THE MINIMAL SO(10) GUT

BORUT BAJC

(JSI, LJUBLJANA, SLOVENIA)

- FLAVOUR IN THE ERA OF LHC (2)
- CERN 06

WORK DONE WITH

[redacted]
C. AULAKH
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MINIMALITY :

THEORY WITH THE MINIMAL ASSUMPTIONS & PARAMETERS

⟶ MAXIMAL PREDICTIVITY

NOT NECESSARILY THE SIMPLEST ONE !

EX. :

SU(5) IS THE SIMPLEST GUT

NOT THE MINIMAL

(NEUTRINO !)

ASSUMPTIONS (WORKING HYPOTHESIS)

- ONLY GUT SYMMETRY - SO(10)

  NO FLAVOUR

  NO SINGLETS

- ONLY RENORMALIZABLE OPERATORS

- LOW ENERGY SUSY

MATTER

$$3 * \quad 16_F$$

$$\hookrightarrow 15 + 1$$

SM $\qquad$ ν_R

HIGGSES — YUKAWA SECTOR

$$16_F * 16_F = 10_H + 126_H + 120_H$$

$$m_{\nu_L} \sim \frac{M_W^2}{M_{\nu_R}} \qquad \text{(SEESAW)}$$

m_{ν_L} ↑ SMALL

M_{ν_R} ↑ LARGE

ν_R = SM SINGLET

⇓

(LARGE) M_{ν_R} = SM SINGLET $\subset \overline{126_H}$

!

IN SUSY LARGE $\langle \overline{126_H} \rangle \sim M_{\nu_R}$

MUST BE CANCELED IN D-TERMS BY 126_H

$(2,1,4) + (1,2,\bar{4})$

$16_F \, Y_{126} \, \overline{126_H} \, 16_F$ (YUKAWA)

3×3 MATRIX IN GENERATION SPACE

$(1,3,10)$ MASS TO ν_R

+

$(2,2,15)$ MASS TO CHARGED FERMIONS

+

$(3,1,\overline{10})$ MASS TO ν_L

+

...

PATI-SALAM DECOMPOSITION

$SU(2)_L \times SU(2)_R \times SU(4)_C$

$\subset SO(10)$

ANOTHER REASON TO USE 126_H :

$\langle (1,3,10) \rangle \neq 0$ HAS $B-L = 2$

$\Downarrow$

R - PARITY UNBROKEN AT HIGH ENERGY

MOHAPATRA, 86
FONT, IBAÑEZ, QUEVEDO, 89
MARTIN, 92

$$R = (-1)^{3(B-L)}$$

IT REMAINS UNBROKEN ALSO AT LOW ENERGY

AULAKH, BENAKLI, SENJANOVIĆ, 97
AULAKH, MELFO, SENJANOVIĆ, 98
AULAKH, MELFO, RAŠIN, SENJANOVIĆ, 99

$\overline{126}_H$:

$\langle (2,2,15) \rangle \Longrightarrow M_E = -3 M_D$

GOOD FOR 2ND GEN.

$m_\mu \approx -3 m_s$

BAD FOR 1ST, 3RD GEN.

NEED FOR 10_H :

$(2,2,1) + \ldots$

$\langle (2,2,1) \rangle \Longrightarrow M_E = M_D$

GOOD FOR 3RD GEN.

$m_\tau \approx m_b$

BAD FOR 1ST, 2ND GEN.

COMBINATION OF 10_H AND $\overline{126}_H$ COULD BE REALISTIC !

- LAZARIDES, SHAFI, WETTERICH, 81
- BABU, MOHAPATRA, 92

$$M_D = v_{10}^d Y_{10} + v_{126}^d Y_{126}$$

$$M_U = v_{10}^u Y_{10} + v_{126}^u Y_{126}$$

$$M_E = v_{10}^d Y_{10} - 3\, v_{126}^d Y_{126}$$

$$M_{\nu_D} = v_{10}^u Y_{10} - 3\, v_{126}^u Y_{126}$$

ONLY 10_H : $M_D = M_E$

ONLY $\overline{126}_H$: $-3 M_D = M_E$

-3 DUE TO $\langle (2,2,15) \rangle \propto \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & -3 \end{pmatrix}$

