News on PHOTOS Monte Carlo: issue of systematic errors.

Z. Was

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talk include contributions from:

P. Golonka CERN IT/CO-BE, Geneva, Institute of Nuclear Physics, Krakow

G. Nanava JINR, Dubna, Russia, Institute of Nuclear Physics, Krakow

E. Barberio Melbourne University, Australia

and:

Other members of BELLE and NA48 Collab.

Web pages: http://wasm.home.cern.ch/wasm/goodies.html
http://piters.home.cern.ch/piters/MC/PHOTOS-MCTESTER/

Z. Was CERN, February 2006
PHOTOS Monte Carlo is widely used by eg. Belle/Babar TEVATRON/LHC.

Its basic design remained nearly unchanged since 1994, however:


Complete NLO corrections in Z decays were introduced into PHOTOS and appropriate tests were completed.

PHOTOS results were compared with respect to prediction of scalar QED for the first time.

Formal aspects of PHOTOS algorithm I have forwarded to March 2nd seminar in CERN-TH.

I may recall some results on PHOTOS for the B-meson decays into final states with leptons.
• PHOTOS (by E. Barberio, B. van Eijk, Z. W., P. Golonka) is used to simulate the effect of radiative corrections in decays, since 1989.

• Events of complicated tree structure: production and subsequent decays fed into PHOTOS, usually though HEPEVT event record of F77.

• This is often source of technical difficulties as standard is often overruled.

• At every event decay branching, PHOTOS intervene. With certain probability extra photon may be added and kinematics of other particles adjusted.

• PHOTOS works on four-momenta four-vectors. Fun with numerical stability.

• I will not talk about those time consuming aspects but about relation of PHOTOS with explicit field theory calculations, n-body phase space, and expansions with respect to leadin-log truncation and eikonal-truncation.

• See my March 2nd talk:
• for many years PHOTOS was exploited as low precision tool.
• Nonetheless it can provide extremely precise and fast simulations.
• This fact may be worth publicising! PHOTOS solution is unique!
• It has no free hidden parameters
• Unless QED lagrangian is replaced with effective one (for hadrons).

Main References

**Why worry about QED?**

Before going into details let me show an example where PHOTOS was used.

Most of you probably know it better than be.

It should illustrate the complex nature of interplay between: (i) QED (ii) experimental cuts (iii) signal-backgound separation

**For me it is just an example to illustrate how QED can produce 10% correction to the rate ...**

At the end of my talk I may return to the point of PHOTOS systematic error for this type of processes.

I did something wrong with encapsulating PDF’s some characters are corrupted.
**electron endpoint**

![Graph showing electron endpoint with signal and background regions.]

**Systematic uncertainty:**
Model dependent signal efficiencies

\[
\Delta Br(X_u 1\nu) = \frac{N(X_u 1\nu)}{2N_{BB} \varepsilon_{MC}}
\]

\[
|V_{ub}| = \sqrt{\frac{(1 + \delta_{rad}) \times \Delta Br(X_u 1\nu)}{\tau_B R}}
\]

*Phys. Lett. B 621, 28 (2005)*
### QED radiative corrections: $b \to Y u$ endpoint

<table>
<thead>
<tr>
<th>Lepton momentum region (GeV)</th>
<th>PHOTOS correction to partial branching fraction measurement of $b \to Y u \ell Y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.9-2.6</td>
<td>$1.060 \pm 0.007$</td>
</tr>
<tr>
<td>2.2-2.6</td>
<td>$1.086 \pm 0.010$</td>
</tr>
<tr>
<td>2.3-2.6</td>
<td>$1.096 \pm 0.011$</td>
</tr>
<tr>
<td>2.4-2.6</td>
<td>$1.107 \pm 0.014$</td>
</tr>
</tbody>
</table>
My Talk

