Reconstructing see-saw models from low energy data

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Based on works in collaboration with Alberto Casas, Sacha Davidson and Graham Ross

Flavour in the era of the LHC

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1-Motivation

Why reconstruct the see-saw? Why determining $\mathbf{Y}_\nu$ and $\mathcal{M}$?

⋆ The effective theory provides a good **description** of neutrino observations. But we would like to have a **deeper understanding** of neutrino observations (why tiny masses? why two large angles?)

⋆ The **clues to unravel the flavour puzzle** lie in the fundamental theory.

- Look for patterns. In the quark sector, $m_u : m_c : m_t \sim \lambda^8 : \lambda^4 : 1$,
  $m_d : m_s : m_b \sim \lambda^4 : \lambda^2 : 1$, λ being the Cabibbo angle.
- Look for similarities with the charged lepton sector
- Look for similarities with the quark sector (GUT symmetries?)

⋆ Determine the **scale of new physics** (masses of the right-handed neutrinos).
Implications for

- GUTs
- leptogenesis

2- Approaches to determine the see-saw parameters

⋆ **top-down**: Start with a concrete model (GUT, Froggatt-Nielsen, strings...) and compare the predictions with the experiments.

Many different possibilities.

Unfortunately, the simplest ideas do not seem to work... May be we are being mislead by theoretical prejudices?

⋆ **bottom-up**: Exploit all the information available at low energies on the leptonic sector, in order to reconstruct the high-energy theory.

Completely phenomenological. Impossible to get mislead by aesthetics, but

very difficult in practice

★ In the Standard Model it is **hopeless**

$\{Y_\nu, \mathcal{M}\}$ depend on 18 parameters — 12 real, 6 phases

$\{\mathcal{M}_\nu\}$ depends on 9 parameters — 6 real, 3 phases

Part of the information is lost in the decoupling process

★ In the Minimal Supersymmetric Standard Model, radiative corrections on slepton parameters provide additional information about the see-saw mechanism, through the combination $Y_\nu^\dagger Y_\nu$

Davidson, A.I.
Ellis, Hisano, Raidal, Shimizu

Assume universality at M_X:

$$m_L^2(M_X) = m^2 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \longrightarrow m_L^2(M_Z) \sim m^2 \begin{pmatrix} \times & \times & \times \\ \times & \times & \times \\ \times & \times & \times \end{pmatrix}$$

$$16\pi^2 \frac{d\mathbf{m_L^2}}{dt} = \left(\mathbf{m_L^2 Y_e^\dagger Y_e} + \mathbf{Y_e^\dagger Y_e m_L^2}\right) + \left(\mathbf{m_L^2 Y_\nu^\dagger Y_\nu} + \mathbf{Y_\nu^\dagger Y_\nu m_L^2}\right)$$
$$+2\left(\mathbf{Y_e^\dagger m_e^2 Y_e} + m_{H_1}^2 \mathbf{Y_e^\dagger Y_e} + \mathbf{A_e^\dagger A_e}\right) + 2\left(\mathbf{Y_\nu^\dagger m_\nu^2 Y_\nu} + m_{H_2}^2 \mathbf{Y_\nu^\dagger Y_\nu} + \mathbf{A_\nu^\dagger A_\nu}\right)$$
$$-\left(\frac{6}{5} g_1^2 \left|M_1\right|^2 + 6 g_2^2 \left|M_2\right|^2\right) \mathbf{I_3} - \frac{3}{5} g_1^2 S \mathbf{I_3}$$

In the leading-log approximation:

$$\left(m_L^2\right)_{ij} \simeq \#\mathbf{I_3} - \frac{1}{8\pi^2}(3m_0^2 + A_0^2)(\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu)_{ij} \log \frac{M_X}{M}$$

$\star$ Is it possible to reconstruct the see-saw parameters with the information from $\mathcal{M}$ and $Y_\nu^\dagger Y_\nu$? **YES!!**

In the basis where $\mathcal{M}$=$D_{\mathcal{M}}$=diag(M_1, M_2, M_3), the Yukawa coupling reads $\mathbf{Y}_\nu = V_R^\dagger \text{diag}(Y_1, Y_2, Y_3) V_L$.

