NEXT-TO-LEADING POWER CORRECTION IN PARTICLE SCATTERING NEAR THRESHOLD

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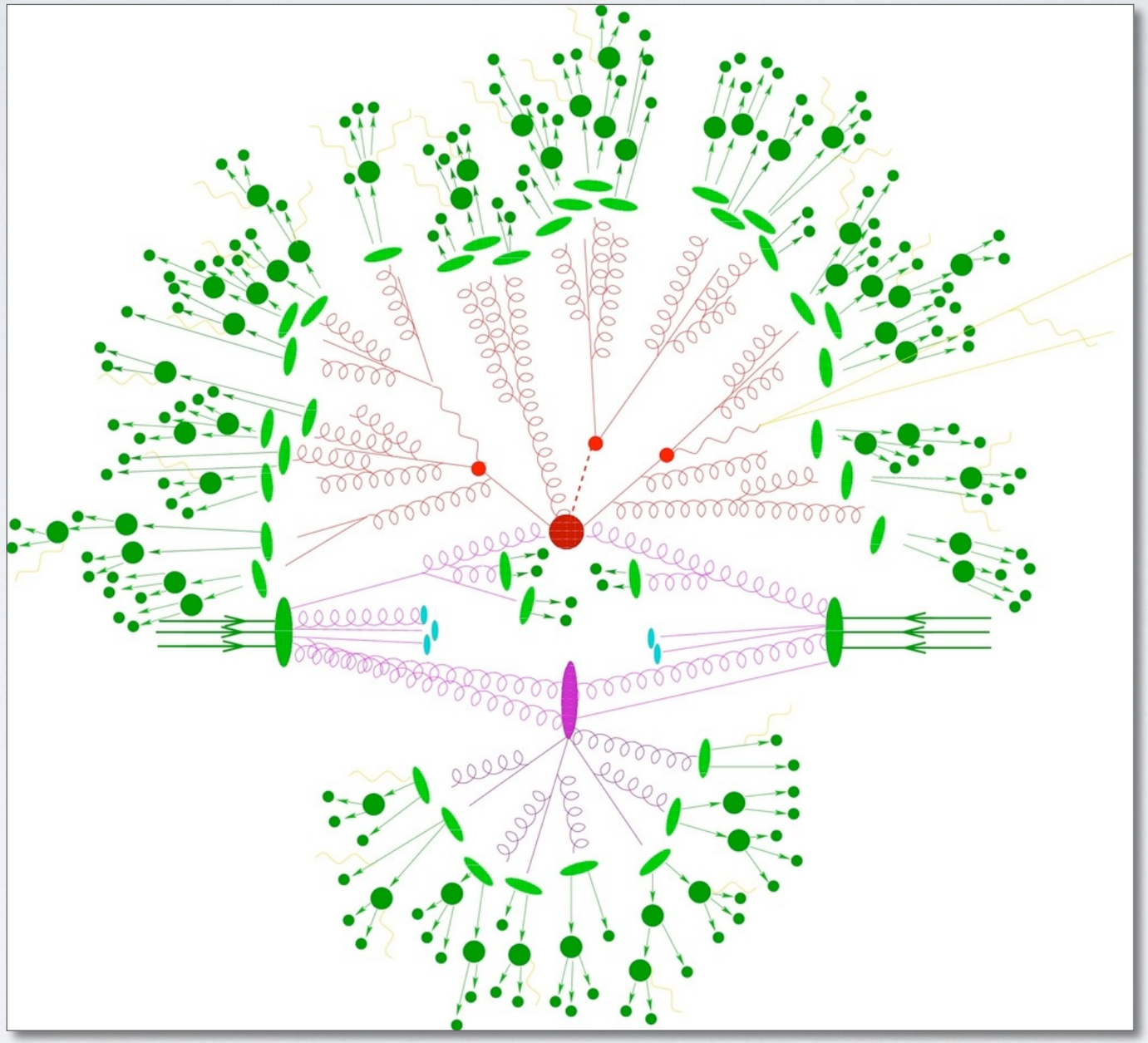
OUTLINE

- **Particle scattering near threshold**
- **Scattering near threshold: NLP logarithms**
- **Next-to-soft and collinear gluons: factorisation theorems**
- **NLP logarithms in Drell Yan with two real soft gluons at N3LO**

- ***Phys. Lett. B 742, (2015), 375, arXiv:1410.6406 [hep-ph],***
- ***JHEP 1506, (2015), 008, arXiv:1503.05156 [hep-ph],***
- ***JHEP 1612 (2016) 121, arXiv:1610.06842 [hep-ph],***
- ***JHEP 1711 (2017) 057, arXiv:1706.04018 [hep-ph],***
- ***arXiv:1807.09246 [hep-ph].***

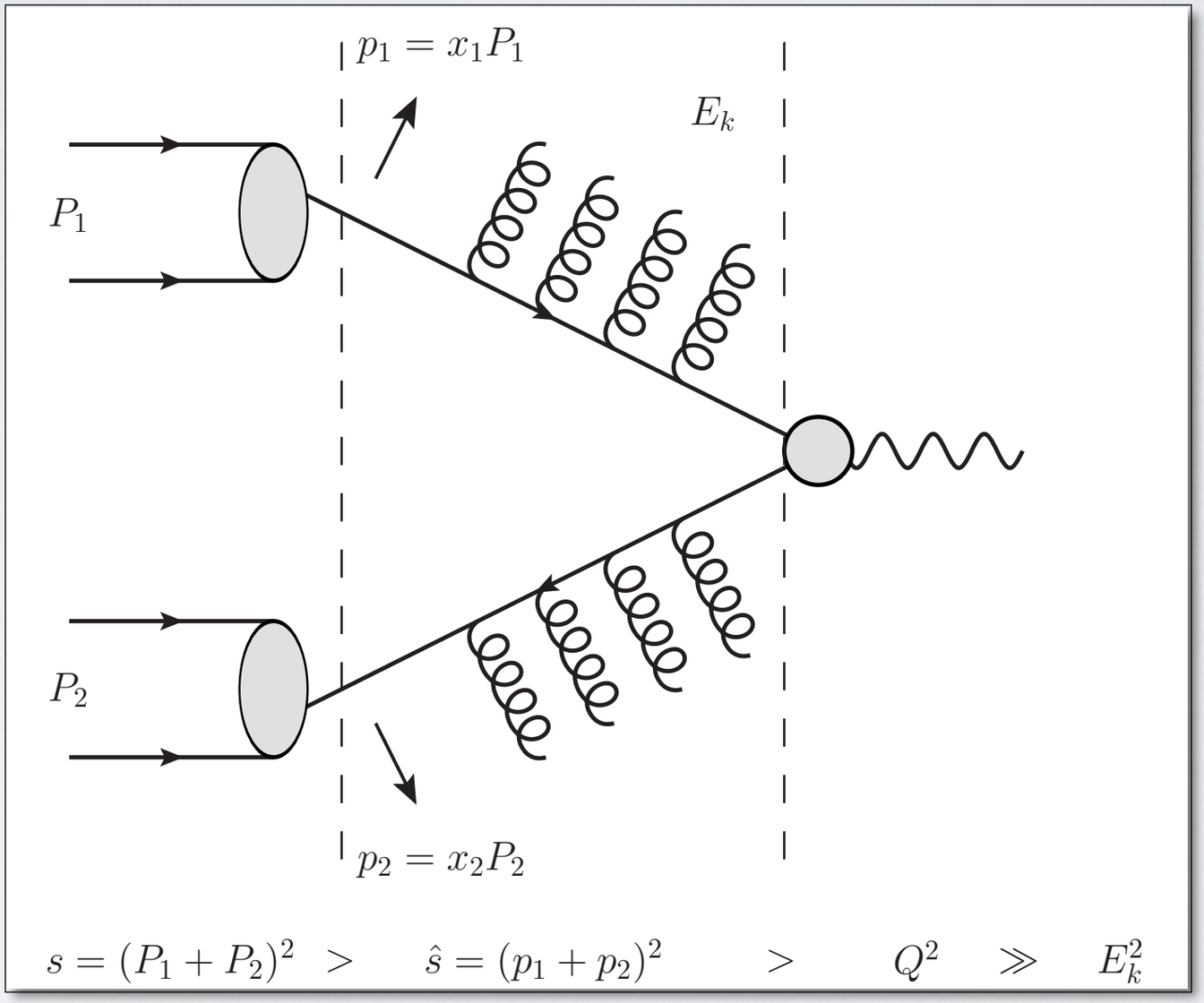
PARTICLE SCATTERING NEAR THRESHOLD



PARTICLE SCATTERING NEAR THRESHOLD

Consider an electroweak annihilation process at the LHC. One has to deal with a multiple scale problem:

$$d\sigma = \sum_{ij} \int dx_1\, dx_2\, f_{i/P_1}(x_1, \mu_f)\, f_{j/P_2}(x_2, \mu_f)\, d\hat{\sigma}_{ij}(x_1, x_2, s, Q^2, \mu_f).$$



Define threshold and partonic threshold variable: $\tau = \dfrac{Q^2}{s}, \qquad z = \dfrac{Q^2}{\hat{s}}, \qquad (z \geq \tau), \qquad z \to 1.$

PARTICLE SCATTERING NEAR THRESHOLD

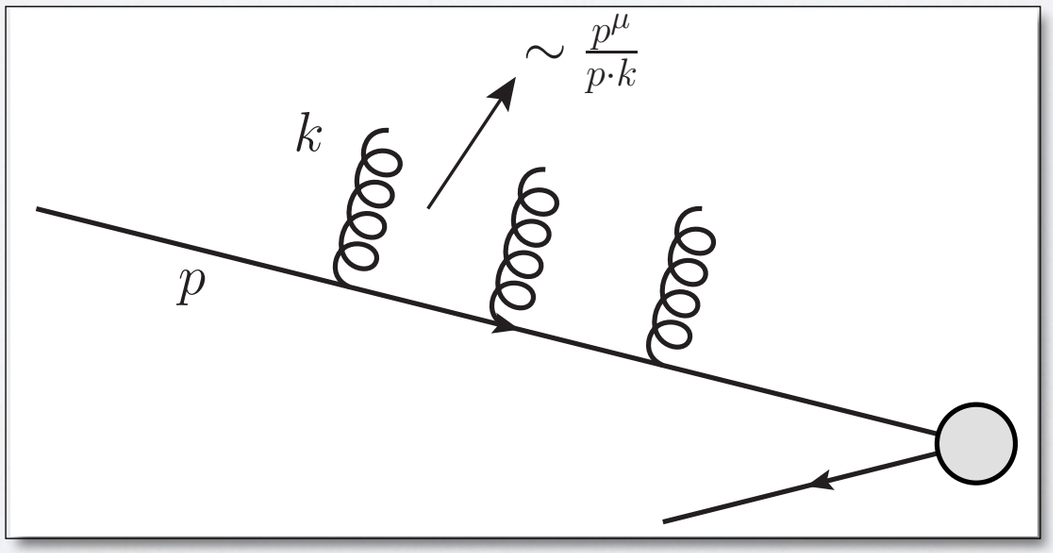
- Partonic cross sections can be calculated as a perturbative expansion in the strong coupling.

