
NLO hard spectator scattering in QCD factorization

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Outline

1. Factorization formula
2. Structure of spectator scattering at NLO
3. $B \rightarrow \pi\pi$ phenomenology

Anatomy of two-body B -decay amplitudes

$$\begin{aligned}
 A(\bar{B} \rightarrow M_1 M_2) & \stackrel{m_b \ll M_W}{=} C_i(\mu_b) \langle M_1 M_2 | \mathcal{O}_i(\mu_b) | \bar{B} \rangle \\
 & \stackrel{\Lambda_{\text{QCD}} \ll m_b}{=} C_i(\mu_b) \left[T_i^{\text{I}}(\mu_b) \phi_{M_2}(\mu_b) F^{BM_1} \right. \\
 & \quad \left. + T_i^{\text{II}}(\mu_b, \mu_{\text{hc}}) \phi_{M_2}(\mu_b) \phi_{M_1}(\mu_{\text{hc}}) \hat{f}_B(\mu_{\text{hc}}) \phi_B(\mu_{\text{hc}}) \right. \\
 & \quad \left. + \mathcal{O}\left(\frac{\Lambda_{\text{QCD}}}{m_b}\right) \right] \quad \text{Beneke, Buchalla, Neubert, Sachrajda 1999,2001}
 \end{aligned}$$

- $C_i(\mu_b)$: virtualities/(masses)² $> \mu_b^2 \sim m_b^2$ (W, Z, t , BSM, hard gluon)
- T_i^{I} : virtualities $\sim \mu_b^2$, T_i^{II} : $\mu_b^2 > k^2 > m_b \Lambda_{\text{QCD}} \sim \mu_{\text{hc}}^2$
- $C_i, T_i^{\text{I,II}} = \sum_n r_n \alpha_s^n$ perturbative, unambiguous (at given order)
- $F^{BM_1}, \hat{f}_B, \phi_M$: soft QCD, **nonperturbative**, real, universal
- $\mathcal{O}(\Lambda_{\text{QCD}}/m_b)$ **power corrections** parametrically small, may be complex, may not factorize

Topological amplitudes, input parameters

$$\mathcal{A}(B \rightarrow M_1 M_2) \propto F^{B M_1} f_{M_2} \left\{ \sum_{i=1,2,3,4,3EW,4EW} V_{ub}^* V_{ud} \alpha_i^u(M_1 M_2) + \sum_{i=3,4,3EW,4EW} V_{cb}^* V_{cd} \alpha_i^c(M_1 M_2) + \sum_i \beta_i(M_1 M_2) \right\}$$

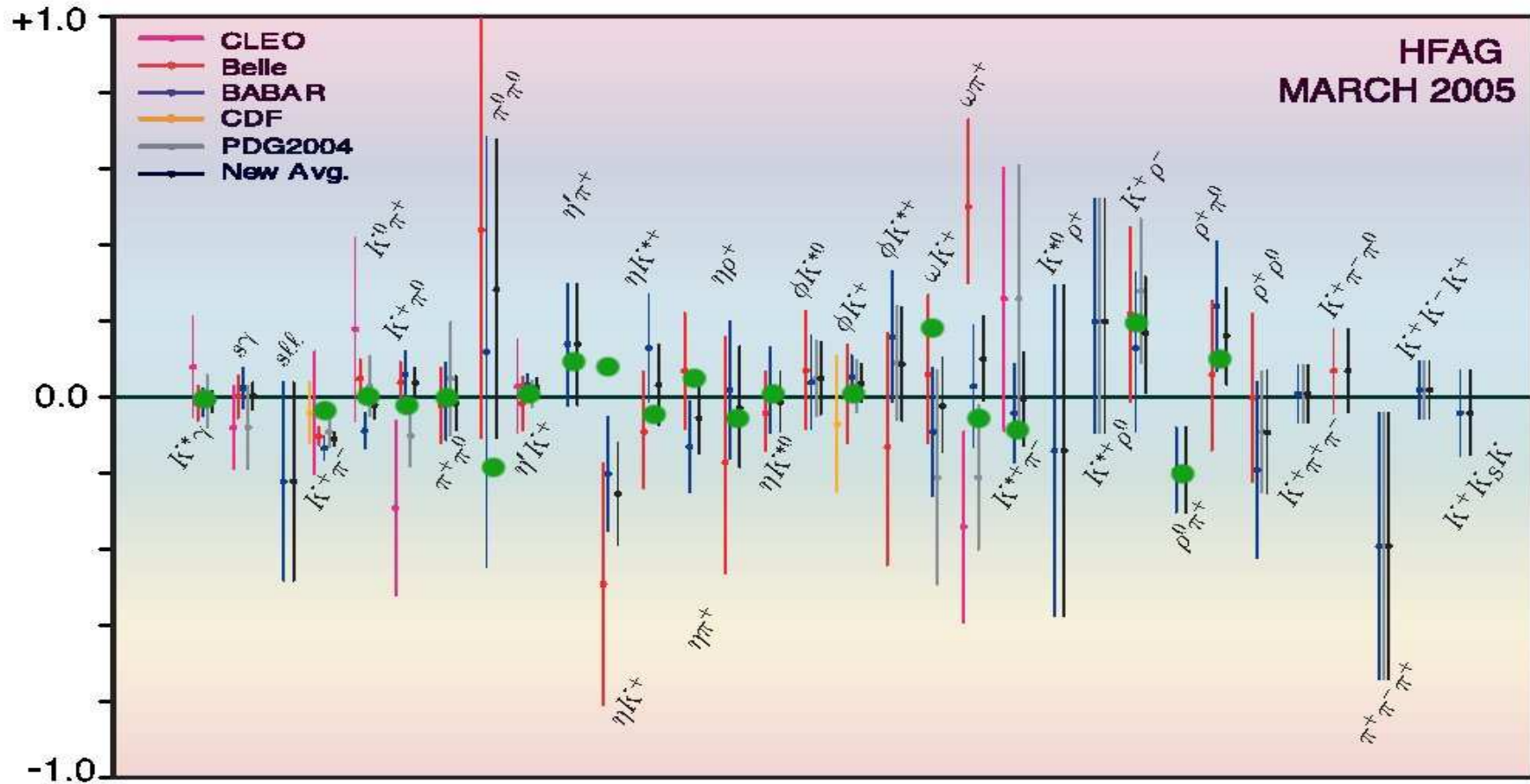
$$\alpha_i^p(M_1 M_2) = \underbrace{\alpha_{i,m}^{p,I} a_m^{M_2}}_{\text{form-factor term}} + \underbrace{\frac{f_{M_2} \hat{f}_B}{m_b F^{B M_1} \lambda_B} \alpha_{i,mn}^{p,II} a_m^{M_2} a_n^{M_1}}_{\text{spectator scattering}}$$

