Reconstructing see-saw models from low energy data

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Based on works in collaboration with Alberto Casas, Sacha Davidson and Graham Ross

Flavour in the era of the LHC
CERN, Feb 2006
1-Motivation

Why reconstruct the see-saw? Why determining $Y_\nu$ and $M$?

- The effective theory provides a good description of neutrino observations. But we would like to have a deeper understanding of neutrino observations (why tiny masses? why two large angles?)

- The clues to unravel the flavour puzzle lie in the fundamental theory.
  - Look for patterns. In the quark sector, $m_u : m_c : m_t \sim \lambda^8 : \lambda^4 : 1$,
    $$m_d : m_s : m_b \sim \lambda^4 : \lambda^2 : 1,$$
    $\lambda$ being the Cabibbo angle.
  - Look for similarities with the charged lepton sector
  - Look for similarities with the quark sector (GUT symmetries?)

- Determine the scale of new physics (masses of the right-handed neutrinos).
  Implications for
  - GUTs
  - leptogenesis
2- Approaches to determine the see-saw parameters

★ **top-down:** Start with a concrete model (GUT, Froggatt-Nielsen, strings...) and compare the predictions with the experiments.

   **Many different possibilities.**

   Unfortunately, the simplest ideas do not seem to work... May be we are being mislead by theoretical prejudices?

★ **bottom-up:** Exploit all the information available at low energies on the leptonic sector, in order to reconstruct the high-energy theory.

   Completely phenomenological. Impossible to get mislead by aesthetics, but

   **very difficult in practice**
In the **Standard Model** it is **hopeless**

\[ \{ Y_\nu, \mathcal{M} \} \text{ depend on 18 parameters} \]

\[ \{ \mathcal{M}_\nu \} \text{ depends on 9 parameters} \]

Part of the information is lost in the decoupling process

In the **Minimal Supersymmetric Standard Model**, radiative corrections on slepton parameters provide additional information about the see-saw mechanism, through the combination \( Y_\nu^\dagger Y_\nu \)

Assume universality at \( M_X \):

\[
m_L^2(M_X) = m^2 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \rightarrow m_L^2(M_Z) \sim m^2 \begin{pmatrix} \times & \times & \times \\ \times & \times & \times \end{pmatrix}
\]

\[
16\pi^2 \frac{dm_L^2}{dt} = \left( m_L^2 Y_\nu^\dagger Y_\nu + Y_e^\dagger Y_e m_L^2 \right) + \left( m_L^2 Y_\nu^\dagger Y_\nu + Y_\nu^\dagger Y_\nu m_L^2 \right) + 2 \left( Y_e^\dagger m_e^2 Y_e + m_{H_1}^2 Y_e^\dagger Y_e + A_e^\dagger A_e \right) + 2 \left( Y_\nu^\dagger m_\nu^2 Y_\nu + m_{H_2}^2 Y_\nu^\dagger Y_\nu + A_\nu^\dagger A_\nu \right) - \left( \frac{6}{5} g_1^2 |M_1|^2 + 6 g_2^2 |M_2|^2 \right) I_3 - \frac{3}{5} g_1^2 S I_3
\]
In the leading-log approximation:

\[
(m_L^2)_{ij} \simeq \# I_3 - \frac{1}{8\pi^2}(3m_0^2 + A_0^2)(Y_\nu^\dagger Y_\nu)_{ij} \log \frac{M_X}{M}
\]

\[
\text{Is it possible to reconstruct the see-saw parameters with the information from } \mathcal{M} \text{ and } Y_\nu^\dagger Y_\nu? \, \text{YES}!!
\]

In the basis where \(\mathcal{M} = D_M = \text{diag}(M_1, M_2, M_3)\), the Yukawa coupling reads \(Y_\nu = V_R^\dagger \text{diag}(Y_1, Y_2, Y_3)V_L\).