- For 15 years PHOTOS was developed as ‘cheap tool’ and its theoretical foundations were left ‘in dark’.
- I apologize.
- Recently its use entered the field of high responsibility: precision CKM, W mass, W coupling etc
- My old approach to rely mainly on comparisons with matrix element calculations to justify PHOTOS with comparisons is succesful, but not anymore sufficient.
- Nonetheless today I will present mainly numerical results:
  – comparisons of PHOTOS running at fixed first order with first order QED matrix element calculations
  – comparisons of PHOTOS running with multiple radiation with precision Monte Carlo KKMC for Z decay, based on second order matrix element and exclusive exponentiation.
An essential Building Block is single emission kernel

In 1991 universal emission kernel was defined for PHOTOS, thanks to approximation for explicit Matrix El. \((Z \rightarrow \mu^+\mu^- (\gamma))\).

Without approximation, kernel is valid only for \(Z \rightarrow \mu^+\mu^-\)

Define transformation from n-body phase space (no bremsstrahlung) to (n+1)-body phase space with photon present. Also no emission probability.

For the phase space parametrisation like in FOWL of F. James (70’s). I used variant of that in TAUOLA too, see references for details.

Exact kernel differs from decay channel to channel. Has to be obtained from orthodox first order calculation.

Let us present new results of tests for Kernel performance; case of scalar charged products in \(B\) meson decays (scalar QED used).
Comparison of SANC with PHOTOS for non-leptonic B Meson Decays
Work Report

Nanava Gizo
Laboratory of Nuclear Problems, Joint Institute for Nuclear Research, Dubna, Russia
(On leave from IHEP, TSU, Tbilisi, Georgia)

- Exercise
- SANC
- Scalar QED Lagrangian
- Radiative Corrections
- Comparison with PHOTOS
- Conclusion
Radiative Corrections

Diagrams of order $O(\alpha)$ contributing to $B^0 \to \pi^- \pi^+ \gamma$
The QED radiative corrections modify the lowest order decay rate

\[ d\Gamma^{\text{One-loop}}_{B^0 \to \pi^+ \pi^- \gamma}(\Lambda_{\text{QED}}) = d\Gamma^{\text{Born}}_{B^0 \to \pi^+ \pi^-}(1 + \delta^{\text{Virt+Soft}}(\Lambda_{\text{QED}}, \omega)) \]

\[ + d\Gamma^{\text{Hard}}_{B^0 \to \pi^+ \pi^- \gamma}(\omega) \]

\[ \delta^{\text{Virt+Soft}}(\Lambda_{\text{QED}}, \omega) = 2 \Re [\delta^{\text{Virt}}(\Lambda_{\text{QED}}, m_{\gamma})] + \delta^{\text{Soft}}(m_{\gamma}, \omega) \]

- IR pole, which is represented here by the photon mass, cancels in the sum
- However, dependence on the Ultraviolet Renormalization Scale remains
- SANC Monte Carlo for radiative B Meson decays is based on this formula
Comparison With PHOTOS

- We used the same methodology as in the case of “W decay” 

- To visualize the usually small differences between SANC and 
  PHOTOS, we plot the ratios of the predictions from the two programs 
  for the certain class of (pseudo-)observables

- These observables are
List of Observables

- **Photon energy in the decaying particle rest frame** – sensitive to the collinear–soft component of the distributions

- **Energy of final state charged particle** – as the previous one

- **Angle of photon with final-state charged particle** - sensitive to the collinear component of the distributions

- **Acollinearity angle of the final-state particles** - sensitive to the non-soft and non-collinear, but non-leading component of the distributions
Single emission, scalar QED

Transparency of G. Nanava

Photon Energy

Pion Energy

Photon angle with res. $\pi^-$

$\pi^+\pi^-$ Acoll. Angle

$\Lambda_{QED} = M_\pi$

$\Lambda_{QED} = M_{B^0}$

$\Lambda_{QED} \rightarrow$ Ultraviolet Renormalization Scale

$B^0 \rightarrow \pi^- \pi^+ \gamma$

$M_\pi = 139.57$ MeV

$M_{B^0} = 5279.4$ MeV

$\omega$ (soft/hard cut-off) = $M_{B^0}$ / 200 MeV

12/12/05

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Single emission, scalar QED

Transparency of G. Nanava

\[ \Lambda_{QED} = M_{B^0} \]
\[ \lambda_{QED} \rightarrow \text{Ultraviolet Renormalization Scale} \]
\[ B^0 \rightarrow \pi^- K^+ \gamma \]