Then, the reconstruction follows in two steps: Davidson,A.I.

- $\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu = V_L^\dagger \text{diag}(Y_1^2, Y_2^2, Y_3^2) V_L$

  From here we extract V_L and diag(Y_1, Y_2, Y_3)

- $\mathcal{M}_\nu = \mathbf{Y}_\nu{}^T \mathcal{M}^{-1} \mathbf{Y}_\nu \langle H_u^0 \rangle^2 = V_L^\dagger D_Y V_R^* D_{\mathcal{M}}^{-1} V_R^\dagger D_Y V_L$

  $\frac{1}{\langle H_u^0 \rangle^2} D_Y^{-1} V_L \mathcal{M}_\nu V_L^\dagger D_Y^{-1} = V_R^* D_{\mathcal{M}}^{-1} V^\dagger$

  From here we extract V_R and $D_{\mathcal{M}}$

We have everything!! However, in practice some of the low energy parameters could be very hard to meassure (if not impossible)

neutrino mass matrix, $\mathcal{M}_\nu$			radiative effects, $P \equiv \mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu$			
m_1	m_1	☺	P_{11}	mass splittings	largest	☺
m_2	Δm^2_{atm}	$\surd$	P_{22}		smallest	☹
m_3	Δm^2_{sol}	$\surd$	P_{33}	absolute scale		☹
θ_{12}	$\surd$		$\vert P_{12}\vert$	rare decays	$\mu \longrightarrow e\gamma$	☺
θ_{13}	☺		$\vert P_{13}\vert$		$\tau \longrightarrow e\gamma$	☺
θ_{23}	$\surd$		$\vert P_{23}\vert$		$\tau \longrightarrow \mu\gamma$	☺
δ		☺	$\arg P_{12}$	e-EDM	one of them	☺
ϕ	$\nu 0\beta\beta$ phase	☹	$\arg P_{13}$	μ-EDM	the other two	☹
ϕ'	orthogonal combination	☹ ☹	$\arg P_{23}$	τ-EDM		

An interesting implication of this procedure:

There is a one to one correspondence between low energy observables and see-saw parameters: $\{\mathbf{Y}_\nu, \mathcal{M}\} \longleftrightarrow \{\mathcal{M}_\nu, \mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu\}$

$\{Y_\nu, \mathcal{M}\}$ depend on 18 parameters — 12 real, 6 phases

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⋆ **Positive lesson**: We can use this trick as an alternative parametrization of the see-saw mechanism, using only low energy observables.

⋆ **Negative lesson**: For any set of observations, there is a see-saw scenario that accommodates them. **The see-saw cannot be ruled out!!**

3- A less radical (and more practical) approach

Both the top-down and the bottom-up approaches are interesting, but have limitations $\longrightarrow$ **try a hybrid approach:**

bottom-up approach with some well-motivated hypotheses about the high-energy theory

- GUT inspired
  - $\mathbf{Y}_\nu$ symmetric
  - $\mathbf{Y}_\nu$ eigenvalues as m_u, m_c, m_t Akhmedov, Frigerio, Smirnov
- texture zeros. Inspired by Gatto-Sartori-Tonin relation, A.I., Ross

$$\sin\theta_c \simeq |\sqrt{\frac{m_d}{m_s}} - e^{i\delta}\sqrt{\frac{m_u}{m_c}}|)$$

- **two right-handed neutrino (2RHN) model**

Kleppe; Ma, Roy, Sarkar; Frampton, Glashow, Yanagida; Raidal, Strumia; Gonzalez-Felipe, Joaquim, Nobre; Raby Appelquist, Shrock; Endoh, Kaneko, Kang, Morozumi, Tanimoto Smirnov; King, Ross; Rodejohann; Dreiner,Murayama, Thormeier A.I., Ross; Altarelli, Feruglio, Masina...

Observations tell us that at least two new mass scales have to be introduced (Δm^2_{sol}, Δm^2_{atm}). Two right-handed neutrinos can do the job.