Then, the reconstruction follows in two steps:

- \(Y_\nu^\dagger Y_\nu = V_L^\dagger \text{diag}(Y_1^2, Y_2^2, Y_3^2)V_L\)
  
  From here we extract \(V_L\) and \(\text{diag}(Y_1, Y_2, Y_3)\)

- \(\mathcal{M}_\nu = Y_\nu^T \mathcal{M}^{-1} Y_\nu \langle H_u^0 \rangle^2 = V_L^\dagger D_Y V_R^* D_M^{-1} V_R^\dagger D_Y V_L\)
  \[
  \frac{1}{\langle H_u^0 \rangle^2} D_Y^{-1} V_L \mathcal{M}_\nu V_L^\dagger D_Y^{-1} = V_R^* D_M^{-1} V^\dagger
  \]

  From here we extract \(V_R\) and \(D_M\)

\textbf{We have everything}!! However, in practice some of the low energy parameters could be very hard to measure (if not impossible)
<table>
<thead>
<tr>
<th>Neutrino mass matrix, $\mathcal{M}_\nu$</th>
<th>Radiative effects, $P \equiv Y^\dagger_\nu Y_\nu$</th>
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</tr>
<tr>
<td>$m_2$ $\Delta m^2_{atm}$ ✓</td>
<td>$P_{22}$ smallest</td>
</tr>
<tr>
<td>$m_3$ $\Delta m^2_{sol}$ ✓</td>
<td>$P_{33}$ absolute scale</td>
</tr>
<tr>
<td>$\theta_{12}$ ✓</td>
<td>$</td>
</tr>
<tr>
<td>$\theta_{13}$ 😂</td>
<td>$</td>
</tr>
<tr>
<td>$\theta_{23}$ ✓</td>
<td>$</td>
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</tr>
<tr>
<td>$\phi$ $\nu 0\beta\beta$ phase 😂</td>
<td>arg$P_{13}$ $\mu$-EDM the other two</td>
</tr>
<tr>
<td>$\phi'$ orthogonal combination 😂</td>
<td>arg$P_{23}$ $\tau$-EDM</td>
</tr>
</tbody>
</table>

Notes:
- 😂 indicates a problematic parameter.
- 😊 indicates a well-behaved parameter.
- 😐 indicates a parameter that is neither problematic nor well-behaved.
An interesting implication of this procedure:

There is a one to one correspondence between low energy observables and see-saw parameters: \( \{ \mathbf{Y}_\nu, \mathcal{M} \} \leftrightarrow \{ \mathcal{M}_\nu, \mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu \} \)

- \( \{ \mathbf{Y}_\nu, \mathcal{M} \} \) depend on 18 parameters: 12 real, 6 phases
- \( \{ \mathcal{M}_\nu, \mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu \} \) depend on 18 parameters: 12 real, 6 phases

★ Positive lesson: We can use this trick as an alternative parametrization of the see-saw mechanism, using only low energy observables.

★ Negative lesson: For any set of observations, there is a see-saw scenario that accommodates them. The see-saw cannot be ruled out!!
3- A less radical (and more practical) approach

Both the top-down and the bottom-up approaches are interesting, but have limitations ——> try a hybrid approach:

**bottom-up approach with some well-motivated hypotheses about the high-energy theory**

- GUT inspired \( Y_\nu \) symmetric
  \( Y_\nu \) eigenvalues as \( m_u, m_c, m_t \)  
  Akhmedov, Frigerio, Smirnov

- texture zeros. Inspired by Gatto-Sartori-Tonin relation, A.I., Ross
  \[
  \sin \theta_c \simeq \left| \sqrt{\frac{m_d}{m_s}} - e^{i\delta} \sqrt{\frac{m_u}{m_c}} \right|
  \]
  Kleppe; Ma, Roy, Sarkar; Frampton, Glashow, Yanagida; Raidal, Strumia; Gonzalez-Felipe, Joaquim, Nobre; Raby Appelquist, Shrock; Endoh, Kaneko, Kang, Morozumi, Tanimoto Smirnov; King, Ross; Rodejohann; Dreiner, Murayama, Thormeier A.I., Ross; Altarelli, Feruglio, Masina...

- **two right-handed neutrino (2RHN) model**

Observations tell us that at least two new mass scales have to be introduced \( (\Delta m_{sol}^2, \Delta m_{atm}^2) \). Two right-handed neutrinos can do the job.
**Further motivation**: there are some interesting situations where the 3RHN model can be well described by the 2RHN model. The mass matrix is:

$$
M_{ij} = \frac{y_{1i}y_{1j}}{M_1} + \frac{y_{2i}y_{2j}}{M_2} + \frac{y_{3i}y_{3j}}{M_3}, \text{ where } y_{ij} = (Y_\nu)_{ij}.
$$

