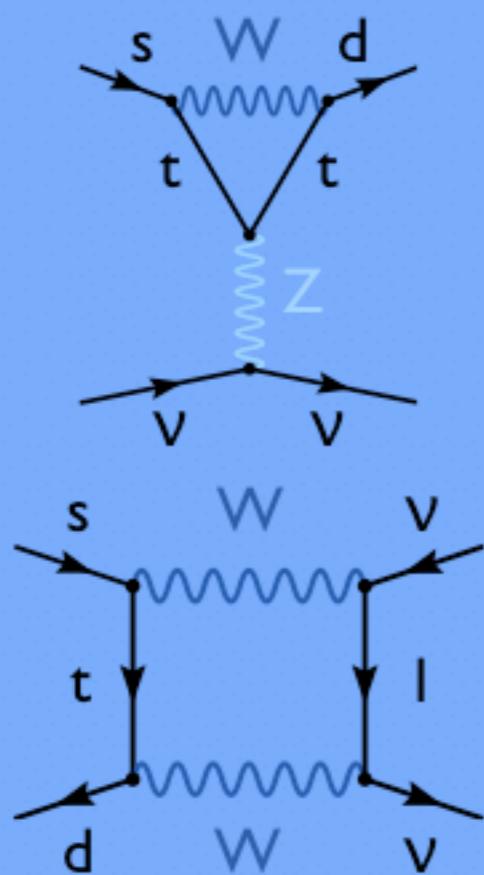




Theoretical Status of $K \rightarrow \pi \nu \bar{\nu}$



Ulrich Haisch
University Zurich

“Flavour in the era of the LHC” workshop,
2nd meeting, 6-8 February, CERN

Next 20 Minutes

- * Introduction
- * Basic Properties of $K \rightarrow \pi\nu\bar{\nu}$
- * Perturbative Effects in $K^+ \rightarrow \pi^+\nu\bar{\nu}$
- * Non-Perturbative Effects in $K^+ \rightarrow \pi^+\nu\bar{\nu}$
- * Unitarity Triangle from $K \rightarrow \pi\nu\bar{\nu}$
- * Conclusions and Outlook

Theorists View of $K \rightarrow \pi \nu \bar{\nu}$

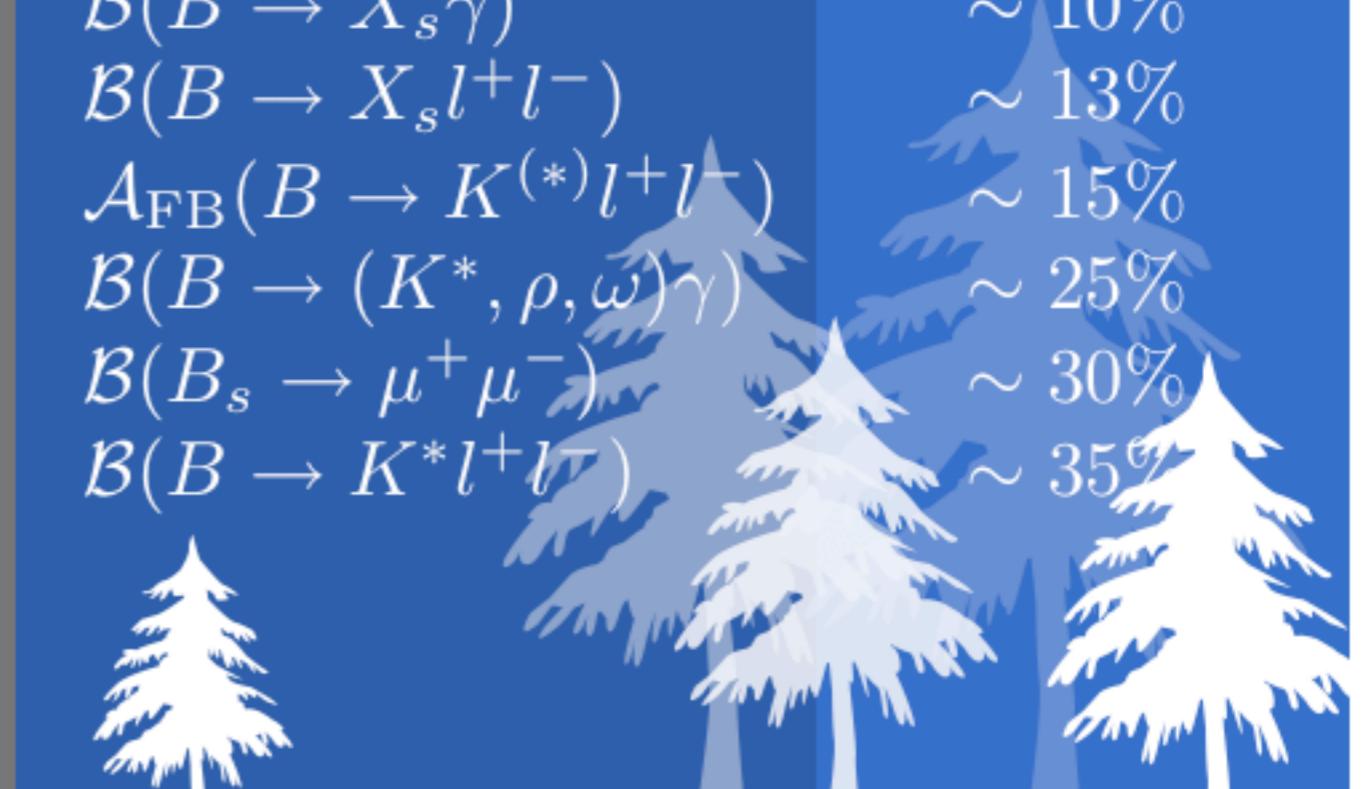
- * FCNC processes strongly suppressed in SM by loop and CKM factors
- * Large sensitivity to short-distance effects calculable with high precision
- * Long-distance effects are small and under theoretical control
- * K_L and K^+ decays would allow a very accurate determination of flavor structure of SM and NP

SM
observable

$$\begin{aligned}\mathcal{B}(K_L \rightarrow \pi^0 \nu \bar{\nu}) \\ \mathcal{B}(K^+ \rightarrow \pi^+ \nu \bar{\nu}) \\ \mathcal{A}_{\text{FB}}(B \rightarrow X_s l^+ l^-) \\ \mathcal{B}(B \rightarrow X_s \gamma) \\ \mathcal{B}(B \rightarrow X_s l^+ l^-) \\ \mathcal{A}_{\text{FB}}(B \rightarrow K^{(*)} l^+ l^-) \\ \mathcal{B}(B \rightarrow (K^*, \rho, \omega) \gamma) \\ \mathcal{B}(B_s \rightarrow \mu^+ \mu^-) \\ \mathcal{B}(B \rightarrow K^* l^+ l^-)\end{aligned}$$

theoretical
error

$\sim 3\%$
 $\sim 6\%$
 $\sim 8\%$
 $\sim 10\%$
 $\sim 13\%$
 $\sim 15\%$
 $\sim 25\%$
 $\sim 30\%$
 $\sim 35\%$



Experimental Side of $K \rightarrow \pi \nu \bar{\nu}$

- Because of neutrinos in final state challenging but not impossible to measure
- E391a recently improved old upper limit from KTeV on branching ratio of K_L
- E787 and E949 found three events of K^+ consistent with SM prediction
- Experimentally K_L and K^+ decays are in essence unexplored up to now

SM
observable

$$\mathcal{B}(B \rightarrow K^* \gamma)$$

$$\mathcal{B}(B \rightarrow X_s \gamma)$$

$$\mathcal{B}(B \rightarrow K^{(*)} l^+ l^-)$$

$$\mathcal{B}(B \rightarrow X_s l^+ l^-)$$

$$\mathcal{A}_{\text{FB}}(B \rightarrow K^{(*)} l^+ l^-)$$

$$\mathcal{B}(B \rightarrow (\rho, \omega) \gamma)$$

$$\mathcal{B}(K^+ \rightarrow \pi^+ \nu \bar{\nu})$$

$$\mathcal{A}_{\text{FB}}(B \rightarrow X_s l^+ l^-)$$

$$\mathcal{B}(B_s \rightarrow \mu^+ \mu^-)$$

$$\mathcal{B}(K_L \rightarrow \pi^0 \nu \bar{\nu})$$

experimental
error

$\sim 6\%$

$\sim 9\%$

$\sim 13\%$

$\sim 20\%$

$\sim 30\%$

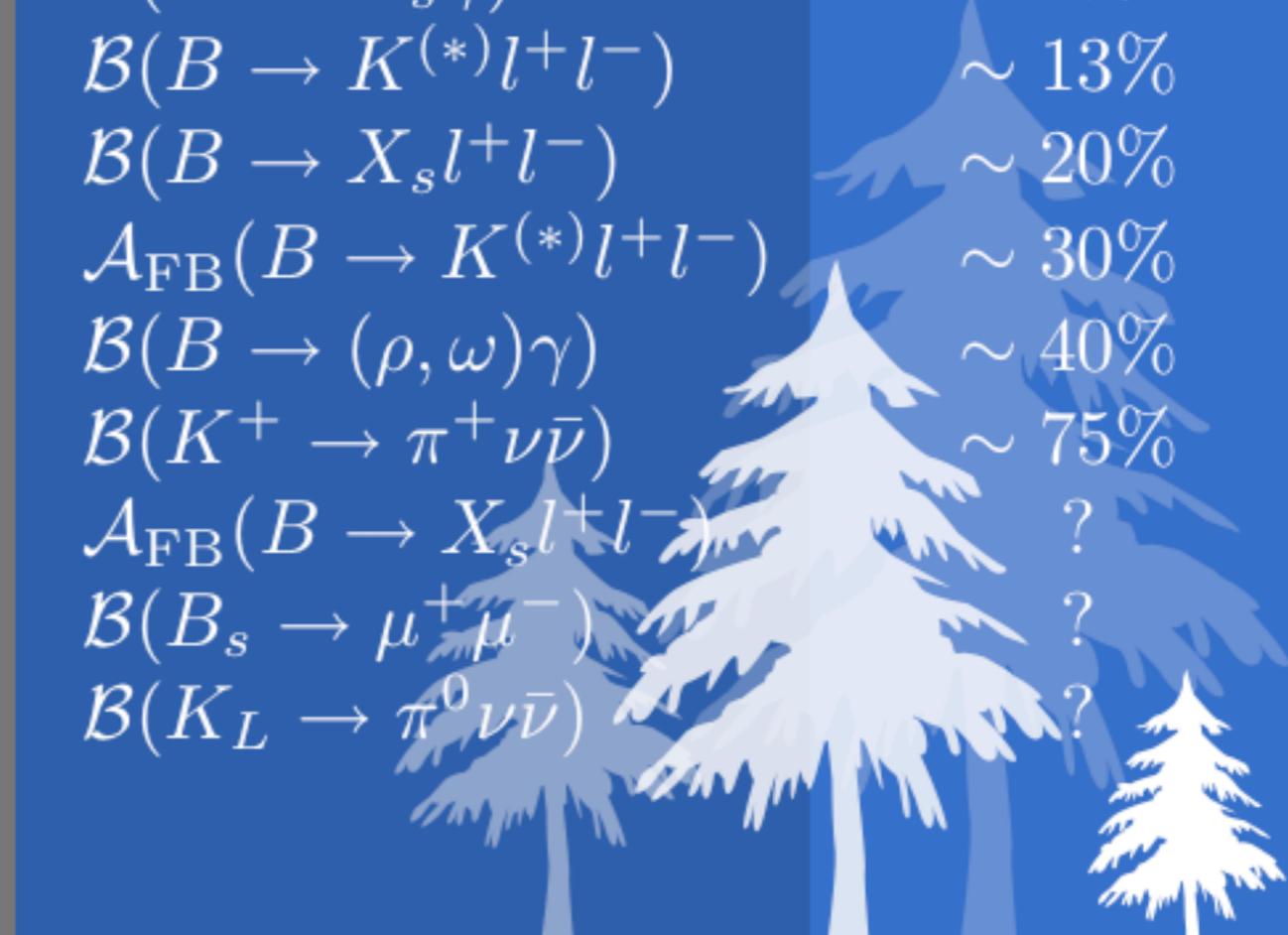
$\sim 40\%$

$\sim 75\%$

?

?

?



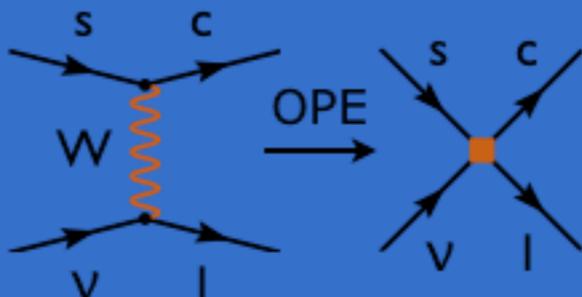
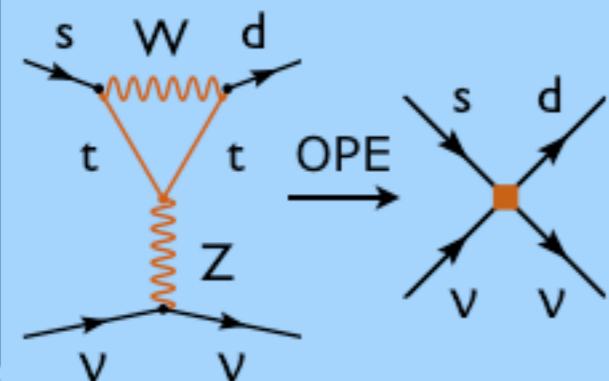
General Properties of $K \rightarrow \pi \nu \bar{\nu}$

$$\mathcal{L}_{\text{eff}}^{(6)} = \frac{4G_F}{\sqrt{2}} \frac{\alpha}{2\pi s_W^2} \sum_{i=u,c,t} C^i(\mu) Q_\nu^{(6)}$$

$$Q_\nu^{(6)} = \sum_{l=e,\mu,\tau} (\bar{s}_L \gamma_\mu d_L)(\bar{\nu}_L \gamma^\mu \nu_L)$$

$$C^i(M_W) \propto m_i^2 V_{is}^* V_{id} \propto \begin{cases} \Lambda_{\text{QCD}}^2 \lambda \\ m_c^2 (\lambda + i\lambda^5) \\ m_t^2 (\lambda^5 + i\lambda^5) \end{cases}$$

u
c
t



...