$$\sigma_{ij} = \sigma_{ij}^{(0)} + \frac{\alpha_s}{\pi}\sigma_{ij}^{(1)} + \left(\frac{\alpha_s}{\pi}\right)^2 \sigma_{ij}^{(2)} + \dots$$

- Calculating additional orders in the perturbative expansion makes the result more precise.
- Is this enough? No! Near threshold, large logarithms of the partonic threshold variable appears:

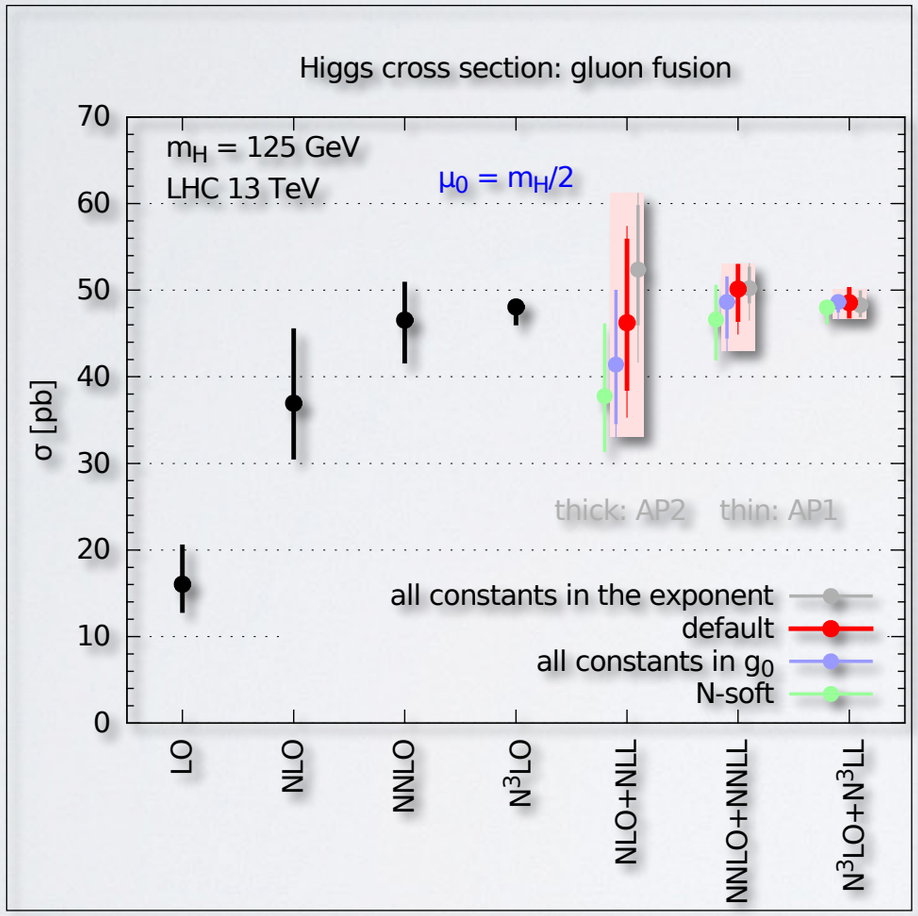
$$\frac{d\sigma}{dz} = \sum_{n=0}^{\infty} \left(\frac{\alpha_s}{\pi}\right)^n \sum_{m=0}^{2n-1} c_{nm}^{(-1)} \left.\frac{\log^m(1-z)}{1-z}\right|_+ + \dots$$

  These logarithms spoil the reliability of the perturbative expansion, and needs to be resummed.
- Resummation of leading-power (LP) logarithms is well established, and relies on the factorisation and exponentiation properties of soft radiation.

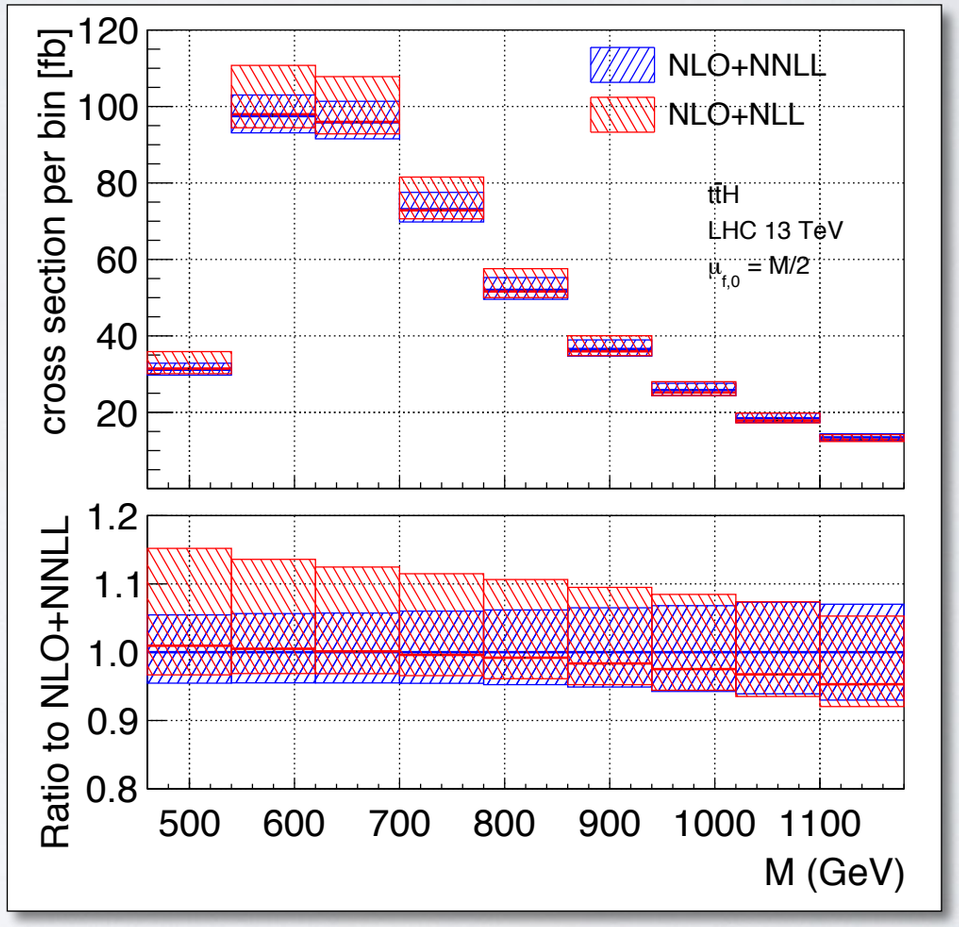


PARTICLE SCATTERING NEAR THRESHOLD

- The contribution of the threshold limit to a physical observables such as the invariant mass distribution can be significant.
- It depends on a phenomenon of dynamical enhancement, which needs to be analysed process by process.
- In general, the resummation of large threshold logarithms leads to a more reliable perturbative expansion.