Two RH neutrinos dominate when

- $\frac{y_{1i}y_{1j}}{M_1} \ll \frac{y_{2i}y_{2j}}{M_2}, \frac{y_{3i}y_{3j}}{M_3}$  
  
  It occurs when $y_{1i} \ll y_{2i}, y_{3i}$. In this case,

$$
M_{ij} \simeq \frac{y_{2i}y_{2j}}{M_2} + \frac{y_{3i}y_{3j}}{M_3} \quad \rightarrow \quad \text{2RHN model!!}
$$

$$
(Y_\nu^\dagger Y_\nu)_{ij} \simeq y_{2i}^*y_{2j} + y_{3i}^*y_{3j}
$$

- $\frac{y_{2i}y_{2j}}{M_2} \ll \frac{y_{1i}y_{1j}}{M_1}, \frac{y_{3i}y_{3j}}{M_3}$  
  
  Not very interesting...

- $\frac{y_{3i}y_{3j}}{M_3} \ll \frac{y_{1i}y_{1j}}{M_1}, \frac{y_{2i}y_{2j}}{M_2}$  
  
  It occurs when $M_3 \gg M_1, M_2$

But it could happen that $(Y_\nu^\dagger Y_\nu)_{ij} = y_{1i}^*y_{1j} + y_{2i}^*y_{2j} + y_{3i}^*y_{3j}$. **Not 2RHN!**

In GMSB models, with messenger masses between $M_2$ and $M_3$,

$$
M_{ij} \simeq \frac{y_{1i}y_{1j}}{M_1} + \frac{y_{2i}y_{2j}}{M_2}
$$

$$
(Y_\nu^\dagger Y_\nu)_{ij} = y_{1i}^*y_{1j} + y_{2i}^*y_{2j} \quad \rightarrow \quad \text{2RHN model!!}
$$
**Parameter counting.** In the basis where the RH mass matrix is diagonal,

\[ \mathcal{M} = D \mathcal{M} \rightarrow 2 \text{ real parameters} \]

\[ Y_\nu \text{ is a } 2 \times 3 \text{ matrix } \rightarrow 6 \text{ real parameters, 3 phases} \]

\[ \{Y_\nu, \mathcal{M}\} \text{ depend on } 11 \text{ parameters} \]

\[ \begin{array}{c}
8 \text{ real} \\
3 \text{ phases}
\end{array} \]

\[ \{\mathcal{M}_\nu, Y^\dagger_\nu Y_\nu\} \text{ depend on } 18 \text{ parameters} \]

\[ \begin{array}{c}
12 \text{ real} \\
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\end{array} \]

**Many predictions!!**

★ **Predictions on the neutrino mass matrix**

- \( m_1 = 0 \) \( (\Rightarrow m_3 = \sqrt{\Delta m^2_{atm}}, \ m_2 = \sqrt{\Delta m^2_{sol}}) \)

- there is only one phase difference between the mass eigenvalues (so, there is only one Majorana phase)

- (still three mixing angles, and a Dirac phase)
\* Predictions on the radiative effects

We want to incorporate in these predictions the information from the neutrino mass matrix. Working in the basis where \( \mathcal{M} = \text{diag}(M_1, M_2) \), it can be checked that

\[
\mathbf{Y}_\nu = D \sqrt{\mathcal{M}} R D \sqrt{m} U^\dagger / \langle H_u^0 \rangle
\]

is the most general Yukawa coupling that satisfies

\[
\mathcal{M}_\nu = Y^T_\nu \text{diag}(M_1^{-1}, M_2^{-1}) Y_\nu \langle H_u^0 \rangle^2
\]

Here, \( R \) is an orthogonal complex matrix

\[
R = \begin{pmatrix}
0 & \cos z & \sin z \\
0 & -\sin z & \cos z
\end{pmatrix}
\]

So, the independent parameters in \( \mathbf{Y}_\nu \) are \( M_1, M_2 \) and \( z \) (three real and one phase). On the other hand, \( Y^\dagger_\nu Y_\nu \) depends in general on 6 real parameters and three phases. We can make predictions on three real parameters and two phases in \( Y^\dagger_\nu Y_\nu \).
To compute the predictions, we construct the matrix

\[ U^\dagger Y_\nu^\dagger Y_\nu U = D \sqrt{m} R^\dagger D M RD \sqrt{m}/\langle H^0 \rangle^2 \]

since \( m_1 = 0 \),

\[ (U^\dagger Y_\nu^\dagger Y_\nu U)_{1i} = 0 \quad \text{for} \quad i = 1, 2, 3. \]

\textbf{five equations} \( \rightarrow \) \textbf{five relations/predictions}.