- * power-like GIM mechanism
- * top contribution dominant
- * QCD corrections small
- * large CPV phase

General Properties of $K \rightarrow \pi \nu \bar{\nu}$

$$\mathcal{L}_{\text{eff}}^{(6)} = \frac{4G_F}{\sqrt{2}} \frac{\alpha}{2\pi s_W^2} \sum_{i=u,c,t} C^i(\mu) Q_\nu^{(6)}$$

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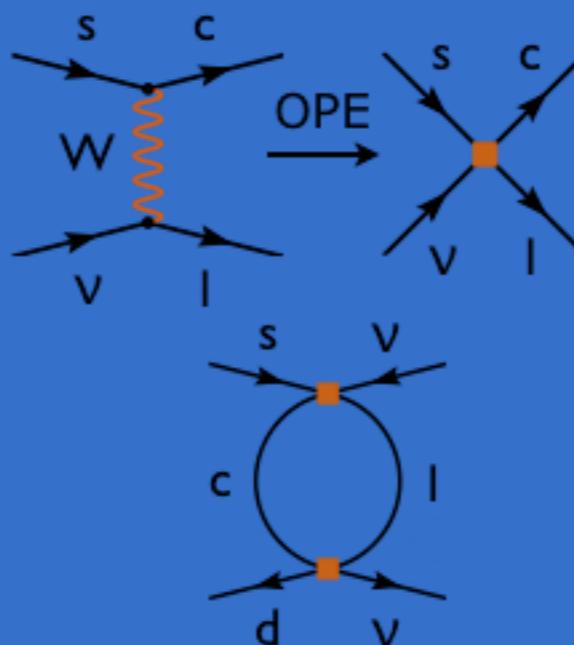
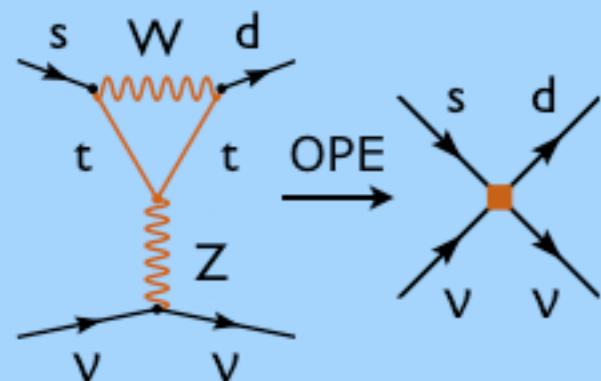
$$\mu \frac{d}{d\mu} C_i(\mu) = \sum_j \underbrace{\gamma_{ji}(\mu)}_{\text{ADM}} C_j(\mu)$$

u

c

M_W

RGE



...

...

- * top contribution does not evolve
- * charm effects negligible for K_L while moderate for K^+

General Properties of $K \rightarrow \pi \nu \bar{\nu}$

$$\mathcal{L}_{\text{eff}}^{(6)} = \frac{4G_F}{\sqrt{2}} \frac{\alpha}{2\pi s_W^2} \sum_{i=u,c,t} C^i(\mu) Q_\nu^{(6)}$$

$$Q_\nu^{(6)} = \sum_{l=e,\mu,\tau} (\bar{s}_L \gamma_\mu d_L)(\bar{\nu}_{lL} \gamma^\mu \nu_{lL})$$

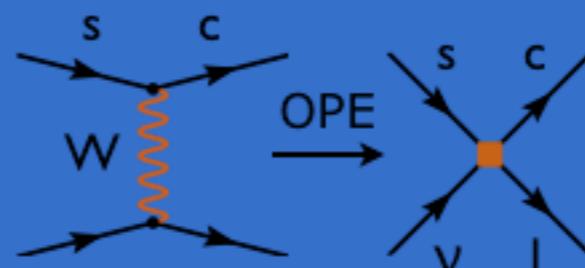
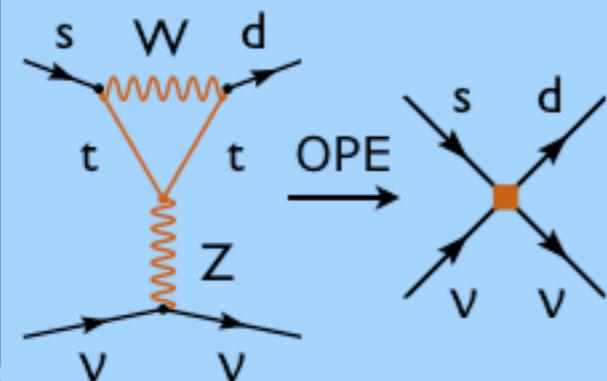
$$\langle \pi | \bar{s}_L \gamma_\mu d_L | K \rangle \propto \langle \pi^0 | \bar{s}_L \gamma_\mu u_L | K^+ \rangle$$

u

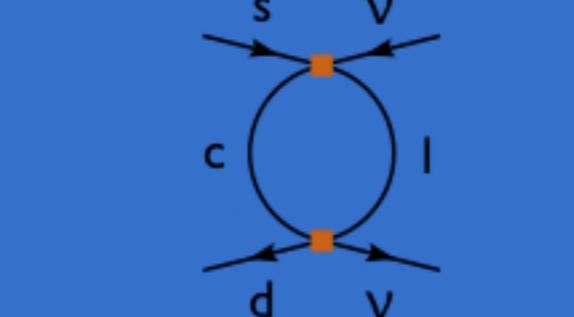
M_W

RGE

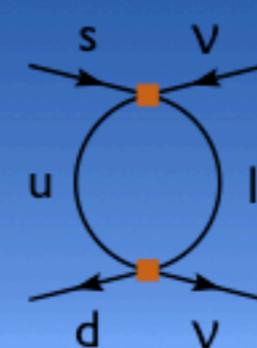
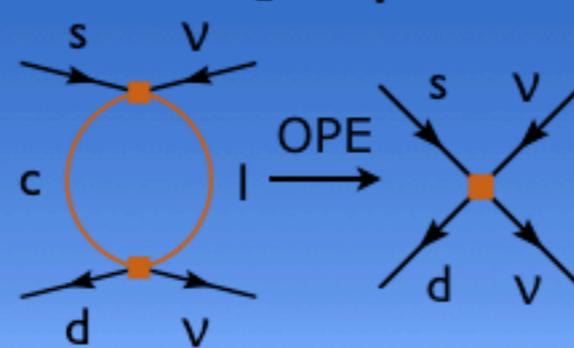
m_c



...



...



- * hadronic matrix elements precisely known from K_{e3}^+

- * neutrino pair in CP eigenstate

- * K_L decay purely CPV

NLO SM Prediction of $K_L \rightarrow \pi^0 \nu \bar{\nu}$

$$\mathcal{B}(K_L \rightarrow \pi^0 \nu \bar{\nu}) = \kappa_L \left[\frac{\text{Im} (V_{ts}^* V_{td})}{\lambda^5} X \right]^2$$

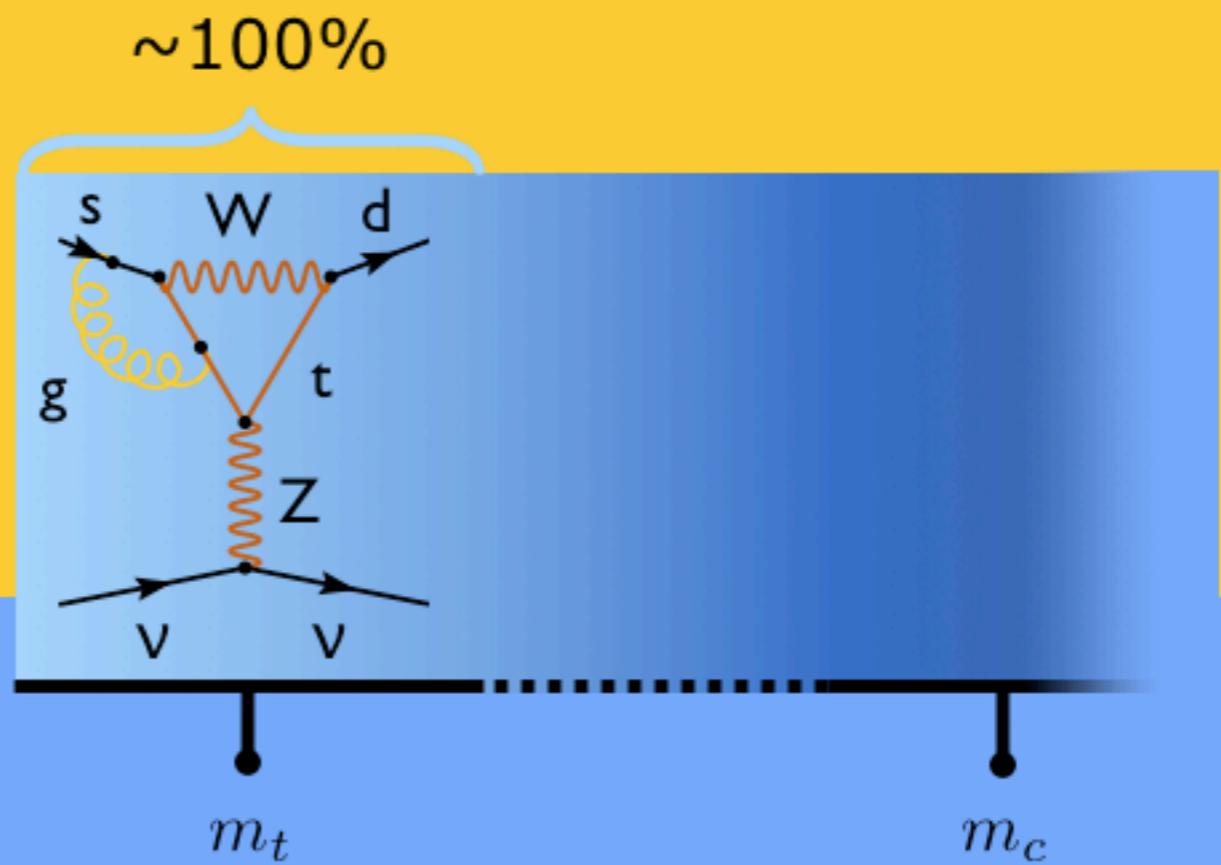
$$\kappa_L = r_{K_L} \frac{3\alpha^2 \mathcal{B}(K^+ \rightarrow \pi^0 e^+ \nu_e)}{2\pi^2 s_W^4} \frac{\tau(K_L)}{\tau(K^+)}$$

$$X = 1.46 \pm 0.04 \quad (\text{NLO}) \quad t$$

Buchalla & Buras '93, '99;
Misiak & Urban '99

$$r_{K_L} = 0.944 \pm 0.028 \quad SU(2)$$

Marciano & Parsa '96



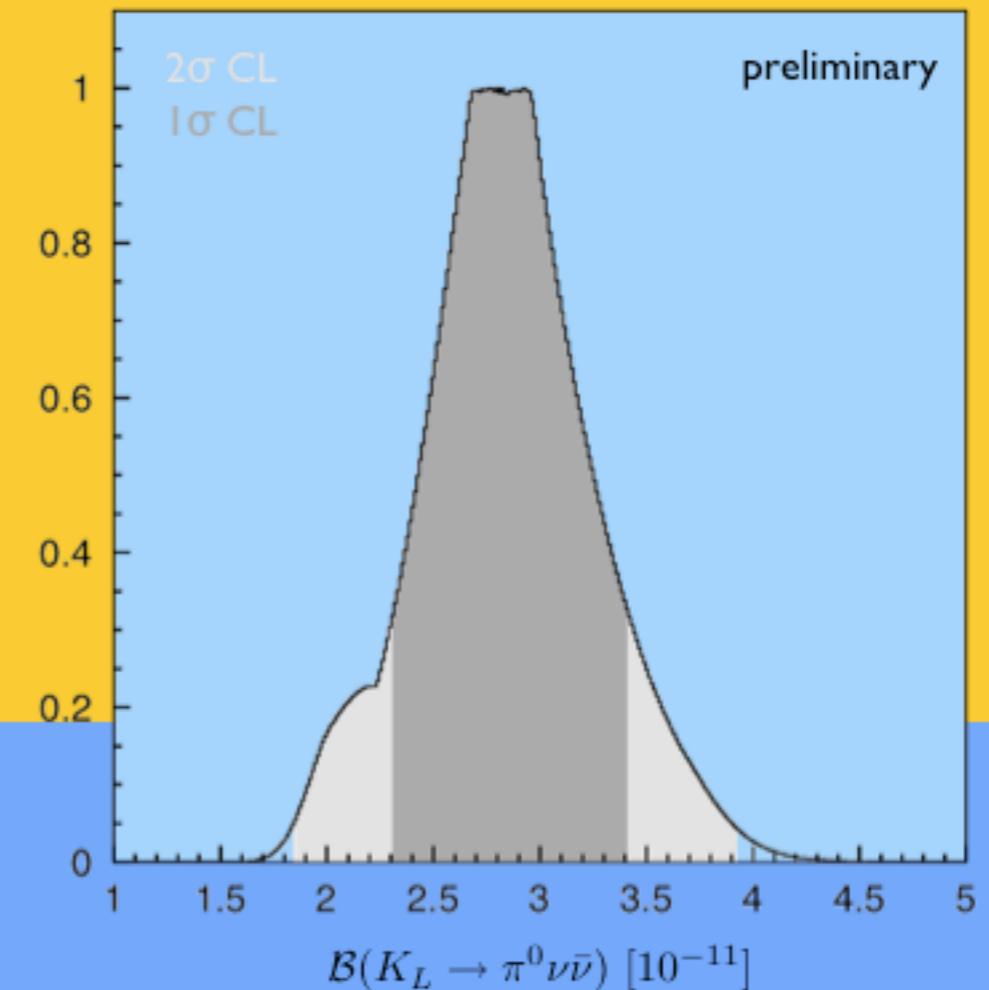
SM Prediction of $K_L \rightarrow \pi^0 \nu \bar{\nu}$

$$\mathcal{B}(K_L \rightarrow \pi^0 \nu \bar{\nu}) = \kappa_L \left[\frac{\text{Im} (V_{ts}^* V_{td})}{\lambda^5} X \right]^2 = (2.81 \pm 0.54) \times 10^{-11}$$

$$\kappa_L = r_{K_L} \frac{3\alpha^2 \mathcal{B}(K^+ \rightarrow \pi^0 e^+ \nu_e)}{2\pi^2 s_W^4} \frac{\tau(K_L)}{\tau(K^+)} = (2.20 \pm 0.07) \times 10^{-10}$$

