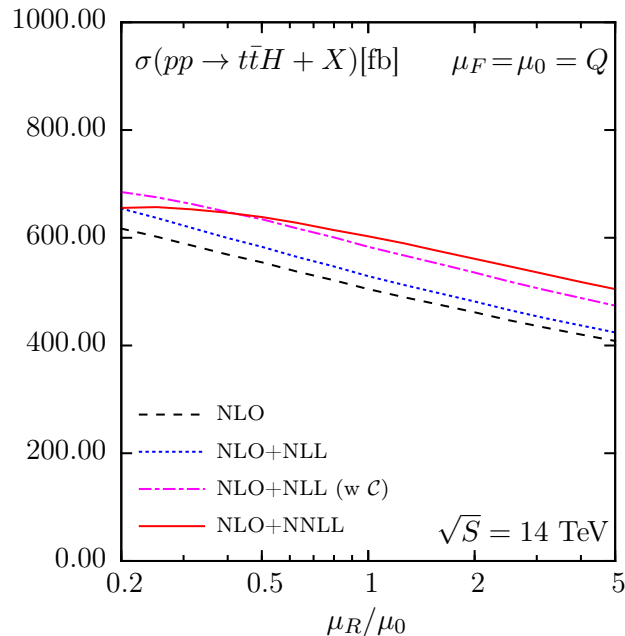


# BACKUP

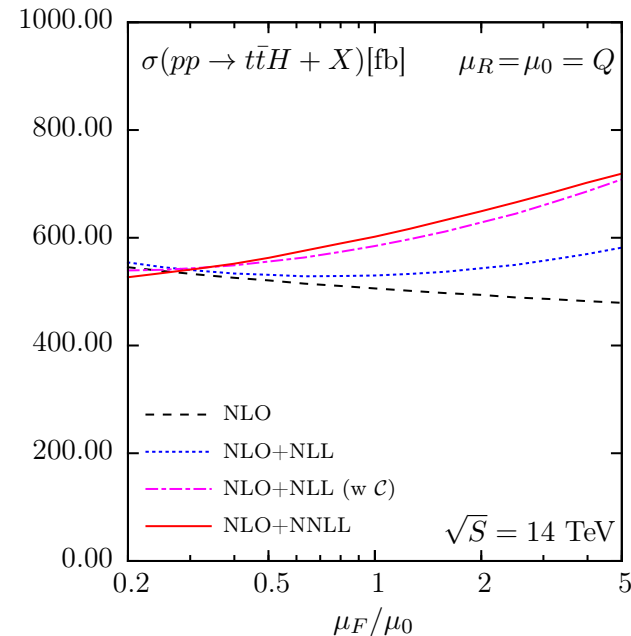
# SCALE DEPENDENCE OF THE TOTAL CROSS SECTION

[AK, Motyka, Stebel, Theeuwes'17]

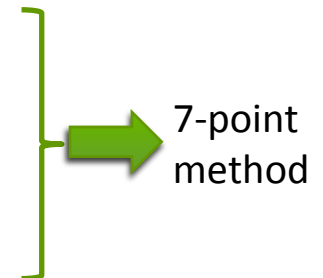
$$\mu_F = \mu_0 = Q$$



$$\mu_R = \mu_0 = Q$$



- Apparent cancellations between  $\mu_F$  and  $\mu_R$  scale dependence
- No significant change in dependence on  $\mu_R$  while increasing the accuracy:  $\alpha_s$  running effect
- $\mu_F$  dependence modified by the hard-matching coefficient



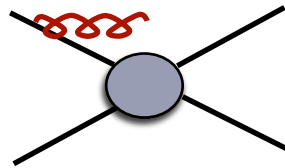
# HIGHER ORDERS AT THRESHOLD

In analogy to top-pair production

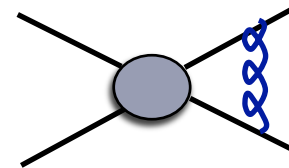
General structure of the NLO correction in the threshold limit  $\beta \rightarrow 0$ ,  $\beta^2 = 1 - 4m^2 / \hat{s}$

$$\Delta \hat{\sigma}_i^{\text{NLO}} \sim \alpha_s \hat{\sigma}_i^{\text{LO}} \left\{ A^{(i)} \log^2(\beta^2) + B^{(i)} \log(\beta^2) + C^{(i)} \frac{1}{\beta} + D^{(i)} \right\}$$