NEUTRINO:

$$M_{\nu_R} = v_R Y_{126} \qquad v_R = \langle (1,3,10) \rangle$$

$$M_{\nu_L} = v_L Y_{126} \qquad v_L = \langle (3,1,\overline{10}) \rangle$$

$$M_N = \underbrace{M_{\nu_D}^T M_{\nu_R}^{-1} M_{\nu_D}}_{\text{TYPE I SEESAW}} + \underbrace{M_{\nu_L}}_{\text{TYPE II SEESAW}}$$

FIRST ATTEMPTS USED ONLY TYPE I

BABU, MOHAPATRA, 92
ODA, TAKASUGI, TANAKA, YOSHIMURA, 98
MATSUDA, KOIDE, FUKUYAMA, NISHIURA, 01
FUKUYAMA, OKADA, 02

NOT VERY SUCCESSFUL

IF ASSUME ONLY TYPE II

$$M_N \propto M_{\nu_L} \propto M_D - M_E$$

- $2^{ND} + 3^{RD}$ GENERATIONS $(m_2 \ll m_3)$
- ASSUME SMALL MIXING ANGLES IN CHARGED SECTOR

$$M_N \propto \begin{pmatrix} \epsilon & \epsilon \\ \epsilon & m_b - m_t \end{pmatrix}$$

IN TYPE II SEESAW LARGE ATMOSPHERIC ANGLE CONNECTED WITH B-TAU UNIFICATION

BAJC, SENJANOVIĆ, VISSANI, 01, 02

SEVERAL ANALYSIS PERFORMED

GOH, MOHAPATRA, NG, 03, 03
DUTTA, MIMURA, MOHAPATRA, 04, 04
BASC, SENJANOVIĆ, VISSANI, 04
GOH, MOHAPATRA, NASRI, 04
BERTOLINI, FRIGERIO, MALINSKY, 04

RESULT :

THE YUKAWA SECTOR

$$16_F \left(10_H Y_{10} + \overline{126}_H Y_{126} \right) 16_F$$

IS REALISTIC BOTH

IN TYPE I
AND TYPE II
SEE-SAW

BERTOLINI, MALINSKY, 05
BABU, MACESANU, 05

WHAT ABOUT THE HIGGS SECTOR?

$$10_H + 126_H + \overline{126}_H$$

- NOT ENOUGH TO BREAK

$$SO(10) \rightarrow SM$$

- NOT ENOUGH TO FINE-TUNE

THE DOUBLET – TRIPLET SPLITTING

$$W_H = M_{10}\, 10_H^2 + M_{126}\, \overline{126}_H \cdot 126_H$$

10_H^2 : $(2,2,1)$; $\overline{126}_H \cdot 126_H$: $(2,2,15)$ $(2,2,15)$

THE DOUBLETS $D, \bar{D}$:

$$\left(\bar{D}_{10}, \bar{D}_{126}, \bar{D}_{\overline{126}}\right) \begin{pmatrix} M_{10} & \circ & \circ \\ \circ & 0 & M_{126} \\ \circ & M_{126} & 0 \end{pmatrix} \begin{pmatrix} D_{10} \\ D_{126} \\ D_{\overline{126}} \end{pmatrix}$$

TO GET

$$\det M_{\text{DOUBLETS}} = 0 \qquad \left(\begin{array}{l}\text{MSSM}\\ \text{HIGGS}!\end{array}\right)$$

ONE NEEDS TO MIX

10_H AND 126_H $\left(\overline{126}_H\right)$

10_H → 1 INDEX

126_H, $\overline{126}_H$ → 5 INDICES

CONNECTED BY

$210_H \rightsquigarrow$ 4 INDICES

=

$(1,1,1)$

+

$(1,1,15)$

+

$(1,3,15)$

TOGETHER WITH $126_H + \overline{126}_H$

BREAK $SO(10) \to SM$

+

$(2,2,10)$

+

$(2,2,\overline{10})$

OTHER DOUBLETS

+

...