\( \star \) Still three independent real parameters and one phase. We choose as independent parameters \( P_{12}, |P_{13}|, |P_{23}| \) (\( P \equiv Y_\nu^\dagger Y_\nu \)). The predictions are:

\[ P_{11} = - \frac{P_{12}^* U_{21}^* + P_{13}^* U_{31}^*}{U_{11}^*} \]

\[ P_{22} = - \frac{P_{12}^* U_{11}^* + P_{23}^* U_{31}^*}{U_{21}^*} \]

\[ P_{33} = - \frac{P_{13}^* U_{11}^* + P_{23}^* U_{21}^*}{U_{31}^*}. \]

\[ e^{i \arg P_{13}} = -i \frac{\text{Im}(P_{12} U_{21} U_{11}^*) \pm \sqrt{|P_{13}|^2 |U_{11}|^2 |U_{31}|^2 - [\text{Im}(P_{12} U_{21} U_{11}^*)]^2}}{|P_{13} U_{31} U_{11}^*|} \]

\[ e^{i \arg P_{23}} = i \frac{\text{Im}(P_{12} U_{21} U_{11}^*) \pm \sqrt{|P_{23}|^2 |U_{21}|^2 |U_{31}|^2 - [\text{Im}(P_{12} U_{21} U_{11}^*)]^2}}{|P_{23} U_{31} U_{21}^*|} \]
4- Reconstructing the 2RHN model

The 2RHN model can be parametrized in terms of

\( \{ Y_\nu, M \} \) depend on 11 parameters 8 real 3 phases

\[ \begin{aligned}
 & \left\{ m_2, m_3, \\
 & \theta_{12}, \theta_{13}, \theta_{23}, \\
 & \delta, \phi, \\
 & |P_{12}|, |P_{13}|, |P_{23}| \\
 & \text{arg} P_{12} \right\} \end{aligned} \]

depend on 11 parameters 8 real 3 phases

It is possible to derive exact expressions for the high-energy parameters in terms of the low-energy parameters:

\[
M_1 = \frac{1}{2} \left[ \sqrt{\left( \frac{Q_{33}}{m_3} + \frac{Q_{22}}{m_2} \right)^2 + \frac{(Q_{23} - Q_{23}^*)^2}{m_2 m_3}} - \sqrt{\left( \frac{Q_{33}}{m_3} - \frac{Q_{22}}{m_2} \right)^2 + \frac{(Q_{23} + Q_{23}^*)^2}{m_2 m_3}} \right] \langle H_u^0 \rangle^2
\]

\[
M_2 = \frac{1}{2} \left[ \sqrt{\left( \frac{Q_{33}}{m_3} + \frac{Q_{22}}{m_2} \right)^2 + \frac{(Q_{23} - Q_{23}^*)^2}{m_2 m_3}} + \sqrt{\left( \frac{Q_{33}}{m_3} - \frac{Q_{22}}{m_2} \right)^2 + \frac{(Q_{23} + Q_{23}^*)^2}{m_2 m_3}} \right] \langle H_u^0 \rangle^2
\]

\[
\cos 2z = \left( \frac{Q_{33}^2}{m_3} - \frac{Q_{22}^2}{m_2} + \frac{(Q_{23} + Q_{23}^*)(Q_{23} - Q_{23}^*)}{m_2 m_3} \right) \frac{\langle H_u^0 \rangle^4}{M_2^2 - M_1^2}
\]

where \( Q = U^\dagger PU = U^\dagger Y_\nu^\dagger Y_\nu U \).