Soft/collinear gluon emission



Coulomb gluons



At higher orders:

$$\sim \alpha_s^n \log^{2n}(\beta)$$

$$\sim \alpha_s^n / \beta^n$$

Both types of corrections can be resummed to all orders

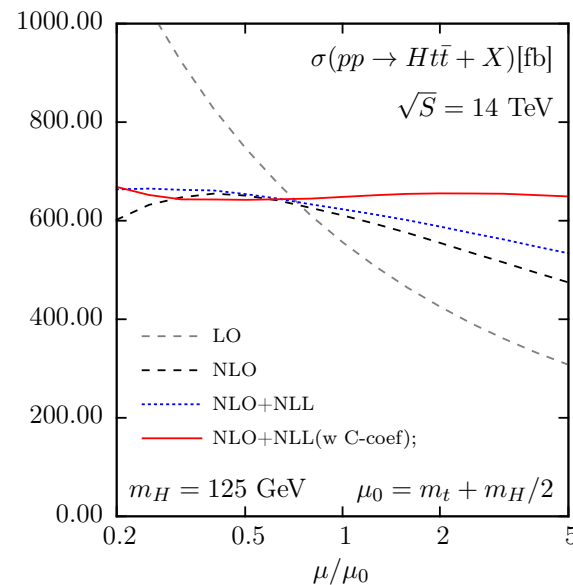
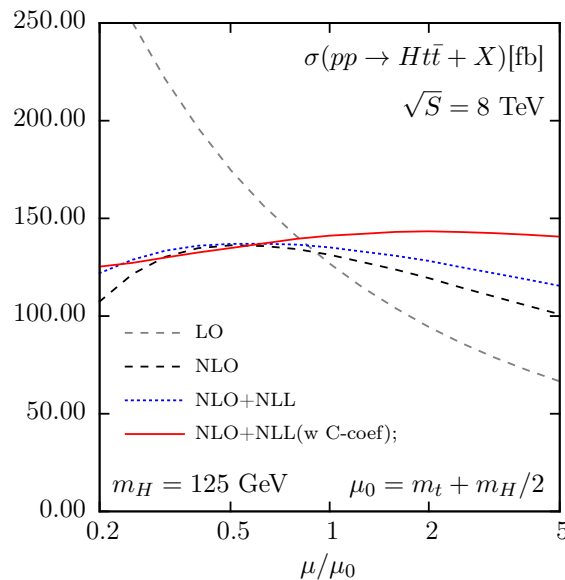
# ASSOCIATED HIGGS PRODUCTION WITH TOP QUARKS, RESUMMATION IN THE ABSOLUTE THRESHOLD LIMIT

➤ Threshold logarithms under scrutiny:

*AK, Motyka, Stebel, Theeuwes'15]*

$$\alpha_s^{2+n} \log^m(1 - \tau_M), \quad m \leq 2n, \quad \tau_M = \frac{M^2}{\hat{s}} = \frac{(M_H + 2M_t)^2}{\hat{s}}$$

➤ NLL accuracy requires knowledge of NLO soft anomalous dimension with 2→3 kinematics and NLO cross section at threshold split into colour channels



NLO obtained with aMC@NLO

MMHT2014NLO

# ASSOCIATED HIGGS PRODUCTION WITH TOP QUARKS, RESUMMATION IN THE ABSOLUTE THRESHOLD LIMIT

[ AK, Motyka, Stebel, Theeuwes'15]

$\sqrt{S}$ [TeV]	NLO [fb]	NLO+NLL		NLO+NLL with $C$		pdf error
		Value [fb]	K-factor	Value [fb]	K-factor	
8	$132^{+3.9\%}_{-9.3\%}$	$135^{+3.0\%}_{-5.9\%}$	1.03	$141^{+7.7\%}_{-4.6\%}$	1.07	$+3.0\%$ $-2.7\%$
13	$506^{+5.9\%}_{-9.4\%}$	$516^{+4.6\%}_{-6.5\%}$	1.02	$537^{+8.2\%}_{-5.5\%}$	1.06	$+2.3\%$ $-2.3\%$
14	$613^{+6.2\%}_{-9.4\%}$	$625^{+4.6\%}_{-6.7\%}$	1.02	$650^{+7.9\%}_{-5.7\%}$	1.06	$+2.3\%$ $-2.2\%$

