Resonance dynamics and transport coefficients in SMASH

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The QCD Phase Diagram

• Main goals of heavy ion research

• Questions to be answered:
  – What is the temperature and the density? What are the relevant degrees of freedom?
  – Phase transition, critical point?
  – What are the transport properties? \((\eta/s)(T,\mu_B)\) and \((\zeta/s)(T,\mu_B)\)

• Understand the structures in the phase diagram
• Investigate the properties of the quark-gluon plasma
• Focus in this talk: Hadron/Resonance dynamics and transport coefficients (\(\eta/s\) and electric conductivity)
Outline

• Transport coefficients in heavy ion collisions
  – Viscosity in the hadron gas
• New hadronic transport approach
  – **SMASH**: content and validation
  – Bulk observables at GSI-SIS energies
• Green-Kubo formalism and its application
• Shear viscosity over entropy ratio
  – Systematic studies of simple pion gases
  – Results for the hadron-resonance gas
• Electric conductivity in simple systems
• Summary and Outlook
Transport Coefficients

- Within hydrodynamics/hybrid approaches the shear viscosity is an input parameter

- Application of Bayesian techniques allows extraction of temperature dependence

Results From Lower Beam Energies

- Viscous UrQMD hybrid fitted to beam energy scan and SPS data allows to extract **effective** shear viscosity of the hydrodynamic stage

**Analysis „by eye“**

**Bayesian framework**

- What is the hadron gas viscosity (relevant at low T)?

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Y. Karpenko, P. Huovinen, HP, M. Bleicher, Phys.Rev. C91 (2015) no.6, 064901

J. Auvinen et al., arXiv:1610.00590
Existing Results - Discrepancy

- Long standing question: Why are the results so different from each other?

SMASH*

- New hadronic transport approach:
  - Includes all mesons and baryons up to ~2 GeV
  - Geometric collision criterion
  - Binary interactions: Inelastic collisions through resonance excitation and decay
  - Infrastructure: C++, Git, Redmine, Doxygen, (ROOT)

* Simulating Many Accelerated Strongly-Interacting Hadrons

J. Weil et al, PRC 94 (2016)
Detailed Balance

- Inverse absorption cross section calculated from production cross section
- Conservation of detailed balance (only 1 $\leftrightarrow$ 2 or 2 $\leftrightarrow$ 2 processes)

Test: Full hadron gas indicating most violating processes
Elementary Cross Sections

- Total cross section for pp collisions
- Parametrised elastic cross section
- Many resonance contributions to inelastic cross section
- Reasonable description of data up to 4 - 4.5 GeV
- String excitation by PYTHIA: work in progress
Analytic Solution

- Comparison to analytic solution of Boltzmann equation within expanding metric

- Perfect agreement proves correct numerical implementation of collision algorithm

Pion Production in Au+Au

- Potentials decrease pion production, while Fermi motion increases yield
- Slightly too high pion multiplicities

J. Weil et al, PRC 94 (2016)
Collective Flow - $v_2$

- Directed and elliptic flow are compared to available data from FOPI and HADES

charged particles, $|y| < 0.1$

- SMASH agrees well with previous UrQMD calculation for $v_2$ excitation function

by Markus Mayer

Dilepton Production

by J. Staudenmaier, paper in preparation

- SMASH and UrQMD compare very similar to data
- Different vector meson thresholds
- Work in progress: Detailed study from elementary collisions to heavy ions and afterburner for RHIC/LHC

HADES, PRL 98 (2007)

Shear Viscosity over Entropy Density

- Box with periodic boundary condition in chemical and thermal equilibrium
- Entropy is calculated via Gibbs formula from thermodynamic properties
- The shear viscosity is extracted following the Green-Kubo formalism:

\[ \eta = \frac{V}{T} \int_0^\infty C^{xy}(t) dt \]

\[ C^{xy}(t) = \frac{1}{N} \sum_s T^{xy}(s)T^{xy}(s + t) \]

\[ T^{\mu\nu} = \frac{1}{V} \sum_i^{N_{part}} \frac{p^{\mu}_i p^{\nu}_i}{p^0_i} \]

\[ C^{xy}(t) \simeq C^{xy}(0) \exp \left( -\frac{t}{\tau} \right) \]

\[ \eta = \frac{V C^{xy}(0) \tau}{T} \]
• Analytic results are well reproduced
**Correlation Function Systematics**

- **Important details:**
  - Fixed intercept and 6% cut-off agree best with analytic expectations

\[ \sigma=20 \text{ mb}, \ T=200 \text{ MeV}, \ \pi \text{ only} \]
Resonance Dynamics