ALL TOGETHER

$$10_H + 126_H + \overline{126}_H + 210_H$$

CLARK, KUO, NAKAGAWA, 82
AULAKH, KOMAPATRA, 82
LEE, 94

$$W_{HIGGS} = m_{210}\, 210_H^2 + \lambda\, 210_H^3$$
$$+ m_{126}\, \overline{126}_H 126_H + \eta\, \overline{126}_H\, 210_H\, 126_H$$
$$+ m_{10}\, 10_H^2 + 210_H\, 10_H \left(\alpha 126_H + \bar{\alpha}\, \overline{126}_H\right)$$

$$W_{YUKAWA} = 16_F \left(Y_{10}\, 10_H + Y_{126}\, \overline{126}_H\right) 16_F$$

$$\mathcal{L}_{GAUGE} = -\frac{1}{4g^2} F^A_{\mu\nu} F^{A\mu\nu} + \ldots$$

TOTAL NUMBER OF MODEL PARAMETERS =

$$26 \quad \rightsquigarrow \text{(AS IN MSSM!)}$$

(+ SUSY BREAKING)

$\Rightarrow$ MINIMAL GUT

[redacted]

AULAKH, BASC, MELFO, SENJANOVIC, VISSANI, 03

SO FAR THE FITTING OF YUKAWA SECTOR HAS BEEN DONE WITHOUT ASSUMING HIGGS SECTOR

WHY SHOULD IT MATTER ?

$$M_{FERMION} = v\, Y$$

v → HIGGS SECTOR

Y → YUKAWA SECTOR

THE 0 VEVS DETERMINED BY THE HIGGS SECTOR PARAMETERS

→ TYPE II SEE-SAW ONLY UNLIKELY

BASC, HELO, SENJANOVIĆ, VISSANI, 05

→ TYPE I SEE-SAW ONLY UNLIKELY

AULAKH, GARG, 05

MORE DETAILS :

FROM SO(10) RELATIONS AT M_{GUT} :

$$M_U = \alpha M_D + \beta M_E$$

$$\frac{\alpha}{\beta} = f(x)$$

ONLY ONE PARAMETER

$x \in \mathbb{C}$

FITTING M_U, M_D, M_E

$\Longrightarrow$ DETERMINES x

(BUT SOLUTION DEPENDENT !)

THE SAME x WILL ENTER THE NEUTRINO MASS FORMULA

CONCENTRATE ON TYPE II SEESAW:

TYPICALLY $M_N \sim \frac{v^2}{M_{GUT}}$

TOO SMALL (FOR 3RD GENERATION)

MORE PRECISELY:

$$M_N \simeq \frac{v\,(M_D - M_E)}{\sqrt{|\eta|}\; h(x)\, m_T(x)}$$

FROM $\eta\ \overline{126_H}\, 126_H\, 210_H$

SAME x AS BEFORE!

M_N ENHANCED IF

(a) $\eta \to 0$

(b) $h(x) \to 0$

(c) $m_T(x) \to 0$

VERY UNLIKELY!

IMPLIES — EITHER LIGHT MULTIPLETS (SPOILS UNIFICATION)

OR x THAT DO NOT FIT CHARGED FERMIONS

MORE WORK NEEDED !

CONCLUSIONS

(1) MINIMAL RENORMALIZABLE SO(10)

$= 3*16_F + 10_H + 126_H + \overline{126}_H + 210_H$:

ONLY 26 PARAMETERS (+ ~~SUSY~~)

(2) GOOD CHANCES TO DESCRIBE LOW-ENERGY SPECTRUM, MIXINGS, PHASE

(3) POSSIBLE PROBLEMS INCLUDING HIGGS SECTOR CONSTRAINTS

(4) PREDICTIONS :

- $|U_{e3}| > 0.1$, ALL PHASES
- NEUTRINOS HIERARCHICAL
- LOW-ENERGY SUSY WITH EXACT R-PARITY (LSP!)

→ SEARCH FOR OTHER SOLUTIONS

→ MIXED TYPE I + II SEESAW

OTHER CHECKS :

PROTON DECAY (SOFT TERMS!)

LEE, MOHAPATRA, 94
GOH, MOHAPATRA, NASRI, NG, 03
FUKUYAMA, ILAKOVAC, KIKUCHI,
MELJANAC, OKADA, 04, 04
DUTTA, MIMURA, MOHAPATRA, 04

LEPTOGENESIS (CORRECT PREDICTIONS)

BABU, MACESANU, 05

SO(10) GROUP THEORY (SPECTRUM)

HE, MELJANAC, 89
LEE, 94
AULAKH, GIRDHAR, 02
FUKUYAMA, ILAKOVAC, KIKUCHI,
MELJANAC, OKADA, 04
BAJC, MELFO, SENJANOVIĆ, VISSANI, 04

RGE + THRESHOLDS (LARGE CORRECTIONS)

AULAKH, GIRDHAR, 04

LFV (SOFT TERMS!)

ILAKOVAC (FLAVOUR LHC 1)