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**Transparency of G. Nanava**

**Photon Energy**

**\( E_\gamma \text{ MeV} \)**

**\( E_\pi \text{ MeV} \)**

**Photon angle with res. \( \pi^- \)**

**\( \cos \theta_{\gamma} \)**

**\( \pi^0 \pi^- \text{ Acoll. Angle} \)**

**\( \cos \theta_{\text{Acoll}} \)**

\[ \Lambda_{\text{QED}} = M_B \]

\[ \Lambda_{\text{QED}} \rightarrow \text{Ultraviolet Renormalization Scale} \]

\[ B^- \rightarrow \pi^- \pi^0 \gamma \]

\[ \begin{align*}
M_B &= 5279.57 \text{ MeV} \\
M_{\pi^-} &= 134.98 \text{ MeV} \\
M_{\pi^0} &= 139.57 \text{ MeV} \\
\omega \text{ (soft/hard cut-off)} &= M_B / 200 \text{ MeV}
\end{align*} \]
Single emission, scalar QED

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Photon Energy

K\(^{-}\) Energy

Photon angle with res. K

\(\pi^{0}K\) Acoll. Angle

\[ \Lambda_{\text{QED}} = M_{B} \quad \Lambda_{\text{QED}} = 770 \text{ MeV} \]

\( \Lambda \rightarrow \) Ultraviolet Renormalization Scale

\[ B^{-} \rightarrow K^{-} \pi^{0} \gamma \]

12/12/05

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Conclusion

- Comparisons of SANC with PHOTOS Monte Carlo generator were made for non-leptonic B Meson decays within the Scalar QED Lagrangian. The results from both two programs for all observables agree within the expected precision range.
- In future it is possible to study other channels and more complicated Lagrangians.
- Some notes about the work was done will be written (I could not do it now because of other tasks).
We could see that approximated kernel of PHOTOS performed reasonably well once confronted with scalar QED.

Does scalar QED make sense for $B \rightarrow hh$ decays?

It is good to be able to start answering such a question having QED predictions reproduced first. No modelling!

For many decay channels emission kernels will have to be modified with shape-factors

These will have to be obtained from experimental data.

PHOTOS profitted from that a lot, already now.
Single emission in $Z$; NLO kernel

**Kernel and exact matrix element**

- From the point of view of matrix element and choice of internal angular variables in case of $Z$ decays, PHOTOS is very close to MUSTRAAL MC by F. Berends S. Jadach and R. Kleiss, CPC 29 (1983) 185.

- The first step to re-introduce NLO terms for $Z$ decay into PHOTOS was to check relations between 4-vectors and angles.

- For PHOTOS those angles are ill-defined in general. incoming beams (defining $Z$ polarisations) have to be along $z$ axis.

- To match this requirement PHOTOS has to read in events, appropriately orientined Born level events generator.

- The fully differential distribution from MUSTRAAL (used also in KORALZ for
Single emission in $\mathcal{Z}$; NLO kernel

single photon mode) reads:

$$X_f = \frac{Q'^2 \alpha (1 - \Delta)}{4 \pi^2 s} s^2 \left\{ \frac{1}{(k'_+ k'_-)} \left[ \frac{\mathrm{d}\sigma_B}{\mathrm{d}\Omega}(s, t, u') + \frac{\mathrm{d}\sigma_B}{\mathrm{d}\Omega}(s, t', u) \right] \right\}$$

- Here:

  $$s = 2p_+ \cdot p_-, \quad s' = 2q_+ \cdot q_-,$$
  $$t = 2p_+ \cdot q_+, \quad t' = 2p_+ \cdot q_-,$$
  $$u = 2p_+ \cdot q_-, \quad u' = 2_+ \cdot q_+,$$
  $$k'_\pm = q_\pm \cdot k, \quad x_k = 2E_\gamma / \sqrt{s}$$

