The see-saw mechanism, neutrino Yukawa couplings, LFV decays and leptogenesis *

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Overview:

- *MSSM + right-handed Majorana neutrinos (MSSMRN)*
- *Ansätze*
- *Lepton flavour violating processes in MSSMRN*
- *Parameterisation of Dirac Yukawa (def. of CP phases)*
- *Renormalisation group evolution effects*
- *Results*
- *Lepton flavour violation decays and thermal leptogenesis*
- *Conclusion*

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*S. T. Petcov, T. Shindou, Y. T., Nucl. Phys. B **738** (2006) 219 [arXiv:hep-ph/0508243];
S. T. Petcov, W. Rodejohann, T. Shindou, Y. T., arXiv:hep-ph/0510404, to be published in Nucl. Phys. B

MSSM + right-handed Majorana Neutrinos

The superpotential with three right-handed Majorana neutrinos:

$$
\begin{aligned}
\mathscr{W} &= \sum_{i,j=1,2,3} (Y_e)_{ij}\epsilon_{\alpha\beta}H_d^\alpha E_i^c L_j^\beta + (Y_d)_{ij}\epsilon_{\alpha\beta}H_d^\alpha D_i^c Q_j^\beta \\
&\quad + (Y_u)_{ij}\epsilon_{\alpha\beta}H_u^\alpha U_i^c Q_j^\beta + \mu\epsilon_{\alpha\beta}H_1^\alpha H_2^\beta \\
&\quad + (Y_\nu)_{ij}\epsilon_{\alpha\beta}H_u^\alpha N_i^c L_j^\beta + \frac{1}{2}(M_R)_{ij}N_i^c N_j^c \,,
\end{aligned}
$$

where

- L_i: *the chiral multiplet of a* $SU(2)_L$ *doublet lepton*
- E_i^c: *a* $SU(2)_L$ *singlet charged lepton*
- N_i^c *a right-handed Majorana neutrino*
- H_d *and* H_u *two Higgs doublets with opposite hypercharge*
- Q, U *and* D *chiral multiplets of quarks of a* $SU(2)_L$ *doublet and two singlets with different* $U(1)_Y$ *charges.*
- $\epsilon_{\alpha\beta}$ *is an anti-symmetric tensor with* $\epsilon_{12} = 1$.

Using see-saw mechanism (ignoring left-handed Majorana term) effective neutrino mass matrix becomes:

$$\begin{aligned} M_{\mathrm{eff}} &\approx M_\nu^{\mathsf{T}}\, M_R^{-1}\, M_\nu \\ &= v_u^2 \sin^2\beta\; {Y_\nu}^{\mathsf{T}} M_R^{-1} Y_\nu \end{aligned}$$

This mass matrix is diagonalised by a single unitary matrix by

$$\begin{aligned} M_{\mathrm{eff}}^{\mathrm{diag}} &= \mathrm{diag}(m_1, m_2, m_3) \\ &= U_{\mathrm{eff}}^{\mathsf{T}}\, M_{\mathrm{eff}}\, U_{\mathrm{eff}} \end{aligned}$$

$$U_{\mathrm{MNS}} = \begin{pmatrix} c_{13}c_{12} & c_{13}s_{12} & s_{13}e^{-i\delta} \\ -c_{23}s_{12} - s_{13}s_{23}c_{12}e^{i\delta} & c_{23}c_{12} - s_{13}s_{23}s_{12}e^{i\delta} & c_{13}s_{23} \\ s_{23}s_{12} - s_{13}c_{23}c_{12}e^{i\delta} & -s_{23}c_{12} - s_{13}c_{23}s_{12}e^{i\delta} & c_{13}c_{23} \end{pmatrix} \times \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\frac{\alpha}{2}} & 0 \\ 0 & 0 & e^{i\frac{\beta_M}{2}} \end{pmatrix},$$

where $c_{ij} = \cos\theta_{ij}$, $s_{ij} = \sin\theta_{ij}$, the angles $\theta_{ij} = [0, \pi/2]$, $\delta = [0, 2\pi]$ is the Dirac CP-violating phase and α and β_M are two Majorana CP-violation phases.

Our Assumptions

- *Gauge unification at GUT scale ($\sim 2.3 \times 10^{16}$ GeV)*

  $g_1 = g_2 = g_3 \sim 0.71$
- *All fermion masses and mixing angles are positive defined quantities. (Values from PDG)*
- $\Delta m^2_{21} = 8.0 \times 10^{-5}\mathrm{eV}^2$, $\sin^2\theta_{12} = 0.31$ *at* M_Z.
- $|\Delta m^2_{31}| = 2.2 \times 10^{-3}\mathrm{eV}^2$, $\sin^2 2\theta_{23} = 1.0$ *at* M_Z.
- *The first- and third-generation mixing angle in the neutrino sector at* M_Z*:* $\tan\theta_{13} = 0.0$.
- *soft SUSY breaking*
- *mSUGRA at GUT scale*

  Gauginos: $M_1 = M_2 = M_3$

  Scalers: $(m^2_{\widetilde{L}})_{ij}\ etc. = \delta_{ij} m_0^2 \;\; \mathrm{and} \;\; m^2_{Hu} = m^2_{H_d} = m_0^2$
- Y_d *and* Y_e *are diagonal at GUT scale*