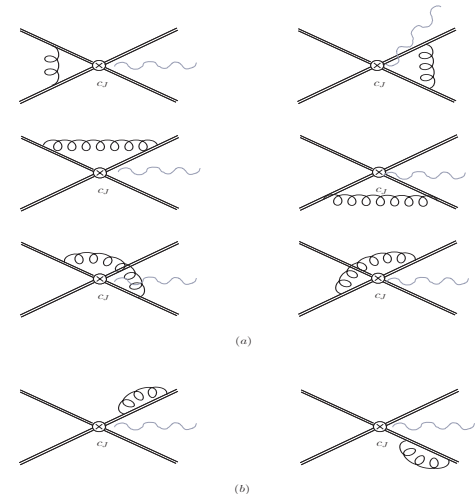
- Shows also a strong impact of the hard-matching coefficient  $C$  on the predictions → contributions away from the *absolute* threshold matter!
- Part of these contributions can be accounted for if instead of resummation for total cross section, resummation for invariant mass of the  $t\bar{t}b\bar{b}H$  system is considered

# SOFT ANOMALOUS DIMENSION

- Soft anomalous dimensions known at two loops for any number of massless/massive legs [Mert-Aybat, Dixon, Sterman'06] [Becher, Neubert'09] [Mitov, Sterman, Sung'09-'10] [Ferrogli, Neubert, Pecjak, Yang'09] [Beneke, Falgari, Schwinn'09], [Czakon, Mitov, Sterman'09] [Kidonakis'10]

- For NLL need only 1-loop

$$\Gamma(g) = -\frac{g}{2} \frac{\partial}{\partial g} \text{Res}_{\epsilon \rightarrow 0} Z(g, \epsilon)$$

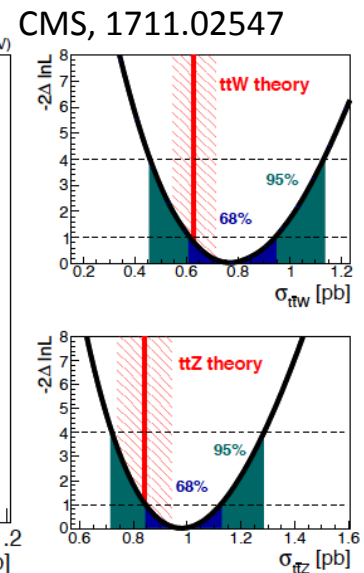
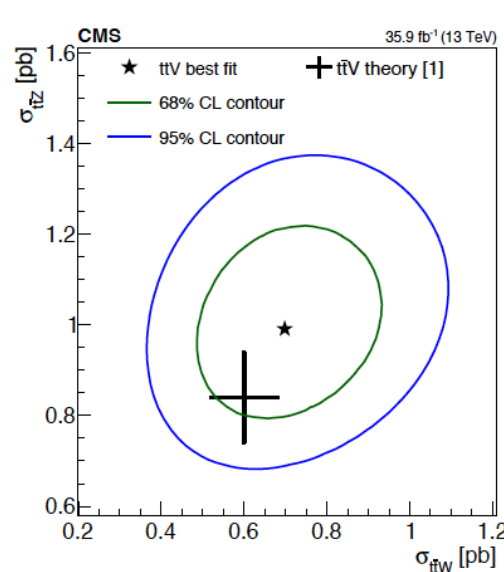
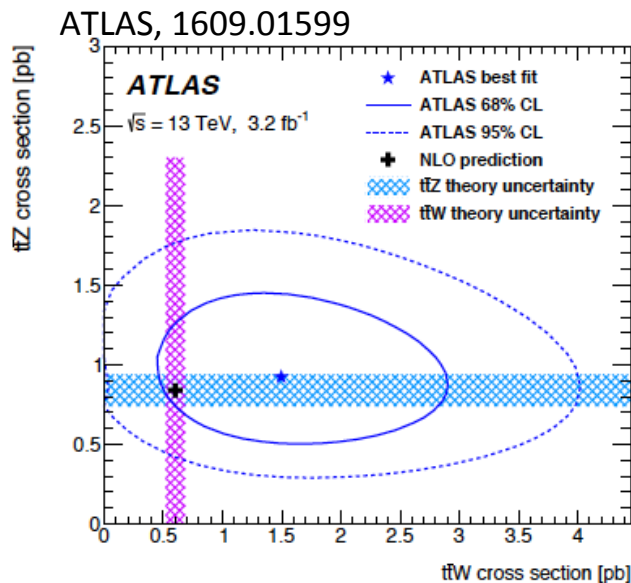