CKMFitter, Buras, Gorbahn, Nierste & UH '06

- * very small theoretical error of only $\sim 3\%$
- * $\sim 85\%$ of total error of $\sim 20\%$ due to CKM parameters
- * within SM amount of CPV can in principle be determined with unmatched precision



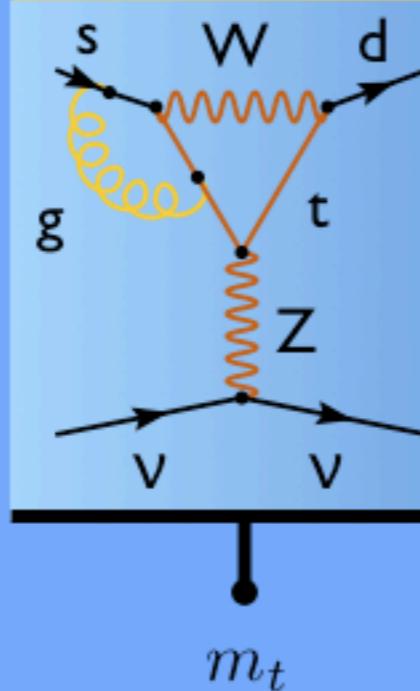
$$\mathcal{B}(K^+ \rightarrow \pi^+ \nu \bar{\nu}) = \kappa_+ \left[\frac{\text{Im} (V_{ts}^* V_{td})}{\lambda^5} X + \left(\frac{\text{Re} (V_{ts}^* V_{td})}{\lambda^5} X + \frac{\text{Im} (V_{cs}^* V_{cd})}{\lambda} (P_c + \delta P_c) \right)^2 \right]$$

$$\kappa_+ = r_{K^+} \frac{3\alpha^2 \mathcal{B}(K^+ \rightarrow \pi^0 e^+ \nu_e)}{2\pi^2 s_W^4} = (5.04 \pm 0.17) \times 10^{-11}$$

$r_{K^+} = 0.901 \pm 0.027$

~~SU(2)~~

Marciano & Parsa '96



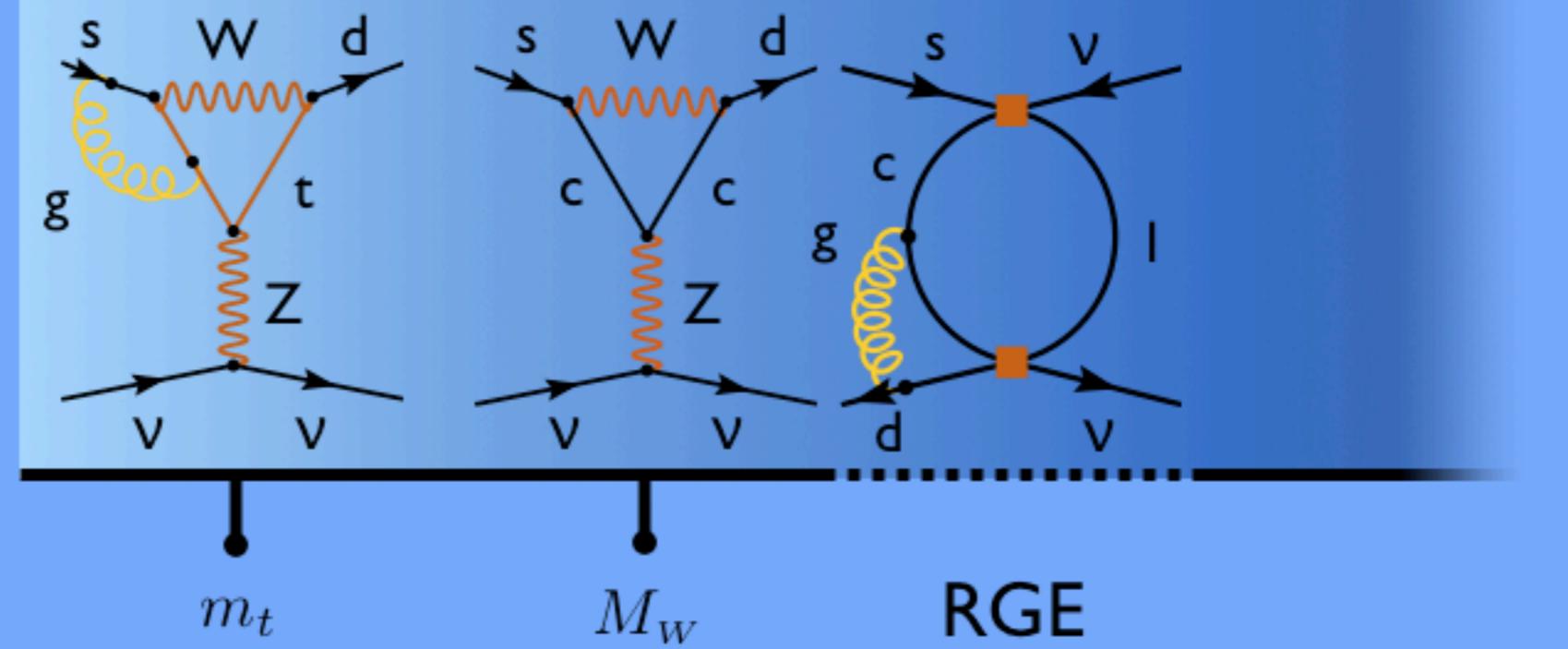
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$P_c = 0.37 \pm 0.06$ (NLO)

C

Buchalla & Buras '94, '99;
Buras, Gorbahn, Nierste & UH '05



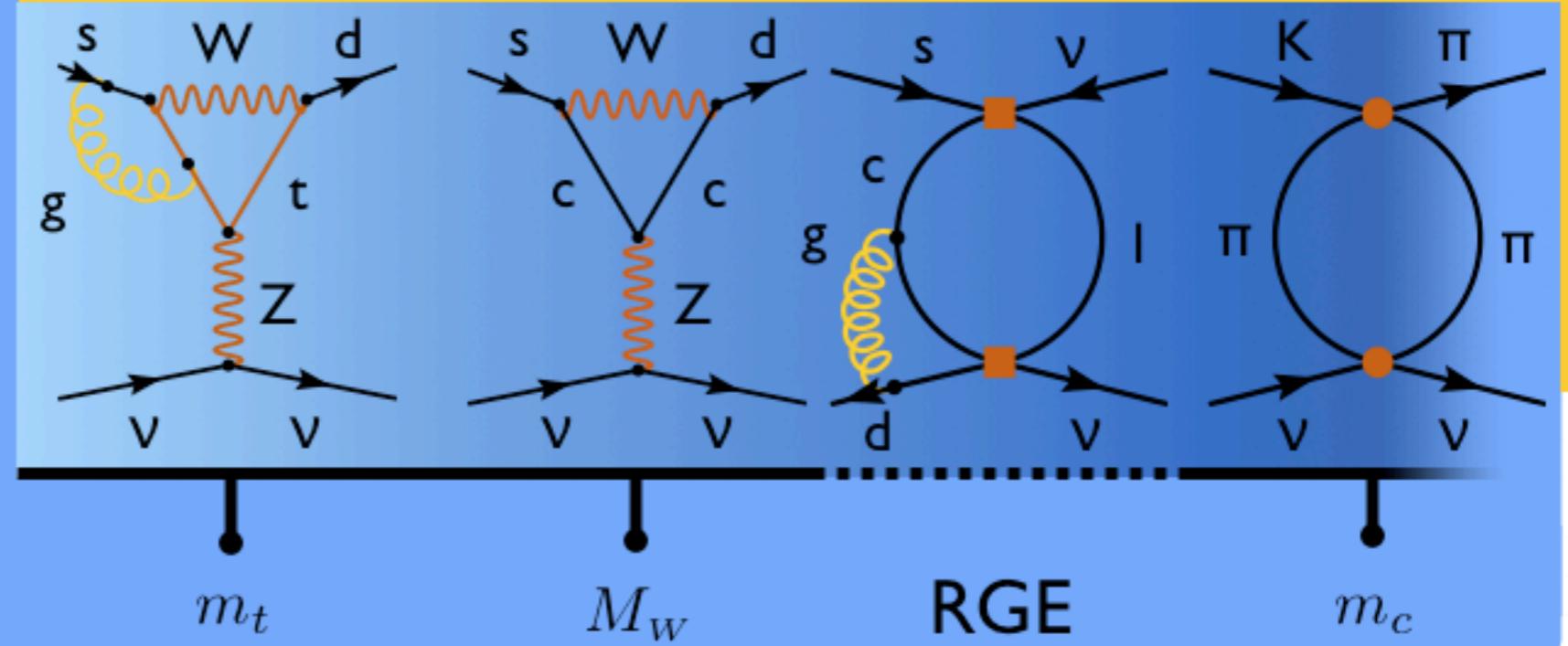
NLO SM Prediction of $K^+ \rightarrow \pi^+ \nu \bar{\nu}$

$$\mathcal{B}(K^+ \rightarrow \pi^+ \nu \bar{\nu}) = \kappa_+ \left[\frac{\text{Im} (V_{ts}^* V_{td})}{\lambda^5} X + \left(\frac{\text{Re} (V_{ts}^* V_{td})}{\lambda^5} X + \frac{\text{Im} (V_{cs}^* V_{cd})}{\lambda} (P_c + \delta P_c) \right)^2 \right]$$

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$$\delta P_c = 0.04 \pm 0.02 \quad (\chi\text{PT}) \quad \text{u}$$

Isidori, Mescia & Smith '05

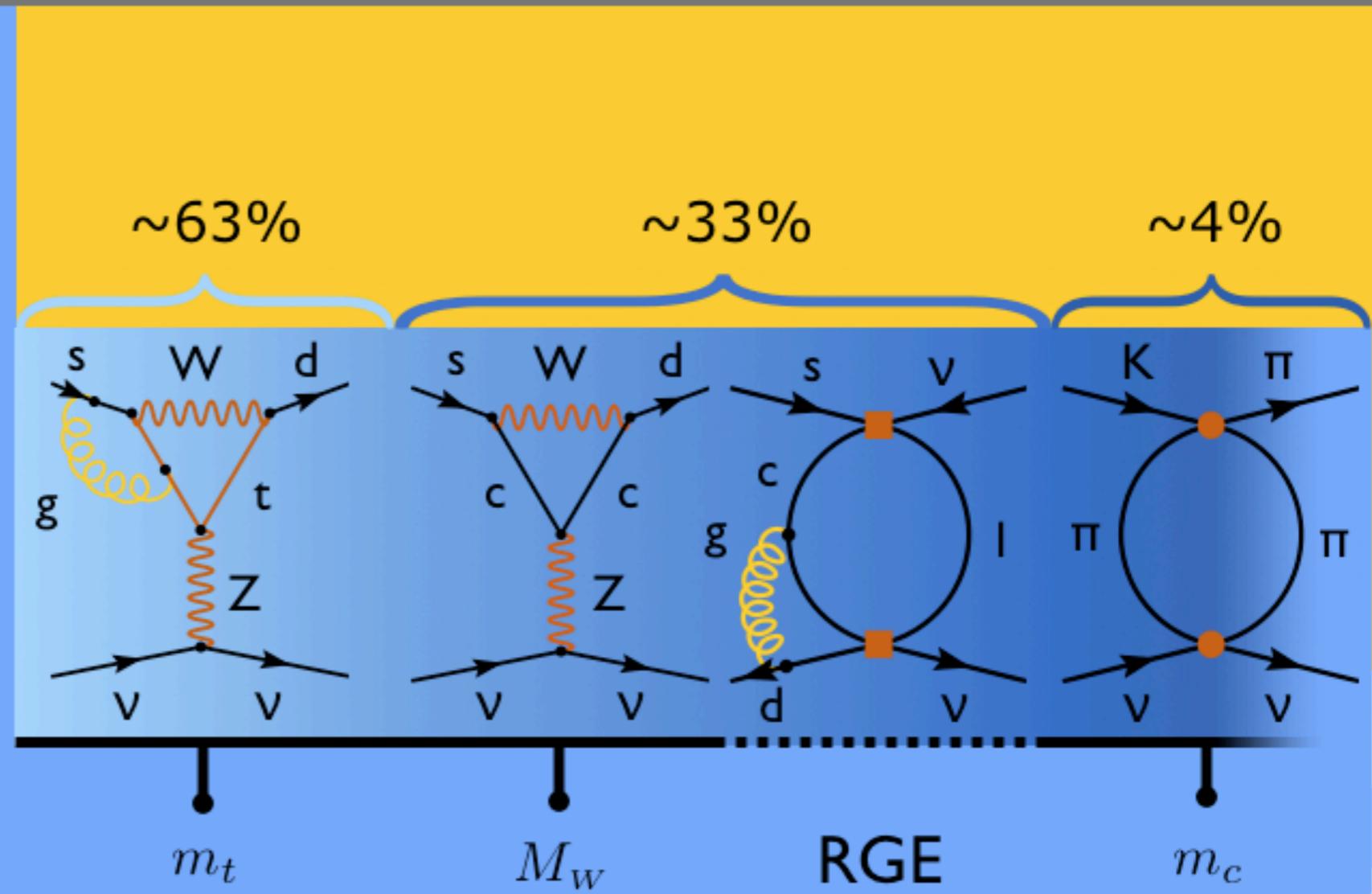


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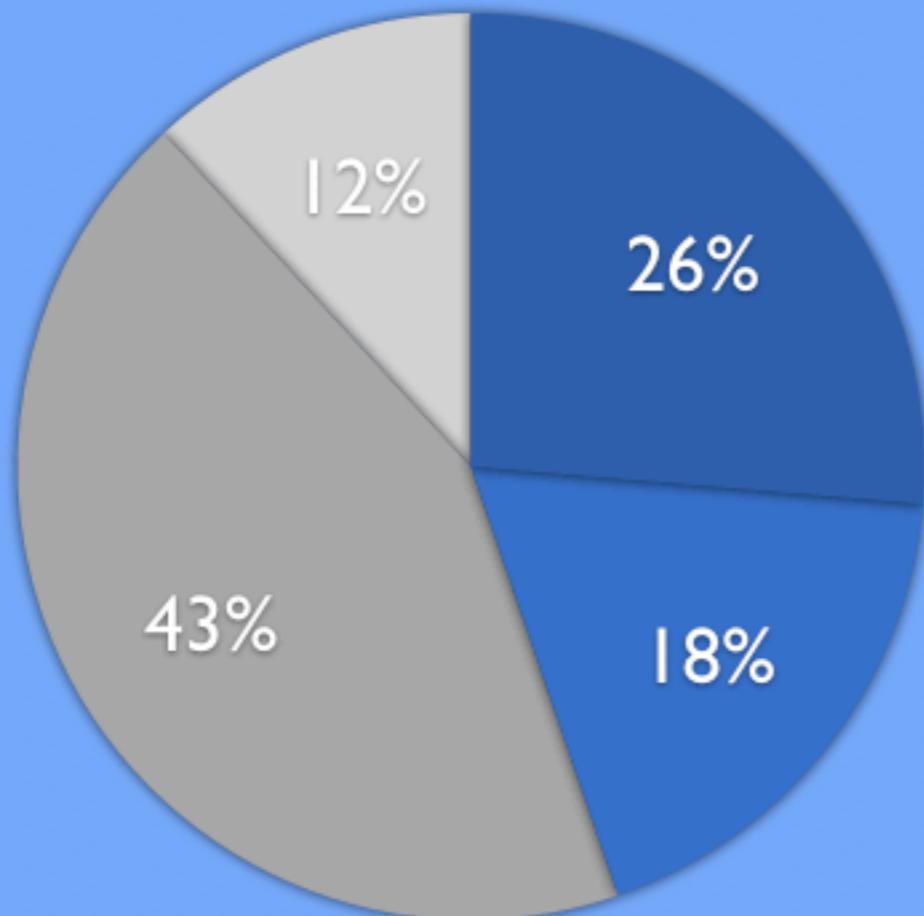
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- * theoretical error at NLO dominated by uncertainty in P_c
- * only $\sim 50\%$ of error in branching ratio is due to m_c and CKM elements