- $\alpha_{m(n)}^{I,II}$ contain C_i and $T_i^{I,II}$ convoluted with “test LCDAs” (Gegenbauer polynomials). Perturbative and $SU(3)_F$ symmetric.
- β_i model power corrections (later partly included into α_i)
- Moments a_i^M and λ_B specify shape of LCDAs.
- Leading-power sources of $SU(3)_F$ violation: $a_i^M, F^{B M_1}, f_M, \hat{f}_B, \lambda_B$

Predictions and uncertainties

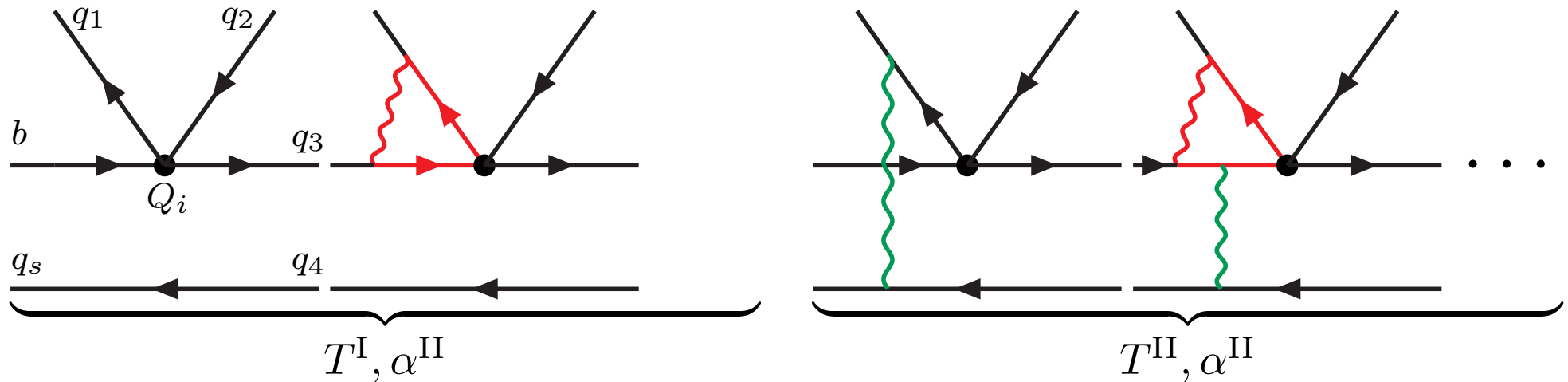
- Form-factor kernel $\alpha^I \propto \overbrace{1}^{\text{naive fact.}} + \overbrace{\mathcal{O}(\alpha_s)}^{\text{BBNS 1999,2001}} + \dots$
- Spectator scattering $\alpha^{II} = \overbrace{\mathcal{O}(\alpha_s)}^{\text{BBNS (tree)}} + \overbrace{\mathcal{O}(\alpha_s^2)}^{\text{new}} + \dots$, acquires complex phase first at NLO ($\mathcal{O}(\alpha_s^2)$)
- Not much known about $\Lambda_{\text{QCD}}/m_b \sim \mathcal{O}(\alpha_s)$ power corrections
- Small strong phases \Rightarrow generically, small CP asymmetries
- Factorization implies evolution. Motivates NLO computation (this talk) to study/reduce residual scale dependence
- Specifically, color-suppressed tree amplitude α_2 ($B \rightarrow \pi^0 \pi^0, \pi^0 \rho^0, \rho_{\parallel}^0 \rho_{\parallel}^0$) extremely sensitive to spectator scattering because of numerical cancellation between LO ($\mathcal{O}(1)$) and NLO ($\mathcal{O}(\alpha_s)$) form factor contributions.

CP asymmetries ($\mathcal{O}(\alpha_s)$)



CP symmetries are small and mostly reproduced, need to include some power corrections.