Finally, the Yukawa coupling can be reconstructed \( Y_\nu = D \sqrt{M} R D \sqrt{M} U^\dagger / \langle H_u^0 \rangle \)
★ Is it feasible? In the 3RHN model

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An example of reconstruction: the case with $|P_{12}| \ll |P_{13}|, |P_{23}|$

Assume for simplicity that all the parameters are real and $\theta_{13} = 0$
\[ |P_{13}| \ll |P_{23}| \implies P \simeq |P_{23}| \begin{pmatrix} \lambda/\sqrt{6} & 0 & -\lambda \\ 0 & 1 & 1 \\ -\lambda & 1 & 1 \end{pmatrix}, \text{ with } \lambda = \frac{|P_{13}|}{|P_{23}|} \ll 1 \]

\[ M_1 \simeq 2 \sqrt{\frac{2}{3} \frac{|P_{13}|}{m_2} \langle H_u^0 \rangle^2} \]

\[ M_2 \simeq \frac{2|P_{23}|}{m_3} \langle H_u^0 \rangle^2 \]

\[ Y_\nu \simeq \sqrt{|P_{23}|} \begin{pmatrix} \sqrt{\frac{|P_{13}|}{\sqrt{6}|P_{23}|}} & \sqrt{\frac{3}{8} \frac{|P_{13}|}{|P_{23}|}} & -\sqrt{\frac{3}{8} \frac{|P_{13}|}{|P_{23}|}} \\ \sqrt{\frac{|P_{13}|}{2|P_{23}|}} & 1 & 1 \end{pmatrix} \]
\[ P \simeq |P_{13}| \begin{pmatrix} 1/\sqrt{6} & 0 & -1 \\ 0 & \lambda & \lambda \\ -1 & \lambda & \sqrt{6} \end{pmatrix}, \text{ with } \lambda = \frac{|P_{23}|}{|P_{13}|} \ll 1 \]

- When the light neutrinos have the same CP parities \( (\phi = 0) \)

\[
M_1 \simeq \frac{8|P_{23}|}{3m_2+4m_3} \langle H_u^0 \rangle^2 \\
M_2 \simeq \frac{(3m_2+4m_3)|P_{13}|}{\sqrt{6}m_2m_3} \langle H_u^0 \rangle^2
\]

\[
Y_\nu \simeq \sqrt{6}|P_{13}| \begin{pmatrix} \sqrt{\frac{\sqrt{6}|P_{23}|}{|P_{13}|} m_2} & \sqrt{\frac{|P_{23}|}{\sqrt{6}|P_{13}|}} & \sqrt{\frac{|P_{23}|}{\sqrt{6}|P_{13}|}} \\ -\frac{1}{\sqrt{6}} \sqrt{\frac{6m_2}{3m_2+4m_3}} & \frac{m_2}{3m_2+4m_3} |P_{23}| & \frac{-3m_2+4m_3}{3m_2+4m_3} \end{pmatrix}
\]

- When they have opposite parities \( (\phi = \pi) \)

\[
M_1 \simeq \frac{8|P_{23}|}{-3m_2+4m_3} \langle H_u^0 \rangle^2 \\
M_2 \simeq \frac{(-3m_2+4m_3)|P_{13}|}{\sqrt{6}m_2m_3} \langle H_u^0 \rangle^2
\]

\[
Y_\nu \simeq \sqrt{6}|P_{13}| \begin{pmatrix} -\sqrt{\frac{\sqrt{6}|P_{23}|}{|P_{13}|} m_2} & \sqrt{\frac{|P_{23}|}{\sqrt{6}|P_{13}|}} & \sqrt{\frac{|P_{23}|}{\sqrt{6}|P_{13}|}} \\ -\frac{i}{\sqrt{6}} \frac{-i\sqrt{6m_2}}{-3m_2+4m_3} & \frac{m_2}{-3m_2+4m_3} |P_{23}| & \frac{3m_2+4m_3}{-3m_2+4m_3} \end{pmatrix}
\]
★ Effect of a non-vanishing $\theta_{13}$, $\delta$ and $\phi$

The reconstruction of the Yukawa coupling requires to know $\delta$ and $\phi$, but the order of magnitude of the right-handed masses could be estimated.

$$\theta_{13} = 0.1, \delta, \phi \text{ random}$$
5- Conclusions

★ The reconstruction of the full see-saw model with three right-handed neutrinos is possible in theory, but extremely difficult in practice.

★ The model with two right-handed neutrinos is the **most simple see-saw scenario that can accommodate the observations**. Furthermore, there are interesting limits of the 3RHN model that resemble the 2RHN model.

★ In the 2RHN model, there are correlations among the elements of $Y_\nu^\dagger Y_\nu$ → they could give rise to correlations among slepton parameters.

★ It could be possible to reconstruct the Yukawa coupling of the 2RHN model in terms of low energy observables **(but only after an enormous experimental effort!)**. On the other hand, the order of magnitude of the right-handed masses could be estimated if SUSY and rare decays are observed.