Bonvini, Marzani, Muselli, Rottoli 2016



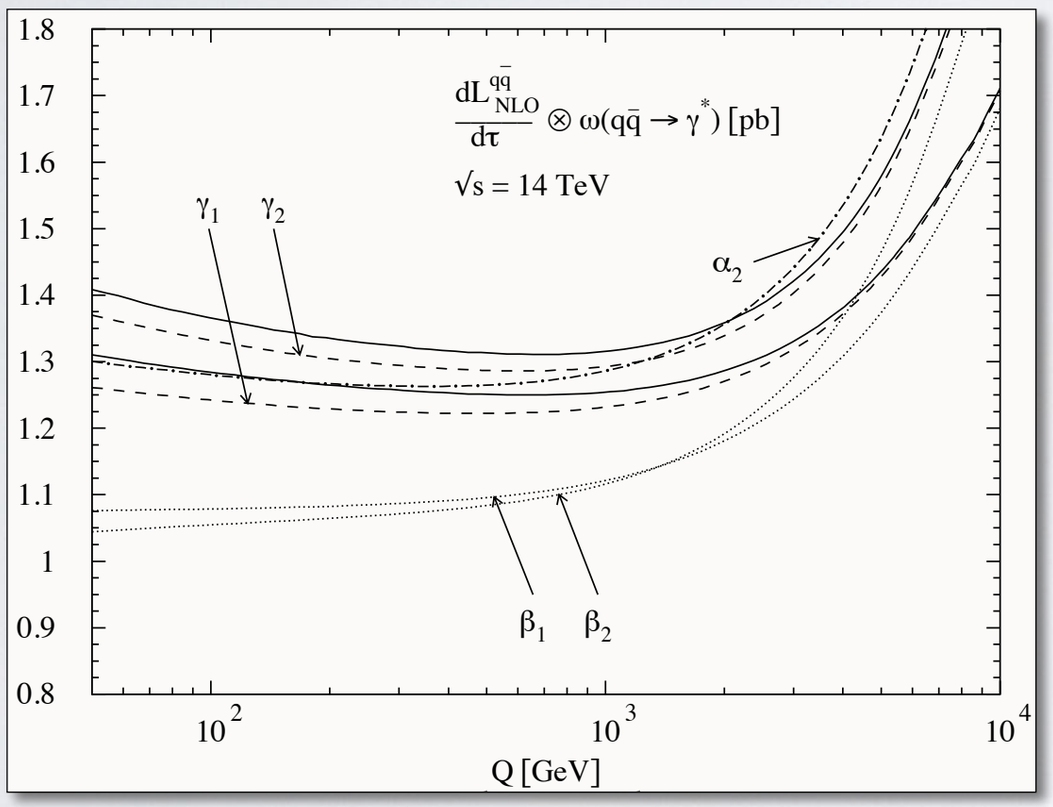
Broggio, Ferroglia, Pecjak, Yang 2016

SCATTERING NEAR THRESHOLD: NLP LOGARITHMS

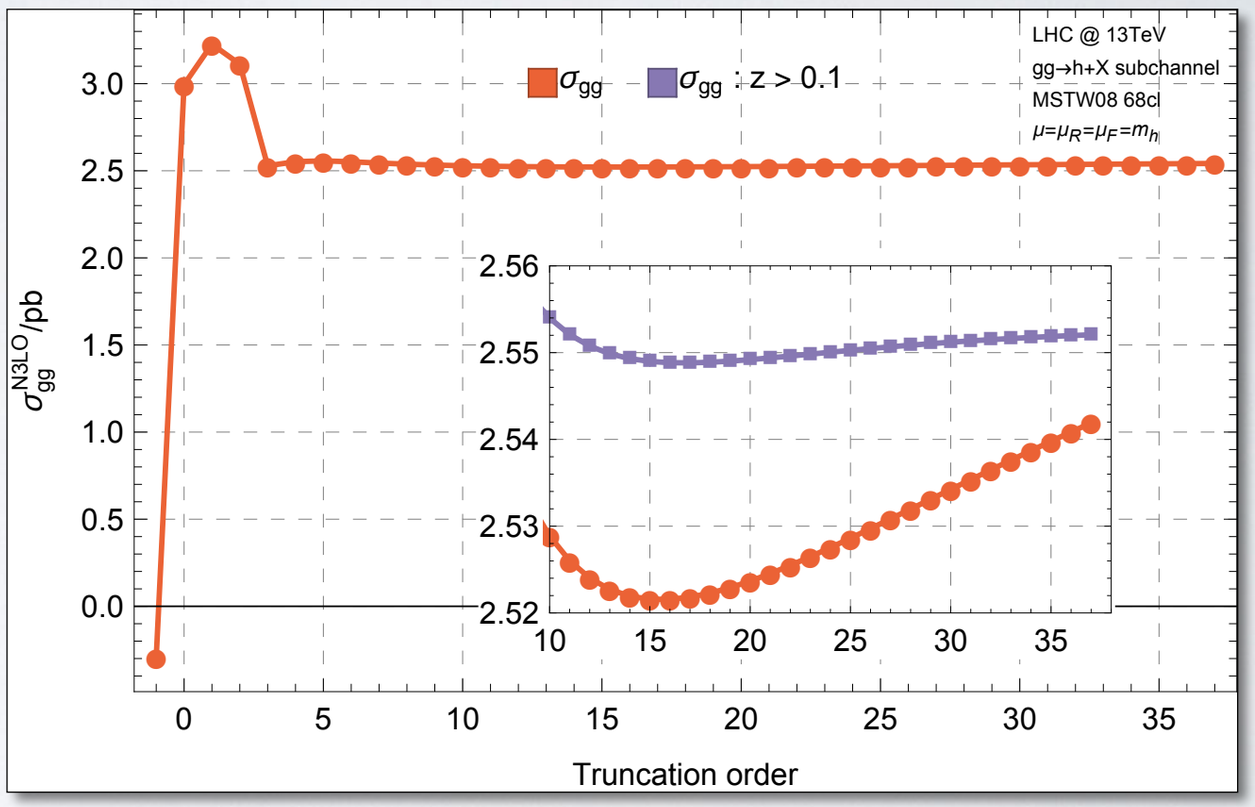
- Is this enough? Examining the DY threshold expansion, one realises that the logarithms $\propto c_{nm}^{(-1)}$ are only the leading power terms in a series expansion:

$$\frac{d\sigma}{dz} = \sum_{n=0}^{\infty} \left(\frac{\alpha_s}{\pi}\right)^n \sum_{m=0}^{2n-1} \left\{ c_{nm}^{(-1)} \left. \frac{\log^m(1-z)}{1-z} \right|_+ + c_{nm}^{(0)} \log^m(1-z) + \dots \right\}$$

where we have written explicitly the next-to-leading power (NLP) logarithms, $\propto c_{nm}^{(0)}$.



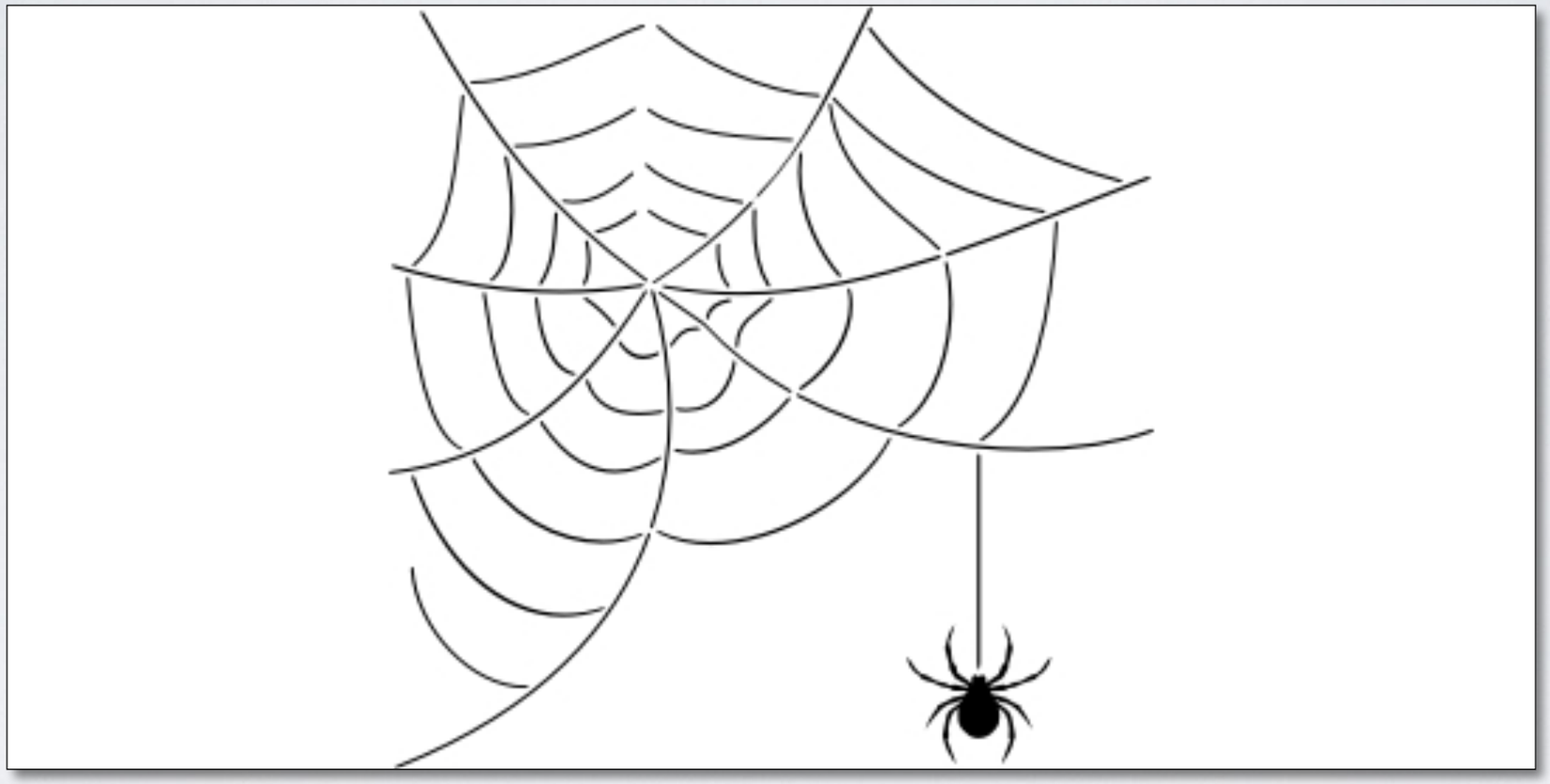
Kramer, Laenen, Spira, 1998



Anastasiou, Duhr, Dulat, Herzog, Mistlberger, 2015

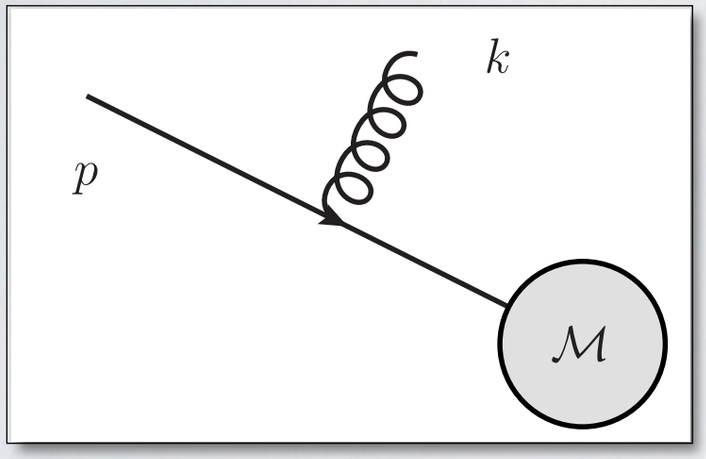
- Resummation beyond leading power: interesting theoretical problem & useful for precision physics.

SCATTERING NEAR THRESHOLD: SOFT AND NEXT-TO-SOFT GLUONS



FACTORISATION OF (NEXT-TO-)SOFT GLUONS

- Let's examine the emission of soft gluons from an energetic parton: (quark line):

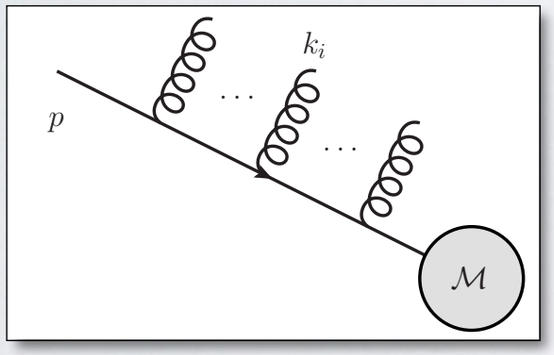


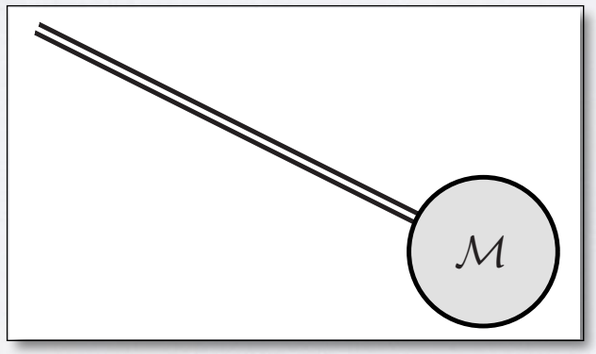
$$\sim \mathcal{M}\,\frac{\not p - \not k}{2p\cdot k}\,\gamma^\mu\,T^A\,u(p)$$

$$= \mathcal{M}\left[\frac{p^\mu}{p\cdot k} + \frac{k^\mu}{2p\cdot k} + \frac{i\,k^\alpha\,\Sigma^{\alpha\mu}}{p\cdot k}\right]T^A\,u(p), \qquad \Sigma^{\alpha\mu} = \frac{i}{4}[\gamma^\alpha,\gamma^\mu].$$

LP ***NLP***

- Emission of soft gluons at leading power (LP) factorises:

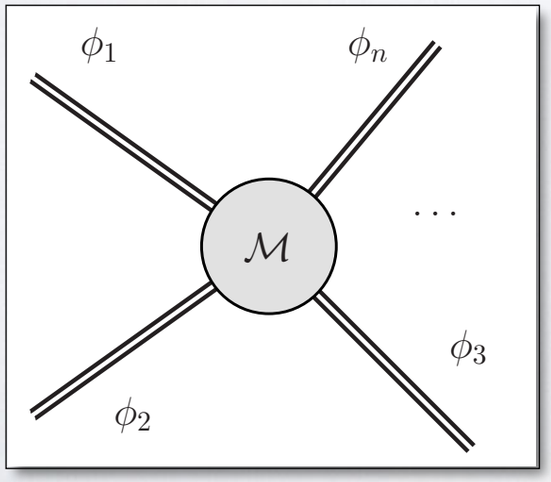




$$\sim \mathcal{M}\,\mathcal{S}\,u(p), \qquad \mathcal{S} = \langle 0|\Phi_\beta(-\infty,0)|0\rangle,$$

$$\Phi_\beta(\lambda_1,\lambda_2) = \mathcal{P}\exp\left\{i\,g_s\int_{\lambda_1}^{\lambda_2} d\lambda\;\beta\cdot A(\lambda\beta)\right\}.$$

- In general



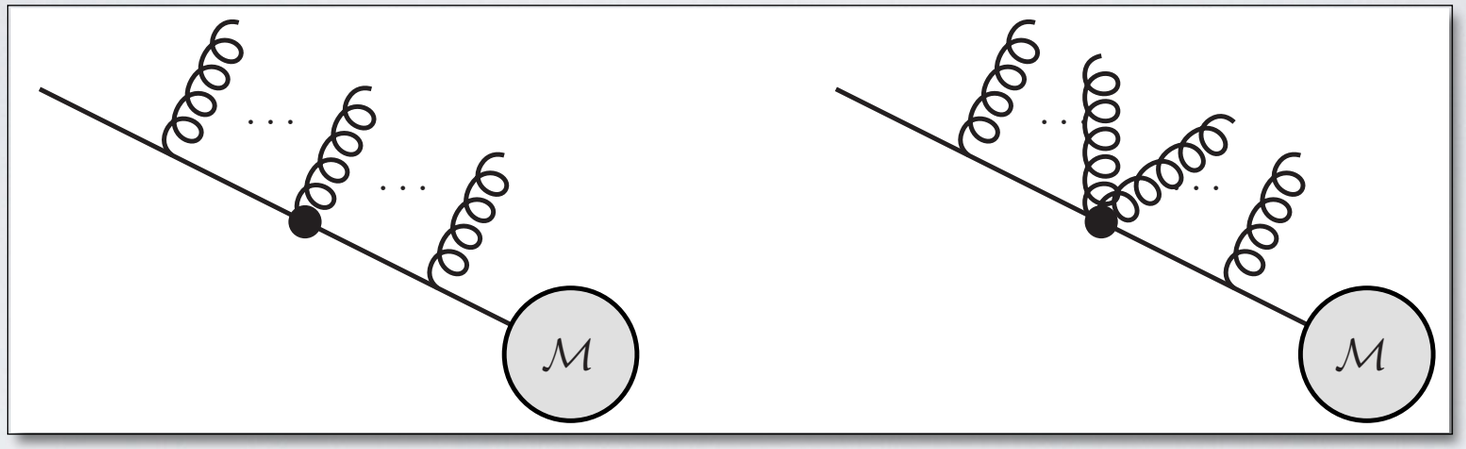
$$\sim \mathcal{M}\,\mathcal{S}\,u(p_1)\bar v(p_2)\ldots\bar u(p_n),$$

$$\mathcal{S} = \langle 0|\Phi_1\ldots\Phi_n|0\rangle \sim e^{\sum_G C_G\,\mathcal{W}_E}.$$

Gardi, Laenen, Stavenga, White, 2010; Gardi, Smillie, White, 2013

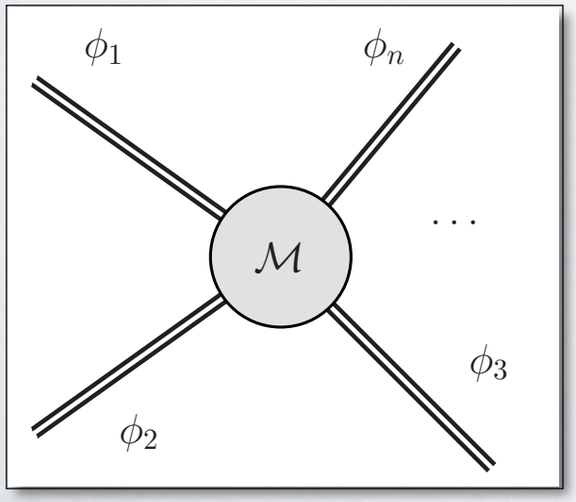
FACTORISATION OF (NEXT-TO-)SOFT GLUONS

- What happens at next-to-leading power (NLP)?



- Emission of soft gluon at NLP described in terms of "NLP" Wilson lines:

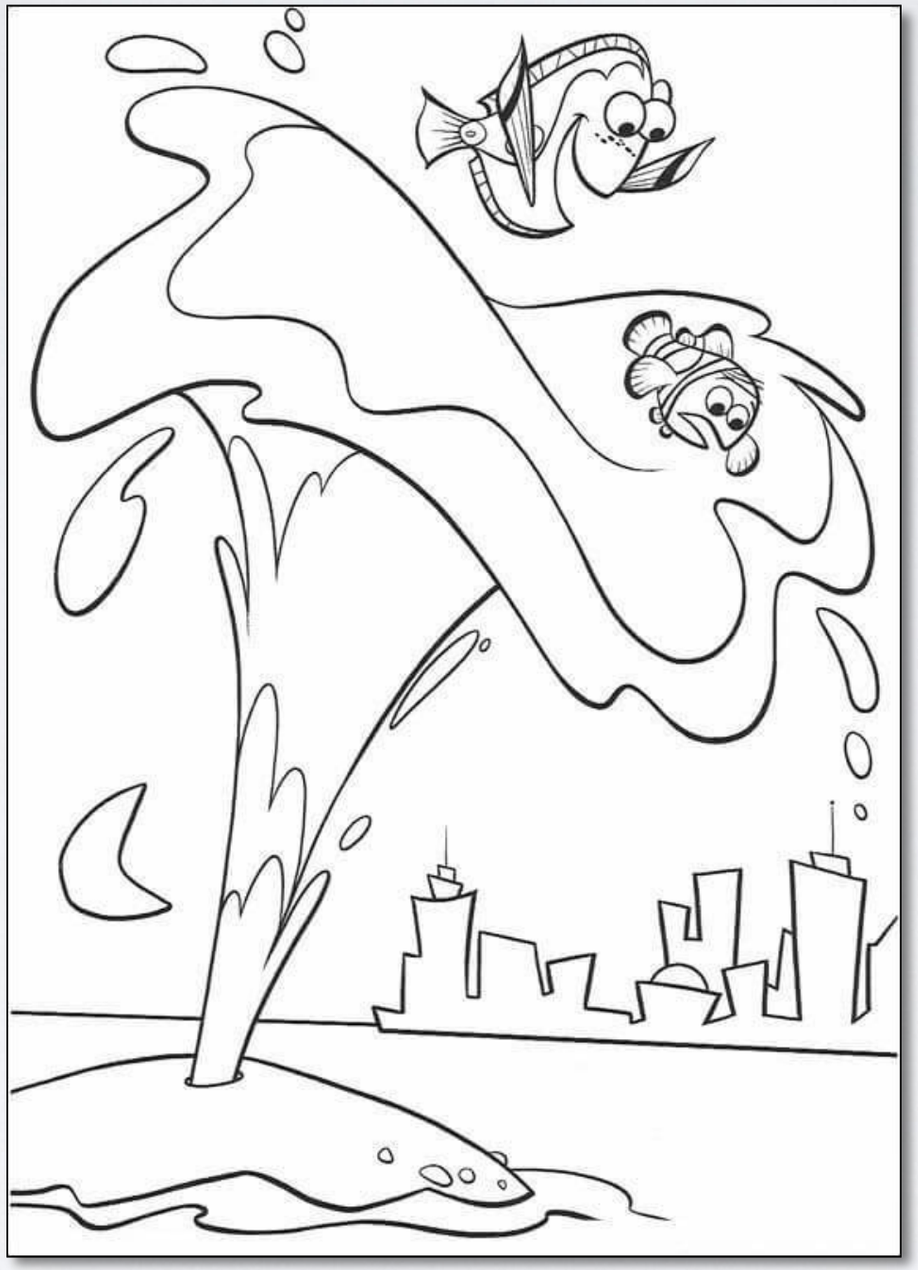
$$F_p(-\infty,0) = \mathcal{P}\exp\left[g\int\frac{d^dk}{(2\pi)^d}A_\mu(k)\left(-\frac{p^\mu}{p\cdot k}+\frac{k^\mu}{2p\cdot k}-k^2\frac{p^\mu}{2(p\cdot k)^2}-\frac{\mathrm{i}k_\nu\Sigma^{\nu\mu}}{p\cdot k}\right)\right.$$
$$+\int\frac{d^dk}{(2\pi)^d}\int\frac{d^dl}{(2\pi)^d}A_\mu(k)A_\nu(l)\left(\frac{\eta^{\mu\nu}}{2p\cdot(k+l)}-\frac{p^\nu l^\mu p\cdot k+p^\mu k^\nu p\cdot l}{2(p\cdot l)(p\cdot k)\left[p\cdot(k+l)\right]}\right.$$
$$\left.\left.+\frac{(k\cdot l)p^\mu p^\nu}{2(p\cdot l)(p\cdot k)\left[p\cdot(k+l)\right]}-\frac{\mathrm{i}\Sigma^{\mu\nu}}{p\cdot(k+l)}\right)\right].$$



$$\tilde{\mathcal{S}} = \langle 0|F_1\ldots F_n|0\rangle \sim e^{\sum_{G_e} C_{G_e}\mathcal{W}_e + \sum_{G_{ne}} C_{G_{ne}}\mathcal{W}_{ne}}.$$

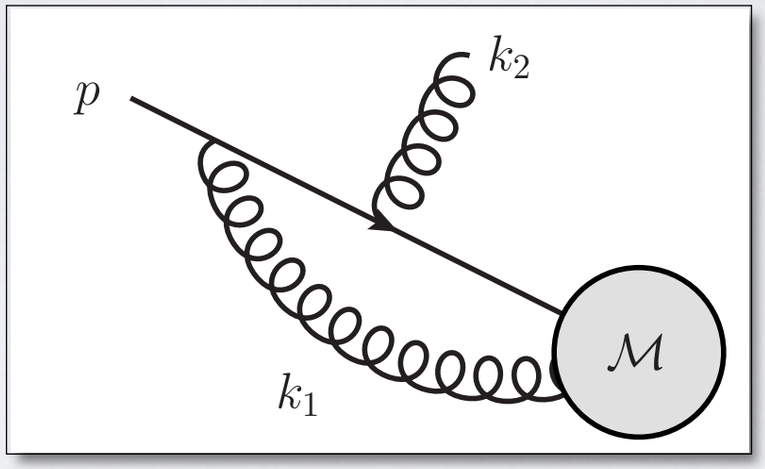
Laenen, Magnea, Stavenga, White, 2009, 2010