Further motivation: there are some interesting situations where the 3RHN model can be well described by the 2RHN model. The mass matrix is:

$$\mathcal{M}_{ij} = \frac{y_{1i}y_{1j}}{M_1} + \frac{y_{2i}y_{2j}}{M_2} + \frac{y_{3i}y_{3j}}{M_3}, \text{ where } y_{ij} = (\mathbf{Y}_\nu)_{ij}.$$

Two RH neutrinos dominate when

- $\frac{y_{1i}y_{1j}}{M_1} \ll \frac{y_{2i}y_{2j}}{M_2}, \frac{y_{3i}y_{3j}}{M_3}$ It occurs when $y_{1i} \ll y_{2i}, y_{3i}$. In this case,

$$\mathcal{M}_{ij} \simeq \frac{y_{2i}y_{2j}}{M_2} + \frac{y_{3i}y_{3j}}{M_3}$$
$$(\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu)_{ij} \simeq y_{2i}^* y_{2j} + y_{3i}^* y_{3j}$$
$\longrightarrow$ **2RHN model!!**

- $\frac{y_{2i}y_{2j}}{M_2} \ll \frac{y_{1i}y_{1j}}{M_1}, \frac{y_{3i}y_{3j}}{M_3}$ Not very interesting...

- $\frac{y_{3i}y_{3j}}{M_3} \ll \frac{y_{1i}y_{1j}}{M_1}, \frac{y_{2i}y_{2j}}{M_2}$ It occurs when $M_3 \gg M_1, M_2$

But it could happen that $(\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu)_{ij} = y_{1i}^* y_{1j} + y_{2i}^* y_{2j} + y_{3i}^* y_{3j}$. **Not 2RHN!**

In GMSB models, with messenger masses between M_2 and M_3,

$$\mathcal{M}_{ij} \simeq \frac{y_{1i}y_{1j}}{M_1} + \frac{y_{2i}y_{2j}}{M_2}$$
$$(\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu)_{ij} = y_{1i}^* y_{1j} + y_{2i}^* y_{2j}$$
$\longrightarrow$ **2RHN model!!**

Parameter counting. In the basis where the RH mass matrix is diagonal,

$\mathcal{M} = D_{\mathcal{M}} \longrightarrow$ 2 real parameters

Y_ν is a 2×3 matrix $\longrightarrow$ 6 real parameters, 3 phases

$\{Y_\nu, \mathcal{M}\}$ depend on 11 parameters — 8 real, 3 phases

$\{\mathcal{M}_\nu, \mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu\}$ depend on 18 parameters — 12 real, 6 phases

Many predictions!!

⋆ Predictions on the neutrino mass matrix

- $m_1 = 0$ ($\Rightarrow m_3 = \sqrt{\Delta m^2_{atm}},\ m_2 = \sqrt{\Delta m^2_{sol}}$)
- there is only one phase difference between the mass eigenvalues (so, there is only one Majorana phase)
- (still three mixing angles, and a Dirac phase)

⋆ Predictions on the radiative effects

We want to incorporate in these predictions the information from the neutrino mass matrix. Working in the basis where $\mathcal{M} = \mathrm{diag}(M_1, M_2)$, it can be checked that

$$\boxed{\mathbf{Y}_\nu = D_{\sqrt{\mathcal{M}}} R D_{\sqrt{m}} U^\dagger / \langle H_u^0 \rangle}$$

Casas, A.I.

is the most general Yukawa coupling that satisfies

$$\mathcal{M}_\nu = \mathbf{Y}_\nu^T \mathrm{diag}(M_1^{-1}, M_2^{-1}) \mathbf{Y}_\nu \langle H_u^0 \rangle^2$$

Here, R is an orthogonal **complex** matrix

$$R = \begin{pmatrix} 0 & \cos z & \sin z \\ 0 & -\sin z & \cos z \end{pmatrix}$$

A.I., Ross

So, the independent parameters in $\mathbf{Y}_\nu$ are M_1, M_2 and z (three real and one phase). On the other hand, $Y_\nu^\dagger Y_\nu$ depends in general on 6 real parameters and three phases. **We can make predictions on three real parameters and two phases in $Y_\nu^\dagger Y_\nu$.**

To compute the predictions, we construct the matrix

$$U^\dagger \mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu U = D_{\sqrt{m}} R^\dagger D_{\mathcal{M}} R D_{\sqrt{m}} / \langle H^0 \rangle^2$$

since $m_1 = 0$,

$$(U^\dagger \mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu U)_{1i} = 0 \quad \text{for } i = 1, 2, 3.$$ A.I.

five equations → five relations/predictions.