- N.B. color structure also known explicitly for 2→3 [Sjödahl'08]



# TTBAR+W,Z

- Probes of top-quark coupling to an EW gauge boson
- Sensitive to BSM contributions
- Dominant backgrounds to searches and SM precision measurements (ttH included)



- Signal strength (CMS):  $\mu_{ttZ}$   
 $\mu_{ttW}$

$$1.23^{+0.19}_{-0.18} (\text{stat})^{+0.20}_{-0.18} (\text{syst})^{+0.13}_{-0.12} (\text{theo})$$

$$1.17^{+0.11}_{-0.10} (\text{stat})^{+0.14}_{-0.12} (\text{syst})^{+0.11}_{-0.12} (\text{theo})$$

# SOFT GLUON RESUMMATION

Systematic reorganization of perturbative series

( $x=1-\beta$ )

$$\hat{\sigma} \sim c_{00} +$$

$$+ \alpha_s \left( \begin{array}{c} c_{12} \log^2(\beta^2) \\ c_{24} \log^4(\beta^2) \\ \dots \end{array} + \begin{array}{c} c_{11} \log(\beta^2) \\ c_{23} \log^3(\beta^2) \\ \dots \end{array} + \begin{array}{c} c_{10} \\ c_{22} \log^2(\beta^2) \\ \dots \end{array} + \dots \right) \leftarrow \text{NLO}$$

$$+ \alpha_s^2 \left( \dots \right) \leftarrow \text{NNLO}$$

$$\begin{array}{c} \uparrow \\ \alpha_s^n \log^{2n}(\beta^2) \end{array} \quad \begin{array}{c} \uparrow \\ \alpha_s^n \log^{2n-1}(\beta^2) \end{array}$$

$$\log(\beta^2) \leftrightarrow \log(N) \equiv L$$

Factorization at threshold: space of Mellin moments  $N$ , taken wrt.  $M^2/S$  or  $Q^2/S$

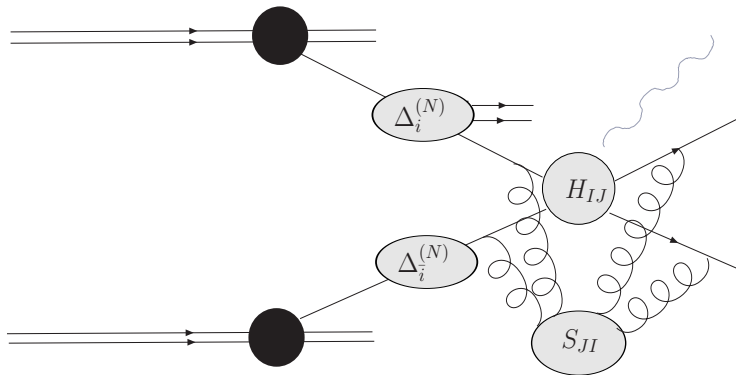
$$\hat{\sigma}^{(N)} \sim \mathcal{C}(\alpha_s) \exp [Lg_1(\alpha_s L) + g_2(\alpha_s L) + \alpha_s g_3(\alpha_s L) + \dots]$$

sums up

$$\text{LL: } \alpha_s^n \log^{n+1}(N)$$

$$\text{NLL: } \alpha_s^n \log^n(N)$$

# ABSOLUTE THRESHOLD RESUMMATION FOR QQB @NLO+NLL



Colour space basis in which  $\Gamma_{II}$  is diagonal in the threshold limit



$$\hat{\sigma}_{ab \rightarrow klB}^{(\text{res}, N)} = \sum_I \underbrace{\hat{\sigma}_{ab \rightarrow klB, I}^{(0, N)} C_{ij \rightarrow klB, I}}_{\text{hard function } H_{ab \rightarrow klB, I}} \Delta_a^{(N+1)} \Delta_b^{(N+1)} \Delta_{ab \rightarrow klB, I}^{(\text{soft}, N)}$$