Lepton Flavour Violation processes

The soft SUSY breaking Lagrangian – lepton part – is a source of lepton flavour violation

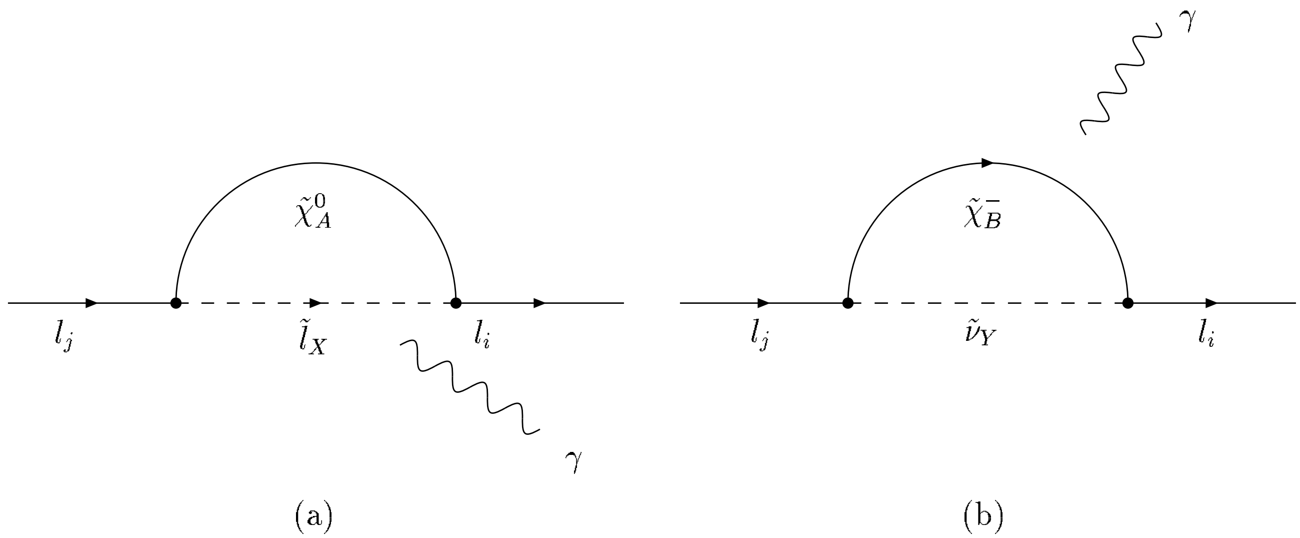
$$
\begin{aligned}
-\mathscr{L}_{\text{soft}} &= (m^2_{\widetilde{L}})_{ij}\widetilde{L}_i^\dagger \widetilde{L}_j + (m^2_{\widetilde{e}})_{ij}\widetilde{e}^*_{Ri}\widetilde{e}_{Rj} + (m^2_{\widetilde{\nu}})_{ij}\widetilde{\nu}^*_{Ri}\widetilde{\nu}_{Rj} \\
&\quad + \Big((A_e)_{ij}H_d\widetilde{e}^*_{Ri}\widetilde{L}_j + (A_\nu)_{ij}H_u\widetilde{\nu}^*_{Ri}\widetilde{L}_j + h.c.\Big)\ .
\end{aligned}
$$

We assume that soft SUSY breaking matrices are diagonal and universal in flavour at GUT scale, but they are affected by renormalisation via Yukawa and gauge interactions $\longrightarrow$ the LFV in the Yukawa couplings will induce LFV in the slepton mass matrix at low energy.

The Renormaisation equation for the left-handed slepton mass matrix is

$$
\begin{aligned}
\mu\frac{d}{d\mu}\left(m^2_{\widetilde{L}}\right)_{ji} &= \mu\frac{d}{d\mu}\left(m^2_{\widetilde{L}}\right)_{ji}\Big|_{\text{MSSM}} \\
&\quad + \frac{1}{16\pi^2}\left[\left(m^2_{\widetilde{L}}Y_\nu^\dagger Y_\nu + Y_\nu^\dagger Y_\nu m^2_{\widetilde{L}}\right)_{ji}\right. \\
&\quad \left. +2\left(Y_\nu^\dagger m^2_{\widetilde{\nu}}Y_\nu + \widetilde{m}^2_{Hu}Y_\nu^\dagger Y_\nu + A_\nu^\dagger A_\nu\right)_{ji}\right]\ ,
\end{aligned}
$$

where the first term does not violate the lepton flavour, while the second one does.



- (a) *slepton-neutralino interaction*
- (b) *sneutrino-chargino interaction*

An approximation for the additional contribution to the off-diagonal element of the slepton mass matrix:

$$(\Delta m^2_{\tilde{L}})_{ji} \approx -\frac{\ln(M_{\rm GUT}/M_R)}{16\pi^2}\left(6m_0^2 + 2A_0^2\right)\left(Y_\nu^\dagger Y_\nu\right)_{ji}\tan^2\beta$$

Note: *These calculations have been performed without employing the reading-log and the mass insertion approximations.*, $i.e.$, full one-loop calculations.

