The see-saw mechanism, neutrino Yukawa couplings, LFV decays and leptogenesis *

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Overview:

- *MSSM + right-handed Majorana neutrinos (MSSMRN)*
- Ansätze
- Lepton flavour violating processes in MSSMRN
- Parameterisation of Dirac Yukawa (def. of CP phases)
- Renormalisation group evolution effects
- Results
- Lepton flavour violation decays and thermal leptogenesis
- Conclusion

MSSM + right-handed Majorana Neutrinos

The superpotential with three right-handed Majorana neutrinos:

\[ \mathcal{W} = \sum_{i,j=1,2,3} (Y_e)_{ij} \epsilon_{\alpha\beta} H_d^\alpha E_i^c L_j^\beta + (Y_d)_{ij} \epsilon_{\alpha\beta} H_d^\alpha D_i^c Q_j^\beta \]

\[ + (Y_u)_{ij} \epsilon_{\alpha\beta} H_u^\alpha U_i^c Q_j^\beta + \mu \epsilon_{\alpha\beta} H_1^\alpha H_2^\beta \]

\[ + (Y_\nu)_{ij} \epsilon_{\alpha\beta} H_u^\alpha N_i^c L_j^\beta + \frac{1}{2} (M_R)_{ij} N_i^c N_j^c, \]

where

- \( L_i \): the chiral multiplet of a \( SU(2)_L \) doublet lepton
- \( E_i^c \): a \( SU(2)_L \) singlet charged lepton
- \( N_i^c \) a right-handed Majorana neutrino
- \( H_d \) and \( H_u \) two Higgs doublets with opposite hypercharge
- \( Q, U \) and \( D \) chiral multiplets of quarks of a \( SU(2)_L \) doublet and two singlets with different \( U(1)_Y \) charges.
- \( \epsilon_{\alpha\beta} \) is an anti-symmetric tensor with \( \epsilon_{12} = 1 \).
Using see-saw mechanism (ignoring left-handed Majorana term) effective neutrino mass matrix becomes:

\[
M_{\text{eff}} \approx M_\nu^T M_{R}^{-1} M_\nu
\]

\[
= \nu_u^2 \sin^2 \beta Y_\nu^T M_{R}^{-1} Y_\nu
\]

This mass matrix is diagonalised by a single unitary matrix by

\[
M_{\text{eff}}^{\text{diag}} = \text{diag}(m_1, m_2, m_3)
\]

\[
= U_{\text{eff}}^T M_{\text{eff}} U_{\text{eff}}
\]

\[
U_{\text{MNS}} = \begin{pmatrix}
    c_{13}c_{12} & c_{13}s_{12} & s_{13}e^{-i\delta} \\
    -c_{23}s_{12} - s_{13}s_{23}c_{12}e^{i\delta} & c_{23}c_{12} - s_{13}s_{23}s_{12}e^{i\delta} & c_{13}s_{23} \\
    s_{23}s_{12} - s_{13}c_{23}c_{12}e^{i\delta} & -s_{23}c_{12} - s_{13}c_{23}s_{12}e^{i\delta} & c_{13}c_{23}
\end{pmatrix} \times \begin{pmatrix}
    1 & 0 & 0 \\
    0 & e^{i\alpha} & 0 \\
    0 & 0 & e^{i\beta_M/2}
\end{pmatrix},
\]

where \(c_{ij} = \cos \theta_{ij}, s_{ij} = \sin \theta_{ij}\), the angles \(\theta_{ij} = [0, \pi/2]\), \(\delta = [0, 2\pi]\) is the Dirac CP-violating phase and \(\alpha\) and \(\beta_M\) are two Majorana CP-violation phases.

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Our Assumptions

- **Gauge unification at GUT scale** ($\sim 2.3 \times 10^{16}$ GeV)
  \[ g_1 = g_2 = g_3 \approx 0.71 \]
- All fermion masses and mixing angles are positive defined quantities. (Values from PDG)
- $\Delta m_{21}^2 = 8.0 \times 10^{-5} \text{eV}^2$, $\sin^2 \theta_{12} = 0.31$ at $M_Z$.
- $|\Delta m_{31}^2| = 2.2 \times 10^{-3} \text{eV}^2$, $\sin^2 2\theta_{23} = 1.0$ at $M_Z$.
- The first- and third-generation mixing angle in the neutrino sector at $M_Z$: $\tan \theta_{13} = 0.0$.
- **soft SUSY breaking**
- **mSUGRA at GUT scale**
  - Gauginos: $M_1 = M_2 = M_3$
  - Scalars: $(m_L^2)_{ij}$ etc. $= \delta_{ij} m_0^2$ and $m_{H_u}^2 = m_{H_d}^2 = m_0^2$
- $Y_d$ and $Y_e$ are diagonal at GUT scale
Lepton Flavour Violation processes