QCD corrections to tree amplitudes (α_1, α_2)



b -quark and spectator in B -meson are soft, quarks representing energetic light mesons collinear. Can form invariants $p_b \cdot p_{q_i} \sim m_b^2$ and $p_{q_s} \cdot p_{q_4} \sim \Lambda m_b$. This results in

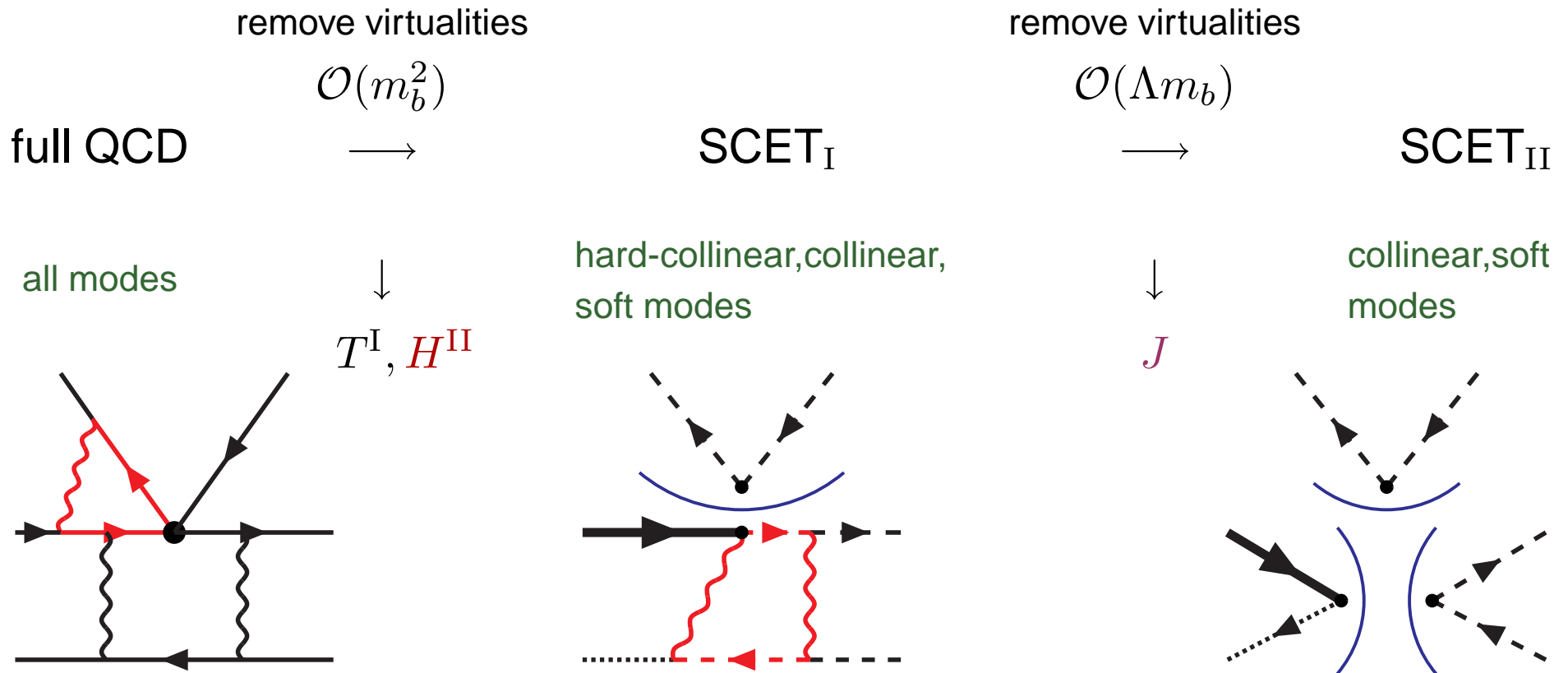
$$\alpha^{\text{I}}(\mu),$$

$$\alpha^{\text{II}}(\mu_b, \mu_{\text{hc}}) = H^{\text{II}}(\mu_b) * J(\mu_{\text{hc}}) \quad (\text{soft-collinear factorization})$$

Soft-collinear effective theory

$T^I, T^{II} = H^{II} * J$ are Wilson coefficients in SCET_I and SCET_{II}.

Bauer, Fleming, Luke 2001; Bauer, Fleming, Pirjol, Stewart 2001,2003;
Beneke, Chapovsky, Diehl, Feldmann 2002



Transition between the scales by RGE - resums large logarithms

QCD factorization in SCET_I

To match diagrams without/with spectator interactions, need

$$\begin{aligned}
 O^{\text{I}} &= \left(\bar{\chi}(tn_-) \frac{\not{n}_-}{2} \gamma_5 \chi(0) \right) \overbrace{\left(\bar{\xi}(sn_+) \not{n}_+ h_v(0) \right)}^{\text{A-type current}} \equiv Q(t) J^A(s) \\
 O^{\text{II}} &= \frac{1}{m_b} \left(\bar{\chi}(tn_-) \frac{\not{n}_-}{2} \gamma_5 \chi(0) \right) \underbrace{\left(\bar{\xi}(s_1 n_+) \not{n}_+ i \not{D}_{\perp c}(s_2 n_+) h_v(0) \right)}_{\text{B-type current}} \equiv Q(t) J^B(s_1, s_2)
 \end{aligned}$$

- $\not{n}_- \xi = \not{n}_+ \chi = 0$ ((hard-)collinear quarks). $iD_{\perp c} = i\partial_{\perp} + gA_{\perp c}$ (transverse (hard-)collinear gluon). h_v HQET heavy quark field
- Nonlocal along n_+, n_- (lightlike directions)
- amplitudes factorize up to $\mathcal{O}(1/m_b)$ (not $1/\sqrt{\Lambda m_b}$) corrections
- [F.T. $\langle \pi | Q(t) | 0 \rangle](u) = \phi_{\pi}(u)$ defines π light-cone distribution amplitude

QCD factorization in SCET_I (2)

$$\mathcal{O}_i \rightarrow \mathcal{H}_{\text{SCET}_I} = \int du C^{\text{I}}(u) Q(u) J_A + \int du dv' C^{\text{II}}(u, v') Q(u) J_B(v')$$