- The $\Delta$ term is responsible for final state mass dependent terms, $p_+, p_-, q_+$, $q_-$, $k$ denote four-momenta of incoming positron, electron beams, outcoming muons and bremsstrahlung photon.
Single emission in $Z$; NLO kernel

- after trivial manipulation it can be written as:

$$X_f = \frac{Q'^2 \alpha (1 - \Delta)}{4\pi^2 s} s^2 \left\{ \frac{1}{(k'_+ + k'_-)} \frac{1}{k'_-} \left[ \frac{d\sigma_B}{d\Omega} (s, t, u') + \frac{d\sigma_B}{d\Omega} (s, t', u) \right] + \frac{1}{(k'_+ + k'_-)} \frac{1}{k'_+} \left[ \frac{d\sigma_B}{d\Omega} (s, t, u') + \frac{d\sigma_B}{d\Omega} (s, t', u) \right] \right\}$$

- In PHOTOS the following expression is used:

$$X_f^{PHOTOS} = \frac{Q'^2 \alpha (1 - \Delta)}{4\pi^2 s} s^2 \left\{ \frac{1}{(k'_+ + k'_-)} \frac{1}{k'_-} \left[ \left( (1 - x_K)^2 \right) \frac{d\sigma_B}{d\Omega} (s, \frac{s(1 - \cos \Theta_+)}{2}, \frac{s(1 + \cos \Theta_+)}{2}) \right] \frac{(1 + \beta \cos \Theta_+)}{2} \right\}$$

$$+ \frac{1}{(k'_+ + k'_-)} \frac{1}{k'_+} \left[ \left( (1 - x_K)^2 \right) \frac{d\sigma_B}{d\Omega} (s, \frac{s(1 - \cos \Theta_-)}{2}, \frac{s(1 + \cos \Theta_-)}{2}) \right] \frac{(1 - \beta \cos \Theta_-)}{2} \right\}$$

where: $\Theta_+ = \langle p_+, q_+ \rangle$, $\Theta_- = \langle p_-, q_- \rangle$

$\Theta_\gamma = \langle \gamma, \mu^- \rangle$ are defined in $(\mu^+, \mu^-)$-pair rest frame

- The two expressions define weight to make out of PHOTOS complete first order.
Found decay modes:

<table>
<thead>
<tr>
<th>Decay channel</th>
<th>Branching Ratio $\pm$ Rough Errors</th>
<th>Max. shape dif. param.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Z^0 \rightarrow \mu^- \mu^+$</td>
<td>$82.5137 \pm 0.0091%$</td>
<td>0.0000</td>
</tr>
<tr>
<td>$Z^0 \rightarrow \mu^- \mu^+\gamma$</td>
<td>$17.4863 \pm 0.0042%$</td>
<td>0.00534</td>
</tr>
</tbody>
</table>

Similarity coefficients: $T1=0.302959\%$, $T2=0.092415\%$
Single emission in $Z$; NLO kernel

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**Standard PHOTOS, (singl em.) 100 Mevts compared!**

1. **Decay Channel: $Z^0 \rightarrow \mu^- \mu^+$**

   Number of events from generator 1: 82513687
   Number of events from generator 2: 82362193

   ![Comparison of Mass(1) of mu- mu+ in channel Z0 => mu- mu+](image)

2. **Decay Channel: $Z^0 \rightarrow \mu^- \mu^+ \gamma$**

   Number of events from generator 1: 17486313
   Number of events from generator 2: 17637807

   ![Comparison of Mass(1) of mu- mu+ in channel Z0 => mu- mu+ gamma](image)

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With NLO, (singl em.) 100MeVts compared!

Discrepancy of 0.1% has to be understood/corrected.