⋆ Still three independent real parameters and one phase. We choose as independent parameters P_{12}, $|P_{13}|$, $|P_{23}|$ $(P \equiv \mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu)$. The predictions are:

$$P_{11} = -\frac{P_{12}^* U_{21}^* + P_{13}^* U_{31}^*}{U_{11}^*}$$

$$P_{22} = -\frac{P_{12} U_{11}^* + P_{23}^* U_{31}^*}{U_{21}^*}$$

$$P_{33} = -\frac{P_{13} U_{11}^* + P_{23} U_{21}^*}{U_{31}^*}.$$

$$e^{i \arg P_{13}} = \frac{-i\ \mathrm{Im}(P_{12} U_{21} U_{11}^*) \pm \sqrt{|P_{13}|^2 |U_{11}|^2 |U_{31}|^2 - [\mathrm{Im}(P_{12} U_{21} U_{11}^*)]^2}}{|P_{13}| U_{31} U_{11}^*}$$

$$e^{i \arg P_{23}} = \frac{i\ \mathrm{Im}(P_{12} U_{21} U_{11}^*) \pm \sqrt{|P_{23}|^2 |U_{21}|^2 |U_{31}|^2 - [\mathrm{Im}(P_{12} U_{21} U_{11}^*)]^2}}{|P_{23}| U_{31} U_{21}^*}$$

4- Reconstructing the 2RHN model

The 2RHN model can be parametrized in terms of

$\{Y_\nu, \mathcal{M}\}$ depend on 11 parameters — 8 real, 3 phases

$$\left\{\begin{matrix} m_2, m_3 & |P_{12}|, |P_{13}|, |P_{23}| \\ \theta_{12}, \theta_{13}, \theta_{23}, & \arg P_{12} \\ \delta, \phi & \end{matrix}\right\}$$

depend on 11 parameters — 8 real, 3 phases

It is possible to derive exact expressions for the high-energy parameters in terms of the low-energy parameters:

$$M_1 = \frac{1}{2}\left[\sqrt{\left(\frac{Q_{33}}{m_3}+\frac{Q_{22}}{m_2}\right)^2+\frac{(Q_{23}-Q_{23}^*)^2}{m_2 m_3}}-\sqrt{\left(\frac{Q_{33}}{m_3}-\frac{Q_{22}}{m_2}\right)^2+\frac{(Q_{23}+Q_{23}^*)^2}{m_2 m_3}}\right]\langle H_u^0\rangle^2$$

$$M_2 = \frac{1}{2}\left[\sqrt{\left(\frac{Q_{33}}{m_3}+\frac{Q_{22}}{m_2}\right)^2+\frac{(Q_{23}-Q_{23}^*)^2}{m_2 m_3}}+\sqrt{\left(\frac{Q_{33}}{m_3}-\frac{Q_{22}}{m_2}\right)^2+\frac{(Q_{23}+Q_{23}^*)^2}{m_2 m_3}}\right]\langle H_u^0\rangle^2$$

$$\cos 2z = \left(\frac{Q_{33}^2}{m_3^2}-\frac{Q_{22}^2}{m_2^2}+\frac{(Q_{23}+Q_{23}^*)(Q_{23}-Q_{23}^*)}{m_2 m_3}\right)\frac{\langle H_u^0\rangle^4}{M_2^2-M_1^2}$$

where $Q = U^\dagger P U = U^\dagger \mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu U$.

Finally, the Yukawa coupling can be reconstructed $\mathbf{Y}_\nu = D_{\sqrt{\mathcal{M}}} R D_{\sqrt{m}} U^\dagger / \langle H_u^0\rangle$