This suggests (cf Bauer,Pirjol,Rothstein,Stewart 2004)

$$\langle \pi\pi | \mathcal{O}_i | \bar{B} \rangle = C^{\text{I}} * \phi_\pi \underbrace{\langle \pi | J_A | \bar{B} \rangle}_{\xi_\pi \text{ (soft form factor)}} + C^{\text{II}} * \phi_\pi * \underbrace{\langle \pi | J_B(v') | \bar{B} \rangle}_{\Xi(v')}$$

but $F^{B \rightarrow \pi} = C_A \xi_\pi + C_B * \underbrace{\Xi(v')}_{\text{same as above}}$ Beneke,Feldmann 2000,2003

redefine $Q(u) J_A \rightarrow Q(u) (C_A J_A + C_B * J_B)$, $C^{\text{I}} \rightarrow T^{\text{I}}$, $C^{\text{II}} \rightarrow H^{\text{II}}$

$$\Rightarrow \langle \pi\pi | \mathcal{O}_i | \bar{B} \rangle = \overbrace{T^{\text{I}}}^{\text{BBNS 1999}} * \phi_\pi F^{B \rightarrow \pi} + \overbrace{H^{\text{II}}}^{\text{NLO is our goal}} * J * \phi_\pi * \phi_\pi * \phi_B$$

Jet function J known to $\mathcal{O}(\alpha_s^2)$

Hill et al 2004; Beneke, Yang 2005; Kirillin 2005

Matching onto SCET_I- current-current diagrams

- Partonic QCD amplitude $\stackrel{!}{=} \text{partonic matrix element}$

$$\langle \bar{q}qqg | \mathcal{O}_{1,2}^u | b \rangle_{\text{QCD}} = T^I * \langle \bar{q}q | Q(u) | 0 \rangle \langle qg | J | b \rangle + H^{II} * \langle \bar{q}q | Q(u) | 0 \rangle * \langle qg | J_B(v') | b \rangle$$

- (IR) divergence structure restricted on the r.h.s, but (a priori) not on l.h.s.
- LHS: compute about 50 diagrams in QCD, with dimensional regularization, as function of large momentum components
- RHS: compute one-loop matrix elements, main issue: evanescent operators
 - $D_2 = \frac{\not{n}_-}{2} \gamma^{\alpha\perp} \otimes \frac{\not{n}_+}{2} \gamma_{\mu\perp} \gamma_{\alpha\perp}$ vanishes in $d = 4$, but appears at tree level
 - must ensure (by finite renormalization) that $\langle D_2 \rangle^{(1-\text{loop})} = 0$
- read off $H^{II(1)}$ from equation

Analytical results

$H^{\text{II}(1)} \propto r_{1,2}$

$$\begin{aligned}
 r_1 = C_F & \left[-\frac{1}{2\bar{u}} \ln^2 \frac{m_b^2}{\mu^2} + \left(6 - \frac{5}{2\bar{u}} + \frac{2}{\bar{u}} \ln \bar{u}\right) \ln \frac{m_b^2}{\mu^2} + \frac{u}{\bar{u}} [V(u) + 18] \right. \\
 & - \frac{2u}{\bar{u}} F(v, u) + \frac{2u^3}{(\bar{v}-u)^3} F(v, \bar{u}) - \frac{2}{\bar{u}} (\ln \bar{u} + \ln \bar{v}) i\pi - \frac{1}{\bar{u}} \left(9 + \frac{5}{12} \pi^2\right) \\
 & + \left(\frac{u}{\bar{v}-u} - \frac{2u^2}{(\bar{v}-u)^2} - \frac{2(3u-2)}{\bar{u}} \right) [\log u - i\pi] - \frac{2(1-uv)}{u\bar{u}\bar{v}} \ln(1-uv) \\
 & - \left(\frac{u(2-4u+u^2)}{\bar{u}^2(\bar{v}-u)} - \frac{2(2-u)u^2}{\bar{u}(\bar{v}-u)^2} + \frac{\bar{u}-v+4u\bar{u}v}{u\bar{u}\bar{v}^2} \right) \ln(1-\bar{u}v) \\
 & + \frac{1}{\bar{u}} (\ln^2 \bar{u} - \ln^2 \bar{v}) + \left(\frac{u}{\bar{v}-u} - \frac{2u^2}{(\bar{v}-u)^2} + \frac{\bar{v}+3uv^2}{uv^2} \right) \ln \bar{v} \\
 & + \left(3 + \frac{2}{u\bar{v}} - 2 \ln v + \left(2 + \frac{2}{\bar{u}}\right) \ln \bar{v} \right) \ln \bar{u} + \left(3 + \frac{2}{\bar{u}} \ln \bar{v} \right) \ln v \\
 & + \left(\frac{1-3u\bar{u}}{\bar{u}^2} + \frac{2(3u-2)}{\bar{u}} + 2 \ln v + \frac{2u}{\bar{u}} \ln \bar{v} \right) \ln u + \frac{2}{\bar{u}} \text{Li}_2(\bar{v}) \Big] \\
 & + \left(C_F - \frac{C_A}{2} \right) \left[-\frac{2}{\bar{u}\bar{v}} \ln v \ln \frac{m_b^2}{\mu^2} + \frac{1}{\bar{u}} V(u) + \frac{2u}{v-u} F(v, u) + \frac{2u^2}{(\bar{v}-u)^2} F(v, \bar{u}) \right. \\
 & + 2i\pi \left(\frac{\bar{v}}{\bar{v}-u} + \frac{u}{\bar{u}} \ln \frac{u}{\bar{u}} + \frac{v}{\bar{u}\bar{v}} \ln v + \frac{1}{\bar{u}} \ln \bar{v} \right) + \frac{1}{\bar{u}} \left[\ln^2 \bar{v} + \frac{\pi^2}{3} \right] + \frac{u}{\bar{u}} (\ln^2 u - \ln^2 \bar{u}) \\
 & - \frac{1+\bar{v}}{\bar{u}\bar{v}} \ln^2 v + \left(\frac{2-3u}{u\bar{u}} + \frac{2}{u\bar{v}} \right) \ln \bar{u} - \frac{2(1-uv)}{u\bar{u}\bar{v}} \ln(1-uv) \\
 & + \left(\frac{2(1-3u^2+u^3)}{u\bar{u}v} + \frac{2(1-\bar{v}^2\bar{u})}{\bar{u}\bar{v}} + \frac{2(2-u)u}{v(\bar{v}-u)} \right) \ln(1-\bar{u}v) \\
 & + \left(\frac{1+2u\bar{u}}{\bar{u}^2} - \frac{2(1-uv)}{\bar{u}^2\bar{v}} + \frac{2u(u\bar{u}-v)}{\bar{u}^2(\bar{v}-u)} \right) \ln u \\
 & + \left(\frac{3(1+\bar{v})}{\bar{u}\bar{v}} + \frac{2}{\bar{u}} \ln u - \frac{2}{\bar{u}\bar{v}} \ln \bar{u} \right) \ln v \\
 & - \left(\frac{2-5u-4u^2+2u^3}{u\bar{u}v} + \frac{(1+2u)\bar{v}}{\bar{u}v} + \frac{2u\bar{u}}{v(\bar{v}-u)} + \frac{2u}{\bar{u}} \ln u \right. \\
 & \quad \left. + 2 \ln \bar{u} + \frac{2(1-2u)}{u\bar{u}\bar{v}} \ln v \right) \ln \bar{v} \\
 & + 2 \left\{ -\frac{1+2u\bar{v}}{\bar{u}\bar{v}} \text{Li}_2(u) + \frac{1-u\bar{v}+2u^2\bar{v}}{u\bar{u}\bar{v}} \text{Li}_2(\bar{u}) + \frac{1+u\bar{v}}{\bar{u}\bar{v}} \text{Li}_2(uv) \right.
 \end{aligned}$$