Found decay modes:

<table>
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<th>Decay channel</th>
<th>Branching Ratio ± Rough Errors</th>
<th>Max. shape dif. param.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Z^0 \rightarrow \gamma\mu^+\mu^-$</td>
<td>21.9118 ± 0.0047%</td>
<td>22.0134 ± 0.0047%</td>
</tr>
<tr>
<td>$Z^0 \rightarrow \mu^+\mu$</td>
<td>78.0882 ± 0.0088%</td>
<td>77.9866 ± 0.0088%</td>
</tr>
</tbody>
</table>

Similarity coefficients: T1=0.203241%, T2=0.000000%
With NLO, (single em.) 100 MeVts compared!

1 Decay Channel: \( Z^0 \rightarrow \gamma \mu^+ \mu^- \)

Number of events from generator 1: 21911762
Number of events from generator 2: 22013395

2 Decay Channel: \( Z^0 \rightarrow \mu^+ \mu^- \)

Number of events from generator 1: 78088238
Number of events from generator 2: 77986605
Also for Higgs decay agreement is to stat. err.!

- Probably PHOTOS formula from transparency 23 is accurate for Higgs.
- Born Cross Section is spin blind for Higgs.
- I do not have analytical, but only numerical support for that statement.
- It suffice by now.
In the numerical tests for Z-decays and for PHOTOS at NLO.vious plots it was relatively simple to define comparison.

All invariant masses, which could be defined from 3 four-vectors were histogramed. Photon 4-momentum both in PHOTOS and KORALZ were generated starting from the same threshold.


MC-TESTER targeting applications for decay packages, such as TAUOLA, more precisely to get a quick test if series generated by two different packages differ and where.

Test had to be automatic and easy to use, P- CP-sensitive effects are ignored in MC-TESTER.
Multiple emission

Results with exponentiation

- Previously presented and tested kernel can be used in multiple photon version of PHOTOS.
- We will use a variant of our universal test, soft photons (of energy below some threshold are ignored: merged with outgoing fermions).
- Also at most 2 (or 1) hardest photons above threshold will be preserved.
- This was necessary for the test to be free from effects of infrared singularities and its regulators.
- Let us show just one example of such a test. (i) PHOTOS working with multiple photon radiation, (ii) test is sensitive to 1 photon at most, (iii) its energy larger than 1 GeV. (iv) Comparison with second order matrix element precision generator of LEP times KKMC.
- Z decay

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Multiple emission

Standard PHOTOS (exp), 100Mevts

Found decay modes:

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<tbody>
<tr>
<td>$Z^0 \rightarrow \mu^- \mu^+$</td>
<td>$83.9176 \pm 0.0092%$</td>
<td>0.00000</td>
</tr>
<tr>
<td>$Z^0 \rightarrow \mu^- \mu^+\gamma$</td>
<td>$16.0824 \pm 0.0040%$</td>
<td>0.00409</td>
</tr>
</tbody>
</table>

Similarity coefficients: T1=0.160659 %, T2=0.065801 %
Multiple emission

Standard PHOTOS (exp), 100Mevts

1 Decay Channel: $Z^0 \rightarrow \mu^- \mu^+$

Number of events from generator 1: 83917588
Number of events from generator 2: 83837243

2 Decay Channel: $Z^0 \rightarrow \mu^- \mu^+ \gamma$

Number of events from generator 1: 16082412
Number of events from generator 2: 16162757
Found decay modes:

<table>
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</tr>
<tr>
<td>$Z^0 \rightarrow \mu^-\mu^+\gamma$</td>
<td>$16.0824 \pm 0.0040%$</td>
<td>$16.0688 \pm 0.0040%$</td>
</tr>
</tbody>
</table>

Similarity coefficients: $T1=0.027109\%$, $T2=0.000482\%$
Multiple emission

NLO in PHOTOS (exp) included, 100MeVts

1 Decay Channel: $Z^0 \rightarrow \mu^- \mu^+$

Number of events from generator 1: 83917588
Number of events from generator 2: 83931158

2 Decay Channel: $Z^0 \rightarrow \mu^- \mu^+ \gamma$

Number of events from generator 1: 16082412
Number of events from generator 2: 16068842

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**SUMMARY: Applications**

- Numerical tests of the bremsstrahlung kernel with scalar QED in B-meson decays available
- Re-implementation of complete first order matrix element into PHOTOS.
- Tests with complete first order kernel in case of Z decays; cases of single photon and exponentiated modes: completed.
- Work on data for possible future form factor tuning initiated (NA48, Belle).
- PHOTOS generates photons over all multiphoton bremsstrahlung phase space!
- Quality of its prediction is determined by the quality of the Kernel calculated from the matrix element
- I have omitted intriguing aspect of how kernel iteration work.