The soft SUSY breaking Lagrangian – lepton part – is a source of lepton flavour violation

\[
-L_{\text{soft}} = (m_{\tilde{e}}^2)_{ij} \tilde{\nu}_R^* \tilde{\nu}_R + (m_{\tilde{\nu}}^2)_{ij} \tilde{\nu}_R^* \tilde{\nu}_R
+ \left( (A_e)_{ij} H_d \tilde{\nu}_R^* \tilde{L}_R + (A_{\nu})_{ij} H_u \tilde{\nu}_R^* \tilde{L}_R \right) + h.c.
\]

We assume that soft SUSY breaking matrices are diagonal and universal in flavour at GUT scale, but they are affected by renormalisation via Yukawa and gauge interactions → the LFV in the Yukawa couplings will induce LFV in the slepton mass matrix at low energy.

The Renormaisation equation for the left-handed slepton mass matrix is

\[
\mu \frac{d}{d\mu} \left( m_{\tilde{L}}^2 \right)_{ji} = \mu \frac{d}{d\mu} \left( m_{\tilde{L}}^2 \right)_{ji} \bigg|_{\text{MSSM}}
+ \frac{1}{16\pi^2} \left[ \left( m_{\tilde{\nu}}^2 Y_\nu^\dagger Y_\nu + Y_\nu^\dagger Y_\nu m_{\tilde{L}}^2 \right)_{ji}
+ 2 \left( Y_\nu^\dagger m_{\tilde{\nu}}^2 Y_\nu + \tilde{m}_{\tilde{\nu}}^2 Y_\nu^\dagger Y_\nu + A_\nu^\dagger A_\nu \right)_{ji} \right]
\]

where the first term does not violate the lepton flavour, while the second one does.
(a) slepton-neutralino interaction
(b) sneutrino-chargino interaction

An approximation for the additional contribution to the off-diagonal element of the slepton mass matrix:

\[
(\Delta m^2_L)^{ji} \approx -\frac{\ln(M_{\text{GUT}}/M_R)}{16\pi^2} \left(6m_0^2 + 2A_0^2\right) \left(Y^\dagger Y\right)^{ji} \tan^2 \beta
\]

Note: These calculations have been performed without employing the reading-log and the mass insertion approximations., i.e., full one-loop calculations.
Parametrisation of Dirac neutrion Yukawa

We parametrise the Dirac neutrino Yukawa coupling constant using a complex orthogonal matrix, \( R \) which satisfies \( R^\dagger R = 1 \)

\[
Y_\nu(M_R) = \frac{1}{v_u} \sqrt{D_N(M_R)} \; R \; \sqrt{D_\nu(M_R)} \; U^\dagger(M_R) ,
\]

in the \( Y_e \) and \( Y_\nu \) diagonal basis. Here

\[
\sqrt{M_R} = \text{diag}(\sqrt{M_1}, \sqrt{M_2}, \sqrt{M_3}) , \quad \sqrt{D_\nu} = \text{diag}(\sqrt{m_1}, \sqrt{m_2}, \sqrt{m_3}) .
\]

We use in our study \( R = O e^{iA} \), where \( O \) is a real orthogonal matrix, and \( A \) is a real antisymmetric matrix, \( A^\dagger = -A \), that is

\[
A = \begin{pmatrix}
0 & a & b \\
-a & 0 & c \\
-b & -c & 0 \\
\end{pmatrix},
\]

where \( a, b, c \) are real parameters.

Note: \( Y_\nu \) is expressed by an “artificial” orthogonal matrix \( R \) which is defined at the scale of \( M_R \), therefore all quantities – \( \nu \)-masses, \( \nu \)-mixing angles – should evaluate in that scale with help of renormalisation group equations.
The renormalisation group evolution effects

For simplicity we have consider further assumptions.

- **Neutrino mass spectrum is normal hierarchical** $m_1 < m_2 < m_3$
- **The right-handed Majorana Yukawa coupling constants are one at GUT scale**: degenerate $M_R$: $M_{R_1} = M_{R_2} = M_{R_3}$.

We summarise the important results here (roughly speaking...):

- The corrections can become significant for relatively large $\tan \beta$.
- The mixing angles are stable for **normal hierarchical** neutrino mass spectrum.
- For $m_1 \gtrsim 0.05$ eV, the RG running effects can change significantly the mixing angles. The corrections are relatively small even for large $\tan \beta$ if $m_1 \lesssim 0.05$ eV.
- The neutrino mixing angles depend strongly on the Majorana phase $\alpha$.
- The RG running effects depend weakly on the Majorana phase $\beta_M$.
- Mass squared differences depend strongly on $m_1$ in the interval $m_1 \gtrsim 0.05$ eV, and on $\alpha$ for $m_1 \gtrsim 0.10$ eV.
The dependence of $\tan^2 \theta_{12}$, $\tan^2 \theta_{23}$ and of $\sin \theta_{13}$, evaluated at $M_R$, on the lightest neutrino mass $m_1$ for $\tan \beta = 10$.