$$\left. - \frac{1-u\bar{u}\bar{v}}{u\bar{u}\bar{v}} \text{Li}_2(\bar{u}v) - \frac{1-2u+u\bar{v}}{u\bar{u}\bar{v}} \text{Li}_2(\bar{v}) \right\},$$

$$\begin{aligned}
 r_2 = \frac{1}{2\bar{u}} & [V(u) + 2] + \frac{u\bar{v}}{\bar{u}(v-u)} F(v, u) + \frac{u^2\bar{v}}{(\bar{v}-u)^3} F(v, \bar{u}) + \frac{\bar{v}(\bar{v}-3u)}{2(\bar{v}-u)^2} [\ln \bar{v} - i\pi] \\
 & + \left(\frac{1+u^2}{2\bar{u}^2} + \frac{u}{2\bar{u}^2\bar{v}} - \frac{u^2}{(\bar{v}-u)^2} - \frac{u}{2(\bar{v}-u)} \right) \ln u + \frac{\bar{u}+\bar{v}}{u\bar{u}\bar{v}} \ln \bar{u} \\
 & + \left(\frac{3}{2\bar{u}} - \frac{1}{2\bar{u}\bar{v}} + \frac{\ln u}{\bar{u}} - \frac{\ln \bar{u}}{\bar{u}} \right) \ln v - \left(\frac{1}{u\bar{u}v} + \frac{1}{u\bar{v}} \right) \ln(1-uv) \\
 & + \left(-\frac{1}{2} - \frac{u}{2\bar{u}^2\bar{v}} + \frac{u^2(\bar{u}^2+v)}{\bar{u}^2(\bar{v}-u)^2} + \frac{u(2+u^2)}{2\bar{u}^2(\bar{v}-u)} \right) \ln(1-\bar{u}v),
 \end{aligned}$$

where

$$F(v, w) = 2 \text{Li}_2\left(-\frac{\bar{v}w}{\bar{u}}\right) + 2 \text{Li}_2(w) - \text{Li}_2(vw) + \frac{1}{2} \ln^2 \frac{\bar{w}}{\bar{v}} + i\pi \ln \frac{\bar{w}}{\bar{v}},$$

$$\begin{aligned}
 V(u) = 6 \ln \frac{m_b^2}{\mu^2} - 18 + 3 & \left(\frac{1-2u}{\bar{u}} \ln u - i\pi \right) \\
 & + \left\{ 2 \text{Li}_2(u) - \ln^2 u + \frac{2 \ln u}{\bar{u}} - (3+2i\pi) \ln u - (u \leftrightarrow \bar{u}) \right\}.
 \end{aligned}$$

Tree amplitudes in QCDF/SCET

- Match $\mathcal{H}_W \supset C_1(\bar{u}b)_{V-A}(\bar{D}u)_{V-A} + C_2(\bar{u}^a b^b)_{V-A}(\bar{D}^b u^a)_{V-A}$
- Two ways of contracting with external quarks define two SCET operators $O_{1,2}^{\text{II}}$.
- Their matrix elements (convolutions) define two “topological tree” amplitudes α_1, α_2

$$\alpha_1 = C_1 + \frac{1}{N_c}C_2 + \mathcal{O}(\alpha_s)C_2 + \mathcal{O}(\alpha_s^2)C_1 + \dots \quad \text{color-allowed } (C_1 \sim 1)$$

$$\alpha_2 = C_2 + \frac{1}{N_c}C_1 + \mathcal{O}(\alpha_s)C_1 + \mathcal{O}(\alpha_s^2)C_1 + \dots \quad \text{color-suppressed } (C_2 \sim -0.2)$$

$$\sqrt{2}\mathcal{A}(B^- \rightarrow \pi^- \pi^0) = V_{ub}V_{ud}^* A_{\pi\pi}(\alpha_1 + \alpha_2) + \text{annihilation}$$