SCATTERING NEAR THRESHOLD: COLLINEAR MODES



SCATTERING NEAR THRESHOLD: COLLINEAR MODES

- Is this all? No, we did not take into account virtual gluons:



- The loop momentum runs over all scaling, cannot be treated as soft:

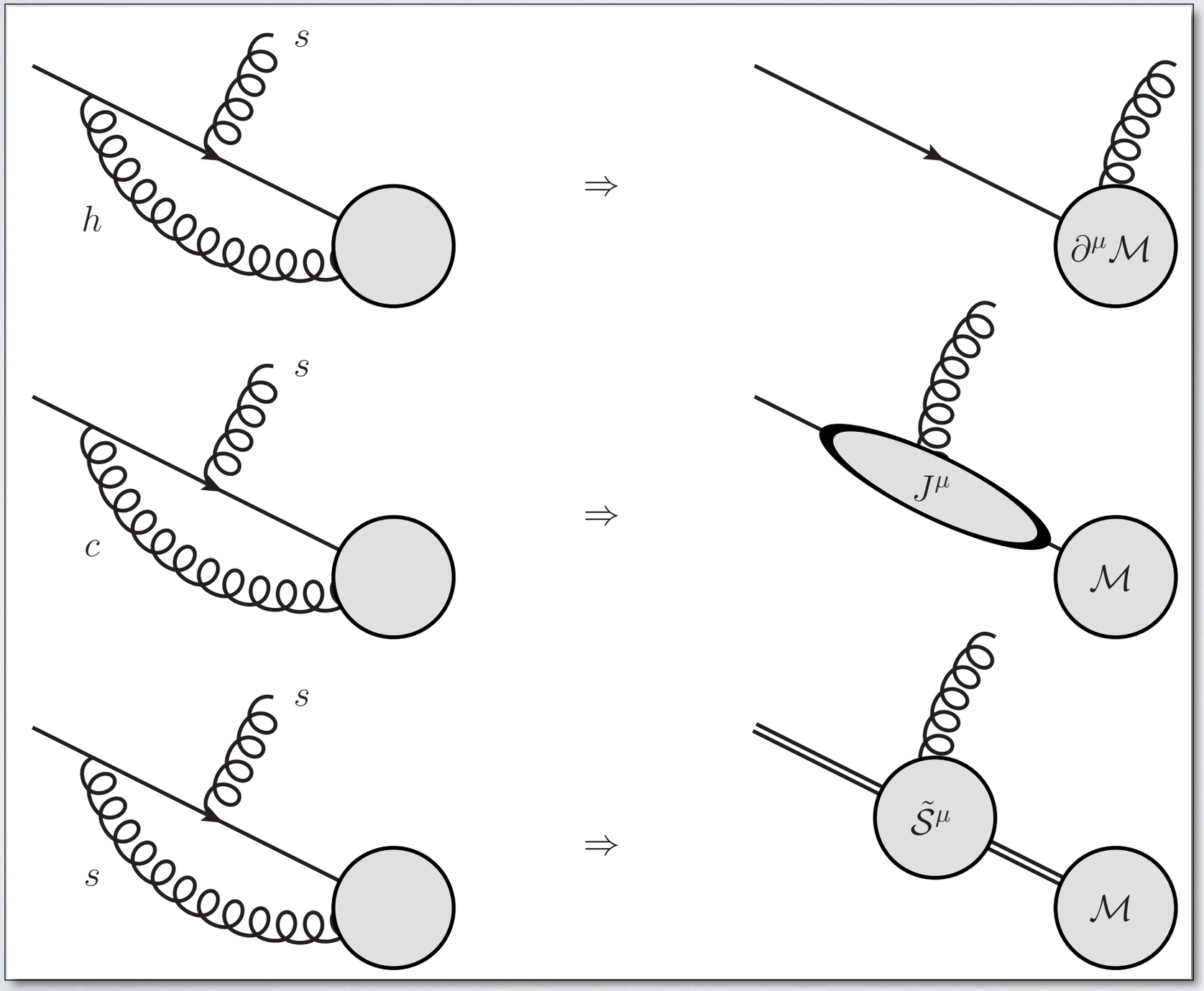
$$k = n_+ \cdot k \frac{n_-}{2} + n_- \cdot k \frac{n_+}{2} + k_\perp,$$

$$n_\pm^2 = 0, \qquad n_- \cdot n_+ = 2, \quad n_- \sim \frac{p}{s},$$

$$\begin{aligned}
\text{hard}: &\quad k \sim (1,1,1) \\
\text{collinear}: &\quad k \sim (1,\lambda,\lambda^2) \\
\text{hard}: &\quad k \sim (\lambda^2,\lambda,1) \\
\text{hard}: &\quad k \sim (\lambda^2,\lambda^2,\lambda^2), \qquad \lambda \ll 1.
\end{aligned}$$

AMPLITUDE FACTORISATION AT NLP

- Goal: factorise hard, collinear and soft modes;
- describe them in terms of simpler, universal functions:



Low 1958, Burnett,Kroll 1968

Del Duca 1990, Bonocore, Laenen, Magnea, Melville, LV, White, 2015,2016

Laenen, Magnea, Stavenga, White, 2009, 2010

Bonocore, Laenen, Magnea, LV, White, 2016

AMPLITUDE FACTORISATION AT NLP: COLLINEAR MODES

- Collinear modes can be described by introducing a radiative jet function:

Del Duca 1990

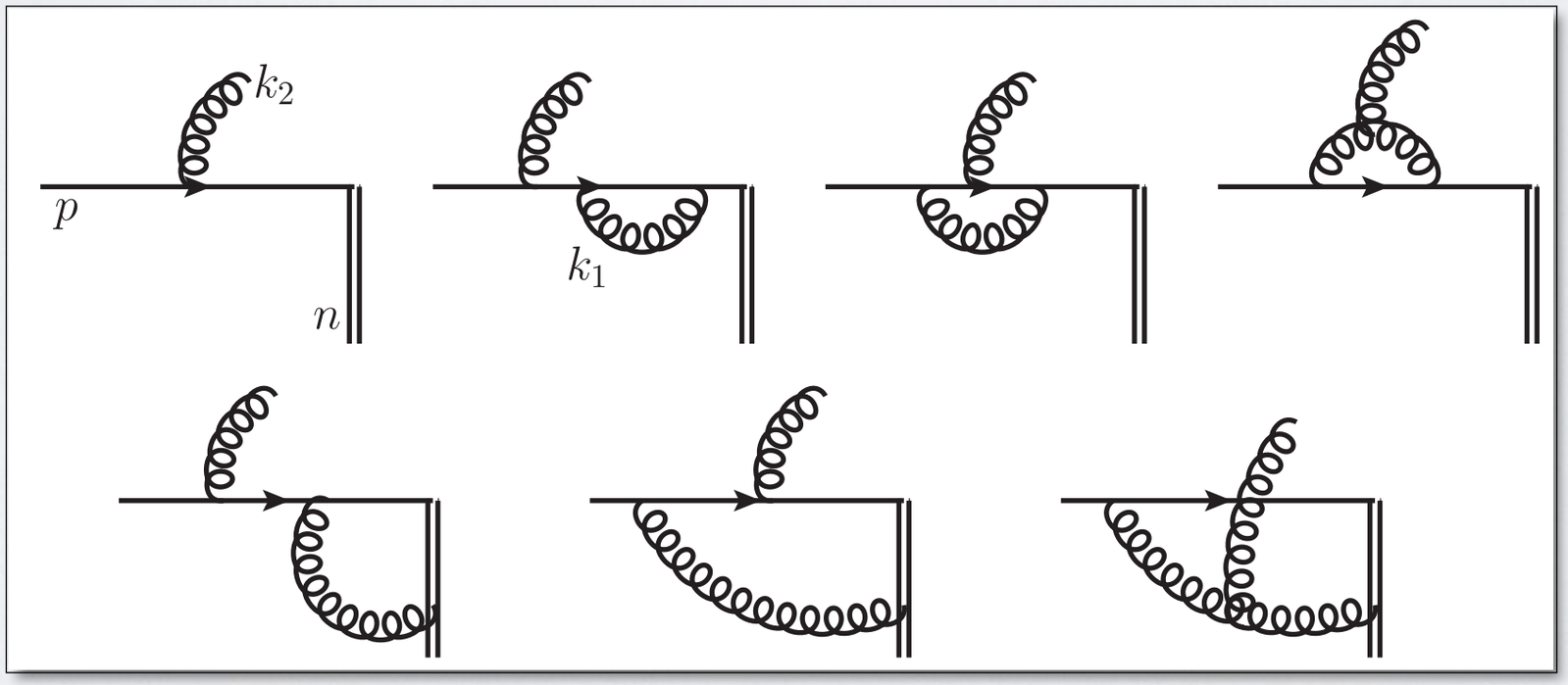
$$J_{\mu,a}\,(p,n,k)\,u(p) \;=\; \int d^d y\; \mathrm{e}^{-\mathrm{i}(p-k)\cdot y}\;\langle 0\,|\,\Phi_n(\infty,y)\,\psi(y)\,j_{\mu,a}(0)\,|\,p\rangle\;,$$

- where

$$j^{\mu}_{a}(x) \;=\; g\,\Big\{-\,\overline{\psi}(x)\,\gamma^{\mu}\,\mathbf{T}_a\,\psi(x)\;+\;f_a{}^{bc}\Big[F^{\mu\nu}_{c}(x)\,A_{\nu\,b}(x)+\partial_\nu\,(A^{\mu}_{b}(x)A^{\nu}_{c}(x))\Big]\Big\}\;.$$

- The radiative jet obeys the Ward identity

$$k_\mu J^{\mu,a}\,(p,n,k) \;=\; g\,\mathbf{T}^a J\,(p,n)\;,\qquad J(p,n)u(p) \;=\; \langle 0\,|\Phi_n(\infty,0)\psi(0)|\,p\rangle\;,$$



Bonocore, Laenen, Magnea, Melville, LV, White, 2015,2016

- Warning: J^μ contains both collinear and soft modes: remove soft modes double counting.