Parametrisation of Dirac neutrion Yukawa

We parametrise the Dirac neutrino Yukawa coupling constant using a complex orthogonal matrix, R which satisfies $R^{\mathsf{T}} R = 1$

$$Y_\nu(M_R) = \frac{1}{v_u}\sqrt{D_N(M_R)}\; R\; \sqrt{D_\nu(M_R)}\; U^\dagger(M_R) \ ,$$

in the Y_e and Y_ν diagonal basis. Here

$$\sqrt{M_R} = \mathrm{diag}(\sqrt{M_1}, \sqrt{M_2}, \sqrt{M_3}) \ , \ \sqrt{D_\nu} = \mathrm{diag}(\sqrt{m_1}, \sqrt{m_2}, \sqrt{m_3}) \ .$$

We use in our study $R = Oe^{iA}$, where O is a real orthogonal matrix, and A is a real antisymmetric matrix, $A^{\mathsf{T}} = -A$, that is

$$A = \begin{pmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{pmatrix} \ ,$$

where a, b, c are real parameters.

Note: Y_ν is expressed by an “artificial” orthogonal matrix R which is defined at the scale of M_R, therefore all quantities – ν-masses, ν-mixing angles – should evaluate in that scale with help of renormalisation group equations.

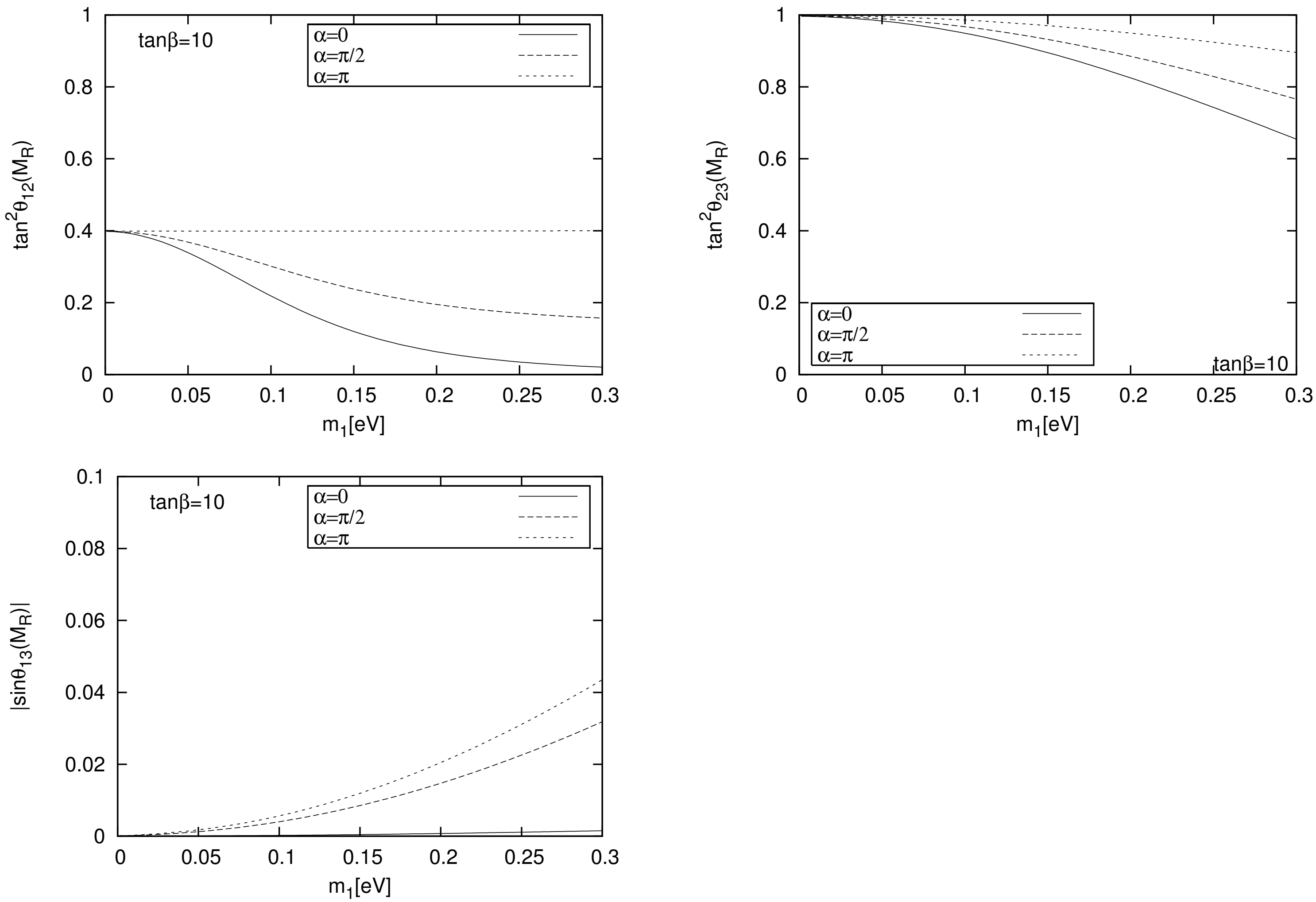
The renormalisation group evolution effects

For simplicity we have consider further assumptions.