incoming jet factors, known
soft-wide angle emission

$$\log \Delta_{ab \rightarrow klB, I}^{(\text{soft}, N)} = \int_0^1 dz \frac{z^{N-1} - 1}{1-z} D_{ij \rightarrow klB, I}(\alpha_s(Q^2(1-z)^2)) \quad D_{ij \rightarrow klB, I} = \lim_{\beta \rightarrow 0} \frac{\pi}{\alpha_s} 2\text{Re}(\bar{\Gamma}_{II})$$

At NLL accuracy  $C_{ab \rightarrow klB, I} = 1$

# INVARIANT MASS KINEMATICS CTND.

- Extending resummation in this kinematics to NNLL requires:
  - Knowledge of the two-loop soft anomalous dimension
  - Amended treatment of the path-ordered exponential to account for it
  - Knowledge of the one-loop hard-matching coefficient

# INVARIANT MASS KINEMATICS CTND.

➤ Extending resummation in this kinematics to NNLL requires:

➤ Knowledge of the two-loop soft anomalous dimension

$$\begin{aligned} \frac{d\tilde{\sigma}_{ij \rightarrow klB}^{(\text{NNLL})}}{dQ^2}(N, Q^2, \{m^2\}, \mu_R^2) &= \text{Tr} \left[ \mathbf{H}_R(Q^2, \{m^2\}, \mu_F^2, \mu_R^2) \bar{\mathbf{U}}_R(N+1, Q^2, \{m^2\}, \mu_R^2) \right. \\ &\times \tilde{\mathbf{S}}_R(N+1, Q^2, \{m^2\}, \mu_R^2) \left. \mathbf{U}_R(N+1, Q^2, \{m^2\}, \mu_R^2) \right] \\ &\times \Delta^i(N+1, Q^2, \mu_F^2, \mu_R^2) \Delta^j(N+1, Q^2, \mu_F^2, \mu_R^2). \end{aligned}$$

$$\mathbf{U}_{ij \rightarrow klB}(N) = \text{P exp} \left[ \int_{\mu}^{Q/\bar{N}} \frac{dq}{q} \Gamma_{ij \rightarrow klB}(\alpha_s(q^2)) \right] \quad \Gamma_{ij \rightarrow klB} = \left[ \left( \frac{\alpha_s}{\pi} \right) \Gamma^{(1)} + \left( \frac{\alpha_s}{\pi} \right)^2 \Gamma^{(2)} + \dots \right]$$

✓ Soft anomalous dimensions known at two loops for any number of legs [*Mert-Aybat, Dixon, Sterman'06*] [*Becher, Neubert'09*] [*Mitov, Sterman, Sung'09-'10*] [*Ferrogli, Neubert, Pecjak, Yang'09*] [*Beneke, Falgari, Schwinn'09*], [*Czakon, Mitov, Sterman'09*] [*Kidonakis'10*]

# INVARIANT MASS KINEMATICS CTND.

- Extending resummation in this kinematics to NNLL requires:
  - Knowledge of the two-loop soft anomalous dimension
  - Amended treatment of the path-ordered exponential to account for it

$$\begin{aligned} \frac{d\tilde{\sigma}_{ij \rightarrow klB}^{(\text{NNLL})}}{dQ^2}(N, Q^2, \{m^2\}, \mu_R^2) &= \text{Tr} \left[ \mathbf{H}_R(Q^2, \{m^2\}, \mu_F^2, \mu_R^2) \bar{\mathbf{U}}_R(N+1, Q^2, \{m^2\}, \mu_R^2) \right. \\ &\quad \times \tilde{\mathbf{S}}_R(N+1, Q^2, \{m^2\}, \mu_R^2) \left. \mathbf{U}_R(N+1, Q^2, \{m^2\}, \mu_R^2) \right] \\ &\quad \times \Delta^i(N+1, Q^2, \mu_F^2, \mu_R^2) \Delta^j(N+1, Q^2, \mu_F^2, \mu_R^2). \end{aligned}$$