⋆ Is it feasible? In the 3RHN model

neutrino mass matrix, $\mathcal{M}_\nu$			radiative effects, $P \equiv \mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu$			
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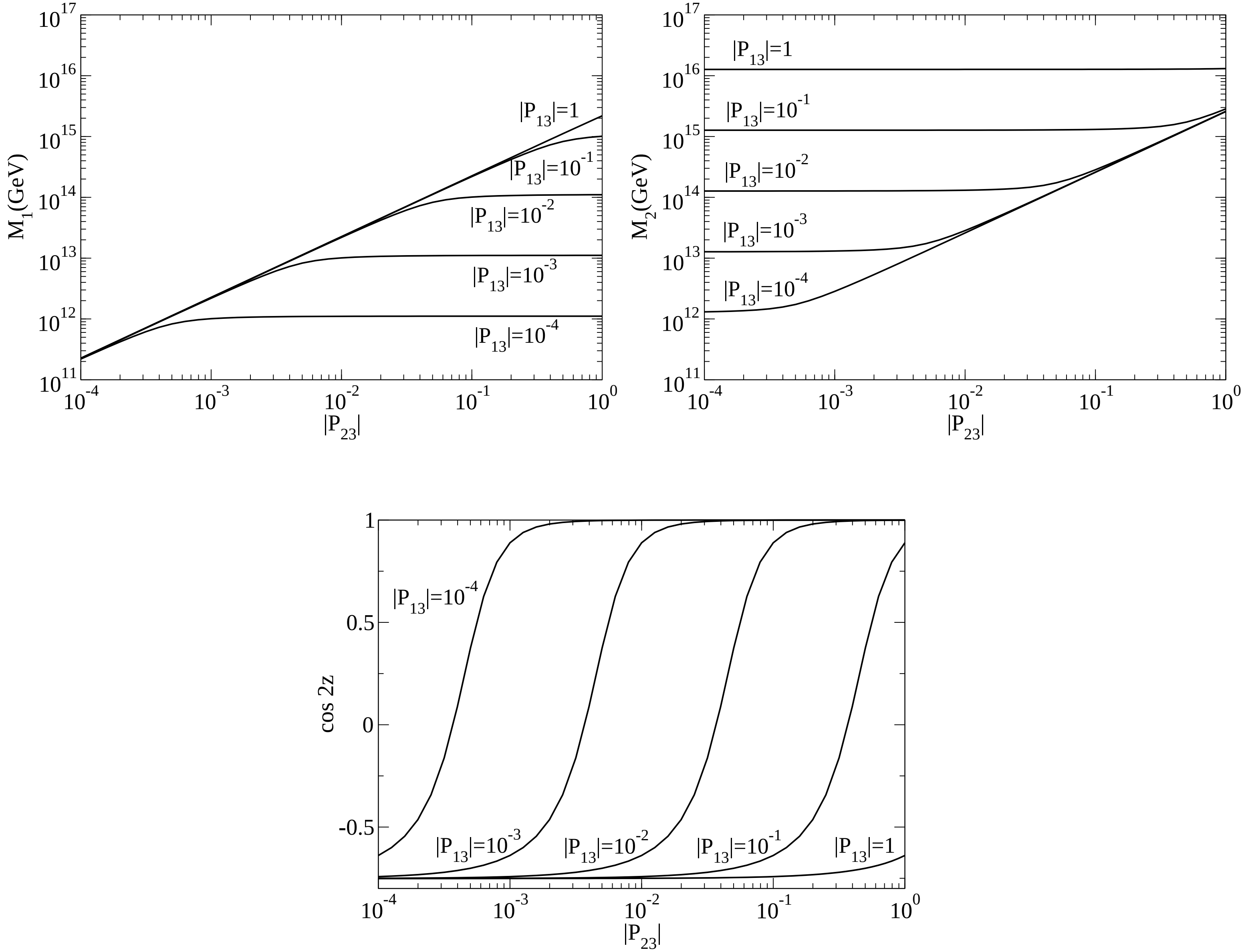
⋆ Is it feasible? In the 2RHN model

neutrino mass matrix, $\mathcal{M}_\nu$			radiative effects, $P \equiv \mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu$			
~~m_1~~	~~m_1~~	~~☺~~	~~P_{11}~~	mass splittings	~~largest~~	~~☺~~
m_2	Δm^2_{atm}	√	~~P_{22}~~		~~smallest~~	~~☹~~
m_3	Δm^2_{sol}	√	~~P_{33}~~	~~absolute scale~~		~~☹~~
θ_{12}	√		$\|P_{12}\|$		$\mu \longrightarrow e\gamma$	☺
θ_{13}	☺		$\|P_{13}\|$	rare decays	$\tau \longrightarrow e\gamma$	☺
θ_{23}	√		$\|P_{23}\|$		$\tau \longrightarrow \mu\gamma$	☺
δ		☺	$\arg P_{12}$	e-EDM	one of them	☺
ϕ	$\nu 0\beta\beta$ phase	☹	~~$\arg P_{13}$~~	~~μ-EDM~~	~~the other two~~	~~☹~~
~~ϕ'~~	~~orthogonal combination~~	~~☹ ☹~~	~~$\arg P_{23}$~~	~~τ-EDM~~		