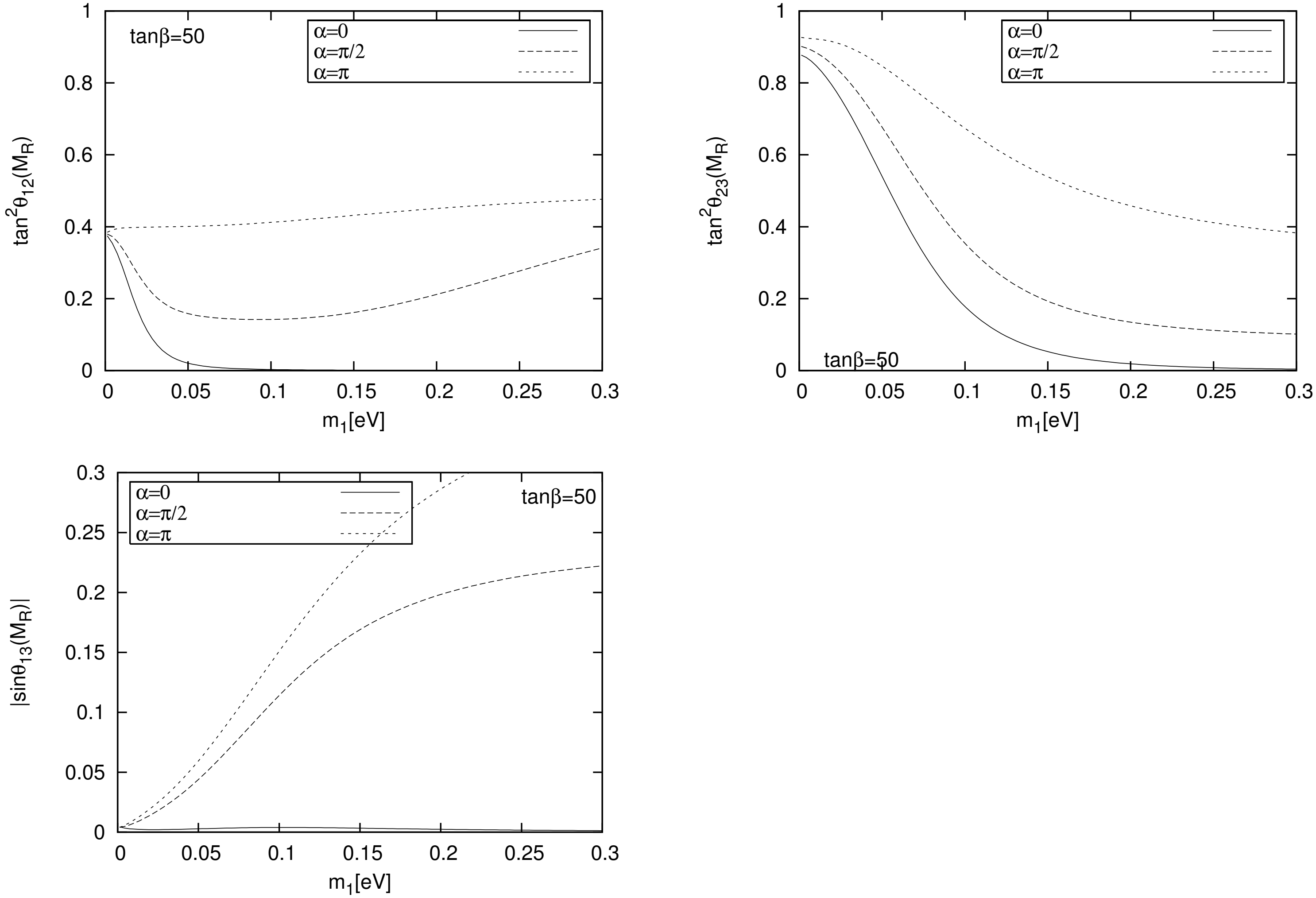
- *Neutrino mass spectrum is normal hierarchical* $m_1 < m_2 < m_3$
- *The right-handed Majorana Yukawa coupling constants are one at GUT scale: degenerate* M_R*:* $M_{R1} = M_{R2} = M_{R3}$.

We summarise the important results here (roughly speaking...):