AMPLITUDE FACTORISATION AT NLP: DRELL YAN

- For Drell Yan we obtain:

$$\frac{k^\nu}{p_l \cdot k}\left[p_{l,\nu}\frac{\partial}{\partial p_l^\mu} - p_{l,\mu}\frac{\partial}{\partial p_l^\nu}\right] = -\frac{\mathrm{i}k^\nu\,\mathrm{L}^{(l)}_{\nu\mu}}{p_i \cdot k}$$

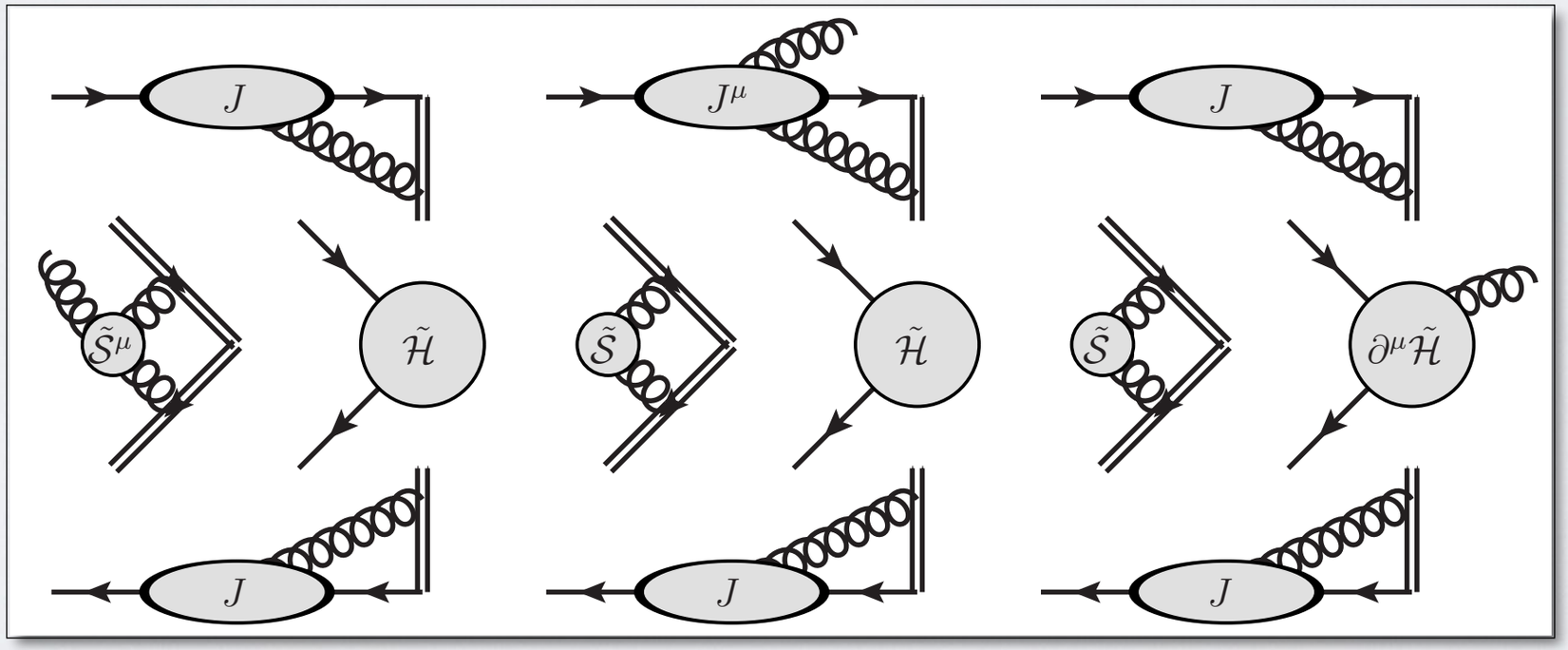
Orbital angular momentum (LBK)

$$\mathcal{A}_{\mu,a}(p_j,k) = \sum_{i=1}^{2}\left(\frac{1}{2}\widetilde{\mathcal{S}}_{\mu,a}(p_j,k) + g\,\mathbf{T}_{i,a}\,G^\nu_{i,\mu}\frac{\partial}{\partial p_i^\nu} + J_{\mu,a}\left(p_i,n_i,k\right)\right)\mathcal{A}(p_j) - \mathcal{A}^{\widetilde{\mathcal{J}}}_{\mu,a}(p_j,k)\,,$$

for $n_1 = p_2$, $n_2 = p_1$.

$$g_s\,T^A\left[\frac{p^\mu}{p\cdot k} - \frac{k^\mu}{2p\cdot k} + \frac{i\,k^\alpha\,\Sigma^{\alpha\mu}}{p\cdot k}\right]$$

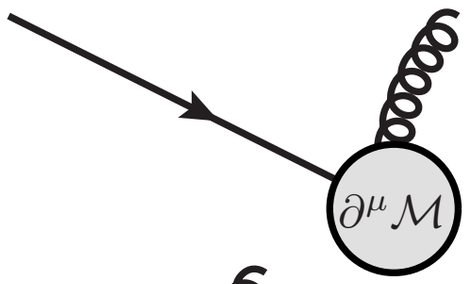
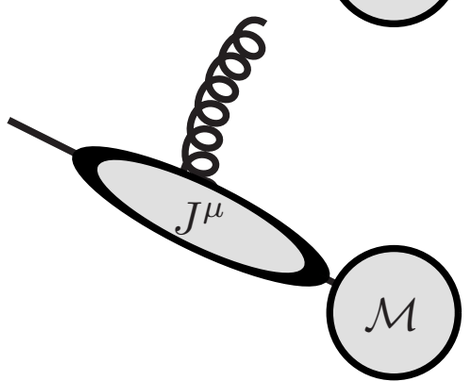
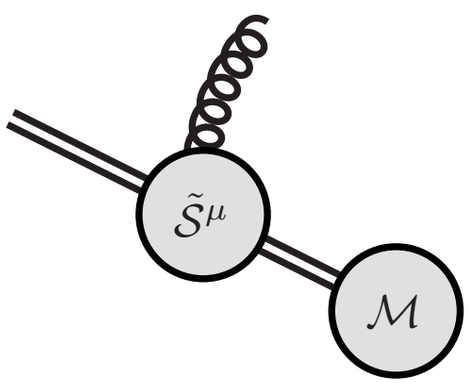
Spin angular momentum (tree level)



Bonocore, Laenen, Magnea, LV, White, 2015,2016

- Reproduces Drell-Yan up to NNLO

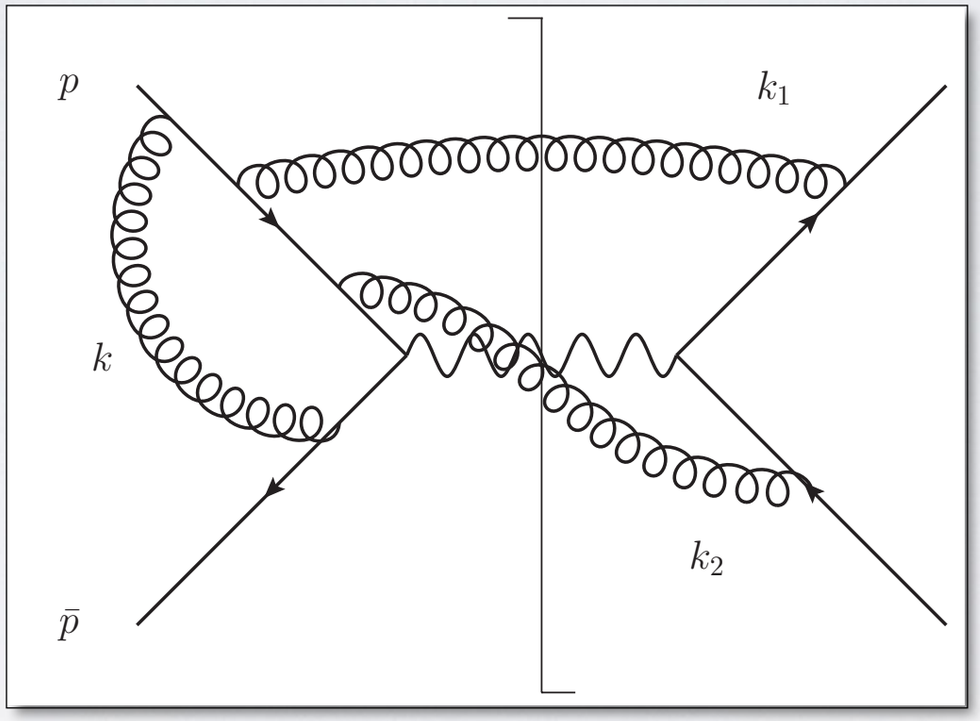
FACTORISATION AT NLP: QCD VS SCET

	"QCD"	"SCET"
	(QCD fields and Wilson lines)	(soft and collinear fields, Wilson lines, current and T-ordered operators)
	LBK theorem $\sim -\frac{\mathrm{i}k^\nu \, \mathrm{L}^{(l)}_{\nu\mu}}{p_i \cdot k} \mathcal{M}$	"multipole expansion" $\sim \partial C_i^{(i)} \, O^{(i)}$,
	radiative jet $\sim \int d^d y \, \mathrm{e}^{-\mathrm{i}(p-k)\cdot y} \, \langle 0 \,\vert\, \Phi_n(\infty, y) \, \psi(y) \, j^\mu(0) \,\vert\, p \rangle$	collinear functions $\sim \langle X_c \vert \mathcal{L}(x) \, O^{(i)}(0) \vert N_i \rangle$
	soft function $\sim \tilde{\mathcal{S}}^\mu$	soft functions $\sim \langle X_s \vert [Y_{\bar{n}}^\dagger Y_n](0) \, inD(x_-) \vert 0 \rangle$

- Better separation of collinear and soft modes (+);
- Immediately generalisable to multiple emission (+);
- Resummation from RGE of operators (+);
- Somewhat more technically involved (-).

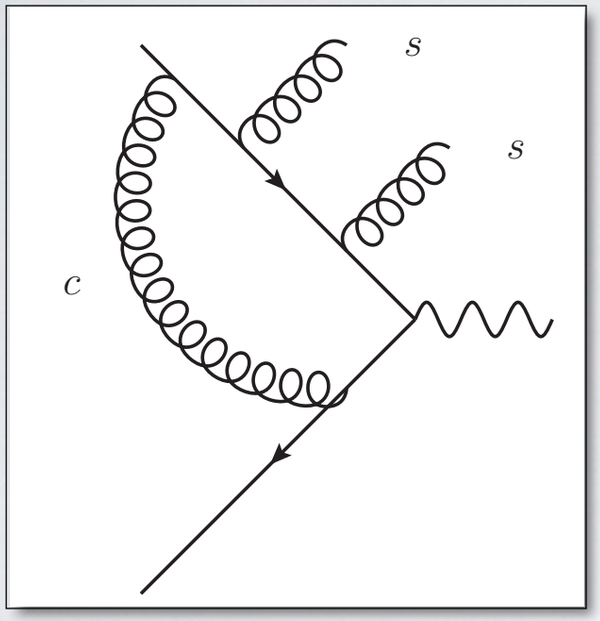
Feige, Kolodrubetz, Larkoski, Moult, Neill, Rothen, Stewart, Tackmann, Zhu, 2014,2015,2016,2017,2018
Beneke,Garny,Szafron,Yang,2017,
Beneke, Broggio, Garny, Jaskiewicz, Szafron, LV, Wang, in progress