$M_R$ is fixed as $M_R = 2 \times 10^{13}$ GeV.
The dependence of $\tan^2 \theta_{12}$, $\tan^2 \theta_{23}$ and of $\sin \theta_{13}$, evaluated at $M_R$, on the lightest neutrino mass $m_1$ for $\tan \beta = 50$. $M_R$ is fixed as $M_R = 2 \times 10^{13}$ GeV.
The light neutrino mass differences at the scale $M_R$ as a functions of $m_1$
for $\alpha = 0, \pi$ and $\tan \beta = 10, 50$:

(a) $\Delta m^2_A(M_R) \equiv m^2_3(M_R) - m^2_1(M_R)$
(b) $\Delta m^2_\od(M_R) \equiv m^2_2(M_R) - m^2_1(M_R)$. 

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The branching ratios of the LFV decay $\mu \rightarrow e + \gamma$ versus $m_1$ in the case of $R = 1$ for $\alpha = 0, \pi/2, \pi$. $\tan \beta = 10$ (left) and $\tan \beta = 50$ (right).
The branching ratios of the LFV decay \( \mu \to e + \gamma \) versus \( m_1 \) in the case of complex matrix \( R \neq 1 \) for \( \alpha = 0, \pi/2, \pi \). \( \tan \beta = 10 \) (left) and \( \tan \beta = 50 \) (right). The three constants \( a, b, c \) are assumed to be \(-0.1 \leq a, b, c \leq 0.1\).
Lepton flavour violation decays and Leptogenesis

The cosmological aspect of the connection see-saw mechanism and low energy neutrino physics; baryon asymmetry of the Universe via the leptogenesis mechanism.

\[ Y_B = (6.15 \pm 0.25) \times 10^{-10}, \]

The right-handed Neutrino masses are \( M_1 \ll M_2 \ll M_3 \), (natural!) the baryon asymmetry is given by

\[ Y_B \simeq -10^{-2} \kappa \epsilon_1, \]

where \( \epsilon_1 \) is the CP-violating asymmetry in the decay of \( N_1 \) (mass \( M_1 \)), and \( \kappa \) is a dilution factor.

The CP-violating decay asymmetry \( \epsilon_1 \):

\[ \epsilon_1 \simeq \frac{1}{8\pi (Y_{\nu}Y_{\nu}^\dagger)_{11}} \sum_{i=2,3} \text{Im} \left[ \left\{ (Y_{\nu}Y_{\nu}^\dagger)_{1i} \right\}^2 \right] f(\chi_i), \]

where \( \chi_i = M_i^2/M_1^2 \) and \( f(\chi_i) \) is defined by

\[ f(\chi_i) \equiv -\sqrt{\chi_i} \ln \left( 1 + \frac{1}{\chi_i} \right) - \frac{2\sqrt{\chi_i}}{\chi_i - 1}. \]
The correlation between the predicted baryon asymmetry $Y_B$ and $\text{Br}(\mu \rightarrow e + \gamma)$ in the case of inverted hierarchical light neutrino mass spectrum and for $M_1 = 7.0 \times 10^{12}$ GeV and $M_2 = 4.0 \times 10^{13}$ GeV. The values of SUSY parameters are $m_0 = m_{1/2} = 250$ GeV, $A_0 = -100$ GeV and $\tan \beta = 5$. 
The correlation between the predicted baryon asymmetry $Y_B$ and $\text{Br}(\mu \rightarrow e + \gamma)$ in the case of quasi-degenerate light neutrino mass spectrum and $M_1 = 3.0 \times 10^{13}$ GeV, $M_2 = 1.2 \times 10^{14}$ GeV, $M_2 = 4.8 \times 10^{14}$ GeV. The values of SUSY parameters are $m_0 = 300$ GeV, $m_{1/2} = 1400$ GeV, $A_0 = 0$ and $\tan \beta = 5$. 

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Conclusion

- **We have calculated lepton flavour violating processes using full RGEs including non-renormalisable RGE for the $M_{\text{eff}}$ under many assumptions (too much restrictive!)**

- **We considered the effects of Majorana CP-violation phases in $U_{\text{MNS}}$, and of the RG “running” of light neutrino masses and mixing angles from $M_Z$ to the right-handed Majorana neutrino mass scale $M_R$. These quantities can run but not so significantly (some of cases, we should say a lot.)**

- **Furthermore, we find that the effects of the CP-violation phases in the Dirac neutrino Yukawa coupling can change by few orders of magnitude the predicted rates of the LFV decays $\mu \rightarrow e + \gamma$ etc. So we really need very careful treatments for phases!**
Satisfying the combined constraints from the existing upper limit on \( \text{Br}(\mu \rightarrow e + \gamma) \) etc. and the requirement of successful thermal leptogenesis proves to be a powerful tool to test the viability of supersymmetric theories with see-saw mechanism and soft flavour-universal SUSY breaking scenario.