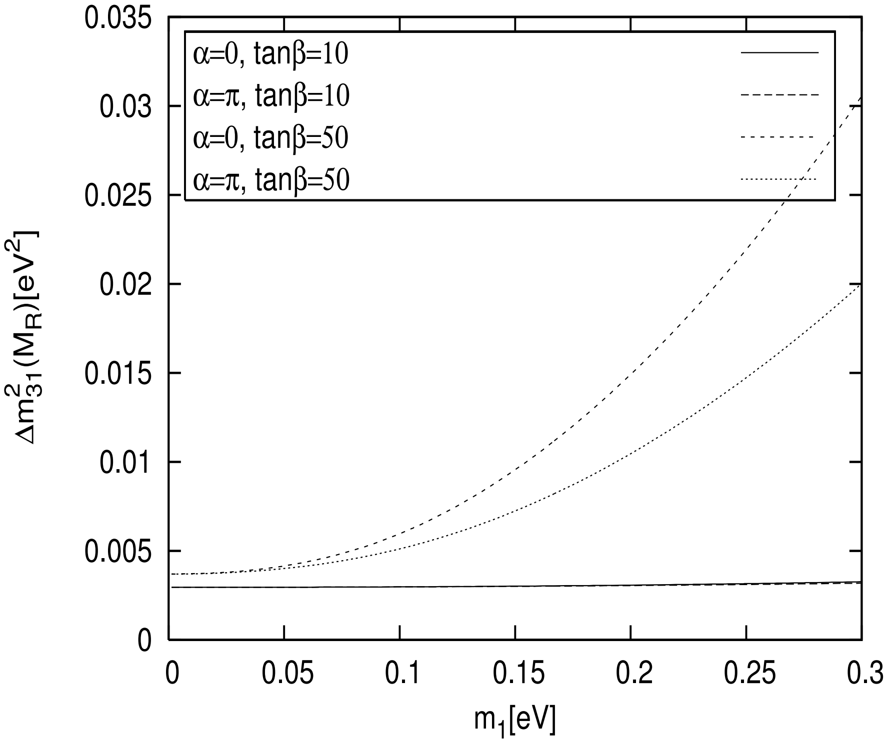
- The corrections can become significant for relatively large $\tan\beta$.
- The mixing angles are stable for $normal\ hierarchical$ neutrino mass spectrum.
- For $m_1 \gtrsim 0.05$ eV, the RG running effects can change significantly the mixing angles. The corrections are relatively small even for large $\tan\beta$ if $m_1 \lesssim 0.05$ eV.
- The neutrino mixing angles depend strongly on the Majorana phase α.
- The RG running effects depend weakly on the Majorana phase β_M.
- Mass squared differences depend strongly on m_1 in the interval $m_1 \gtrsim 0.05$ eV, and on α for $m_1 \gtrsim 0.10$ eV.



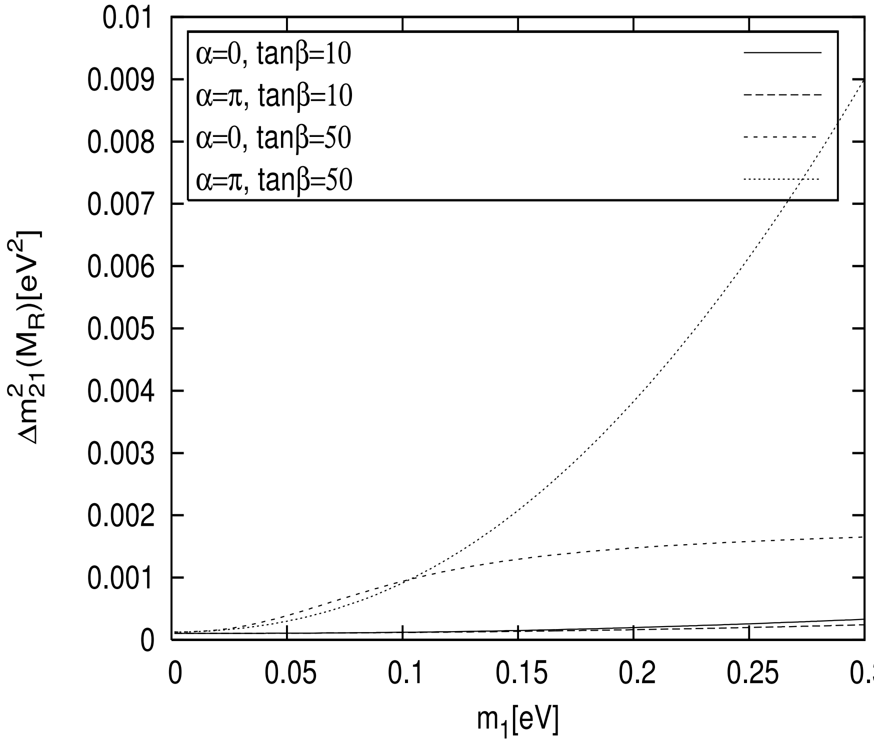
The dependence of $\tan^2\theta_{12}$, $\tan^2\theta_{23}$ and of $\sin\theta_{13}$, evaluated at M_R, on the lightest neutrino mass m_1 for $\tan\beta = 10$.
M_R is fixed as $M_R = 2\times10^{13}$ GeV.



The dependence of $\tan^2\theta_{12}$, $\tan^2\theta_{23}$ and of $\sin\theta_{13}$, evaluated at M_R, on the lightest neutrino mass m_1 for $\tan\beta = 50$.
M_R is fixed as $M_R = 2 \times 10^{13}$ GeV.