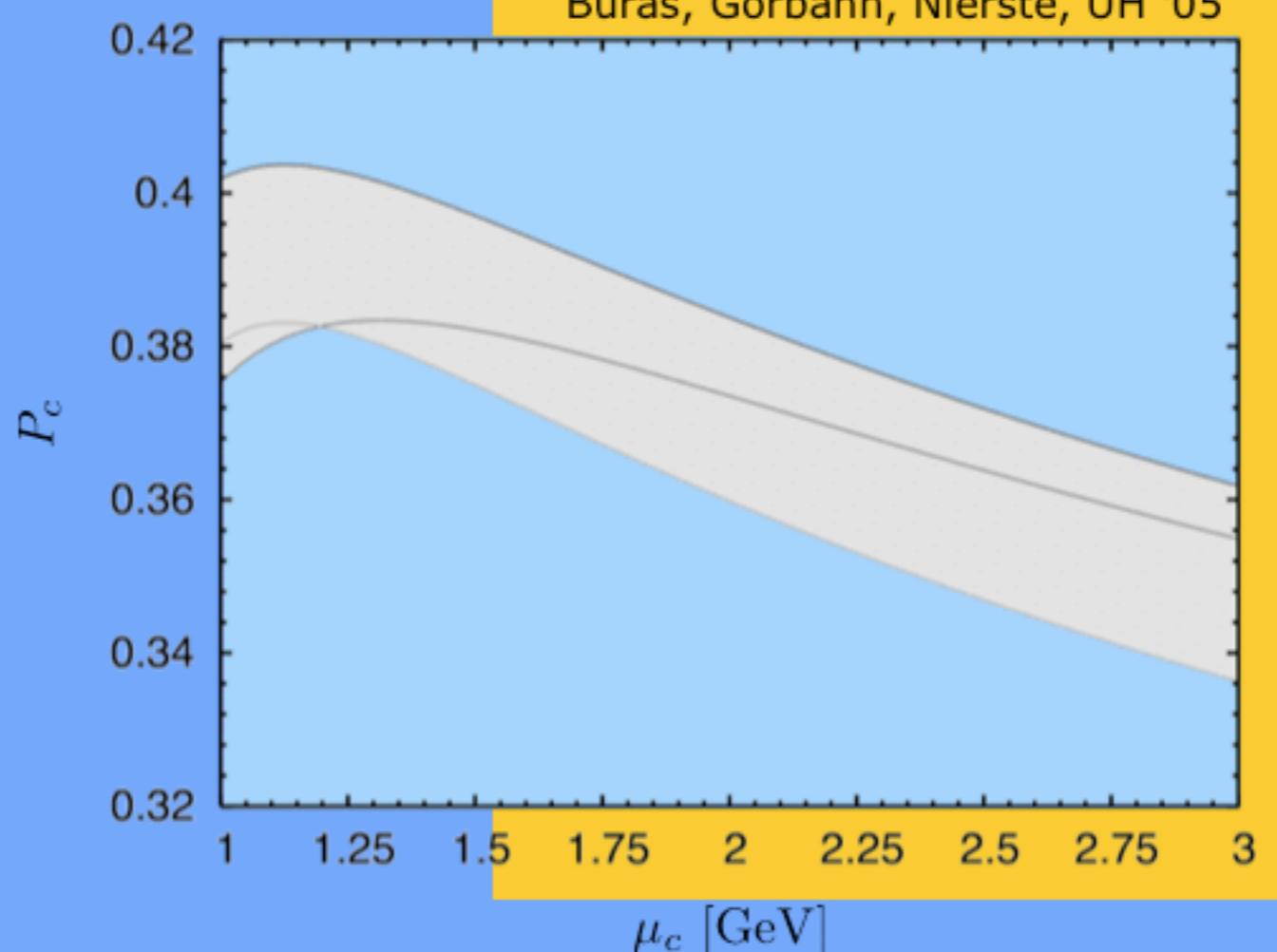


Error Budget of P_c at NLO

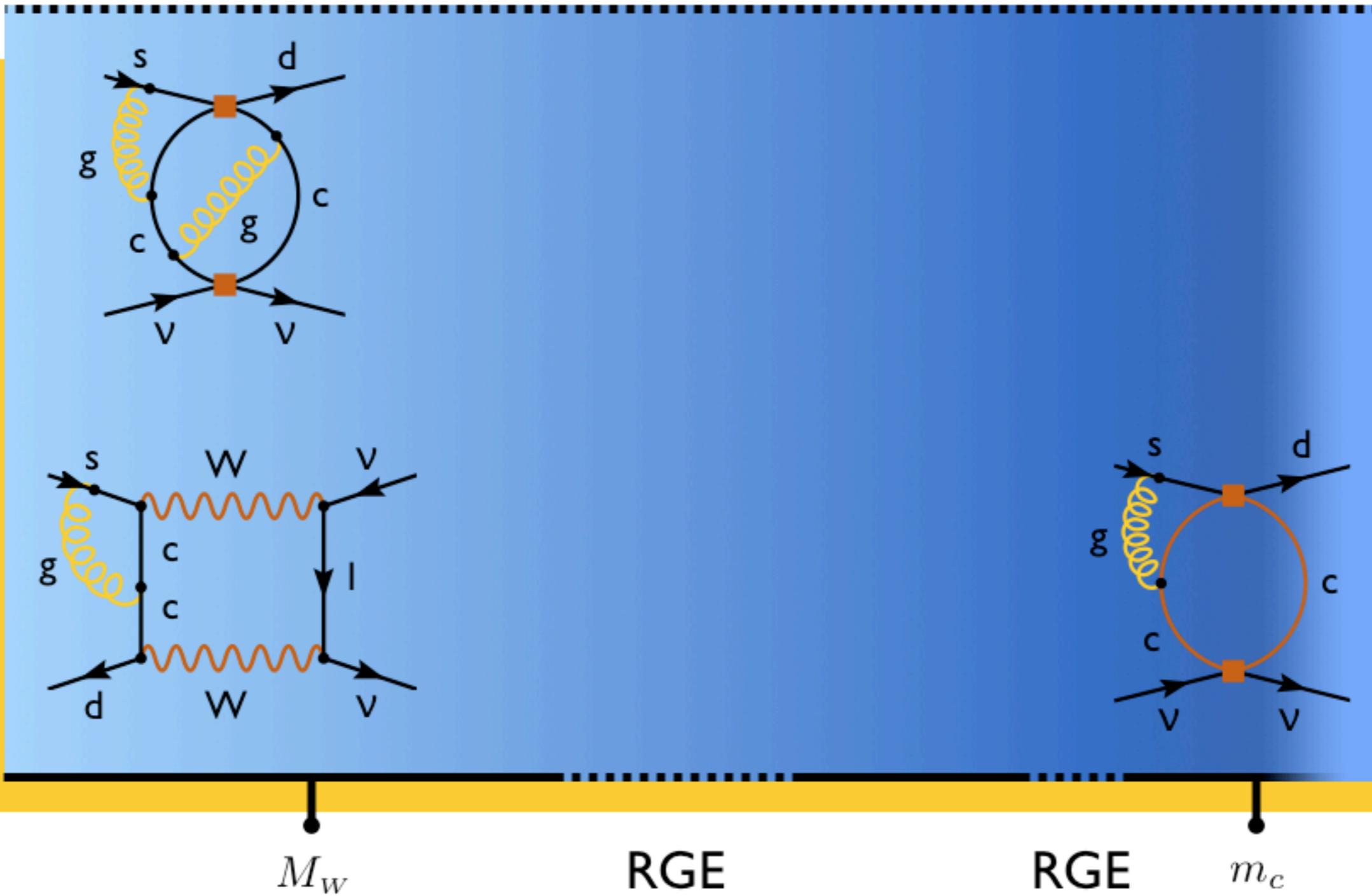
$$P_c = 0.369 \pm 0.036 \pm 0.033 \pm 0.009$$



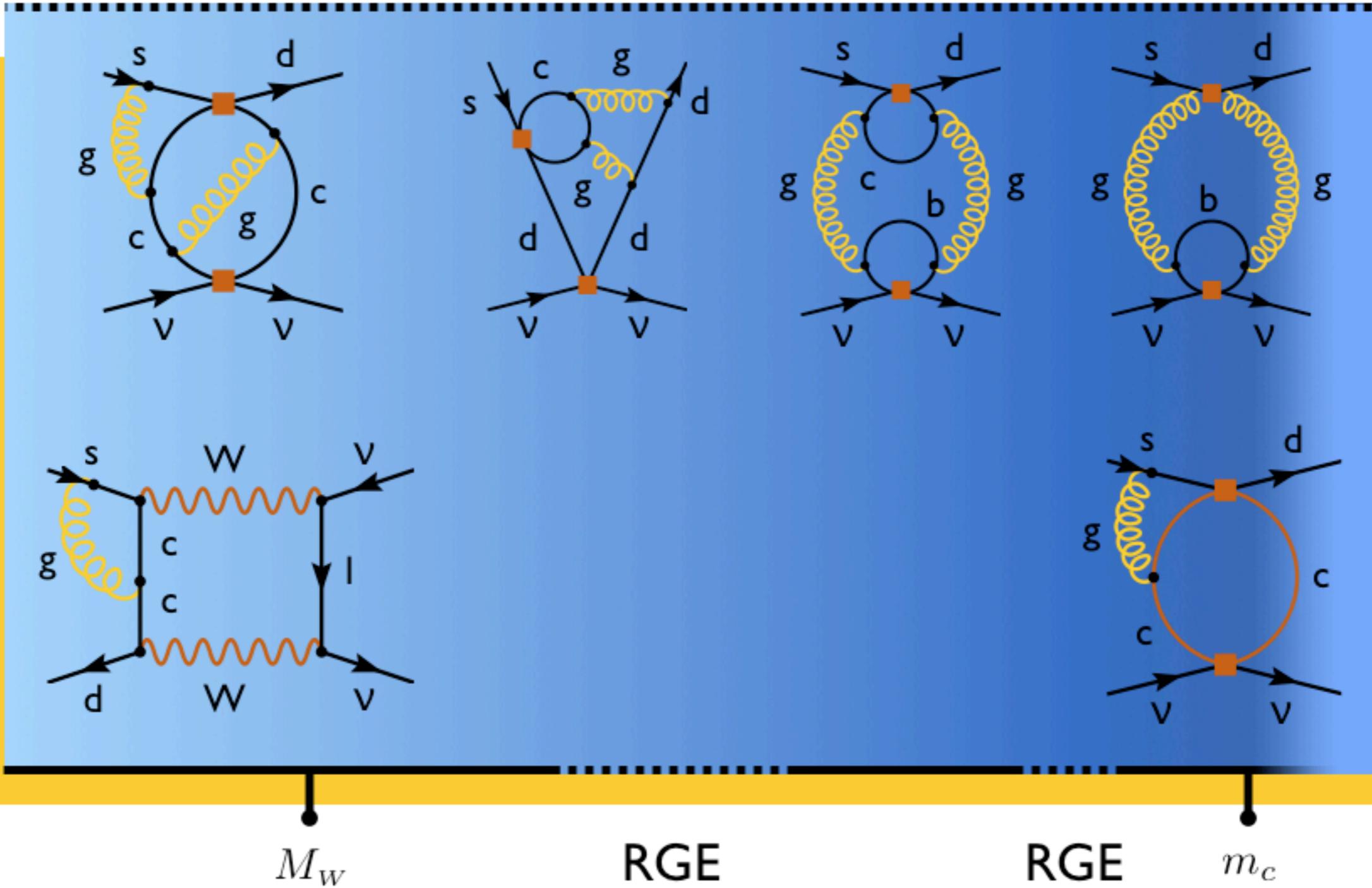
- μ_c
- $m_c(m_c)$
- $\alpha_s(\mu_c)$
- $\alpha_s(M_Z)$