DRELL-YAN AT N3LO AND NLP: DOUBLE REAL EMISSION

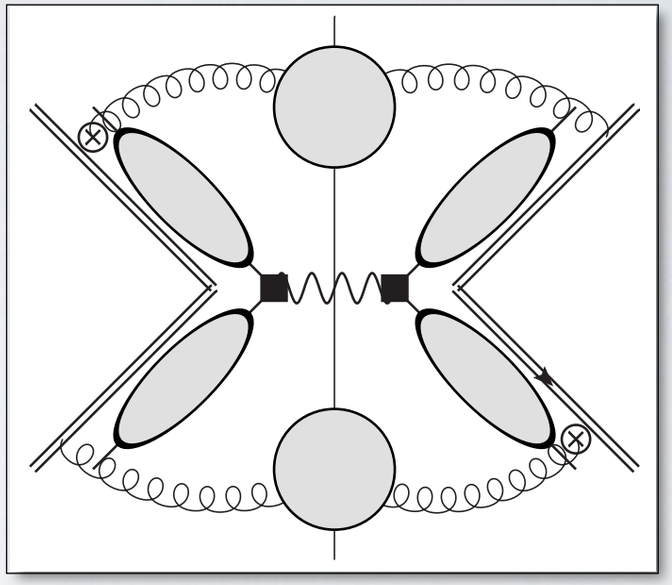


DOUBLE REAL EMISSION AT N3LO AND NLP

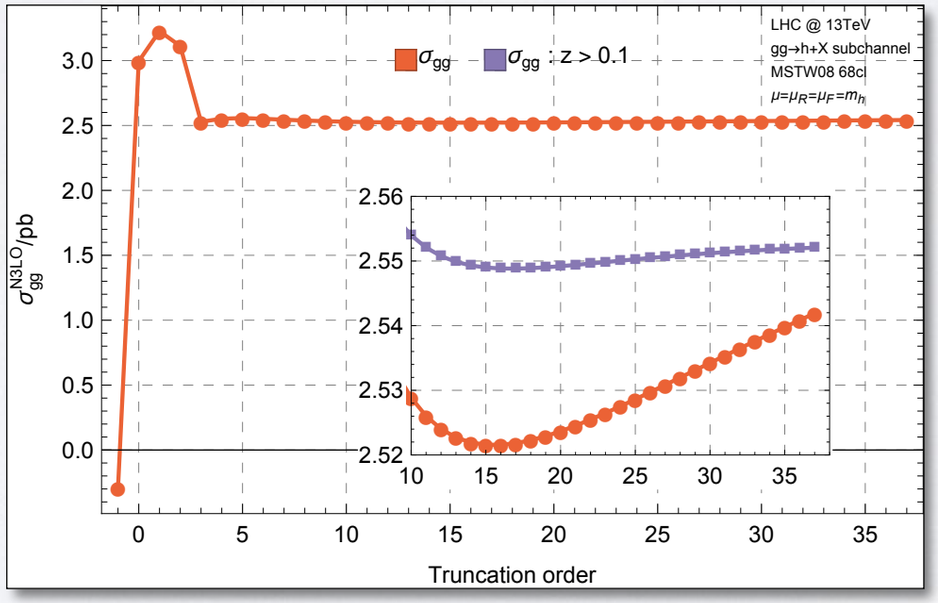
- Motivation: several reasons:



generalisation of J^μ: $J^\mu S^\nu$, $J^{\mu\nu}$?



comparison data for resummation



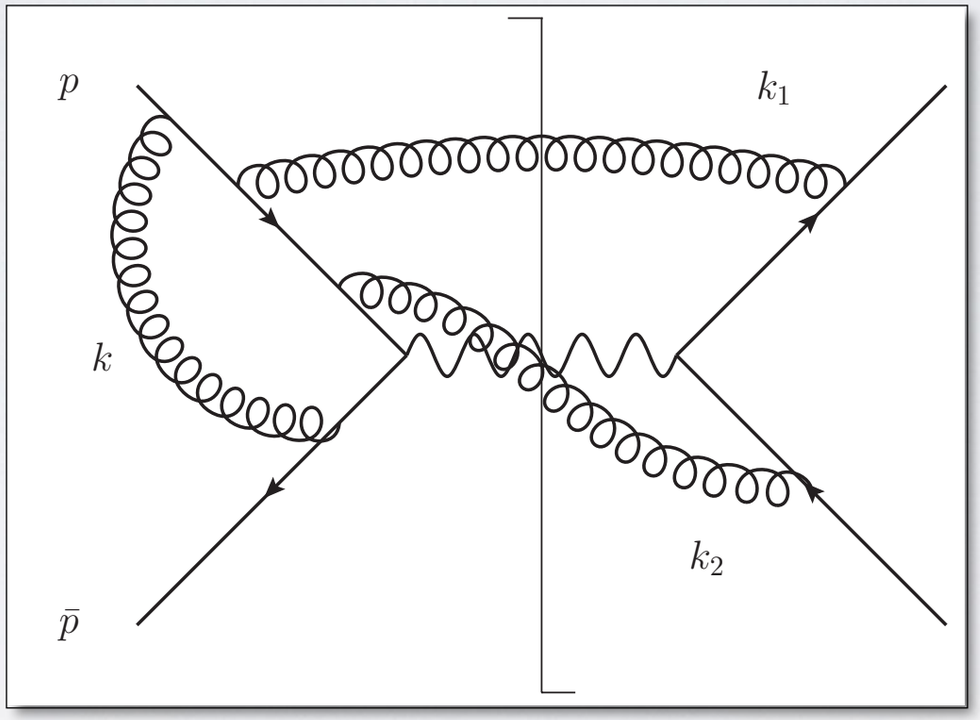
Drell Yan at N3LO unknown beyond leading power

DOUBLE REAL EMISSION AT N3LO AND NLP

- Consider the double real emission:

$$\mathcal{M} = \int \frac{d^d k}{(2\pi)^d} \mathcal{A}_{2\mathrm{r},1\mathrm{v}}(p, \bar{p}, k_1, k_2, k)\, \mathcal{A}^\dagger_{2r}(p, \bar{p}, k_1, k_2),$$

i.e. diagram such as



- What can we learn?

- In the following use invariants

$$t_2 = (p - k_1)^2 = -2p \cdot k_1, \qquad t_3 = (p - k_2)^2 = -2p \cdot k_2,$$
$$u_2 = (\bar{p} - k_1)^2 = -2\bar{p} \cdot k_1, \qquad u_3 = (\bar{p} - k_2)^2 = -2\bar{p} \cdot k_2,$$
$$s_{12} = (k_1 + k_2)^2 = 2k_1 \cdot k_2.$$

DOUBLE REAL EMISSION AT N3LO AND NLP

- The hard region is completely given in terms of lower-orders information:

Bahjat-Abbas, Sinninghe Damsté, LV, White, 2018

$$\mathcal{M}^{\rm LP}_{2r1v,\,\rm h} \propto \left(\frac{\mu^2_{\overline{\rm MS}}}{-s}\right)^{\epsilon} f_1^{\rm H}(\epsilon)\frac{s^3}{t_2\,t_3\,u_2\,u_3},$$

$$\mathcal{M}^{\rm NLP}_{2r1v,\,\rm h} \propto \left(\frac{\mu^2_{\overline{\rm MS}}}{-s}\right)^{\epsilon} \frac{s^2(t_2+t_3+u_2+u_3)}{t_2\,t_3\,u_2\,u_3}\left[f_2^{\rm H}(\epsilon)+\frac{t_2\,u_3+t_3\,u_2-s_{12}\,s}{2(t_2+t_3)(u_2+u_3)}f_1^{\rm H}(\epsilon)\right],$$

- compare with single-

$$\mathcal{M}^{\rm LP}_{1r1v,\,\rm h} \propto \left(\frac{\mu^2_{\overline{\rm MS}}}{-s}\right)^{\epsilon}\frac{s^2}{tu}f_{h_1}(\epsilon), \qquad \mathcal{M}^{\rm NLP}_{1r1v,\,\rm h} \propto -\left(\frac{\mu^2_{\overline{\rm MS}}}{-s}\right)^{\epsilon}\frac{s(t+u)}{tu}f_{h_2}(\epsilon)$$

- and double-real emission at NNLO:

$$\mathcal{M}^{\rm LP}_{2r0v} \propto \frac{s^3}{t_2\,t_3\,u_2\,u_3}, \quad \mathcal{M}^{\rm NLP}_{2r0v} \propto \frac{s^2(t_2+t_3+u_2+u_3)}{t_2\,t_3\,u_2\,u_3}\left[1+\frac{t_2\,u_3+t_3\,u_2-s_{12}\,s}{2(t_2+t_3)(u_2+u_3)}\right].$$

- ⇒ Hard region most likely given in terms of the Low-burnett-Kroll theorem: indeed,

$$f_{h_2}(\epsilon) = s^{\epsilon} f_{h_1}(\epsilon)\,\frac{\partial}{\partial s}s^{1-\epsilon} = (1-\epsilon)\,f_{h_1}(\epsilon),$$

where $f_{h1}(\epsilon)$ is the one-loop function associated with the quark form factor.

DOUBLE REAL EMISSION AT N3LO AND NLP

- Integrating over the phase space we get the differential cross section

$$\frac{d\sigma_{2r,1v}}{dz} = \frac{1}{4N_c^2}\frac{1}{2s}\left\{2\mathrm{Re}\left[\int \frac{d^dk}{(2\pi)^d}\int d\Phi^{(3)}\delta\left(z-\frac{Q^2}{s}\right)\mathcal{A}_{2\mathrm{r},1\mathrm{v}}(p,\bar{p},k_1,k_2,k)\,\mathcal{A}^{\dagger}_{2r}(p,\bar{p},k_1,k_2)\right]\right\}.$$

- In terms of the *K* factor:

$$\left(\frac{\alpha_s}{4\pi}\right)^n K^{(n)}(z) = \frac{1}{\sigma_0}\frac{d\sigma^{(n)}(z)}{dz},$$