$$\mathcal{A}(\bar{B}^0 \rightarrow \pi^+ \pi^-) = V_{ub}V_{ud}^* A_{\pi\pi} \alpha_1 + \text{QCD penguins} + \text{annihilation}$$

$$-\mathcal{A}(\bar{B}^0 \rightarrow \pi^0 \pi^0) = V_{ub}V_{ud}^* A_{\pi\pi} \alpha_2 + \text{QCD penguins} + \text{annihilation}$$

$$A_{\pi\pi} = i \frac{G_F}{\sqrt{2}} m_B^2 f_\pi f_+^{B\pi}(0)$$

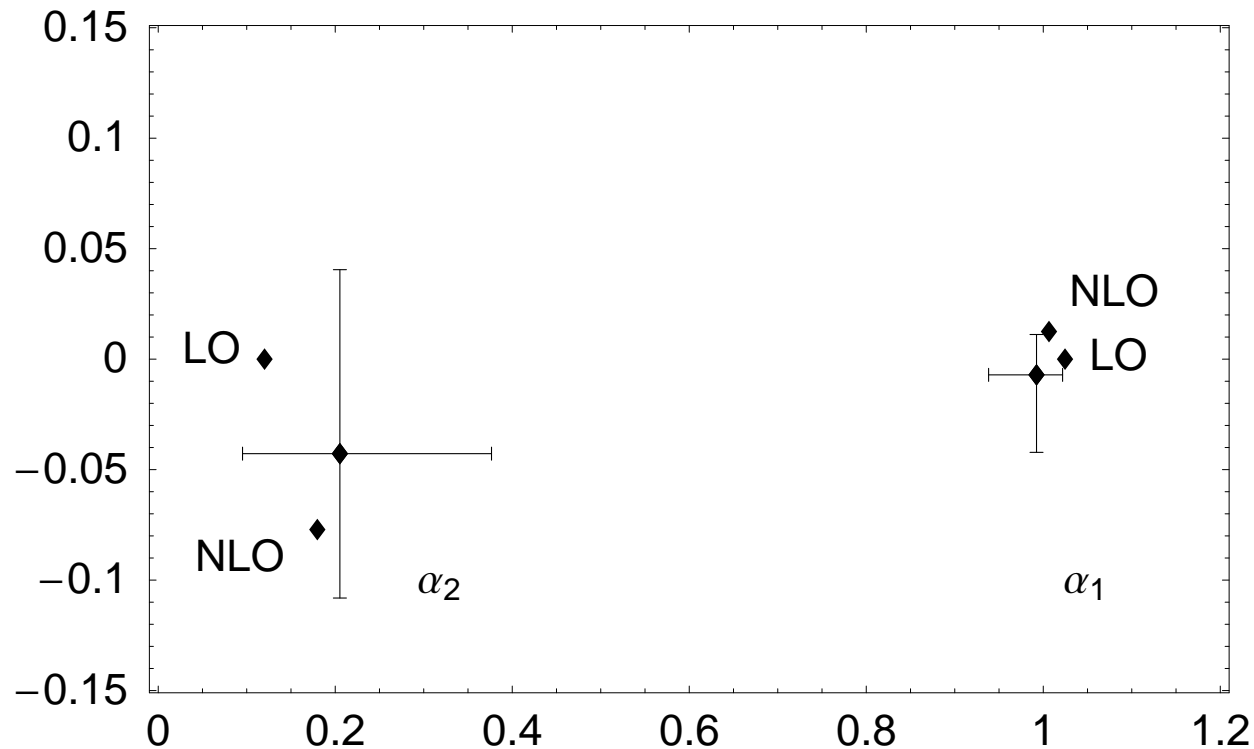
Choice of parameters

Parameter	Value/Range	Parameter	Value/Range
$\Lambda^{\overline{\text{MS}}(5)}$	0.225	μ_b	$4.8^{+4.8}_{-2.4}$
m_c	1.3 ± 0.2	μ_{hc}	1.5 ± 0.6
$m_s(2 \text{ GeV})$	0.09 ± 0.02	f_{B_d}	0.20 ± 0.03
m_b	4.8	$f_+^{B\pi}(0)$	0.28 ± 0.05
$\bar{m}_b(\bar{m}_b)$	4.2	$\lambda_B(1 \text{ GeV})$	0.35 ± 0.15
$ V_{cb} $	0.0415 ± 0.0010	$\sigma_1(1 \text{ GeV})$	1.5 ± 1
$ V_{ub}/V_{cb} $	0.09 ± 0.02	$\sigma_2(1 \text{ GeV})$	3 ± 2
γ	$(70 \pm 20)^\circ$	$a_2^\pi(2 \text{ GeV})$	0.1 ± 0.2

- $\lambda_B, \sigma_{1,2}$ parameterize ϕ_{B_+} , a_2^π the deviation of ϕ_π from asymptotic
- μ_b, μ_{hc} are the hard and hard-collinear factorization (renormalization) scales