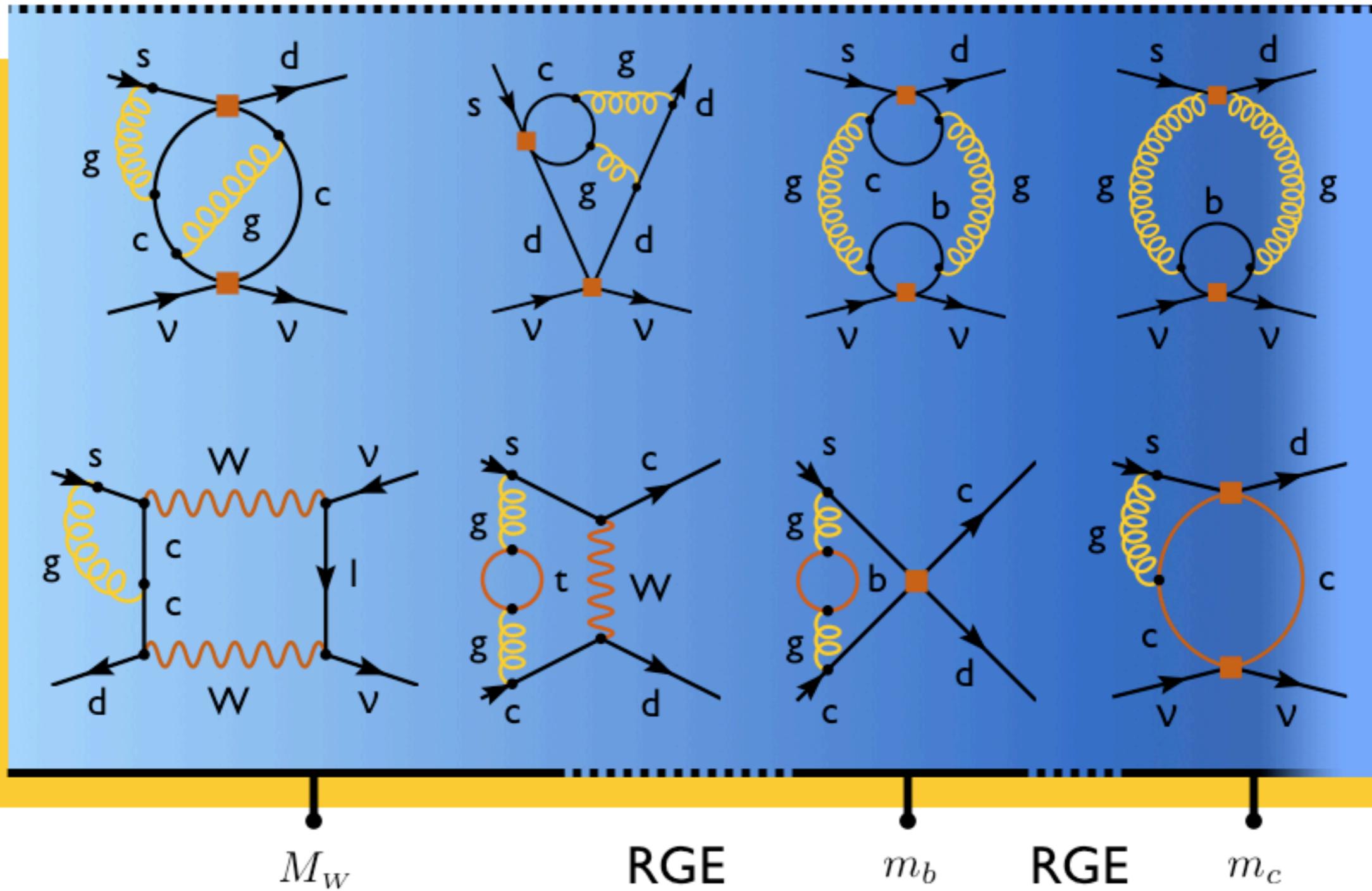
Flavour of NNLO calculation of P_c



Flavour of NNLO calculation of P_c

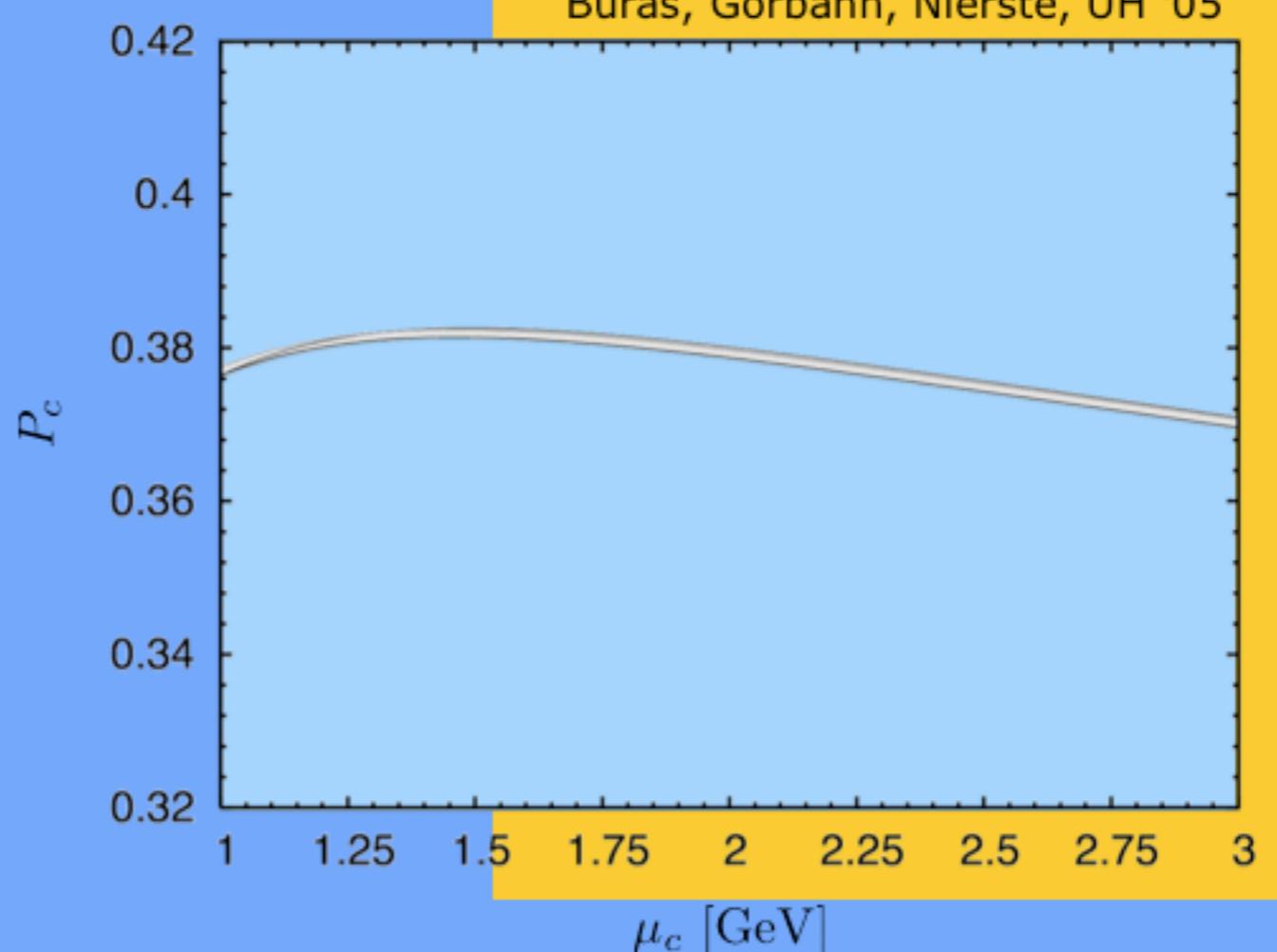
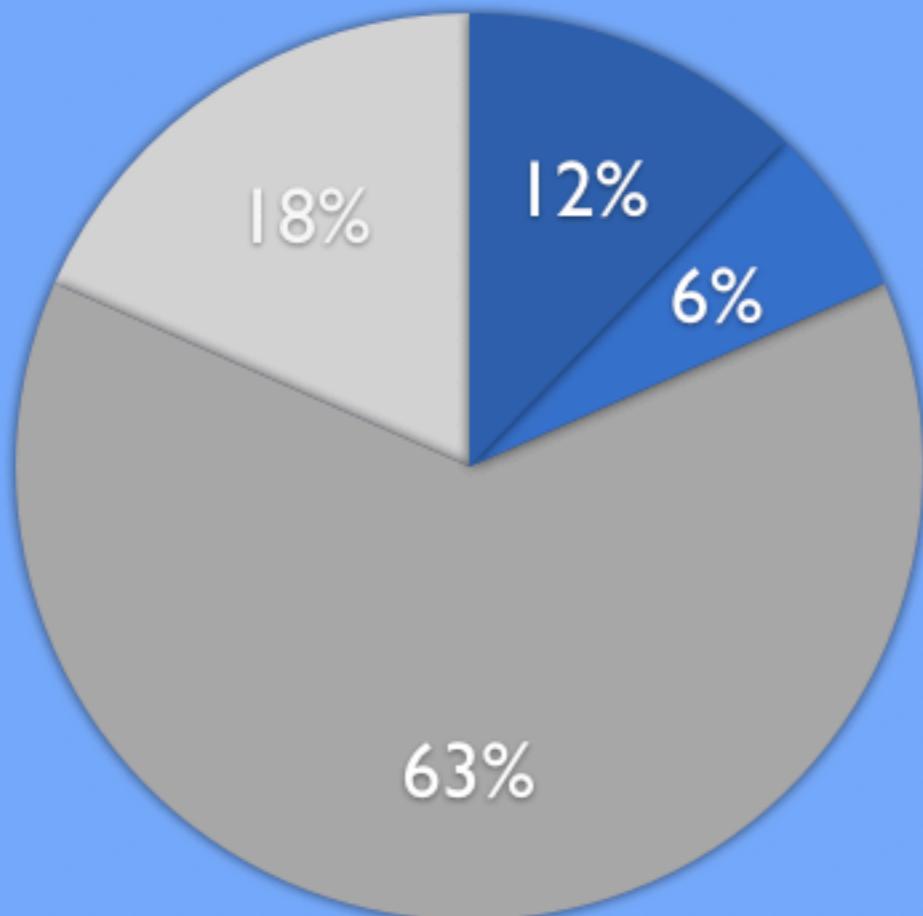


Flavour of NNLO calculation of P_c



Error Budget of P_c at NNLO

$$P_c = 0.375 \pm 0.009 \pm 0.031 \pm 0.009$$

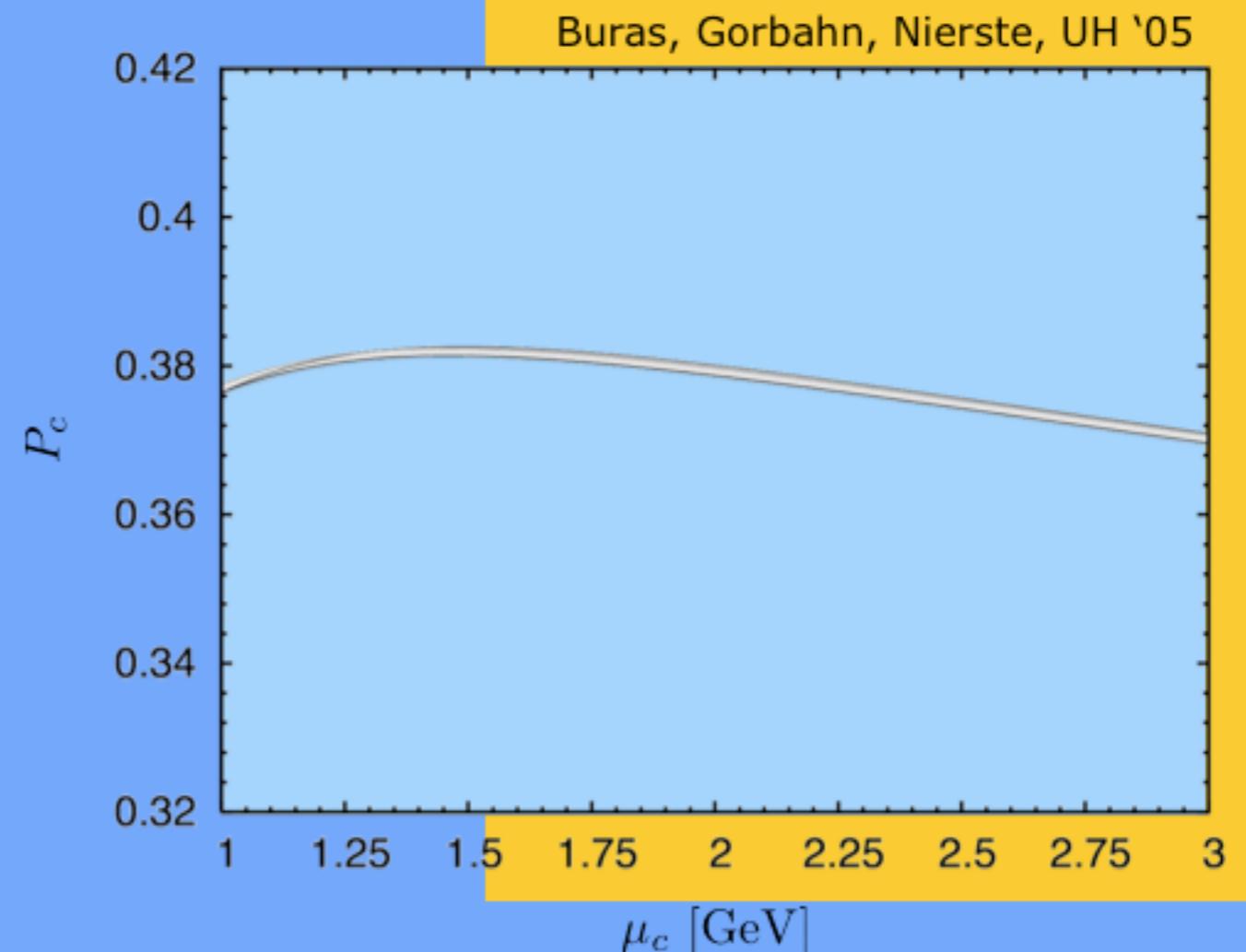


- μ_c
- $m_c(m_c)$
- $\alpha_s(\mu_c)$
- $\alpha_s(M_Z)$

Error Budget of P_c at NNLO

$$P_c = 0.375 \pm 0.009 \pm 0.031 \pm 0.009$$

- * NNLO calculation improves theoretical uncertainty in P_c by factor ~ 4
- * good convergence of perturbation series at charm scale
- * uncertainty of P_c now dominated by error in m_c



Progress and Prospects in δP_c

$$\mathcal{L}_{\text{eff}}^{(8)} = \frac{4G_F}{\sqrt{2}} \frac{\alpha}{2\pi s_W^2} \frac{V_{cs}^* V_{cd}}{M_W^2} \sum_{l=e,\mu,\tau} \sum_i C_i^l(\mu) Q_i^{l(8)}$$