**SUMMARY: Algorithm**

- Skipped to my March 2nd talk.
- Eg. I have omitted intriguing aspect of how kernel iteration work.
There is nothing like systematic error for theoretical calculation: Monte Carlo or not.
All depends on choice of particular observable.
PHOTOS predictions are ready for such analysis.
It requires contributions from experiments and from people working on phenomenology of decays.
Exactly as it was done at LEP times.
SUPPLEMENTARY TRANSPARENCIES
Phase space–near factorization

\[ d\text{Lip}_{n+1}(P) = \]
\[ \frac{d^3 k_1}{2k_1^0(2\pi)^3} \cdots \frac{d^3 k_n}{2k_n^0(2\pi)^3} \frac{d^3 q}{2q^0(2\pi)^3} (2\pi)^4 \delta^4 \left( P - \sum_{i=1}^{n} k_i - q \right) \]
\[ = d^4 p \delta^4 (P - p - q) \frac{d^3 q}{2q^0(2\pi)^3} \frac{d^3 k_1}{2k_1^0(2\pi)^3} \cdots \frac{d^3 k_n}{2k_n^0(2\pi)^3} (2\pi)^4 \delta^4 \left( p - \sum_{i=1}^{n} k_i \right) \]
\[ = d^4 p \delta^4 (P - p - q) \frac{d^3 q}{2q^0(2\pi)^3} d\text{Lip}_{n}(p \rightarrow k_1 \cdots k_n). \]

Let us now recall the two most interesting forms of the \((n + 1)\)-body phase space:

\[ d\text{Lip}_{n+1}(P) = \]
\[ \frac{dM_2^2}{(2\pi)} d\text{Lip}_2(p \rightarrow p \ q) \times d\text{Lip}_{n}(p \rightarrow k_1 \cdots k_n) \]
\[ = dM_1^2 \left[ d\cos \theta d\phi \frac{1}{8(2\pi)^3} \frac{1}{M^2} \left( \frac{M^2}{M_1^2}, \frac{1}{M^2} \right) \right] \times d\text{Lip}_{n}(p \rightarrow k_1 \cdots k_n). \]

This is basically what is underneath of FOWL, TAUOLA PHOTOS and many other generators for phase space.

The definitions of angular orientations can be quite important and non-trivial.
Phase space–near factorization

\[ dLip_{n+1}(P) = \left[ \frac{2k}{M} d k \gamma \frac{1}{8(2\pi)^3} \right] \times dLip_{n}(p \to k_1 \ldots k_n) \]

\[ = \left[ k \gamma dk \gamma d \cos \theta d\phi \frac{1}{2(2\pi)^3} \right] \times dLip_{n}(p \to k_1 \ldots k_n), \quad (3) \]

If we have \( l \) photons accompanying \( n \) other particles, then the factor in brackets become iterated. Statistical factor \( \frac{1}{l!} \) has to be introduced:

\[ dLip_{n+1}(P) = \frac{1}{l!} \prod_{i=1}^{l} \left[ k \gamma_i dk \gamma_i d \cos \theta_i d\phi_i \frac{1}{2(2\pi)^3} \right] \times dLip_{n}(p \to k_1 \ldots k_n). \quad (4) \]

If we do not ask questions about phase-space limit this look like perfect term in expansion of exponent\(^a\).

As the leading parts of the Matrix elements squared, have the same factorization property, the exponents of the YFS expansion form.