Findings - α_1 and α_2

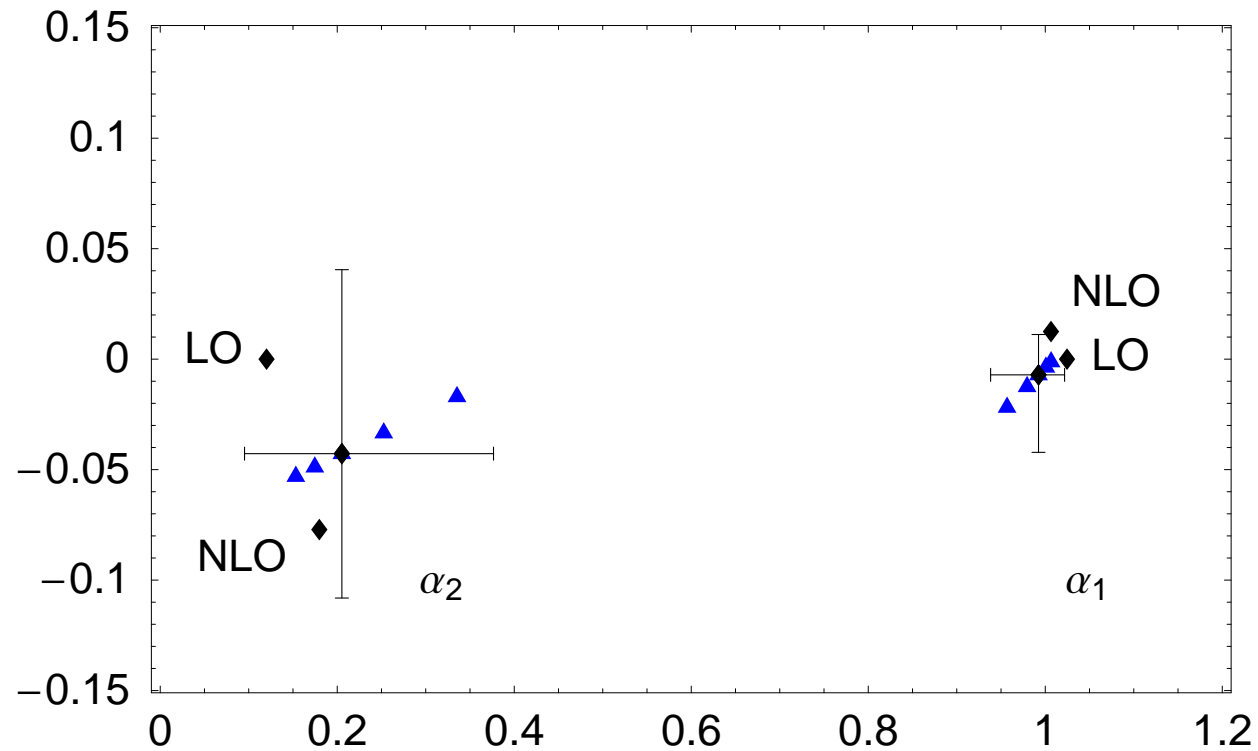
Vary all input parameters and the two factorization scales



- Closeness of LO, NLO, NNLO validates perturbative approach
- For NNLO, errors combined in quadrature; dominant: hard-collinear factorization scale, power corrections

Findings - α_1 and α_2

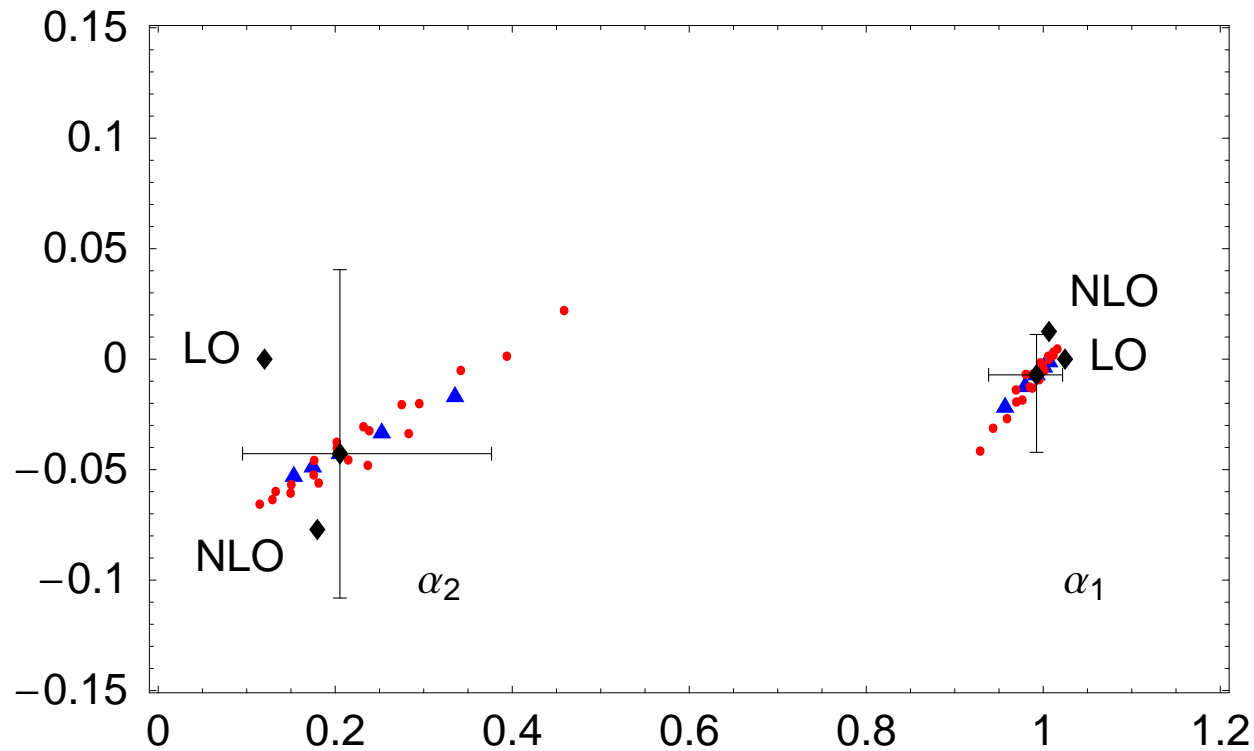
Vary all input parameters and the two factorization scales