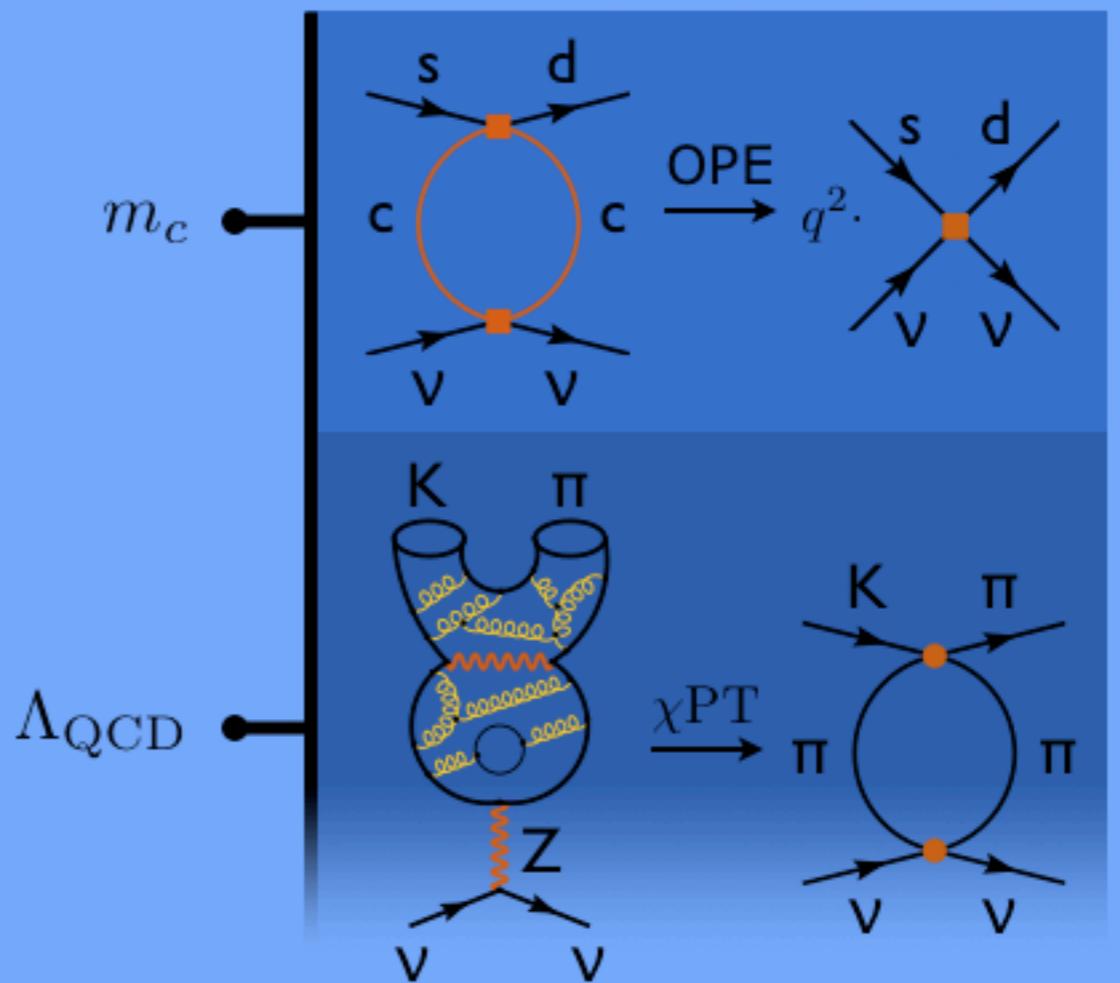
$$Q_1^{l(8)} = (\bar{s}_L \gamma_\mu d_L) \partial^2 (\bar{\nu}_L \gamma^\mu \nu_L), Q_2^{l(8)} = (\bar{s}_L \overleftarrow{D}_\nu \gamma_\mu \overrightarrow{D}^\nu d_L) (\bar{\nu}_L \gamma^\mu \nu_L), \dots$$

u
c

Falk, Lewandowski & Petrov '00

- * local charm effects due to dimension-eight operators naively of $O(m_K^2/m_c^2) \sim 15\%$

- * genuine long-distance up effects of $O(\Lambda_{\text{QCD}}^2/m_c^2) \sim 10\%$



Progress and Prospects in δP_c

$$\mathcal{L}_{\text{eff}}^{(8)} = \frac{4G_F}{\sqrt{2}} \frac{\alpha}{2\pi s_W^2} \frac{V_{cs}^* V_{cd}}{M_W^2} \sum_{l=e,\mu,\tau} \sum_i C_i^l(\mu) Q_i^{l(8)}$$

$$Q_1^{l(8)} = (\bar{s}_L \gamma_\mu d_L) \partial^2 (\bar{\nu}_l L \gamma^\mu \nu_L), Q_2^{l(8)} = (\bar{s}_L \overleftarrow{D}_\nu \gamma_\mu \overrightarrow{D}^\nu d_L) (\bar{\nu}_l L \gamma^\mu \nu_L), \dots$$

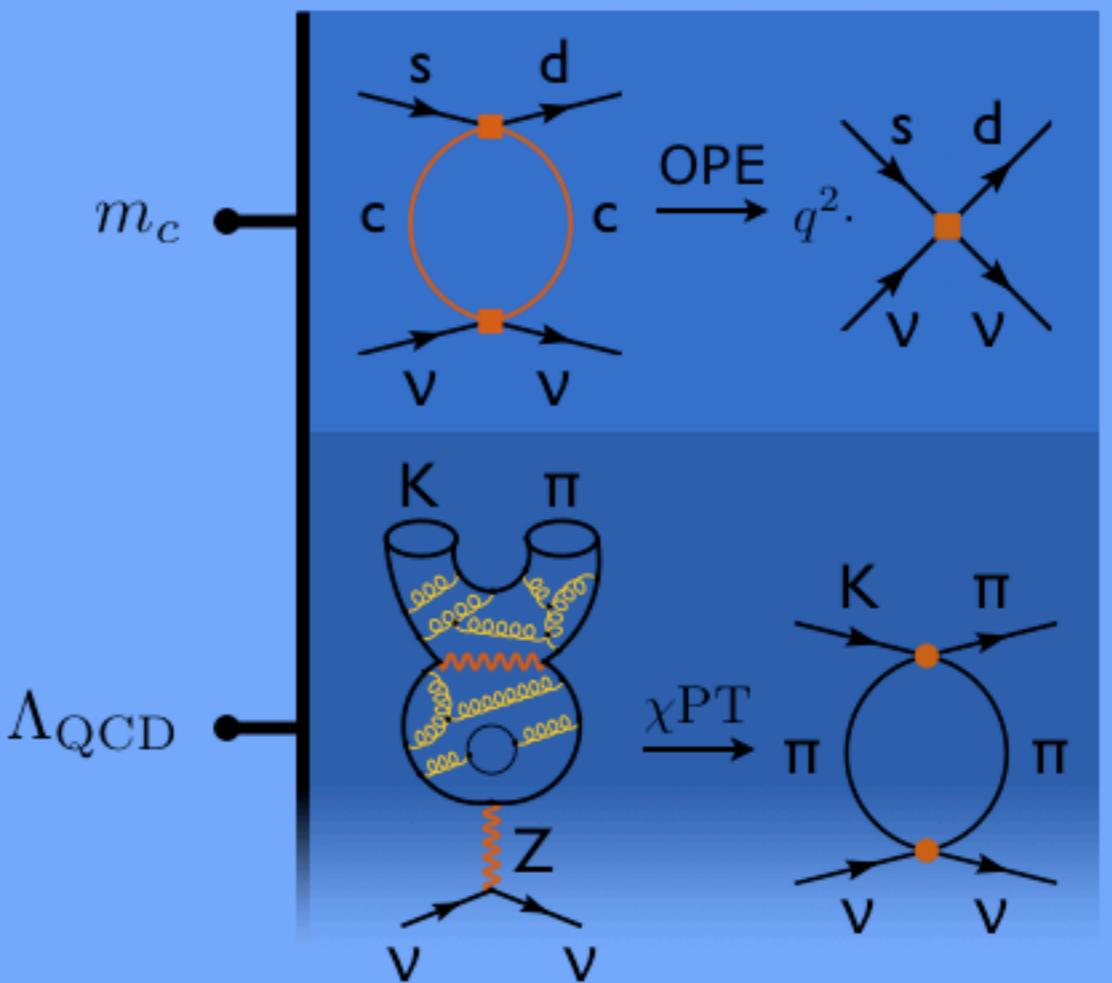
u
c

Falk, Lewandowski & Petrov '00

- * effects scale as $O(\pi^2 F_\pi^2 / m_c^2)$
 $\sim 5\%$ and increase SM branching ratio by $\sim 6\%$

- * theoretical errors associated to non-perturbative effects can be reduced with LQCD

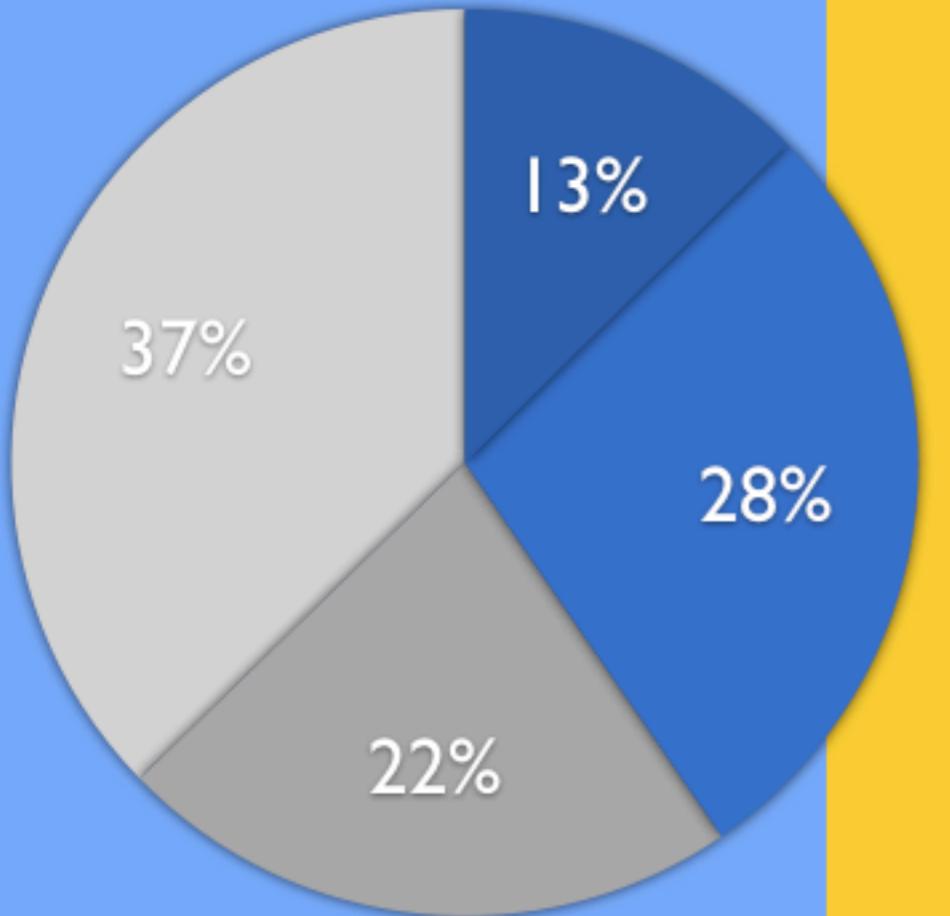
Isidori, Mescia & Smith '05;
 Isidori, Martinelli & Turchetti '05



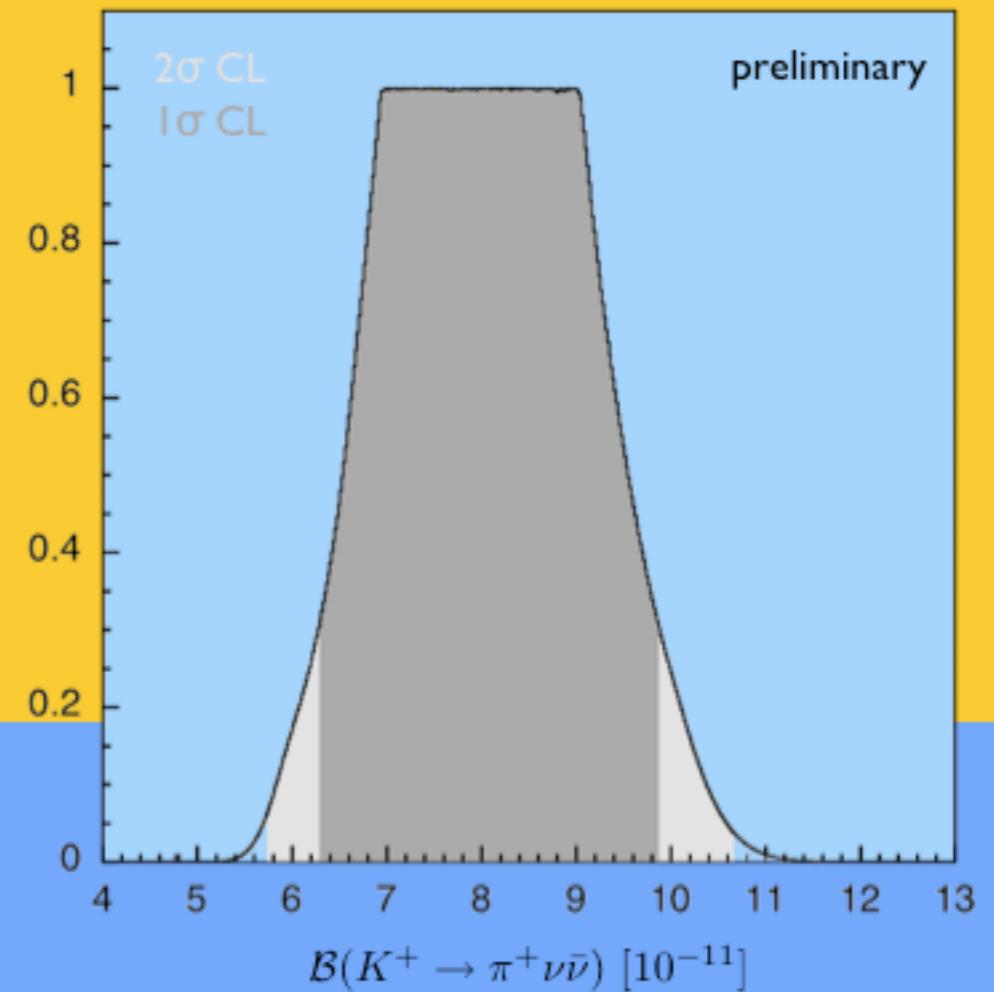
NLO SM Prediction of $K^+ \rightarrow \pi^+ \nu \bar{\nu}$

$$\mathcal{B}(K^+ \rightarrow \pi^+ \nu \bar{\nu}) = (8.0 \pm 0.2 \pm 0.5 \pm 0.4 \pm 0.7) \times 10^{-11}$$