An example of reconstruction: the case with $|P_{12}| \ll |P_{13}|, |P_{23}|$

★ Assume for simplicity that all the parameters are real and $\theta_{13} = 0$



- $\boxed{|P_{13}| \ll |P_{23}|} \Longrightarrow P \simeq |P_{23}| \begin{pmatrix} \lambda/\sqrt{6} & 0 & -\lambda \\ 0 & 1 & 1 \\ -\lambda & 1 & 1 \end{pmatrix}$, with $\lambda = \frac{|P_{13}|}{|P_{23}|} \ll 1$

$$M_1 \simeq 2\sqrt{\frac{2}{3}}\frac{|P_{13}|}{m_2}\langle H_u^0\rangle^2$$

$$M_2 \simeq \frac{2|P_{23}|}{m_3}\langle H_u^0\rangle^2$$

$$\mathbf{Y}_\nu \simeq \sqrt{|P_{23}|}\begin{pmatrix} \sqrt{\frac{|P_{13}|}{\sqrt{6}|P_{23}|}} & \sqrt{\sqrt{\frac{3}{8}}\frac{|P_{13}|}{|P_{23}|}} & -\sqrt{\sqrt{\frac{3}{8}}\frac{|P_{13}|}{|P_{23}|}} \\ -\frac{|P_{13}|}{2|P_{23}|} & 1 & 1 \end{pmatrix}$$

- $\boxed{|P_{23}| \ll |P_{13}|} \Longrightarrow P \simeq |P_{13}| \begin{pmatrix} 1/\sqrt{6} & 0 & -1 \\ 0 & \lambda & \lambda \\ -1 & \lambda & \sqrt{6} \end{pmatrix}$, with $\lambda = \frac{|P_{23}|}{|P_{13}|} \ll 1$

  – When the light neutrinos have the same CP parities ($\phi = 0$)

$$M_1 \simeq \frac{8|P_{23}|}{3m_2+4m_3} \langle H_u^0 \rangle^2$$

$$M_2 \simeq \frac{(3m_2+4m_3)|P_{13}|}{\sqrt{6}m_2 m_3} \langle H_u^0 \rangle^2$$

$$\mathbf{Y}_\nu \simeq \sqrt{\sqrt{6}|P_{13}|} \begin{pmatrix} \sqrt{\frac{\sqrt{6}|P_{23}|}{|P_{13}|}} \frac{m_2}{3m_2+4m_3} & \sqrt{\frac{|P_{23}|}{\sqrt{6}|P_{13}|}} & \sqrt{\frac{|P_{23}|}{\sqrt{6}|P_{13}|}} \frac{-3m_2+4m_3}{3m_2+4m_3} \\ -\frac{1}{\sqrt{6}} & \frac{\sqrt{6}m_2}{3m_2+4m_3} \frac{|P_{23}|}{|P_{13}|} & 1 \end{pmatrix}$$

  – When they have opposite parities ($\phi = \pi$)