---

\(^a\)In reality it can be realized relatively easy in M.C. because of the scale symmetry present in the expression.
We get:

\[
\sum_{l=0} d\text{Lips}_n(P) = \exp \left[ k_\gamma dk_\gamma d\cos \theta d\phi \frac{1}{2(2\pi)^3} \right]
\times d\text{Lips}_n(p \rightarrow k_1 \ldots k_n) \times \mathcal{J}(P, p, l, \ldots)
\]

Where \(\mathcal{J}\) is:
- complicated,
- depends on everything
- but is calculable
- and \(0 \leq \mathcal{J} \leq \mathcal{J}_{max} \) !!!!

Here we omit in discussion lots of essential points:
- angular singularities,
- initial vs. final state radiation,
- etc.

We got functional exponent
Now we can obtain an exponent for the first time. If we sum the cross sections for the configurations with 0, 1, 2, ..., photons we obtain:

\[
\frac{d\sigma(p_1, p_2 \rightarrow q_1, \ldots, q_n, \text{ and photons})}{|v_1 - v_2|} = \frac{1}{2p_1^0} \left| \mathcal{M}_B \right|^2 d\mathcal{L}_{psn}(p \rightarrow q_1 \ldots q_n)
\]

\[
\times \sum_{l=0}^{\infty} \frac{1}{l!} \prod_{i=1}^{l} \left[ k_i dk_i d\cos \theta_i d\phi_i \frac{1}{2(2\pi)^3} \sum_{\varepsilon_i} e^{2 \left( \frac{\varepsilon_i p_2}{k_i p_2} - \frac{\varepsilon_i p_1}{k_i p_1} \right)^2} \right]
\]

\[
\times \mathcal{J} \times (1 + \text{higher orders})
\]

\[
= \frac{1}{|v_1 - v_2|} \frac{1}{2p_1^0} \left| \mathcal{M}_B \right|^2 d\mathcal{L}_{psn}(p \rightarrow q_1 \ldots q_n)
\]

\[
\times \exp \left[ k dk d\cos \theta d\phi \frac{1}{2(2\pi)^3} \sum_{\varepsilon} e^{2 \left( \frac{\varepsilon p_2}{k p_2} - \frac{\varepsilon p_1}{k p_1} \right)^2} \right]
\]

\[
= \times \mathcal{J} \times (1 + \text{higher orders}).
\]

Important is that the term \( \mathcal{J} \times (1 + \text{higher orders}) \) can be made always positive, and bounded from above.

**Infrared Catastrophy**

It is nearly complete result of our presentation of exclusive exponentiation. However, it is infinite. Let us introduce the fictitious photon mass \( \lambda \) and replace the \( k dk \) factors of integration by:

\[
k^2 dk / \sqrt{k^2 + \lambda^2}, \text{ to regularize the infinity.}
\]
In case of PHOTOS underlying organization of the phase space is basically the same as in exponentiation.

1. The same exact treatment of all n-body phase space
2. Similar tangent space for first crude distribution

There are differences

1. In PHOTOS conformal symmetry of: tangent phase space combined with soft factors is not used to define the projection to realistic space in one step. Instead consecutive transformations are introduced
2. Organization of the non-dominant terms is different, seem to be more efficient, but not necessarily easier to describe and to calculate missing (next-to-included) terms.
3. As conformal symmetry is not used, there is thus hope to extend into application for massive particles
4. PHOTOS is also way faster than KKMC for FSR.
"QED bremsstrahlung in semileptonic $B$ and leptonic $\tau$ decays" by E. Richter-Was.

- agreement up to 1%
- disagreement in the low-$x$ region due to missing sub-leading terms
- study performed in 1993

PHOTOS 1.06
only on 28 December 2004 we realized that PHOTOS is used for K decays and precision is not sufficient. Even though, program works not worse than expected.