CKMFitter, Buras, Gorbahn, Nierste & UH '06



- scales
- $r_{K^+}, \delta P_c$
- $m_c(m_c)$
- $M_t, \alpha_s(M_Z), \text{CKM}$

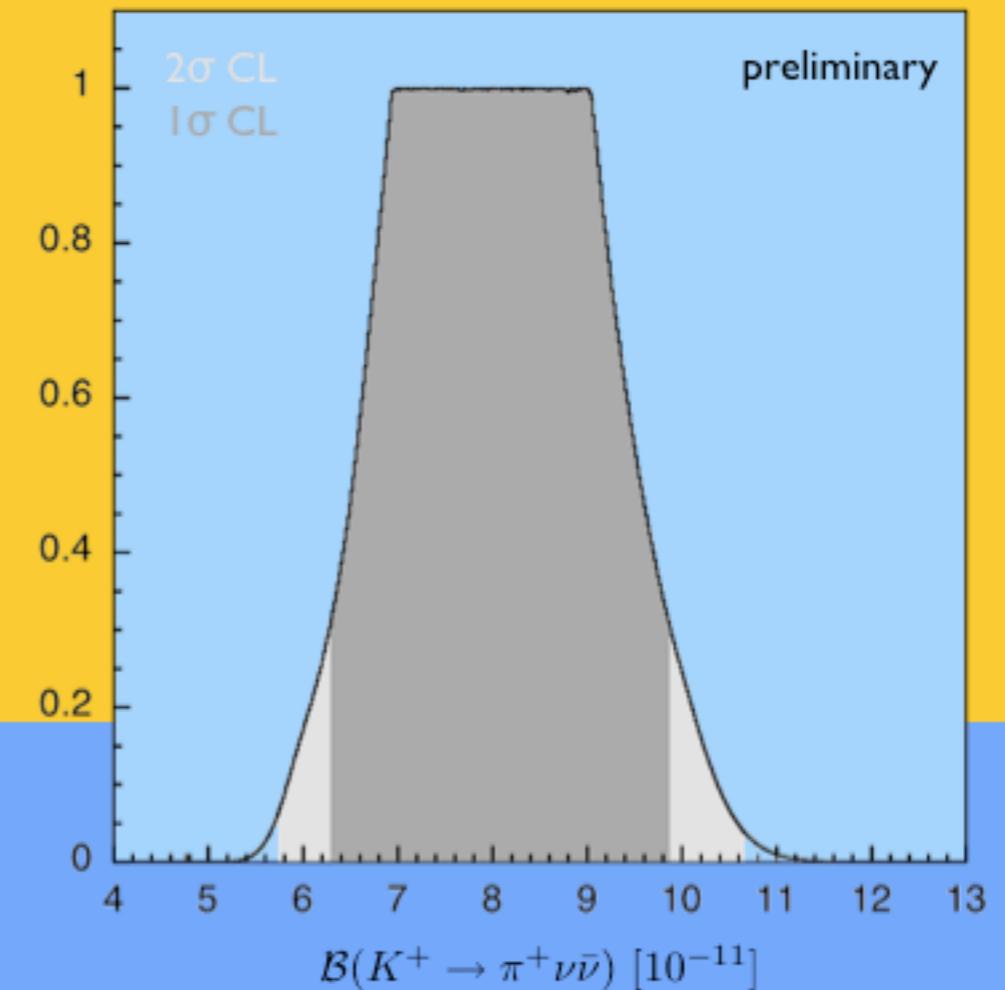


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CKMFitter, Buras, Gorbahn, Nierste & UH '06

- * scale uncertainties are no longer dominant source of error in SM branching ratio
- * intrinsic theoretical errors due to r_{K^+} and δP_c come to fore
- * precise determinations of m_c gain further importance



CKM Fit from $K \rightarrow \pi \nu \bar{\nu}$

$$\mathcal{B}(K_L \rightarrow \pi^0 \nu \bar{\nu}) = (2.67 \pm 0.27) \times 10^{-11}$$

$$\mathcal{B}(K^+ \rightarrow \pi^+ \nu \bar{\nu}) = (7.50 \pm 0.75) \times 10^{-11}$$

$$\frac{\sigma(|V_{td}|)}{|V_{td}|} = \pm 4.0\%$$

$$\sigma(\sin 2\beta) = \pm 0.024$$

$$\sigma(\gamma) = \pm 4.7^\circ$$

$$\frac{\sigma(|V_{td}|)}{|V_{td}|} = \pm 1.0\%$$

$$\sigma(\sin 2\beta) = \pm 0.006$$

$$\sigma(\gamma) = \pm 1.2^\circ$$

Future (?)

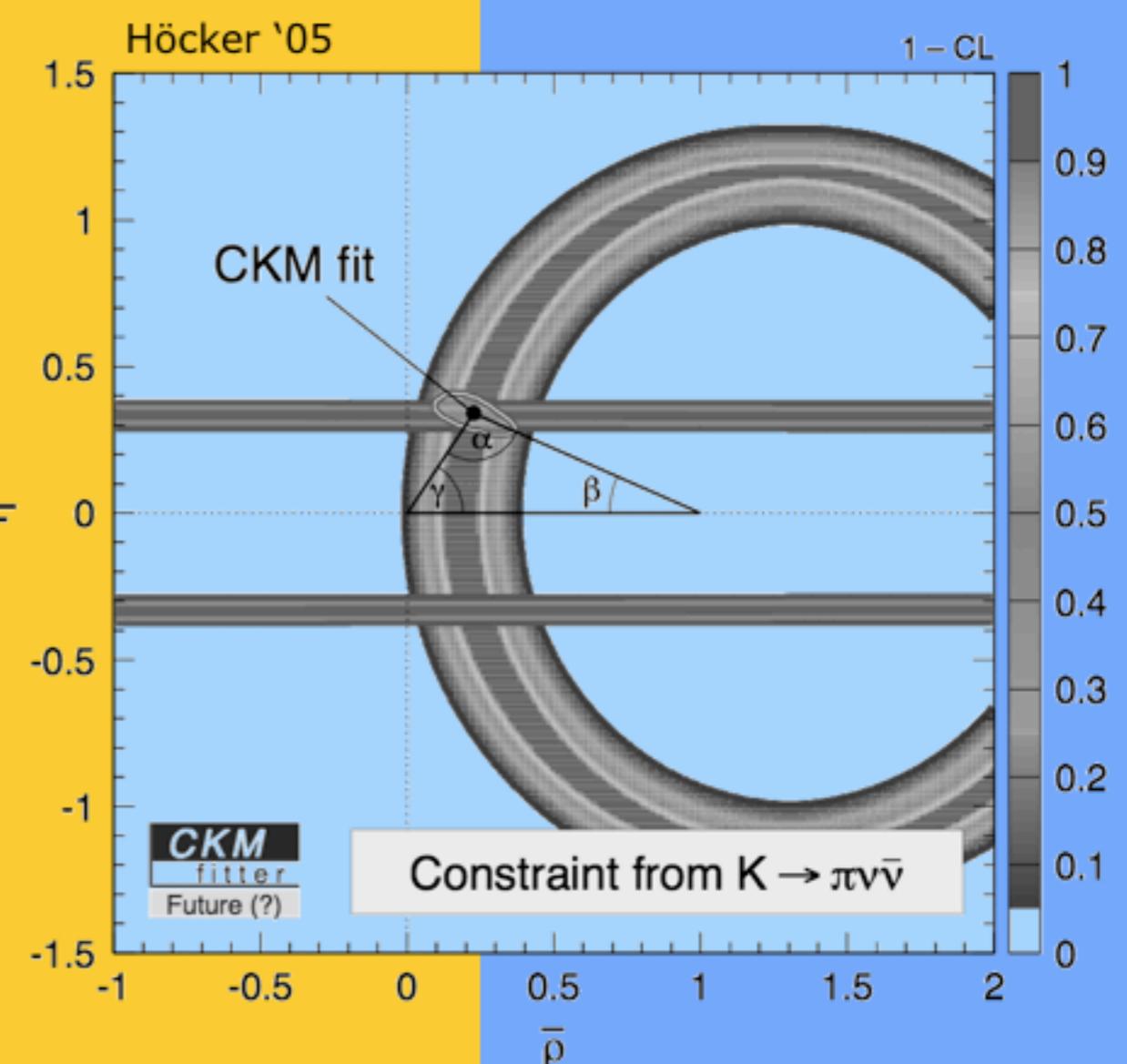
NLO

(theory error in P_c only)

NNLO

(theory error in P_c only)

Buras, Gorbahn, Nierste & UH '05



CKM Fit from $K \rightarrow \pi \nu \bar{\nu}$

$$\mathcal{B}(K_L \rightarrow \pi^0 \nu \bar{\nu}) = (2.67 \pm 0.27) \times 10^{-11}$$

$$\mathcal{B}(K^+ \rightarrow \pi^+ \nu \bar{\nu}) = (7.50 \pm 0.75) \times 10^{-11}$$

$$\frac{\sigma(|V_{td}|)}{|V_{td}|} = \pm 1.0\%$$

$$\sigma(\sin 2\beta) = \pm 0.006$$

$$\sigma(\gamma) = \pm 1.2^\circ$$

$$\frac{\sigma(|V_{td}|)}{|V_{td}|} = \pm 8.4\%$$

$$\sigma(\sin 2\beta) = \pm 0.056$$

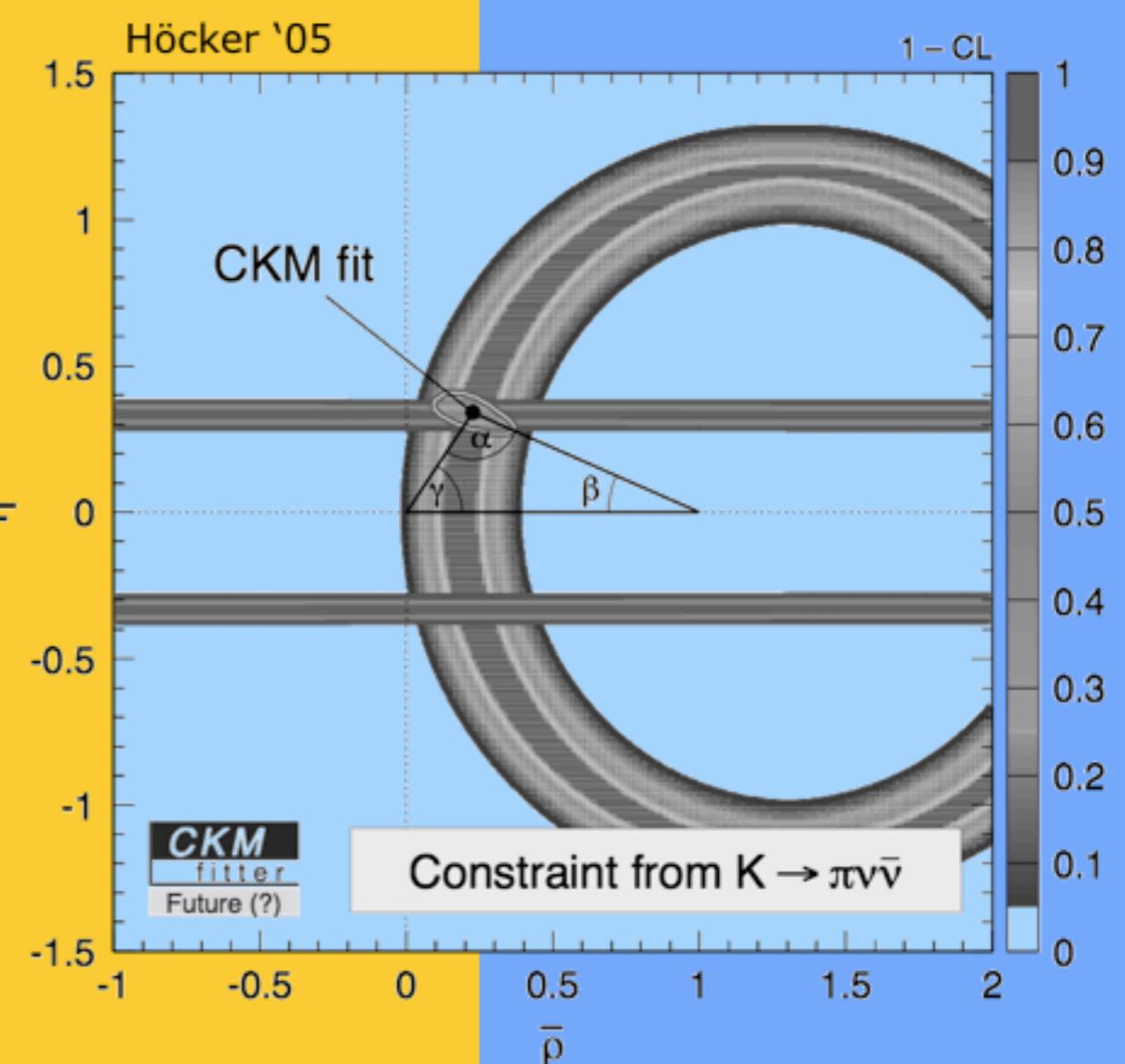
$$\sigma(\gamma) = \pm 11^\circ$$

Future (?)

Future (?)