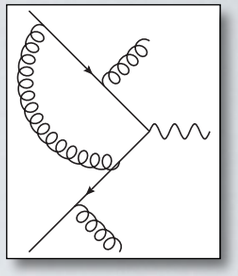
Bahjat-Abbas, Sinninghe Damsté, LV, White, 2018

we have

$$\begin{aligned}
K^{(3),\mathrm{H}}_{q\bar{q}}\Big|_{C_F^3} &= 128\left\{\frac{1}{\epsilon^5}(\mathcal{D}_0-1)+\frac{1}{\epsilon^4}\left(-4\mathcal{D}_1+\frac{3\mathcal{D}_0}{2}+4L-4\right)+\frac{1}{\epsilon^3}\left(8\mathcal{D}_2-6\mathcal{D}_1+\frac{(8-21\zeta_2)}{2}\mathcal{D}_0-8L^2+16L\right.\right.\\
&\left.-\frac{31}{4}+\frac{21}{2}\zeta_2\right)+\frac{1}{\epsilon^2}\left[-\frac{32\mathcal{D}_3}{3}+12\mathcal{D}_2+(-16+42\zeta_2)\mathcal{D}_1+\left(8-\frac{63}{4}\zeta_2-23\zeta_3\right)\mathcal{D}_0+\frac{32}{3}L^3-32L^2\right.\\
&\left.+(31-42\zeta_2)L-18+42\zeta_2+23\zeta_3\right]+\frac{1}{\epsilon}\left[\frac{32}{3}\mathcal{D}_4-16\mathcal{D}_3+(32-84\zeta_2)\mathcal{D}_2+(-32+63\zeta_2+92\zeta_3)\mathcal{D}_1\right.\\
&+\left(16-42\zeta_2-\frac{69}{2}\zeta_3+\frac{1017}{16}\zeta_4\right)\mathcal{D}_0-\frac{32}{3}L^4+\frac{128}{3}L^3+(-62+84\zeta_2)L^2+(72-168\zeta_2-92\zeta_3)\,L-36\\
&\left.+\frac{651}{8}\zeta_2+92\zeta_3-\frac{1017}{16}\zeta_4\right]-\frac{128}{15}\mathcal{D}_5+16\mathcal{D}_4+\left(-\frac{128}{3}+112\zeta_2\right)\mathcal{D}_3+(64-126\zeta_2-184\zeta_3)\mathcal{D}_2\\
&+\left(-64+168\zeta_2+138\zeta_3-\frac{1017}{4}\zeta_4\right)\mathcal{D}_1+\left(32-84\zeta_2-92\zeta_3+\frac{3051}{32}\zeta_4-\frac{1053}{5}\zeta_5+\frac{483}{2}\zeta_3\,\zeta_2\right)\mathcal{D}_0+\frac{128}{15}L^5\\
&\left.-\frac{128}{3}L^4+\left(\frac{248}{3}-112\zeta_2\right)L^3+(-144+336\zeta_2+184\zeta_3)L^2+\left(144-\frac{651}{2}\zeta_2-368\zeta_3+\frac{1017}{4}\zeta_4\right)L\right\}.
\end{aligned}$$

The NLP contribution is a necessary part of information for the calculation of DY at N3LO, and was not known before.

DOUBLE REAL EMISSION AT N3LO AND NLP

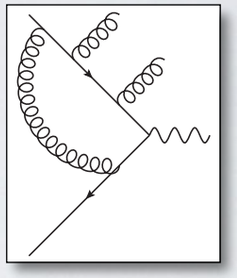


- In the collinear region we get

$$\mathcal{M}^{\rm LP}_{\rm col.} = 0;$$

$$\mathcal{M}^{\rm NLP}_{\rm col.} \propto (\mu^2_{\overline{\rm MS}})^\epsilon \frac{s^2}{t_2 t_3 u_2 u_3}\Bigg\{\Big[u_2(-t_2)^{-\epsilon} + u_3(-t_3)^{-\epsilon}\Big] f_1^{\rm C}(\epsilon)$$
$$+ \frac{t_3 u_2 + t_2 u_3 - s_{12} s}{t_2 + t_3}\Bigg[\Big((-t_2)^{-\epsilon} - 2(-t_2 - t_3)^{-\epsilon} + (-t_3)^{-\epsilon}\Big) f_2^{\rm C}(\epsilon)$$
$$- \Big(\frac{t_2}{t_3}(-t_2)^{-\epsilon} - \frac{(t_2^2 + t_3^2)}{t_2 t_3}(-t_2 - t_3)^{-\epsilon} + \frac{t_3}{t_2}(-t_3)^{\epsilon}\Big) f_3^{C}(\epsilon)\Bigg]\Bigg\},$$

where



$$f_1^{\rm C} = -\frac{5}{2} - \frac{2}{\epsilon} + \epsilon\big(-3 + \zeta_2\big) + \epsilon^2\Big(-4 + \frac{5\zeta_2}{4} + \frac{14\zeta_3}{3}\Big) + \ldots,$$
$$f_2^{\rm C} = -\frac{1}{4\epsilon} + \frac{1}{8} + \epsilon\Big(\frac{3}{4} + \frac{\zeta_2}{8}\Big) + \epsilon^2\Big(2 - \frac{\zeta_2}{16} + \frac{7\zeta_3}{12}\Big) + \ldots,$$
$$f_3^{\rm C} = \frac{1}{4\epsilon^2} - \frac{1}{8\epsilon} - \frac{3}{4} - \frac{\zeta_2}{8} + \epsilon\Big(-2 + \frac{\zeta_2}{16} - \frac{7\zeta_3}{12}\Big) + \epsilon^2\Big(-\frac{9}{2} + \frac{3\zeta_2}{8} + \frac{7\zeta_3}{24} - \frac{47\zeta_4}{64}\Big) + \ldots.$$

- We find some hints of factorisation, as expected; however, the result seems also to point to a contribution due to a genuinely new radiative jet $J^{\mu\nu}$.

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DOUBLE REAL EMISSION AT N3LO AND NLP

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- This integrates to

$$K_{q\bar{q}}^{(3),\mathrm{C}}\Big|_{C_F^3} = 32\left\{-\frac{1}{\epsilon^4}+\frac{1}{\epsilon^3}\left(5L-\frac{5}{4}\right)+\frac{1}{\epsilon^2}\left(-\frac{3}{2}-\frac{25}{2}L^2+\frac{25}{4}L+\frac{21}{2}\zeta_2\right)\right.$$
$$+\frac{1}{\epsilon}\left[\frac{125L^3}{6}-\frac{125L^2}{8}+\left(\frac{15}{2}-\frac{105\zeta_2}{2}\right)L-2+\frac{105}{8}\zeta_2+41\zeta_3\right]-\frac{625}{24}L^4+\frac{625}{24}L^3$$
$$\left.+\left(-\frac{75}{4}+\frac{525\zeta_2}{4}\right)L^2+\left(10-\frac{525}{8}\zeta_2-205\zeta_3\right)L\right\},$$

which is consistent with the expectation that virtual collinear radiation contributes only starting at NLL, at NLP.

- This is however not the end of the story..

DOUBLE REAL EMISSION AT N3LO AND NLP

- The double real emission gets a (rather involved) contribution from the soft region, too!

$$\mathcal{M}^{\rm LP}_{\rm soft} = 0;$$

$$\begin{aligned}
\mathcal{M}^{\rm NLP}_{\rm soft} \propto & \left(\frac{\mu^2_{\overline{\rm MS}}}{-s_{12}}\right)^{\epsilon} \frac{s^2}{t_2 t_3 u_2 u_3} \\
& \times \Bigg\{ \frac{t_3\, f_1^{\rm S}(\epsilon)}{t_2(t_2+t_3)^2} \Bigg[(s_{12}s - t_2u_3 - t_3u_2)\Big(t_2+t_3-t_3\, {}_2F_1\Big(1,1,1-\epsilon,\tfrac{t_2}{t_2+t_3}\Big)\Big)\Bigg] \\
& + \frac{f_2^{\rm S}(\epsilon)}{s\, s_{12}(t_2+t_3)} \left[(t_2u_3-t_3u_2)^2 - s_{12}s(t_2u_3+t_3u_2)\right] \\
& + \frac{f_3^{\rm S}(\epsilon)}{s\, s_{12} t_2 (t_2+t_3)^2} \Bigg[s_{12}^2 s^2 t_3(t_2-t_3) + t_3(t_2+t_3)(t_2u_3-t_3u_2)^2 \\
& + s_{12}st_2(t_2+t_3)(t_2u_3-3t_3u_2) - t_3\Big(s_{12}^2s^2(t_2-t_3) + (t_2+t_3)(t_2u_3-t_3u_2)^2 \\
& - 2s_{12}st_2(t_2u_3+t_3u_2)\Big)\, {}_2F_1\Big(1,1,1-\epsilon,\tfrac{t_2}{t_2+t_3}\Big)\Bigg] \\
& + \{t_2,t_3 \leftrightarrow u_2,u_3\} + \{t_2,t_3 \leftrightarrow u_3,u_2\} + \{t_2,u_2 \leftrightarrow t_3,u_3\} \Bigg\},
\end{aligned}$$

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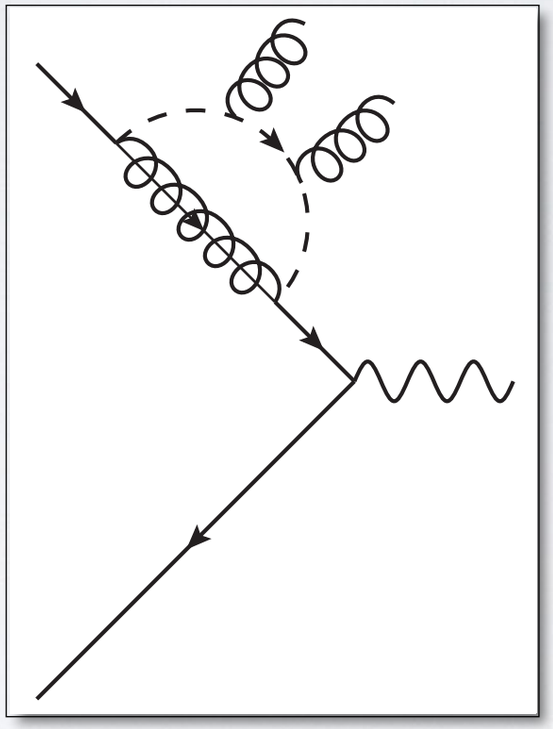
where the loop functions f^{S}_i are of the type

$$f_1^{\rm S}(\epsilon) = \frac{1}{4\epsilon^2} + \frac{1}{4\epsilon} + \ldots, \qquad f_2^{\rm S}(\epsilon) = \frac{1}{4\epsilon} + \ldots, \qquad f_3^{\rm S}(\epsilon) = \frac{1}{4\epsilon} + \ldots.$$

- This result is somehow unexpected at first, because at NNLO there is no soft region contribution, in the abelian-like correction.

DOUBLE REAL EMISSION AT N3LO AND NLP

- The soft region gets a contribution starting at N3LO, because only with two emissions one has a genuinely soft scale $s_{12} = 2k_1.k_2$, absent in case of single emission ($k^2 = 0$).
- The soft region arises from a single diagram:



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- The integration over the phase space gives a rather simple result:

$$K_{q\bar{q}}^{(3),\mathrm{S}}\Big|_{C_F^3} = 32\left\{\frac{1}{\epsilon}\left(\frac{2}{3}\zeta_2 + \frac{1}{3}\zeta_3\right) - (4\zeta_2 + 2\zeta_3)L\right\}.$$

CONCLUSION

Differential distributions near threshold develop large logarithms, needs special attention → resummation

Resummation is well known at leading power. However the precision goal set by the LHC requires to study NLP logarithms as well.

NLP logarithms are interesting: they allow us to access all-order properties of the amplitude beyond the semi-classical approximation.

Drell-Yan amplitude factorises into universal functions, such as a radiative jet function, and a next-to-soft function.

We have further considered two (next-to-)soft real emission, with the method of regions, to gain insight into the factorisation structure in presence of multiple emissions. The result suggest that a factorised structure hides also at higher orders.

Factorisation theorem for DY at subleading power can be derived as well in SCET; study concerning the resummation of leading NLP logarithms is ongoing.