Sensitivity to λ_B

Findings - α_1 and α_2

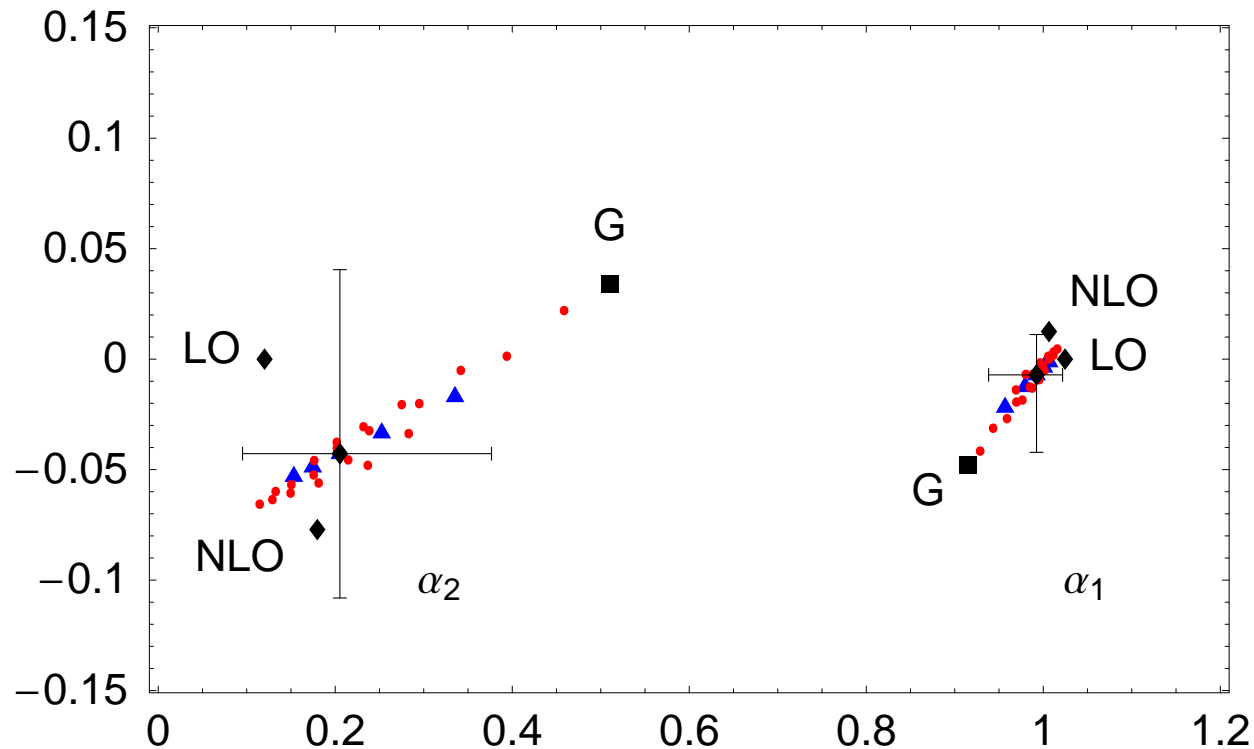
Vary all input parameters and the two factorization scales



Sensitivity to λ_B combines almost linearly with Gegenbauer moment one!

Findings - α_1 and α_2

Vary all input parameters and the two factorization scales



“G” shows result of two-parameter fit of $f_+(0)\lambda_B$ and a_2^π (fit to branching ratios).

Numerical results

$$\alpha_1(\pi\pi) = 0.992_{-0.054}^{+0.029} + (-0.007_{-0.035}^{+0.018})i$$

$$\alpha_2(\pi\pi) = 0.205_{-0.110}^{+0.171} + (-0.043_{-0.065}^{+0.083})i$$

- no large complex phases
- some enhancement of color-suppressed tree possible

For BRs, use fit values for $f_+(0)$, λ_B , a_2^π (parameter set “G” \approx S4)

$$10^6 \text{Br}(B^- \rightarrow \pi^- \pi^0) = 5.5_{-0.3}^{+0.3}(\text{CKM})_{-0.4}^{+0.5}(\text{hadr.})_{-0.8}^{+0.9}(\text{pow.}) \quad [\text{Exp: } 5.5 \pm 0.6]$$

$$10^6 \text{Br}(\bar{B}^0 \rightarrow \pi^+ \pi^-) = 5.0_{-0.9}^{+0.8}(\text{CKM})_{-0.5}^{+0.3}(\text{hadr.})_{-0.5}^{+1.0}(\text{pow.}) \quad [\text{Exp: } 5.0 \pm 0.4]$$

$$10^6 \text{Br}(\bar{B}^0 \rightarrow \pi^0 \pi^0) = 0.73_{-0.24}^{+0.27}(\text{CKM})_{-0.21}^{+0.52}(\text{hadr.})_{-0.25}^{+0.35}(\text{pow.}) \quad [\text{Exp: } 1.45 \pm 0.29]$$

- CKM “error” may be interpreted as sensitivity
- Dominant “hadronic” error: hard-collinear factorization scale μ_{hc}

Conclusion and outlook

- Part of NNLO contributions to the QCD factorization formula available, factorization holds.
- Perturbation expansion well behaved also at the hard-collinear scale, scale uncertainties are moderate
- Enhancement of the color-suppressed tree, also with respect to the color-allowed tree, due to 1-loop hard spectator scattering
- $BR(B \rightarrow \pi\pi)$ in better agreement with experiment. Better determinations of hadronic parameters ($F^{B \rightarrow \pi}, a_2^\pi, \lambda_B$) desirable. May fit from data with enough modes.
- Penguin contributions important for CP asymmetries and many branching fractions, but not yet available at NNLO