- ✓ Perturbative expansion [Buchalla, Buras, Lautenbacher'96] [Ahrens, Neubert, Pecjak, Yang'10]

$$\mathbf{U}_R(N, Q^2, \{m^2\}, \mu_F^2, \mu_R^2) = \left( \mathbf{1} + \frac{\alpha_s(Q^2/\bar{N}^2)}{\pi} \mathbf{K} \right) \left[ \left( \frac{\alpha_s(\mu_F^2)}{\alpha_s(Q^2/\bar{N}^2)} \right)^{\frac{\vec{\lambda}^{(1)}}{2\pi b_0}} \right]_D \left( \mathbf{1} - \frac{\alpha_s(\mu_F^2)}{\pi} \mathbf{K} \right)$$

$$K_{IJ} = \delta_{IJ} \lambda_I^{(1)} \frac{b_1}{2b_0^2} - \frac{(\Gamma_R^{(2)})_{IJ}}{2\pi b_0 + \lambda_I^{(1)} - \lambda_J^{(1)}}$$

$$\vec{\lambda}^{(1)} = \left\{ \lambda_1^{(1)}, \dots, \lambda_D^{(1)} \right\}$$

eigenvalues of  $\Gamma^{(1)}$

# INVARIANT MASS KINEMATICS CTND.

- Extending resummation in this kinematics to NNLL requires:
  - Knowledge of the two-loop soft anomalous dimension
  - Amended treatment of the path-ordered exponential to account for it
  - Knowledge of the one-loop hard function  $H_{ij}$

$$\frac{d\tilde{\sigma}_{ij \rightarrow klB}^{(\text{NNLL})}}{dQ^2}(N, Q^2, \{m^2\}, \mu_R^2) = \text{Tr} \left[ \mathbf{H}_R(Q^2, \{m^2\}, \mu_F^2, \mu_R^2) \bar{\mathbf{U}}_R(N+1, Q^2, \{m^2\}, \mu_R^2) \right. \\ \left. \times \tilde{\mathbf{S}}_R(N+1, Q^2, \{m^2\}, \mu_R^2) \mathbf{U}_R(N+1, Q^2, \{m^2\}, \mu_R^2) \right] \\ \times \Delta^i(N+1, Q^2, \mu_F^2, \mu_R^2) \Delta^j(N+1, Q^2, \mu_F^2, \mu_R^2).$$

- needs access to colour structure of virtual corrections
- ✓ extracted from the results provided by the PowHel (HELAC-NLO [Bevilacqua et al.'11] + POWHEG-Box) package [Garzelli, Kardos, Papadopoulos, Trocksanyi'11] Debrece
- translation between the colour flow and singlet-octet basis
- implementation checked against colour-averaged virtual corrections obtained from public POWHEG [Hartanto, Jäger, Reina, Wackerath'15] and aMC@NLO implementations [Frederix, Frixione, Hirschi, Maltoni, Pittau, Torrielli'11]

# ADDITIVE VS MULTIPLICATIVE

$$\sigma_{\text{QCD}}^{\text{NLO}} = \sigma^{\text{Born}} + \delta\sigma_{\text{QCD}}^{\text{NLO}}$$

$$\sigma_{\text{EW}}^{\text{NLO}} = \sigma^{\text{Born}} + \delta\sigma_{\text{EW}}^{\text{NLO}}$$

$$\sigma_{\text{QCD+EW}}^{\text{NLO}} = \sigma^{\text{Born}} + \delta\sigma_{\text{QCD}}^{\text{NLO}} + \delta\sigma_{\text{EW}}^{\text{NLO}}$$

$$\sigma_{\text{QCD}\times\text{EW}}^{\text{NLO}} = \sigma_{\text{QCD}}^{\text{NLO}} \left( 1 + \frac{\delta\sigma_{\text{EW}}^{\text{NLO}}}{\sigma^{\text{Born}}} \right) = \sigma_{\text{EW}}^{\text{NLO}} \left( 1 + \frac{\delta\sigma_{\text{QCD}}^{\text{NLO}}}{\sigma^{\text{Born}}} \right)$$