(a)

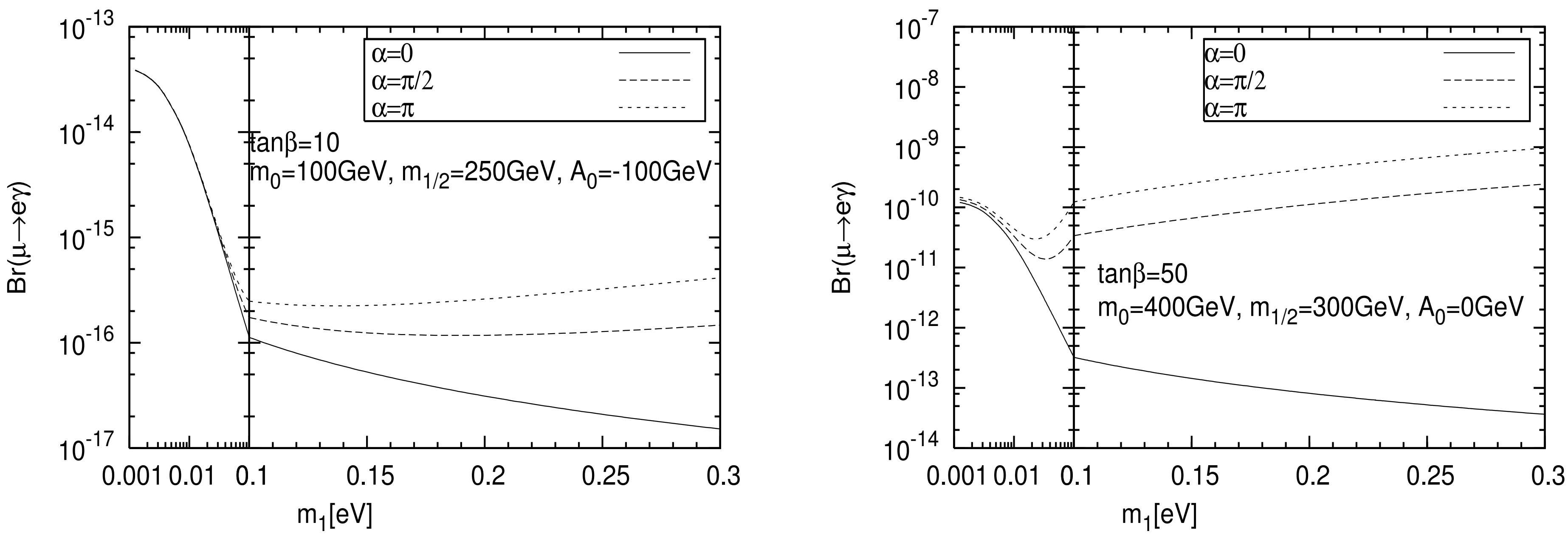


(b)

The light neutrino mass differences at the scale M_R as a functions of m_1 for $\alpha = 0, \pi$ and $\tan\beta = 10, 50$:

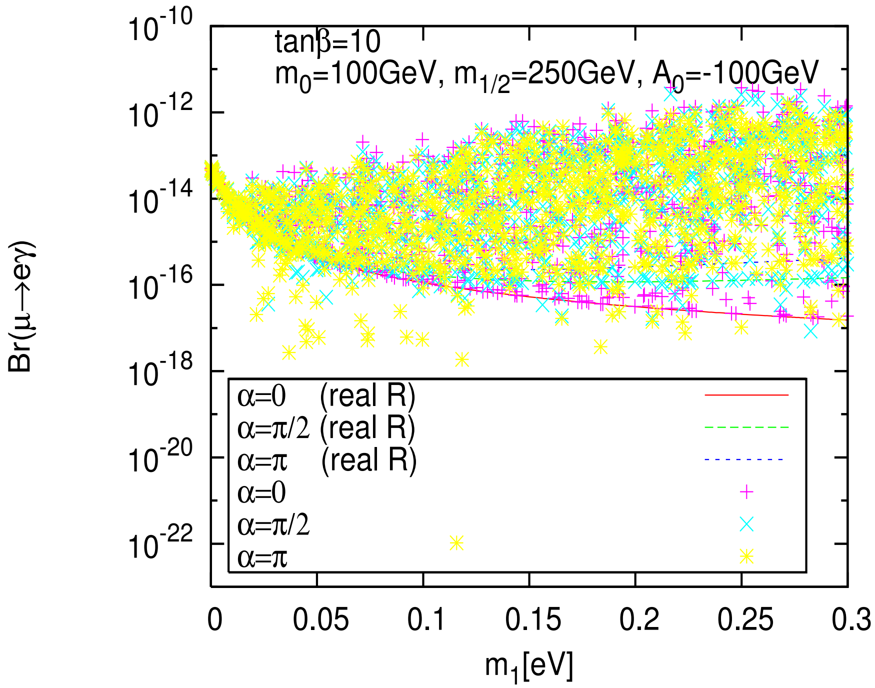
(a) $\Delta m^2_A(M_R) \equiv m^2_3(M_R) - m^2_1(M_R)$
(b) $\Delta m^2_\odot(M_R) \equiv m^2_2(M_R) - m^2_1(M_R)$.