• Energy-dependence of cross-sections is modelled via resonances

• Point-like in analytic calculation and finite lifetime in transport approach

• Agreement recovered by decreasing $\rho$ meson lifetime
• Results for viscosity are affected when the lifetime of resonances is on the same order

• Finite lifetime delays momentum transfer of pions

• Mean free time between collisions which is expected to determine the relaxation time stays constant
• Shear viscosity over enthalpy decreases and saturates as a function of temperature

• Mild dependence on net baryon chemical potential
Comparison to Literature

- Closest similarity to Bass/Demir result as expected
Point-like Interactions

- Adding a constant elastic cross section leads to agreement with B3D result

- Approximately linear relationship between relaxation time and mean free time is recovered
Electric Conductivity

- Comparison to linear response kinetic theory to validate our approach

\[ \sigma_{el} = \frac{V}{T} \int_0^\infty \langle j_i(0) j_i(t) \rangle dt \]

\[ \sigma_{el} = \frac{VC(0)\tau}{T} \]

- Infinite matter with constant \( \sigma = 30 \text{ mb} \)

Effect of Lifetime

- $\rho-\pi$ system is again affected by the lifetime

- Work in progress: Understand the differences between analytic calculation and SMASH results
Summary and Outlook

• Transport coefficients are of major interest to the heavy ion community

• SMASH has been developed as a new hadronic transport approach
  – Bulk observables are in reasonable agreement with experimental data
  – Electromagnetic observables are integrated

• Shear viscosity and electric conductivity have been calculated via Green-Kubo formalism
  – Comparison to analytic results are used for validation
  – Resonance lifetimes have large impact on relaxation dynamics in both cases

• Hadron gas properties can be constrained
Backup
General Setup

- Transport models provide an effective solution of the relativistic Boltzmann equation

\[ p^\mu \partial_\mu f_i(x, p) + m_i F^\alpha \partial_\alpha f_i(x, p) = C_{\text{coll}}^i \]

- Particles represented by Gaussian wave packets
- Geometric collision criterion

\[ d_{\text{trans}} < d_{\text{int}} = \sqrt{\frac{\sigma_{\text{tot}}}{\pi}} \]

\[ d_{\text{trans}}^2 = (r_a - r_b)^2 - \frac{((r_a^* - r_b^*) \cdot (p_a^* - p_b^*))^2}{(p_a^* - p_b^*)^2} \]

- Test particle method

\[ \sigma \rightarrow \sigma \cdot N_{\text{test}}^{-1} \]

\[ N \rightarrow N \cdot N_{\text{test}} \]
Resonances

- **Spectral function**
  - All unstable particles ("resonances") have relativistic Breit-Wigner spectral functions

- **Decay widths**
  - Particles stable, if width < 10 keV
    - \((\pi, \eta, K, \ldots)\)
  - Treatment of Manley et al

\[
A(m) = \frac{2N}{\pi} \frac{m^2\Gamma(m)}{(m^2 - M_0^2)^2 + m^2\Gamma(m)^2}
\]

\[
\Gamma_{R \rightarrow ab} = \frac{\Gamma_{R \rightarrow ab}}{\rho_{ab}(M_0)} \rho_{ab}(m)
\]

Collision Term

• In few GeV energy regime decay and excitation of resonances dominate hadronic cross section

• No string fragmentation yet
Treatment of Manley


- Scaling of on-shell decay width:

\[
\Gamma_{R\rightarrow ab} = \Gamma_{R\rightarrow ab}^0 \frac{\rho_{ab}(m)}{\rho_{ab}(M_0)}
\]

- Definition of rho-function:

\[
\rho_{ab}(m) = \int dm_a dm_b A_a(m_a) A_b(m_b) \times \frac{|\vec{p}_f|}{m} B^2_L(|\vec{p}_f| R) F^2_{ab}(m)
\]


- Hadronic Form Factor:

\[
F_{ab}(m) = \frac{\lambda^4 + 1/4(s_0 - M_0^2)^2}{\lambda^4 + (m^2 - 1/2(s_0 + M_0^2))^2}
\]

Blatt Weisskopf functions

\[
B_0^2 = 1 \quad \quad B_1^2(x) = x^2/(1 + x^2) \quad \quad \ldots
\]

<table>
<thead>
<tr>
<th>Decay</th>
<th>(\lambda) [GeV]</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\pi\rho) unstable mesons (e.g. (\rho N, \sigma N))</td>
<td>0.8</td>
</tr>
<tr>
<td>unstable baryons (e.g. (\pi \Delta))</td>
<td>1.6</td>
</tr>
<tr>
<td>two unstable daughters (e.g. (\rho \rho))</td>
<td>0.6</td>
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