$$M_1 \simeq \frac{8|P_{23}|}{-3m_2+4m_3} \langle H_u^0 \rangle^2$$

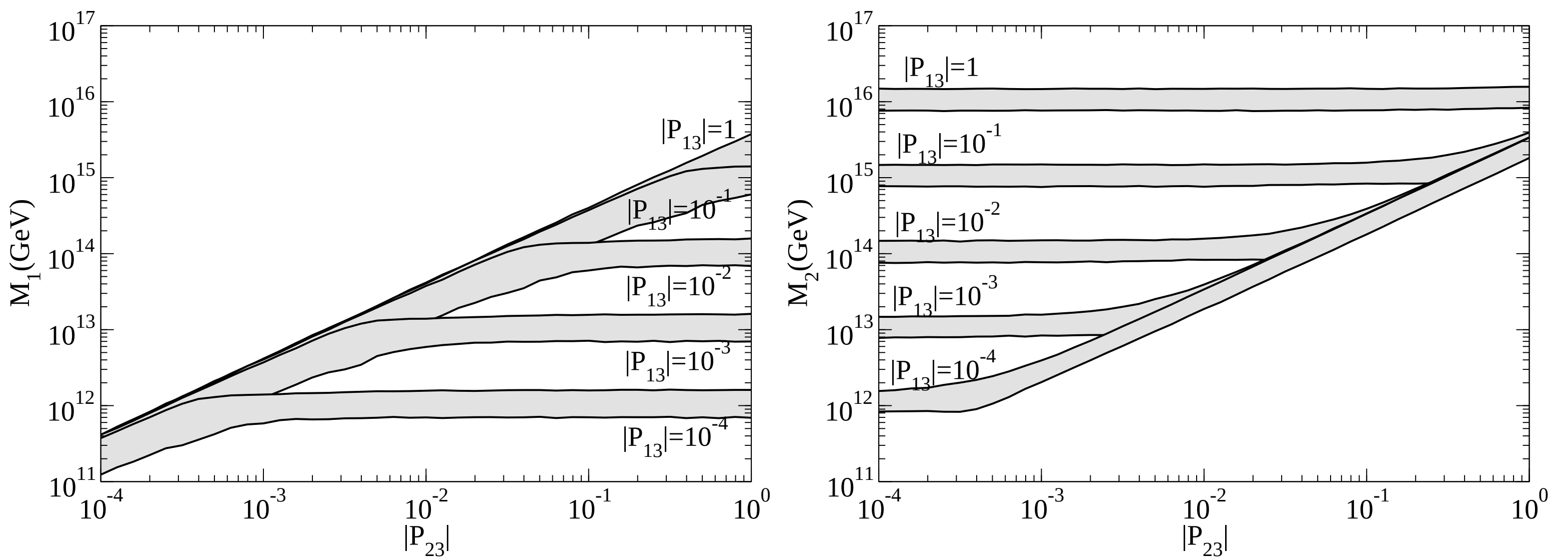
$$M_2 \simeq \frac{(-3m_2+4m_3)|P_{13}|}{\sqrt{6}m_2 m_3} \langle H_u^0 \rangle^2$$

$$\mathbf{Y}_\nu \simeq \sqrt{\sqrt{6}|P_{13}|} \begin{pmatrix} -\sqrt{\frac{\sqrt{6}|P_{23}|}{|P_{13}|}} \frac{m_2}{-3m_2+4m_3} & \sqrt{\frac{|P_{23}|}{\sqrt{6}|P_{13}|}} & \sqrt{\frac{|P_{23}|}{\sqrt{6}|P_{13}|}} \frac{3m_2+4m_3}{-3m_2+4m_3} \\ -\frac{i}{\sqrt{6}} & \frac{-i\sqrt{6}m_2}{-3m_2+4m_3} \frac{|P_{23}|}{|P_{13}|} & i \end{pmatrix}$$

⋆ Effect of a non-vanishing θ_{13}, δ and ϕ

The reconstruction of the Yukawa coupling requires to know δ and ϕ, but the order of magnitude of the right-handed masses could be estimated.

$\theta_{13} = 0.1$, δ,ϕ random



5- Conclusions

⋆ The reconstruction of the full see-saw model with three right-handed neutrinos is possible in theory, but extremely difficult in practice.

⋆ The model with two right-handed neutrinos is the **most simple see-saw scenario that can accommodate the observations**. Furthermore, there are interesting limits of the 3RHN model that resemble the 2RHN model.

⋆ In the 2RHN model, there are correlations among the elements of $Y_\nu^\dagger Y_\nu \longrightarrow$ they could give rise to correlations among slepton parameters.

⋆ It could be possible to reconstruct the Yukawa coupling of the 2RHN model in terms of low energy observables (but only after an enormous experimental effort!). On the other hand, the order of magnitude of the right-handed masses could be estimated if SUSY and rare decays are observed.