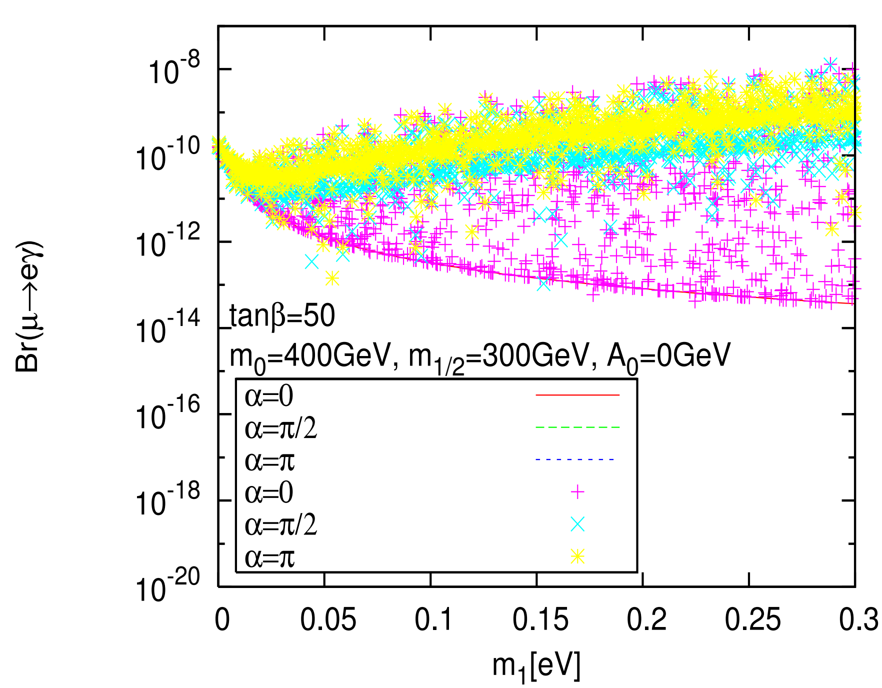
R=1 case



The branching ratios of the LFV decay $\mu \to e + \gamma$ versus m_1 in the case of $\mathbf{R} = \mathbf{1}$ for $\alpha = 0,\ \pi/2,\ \pi$. $\tan\beta = 10$ (left) and $\tan\beta = 50$ (right).

$\mathbf{R} \neq 1$ case





The branching ratios of the LFV decay $\mu \to e + \gamma$ versus m_1 in the case of complex matrix $\mathbf{R} \neq \mathbf{1}$ for $\alpha = 0,\ \pi/2,\ \pi$. $\tan\beta = 10$ (left) and $\tan\beta = 50$ (right). The three constants a, b, c are assumed to be $-0.1 \leq a, b, c \leq 0.1$.

Lepton flavour violation decays and Leptogenesis

The cosmological aspect of the connection see-saw mechanism and low energy neutrino physics; baryon asymmetry of the Universe via the leptogenesis mechanism.

$$Y_B = (6.15 \pm 0.25) \times 10^{-10} \ ,$$

The right-handed Neutrino masses are $M_1 \ll M_2 \ll M_3$, (natural!) the baryon asymmetry is given by

$$Y_B \simeq -10^{-2} \, \kappa \, \epsilon_1 \ ,$$

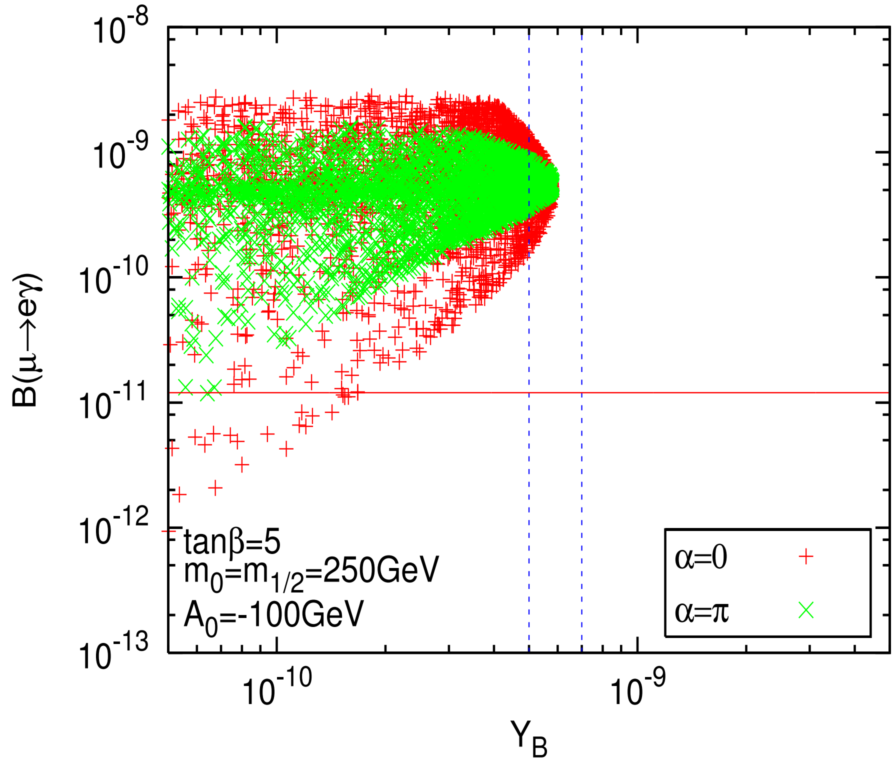
where ϵ_1 is the CP-violating asymmetry in the decay of N_1 (mass M_1), and κ is a dilution factor.