NNLO
(theory error in P_c only)

NNLO
(all uncertainties)

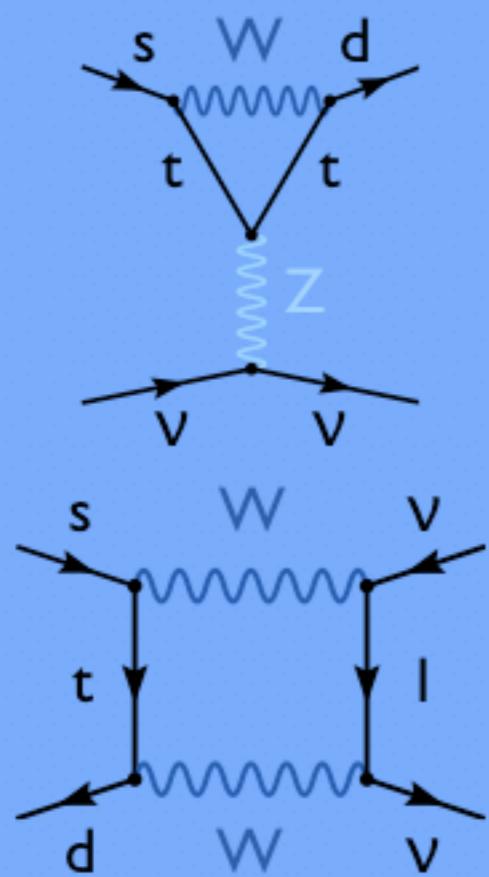


Conclusions and Outlook

- * After NNLO calculation of dimension-six charm contribution scale ambiguities in K^+ no longer dominant source of error
- * Further theoretical improvements concerning isospin-breaking, long-distance, higher-order electroweak effects and charm mass desirable
- * Measurements of branching ratios at $\sim 10\%$ would substantially improve our understanding of flavor dynamics at TeV scale

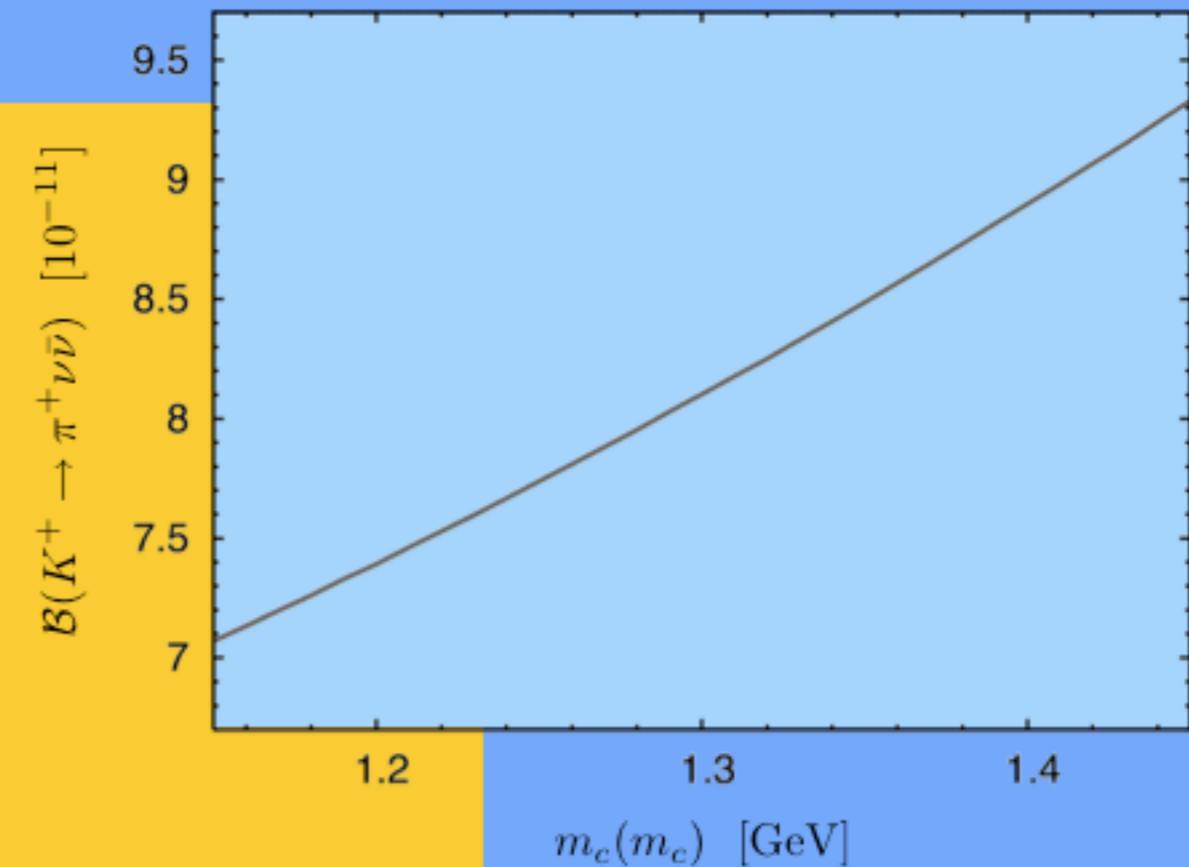
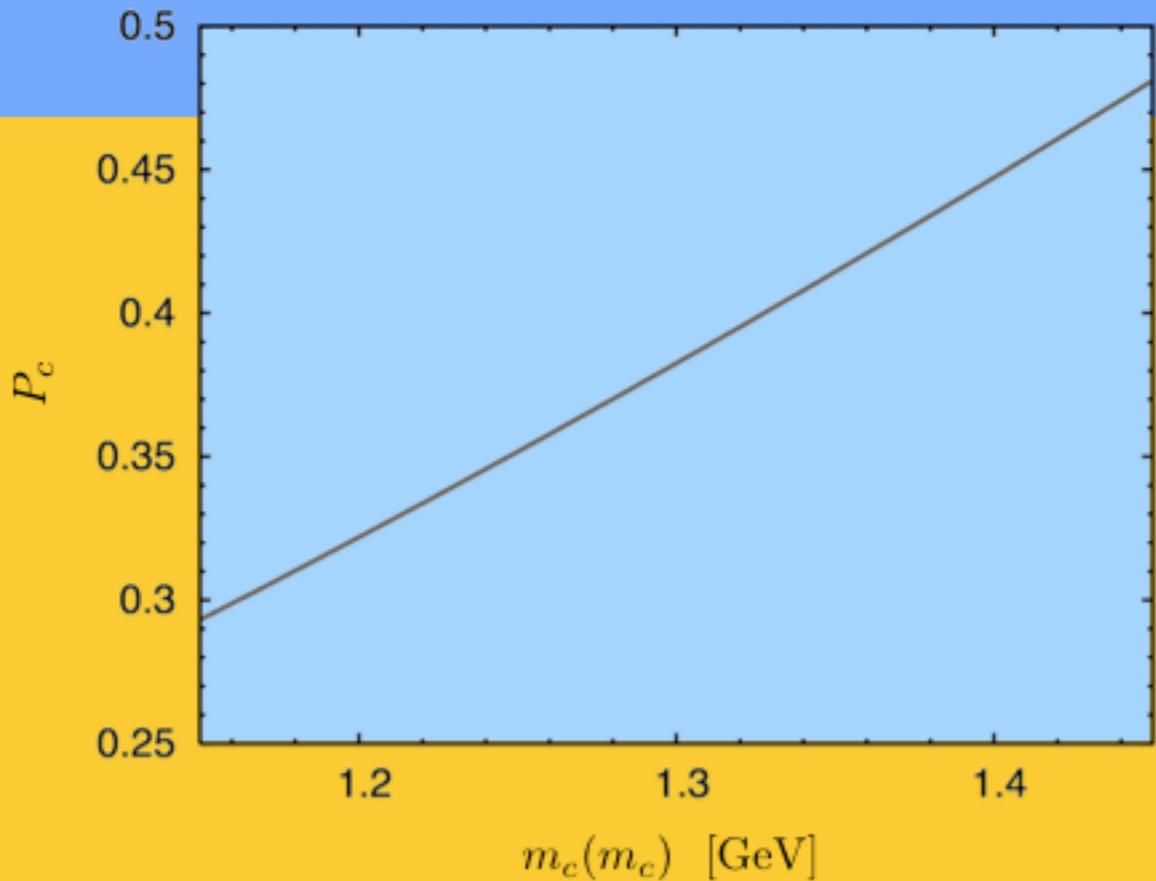


Credits



Andrzej Buras,
Andreas Höcker,
Martin Gorbahn,
Ulrich Nierste &
Jose Ocariz

Dependences of P_c and $\mathcal{B}(K^+ \rightarrow \pi^+ \nu \bar{\nu})$



$$P_c = 0.379 \left(\frac{m_c(m_c)}{1.3 \text{ GeV}} \right)^{2.155} \left(\frac{\alpha_s(M_Z)}{0.1187} \right)^{-1.417}$$

$$\mathcal{B}(K^+ \rightarrow \pi^+ \nu \bar{\nu}) = 8.15 \left(\frac{m_c(m_c)}{1.3 \text{ GeV}} \right)^{1.19} \times 10^{-11}$$

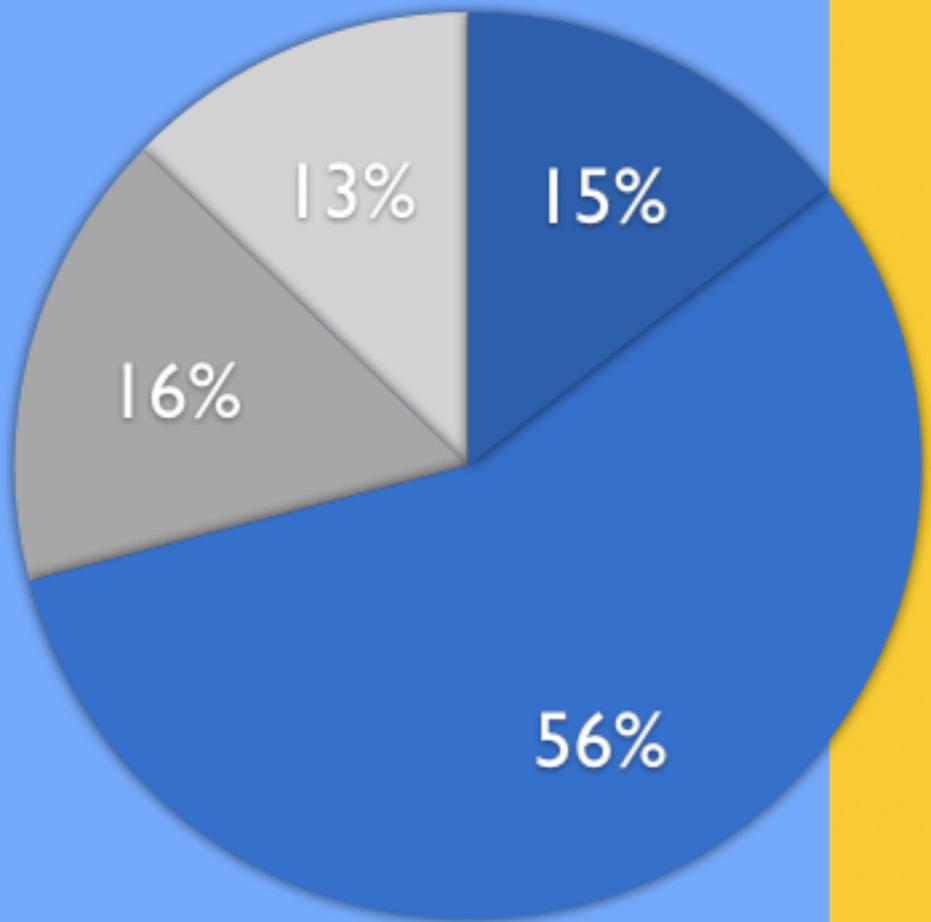
Buras, Gorbahn, Nierste & UH '06



SM Prediction of P_c

$$P_c = 0.367 \pm 0.008 \pm 0.031 \pm 0.009 \pm 0.007$$

CKMFitter, Buras, Gorbahn, Nierste & UH '06

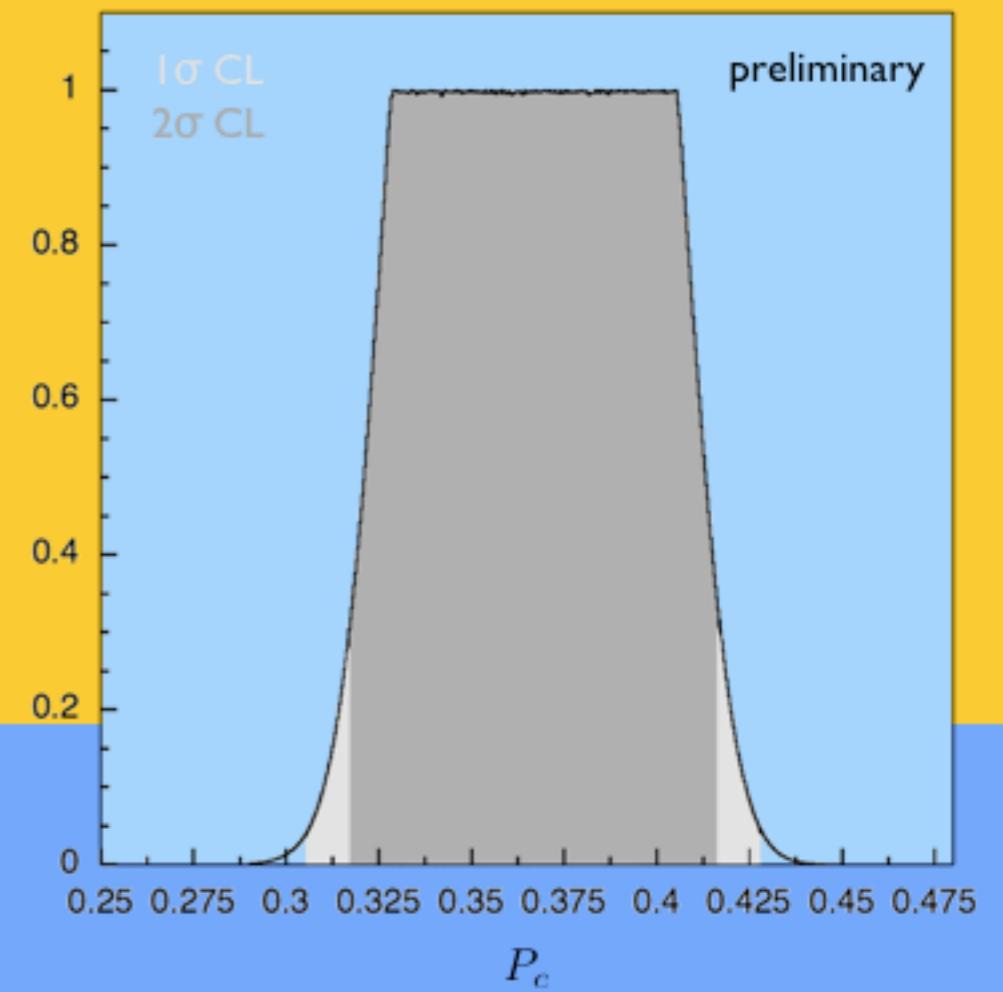


● scales

● $\alpha_s(M_Z)$

● $m_c(m_c)$

● λ



Error Budget of $\mathcal{B}(K^+ \rightarrow \pi^+ \nu \bar{\nu})$

	$\mathcal{B}(K^+ \rightarrow \pi \nu \bar{\nu}) [10^{-11}]$	
method	central \pm CL $\equiv 1\sigma$	\pm CL $\equiv 2\sigma$
all	8.00 ± 1.80	± 2.45
$\mu_c, \alpha_s(\mu_c)$	8.15 ± 1.65	± 2.35
no scales	8.07 ± 1.56	± 2.25
δP_c	8.04 ± 1.32	± 2.01
r_{K^+}	8.01 ± 1.08	± 1.76
$m_c(m_c)$	7.99 ± 0.71	± 1.47

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