The CP-violating decay asymmetry ϵ_1:

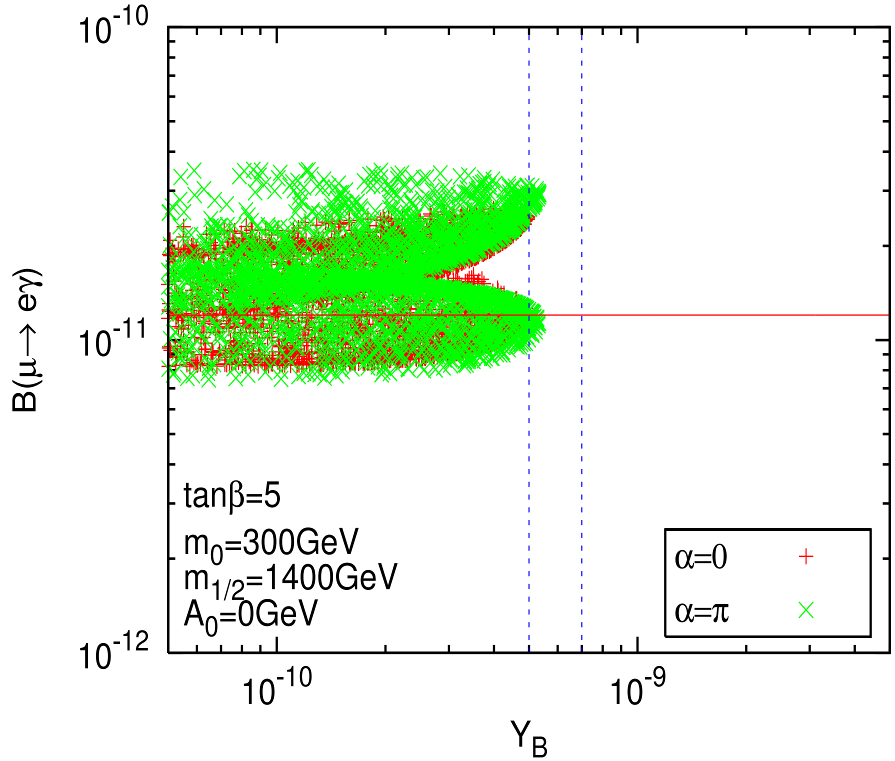
$$\epsilon_1 \simeq \frac{1}{8\pi (\mathbf{Y}_\nu \mathbf{Y}_\nu^\dagger)_{11}} \sum_{i=2,3} \mathrm{Im} \left[\left\{ (\mathbf{Y}_\nu \mathbf{Y}_\nu^\dagger)_{1i} \right\}^2 \right] f(x_i) \ ,$$

where $x_i = M_i^2 / M_1^2$ and $f(x_i)$ is defined by

$$f(x_i) \equiv -\sqrt{x_i} \ln \left(1 + \frac{1}{x_i} \right) - \frac{2\sqrt{x_i}}{x_i - 1} \, .$$



The correlation between the predicted baryon asymmetry Y_B and Br$(\mu \to e + \gamma)$ in the case of inverted hierarchical light neutrino mass spectrum and for $M_1 = 7.0 \times 10^{12}$ GeV and $M_2 = 4.0 \times 10^{13}$ GeV. The values of SUSY parameters are $m_0 = m_{1/2} = 250$ GeV, $A_0 = -100$ GeV and $\tan\beta = 5$.



The correlation between the predicted baryon asymmetry Y_B and Br$(\mu \longrightarrow e + \gamma)$ in the case of quasi-degenerate light neutrino mass spectrum and $M_1 = 3.0 \times 10^{13}$ GeV, $M_2 = 1.2 \times 10^{14}$ GeV, $M_2 = 4.8 \times 10^{14}$ GeV. The values of SUSY parameters are $m_0 = 300$ GeV, $m_{1/2} = 1400$ GeV, $A_0 = 0$ and $\tan\beta = 5$.

Conclusion

- *We have calculated lepton flavour violating processes using full RGEs including non-renormalisable RGE for the $M_{\rm eff}$ under many assumptions (too much restrictive!)*

- *We considered the effects of Majorana CP-violation phases in $U_{\rm MNS}$, and of the RG "running" of light neutrino masses and mixing angles from M_Z to the right-handed Majorana neutrino mass scale M_R. These quantities can run but not so significantly (some of cases, we should say a lot.)*

- *Furthermore, we find that the effects of the CP-violation phases in the Dirac neutrino Yukawa coupling can change by few orders of magnitude the predicted rates of the LFV decays $\mu \to e + \gamma$ etc. So we really need very carful treatments for phases!*

- *Satisfying the combined constraints from the existing upper limit on* $\mathrm{Br}(\mu \to e + \gamma)$ $etc.$ *and the requirement of successful thermal leptogenesis proves to be a powerful tool to test the viability of supersymmetric theories with see-saw mechanism and soft flavour-universal SUSY breaking scenario.*