(a) $\cos(\Theta_{\gamma,l})$ $K_{\mu3}$
(b) $\cos(\Theta_{\gamma,l})$ $K_{e3}$
(c) $\log_{10}(E_\gamma)$ $K_{\mu3}$
(d) $\log_{10}(E_\gamma)$ $K_{e3}$

in KLOR
and PHOTOS
Universal interference weight in PHOTOS

\[ K \rightarrow \pi \mu \nu \] + PHOTOS bremsstrahlung, interference on/off

\[ \cos(\Theta_{\mu\nu}) \text{ in } K^0_L \rightarrow \mu \pi \nu, E_\gamma > 10 \text{ MeV} \]

\[ K^0_L \rightarrow \pi \mu \nu \]

\[ x = \log_{10}(E_\gamma) \text{ [GeV]} \]

Z. Was

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$K \rightarrow \pi e\nu +$ PHOTOS bremsstrahlung, interference on/off

\[ \cos(\Theta_{\gamma e}) \text{ in } K^0_L \rightarrow e\pi\nu, E_\gamma > 10 \text{ MeV} \]

\[ K_L^0 \rightarrow e\pi\nu \]

Seems that the interference weight removed the difference to a large degree, but still some inconsistencies at $\cos \Theta \simeq -1$
• We used published results which indicated improvements in PHOTOS were urgent.

• Fortunately thanks to work for $W$ it was trivial to do.

• After initial success we need to worry about smaller, also possibly technical problems.

• Thanks to NA48 (L. Litov, et al) we proceed with further comparisons with Matrix-Element generators.

• channel $K \rightarrow \pi^\pm e^\mp \nu$

• channel $K \rightarrow \pi^\pm \mu^\mp \nu$
\[ K \to \pi e \nu(\gamma) \] PHOTOS (A.D. 2004) vs Gasser

This looks bad - no surprise, because LL is not sufficient nowadays.

Z. Was

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Universal interference weight in PHOTOS

\[ K \rightarrow \pi e\nu(\gamma) \] PHOTOS w/Interf vs Gasser

This looks better - still straightforward improvements possible

Z. Was

CERN, February 2006
Universal interference weight in PHOTOS

Events with and without photon:

\[ R = \frac{\Gamma_{K_{e3}\gamma}}{\Gamma_{K_{e3}}} \]

<table>
<thead>
<tr>
<th>( 5 &lt; E_{\gamma} &lt; 15 \text{ MeV} )</th>
<th>PHOTOS interf</th>
<th>GASSER</th>
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</thead>
<tbody>
<tr>
<td>2.38</td>
<td>2.42</td>
<td></td>
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</tbody>
</table>

<table>
<thead>
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<th>( 15 &lt; E_{\gamma} &lt; 45 \text{ MeV} )</th>
<th>PHOTOS interf</th>
<th>GASSER</th>
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</thead>
<tbody>
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<td>2.03</td>
<td>2.07</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>( \Theta_{e,\gamma} &gt; 20 )</th>
<th>PHOTOS interf</th>
<th>GASSER</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.876</td>
<td>0.96</td>
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</table>

This table may indicate that residual discrepancy between new PHOTOS and KLOR for e-channel may be not a real problem ...

New PHOTOS (beta version 2.13) is available (as a special patch) from
http://cern.ch/wasm/goodies.html
Goals:

- There is an urgent need to clarify the status of radiative corrections for CKM matrix measurement at Belle
- Gradually superiority of Belle becomes evident
- That means that phenomenological solutions from Cleo and or even BaBar will become less and less adequate.
- PHOTOS is used for related simulation and discussion of its reliability has to be discussed channel by channel
- Good standard for discussion is for the cases of $Z$, $W$ and $H$ decays.
- Some steps in direction of such work was done for NA48 needs, and reported by me 2 weeks ago, but is it enough for Belle?
- I guess YOU have to know. The famous question of Alex Bondar remains not answered.
Outcome of mine and Melbourne group activity:

- Does the work done last year for BaBar is enough?
- If not, do you have a strategy?
- Is PHOTOS part of the solution?
- We tried to convince ourselves that it is possibly part of the solution, with well defined methods how to proceed. Examples of $W$ and $Z$.
- We remain convinced that such work was not completed for any channel of $B$ decays.
- We tried in Melbourne to:
  - Prepare framework for tests: training and how to add new options.
  - execute some tests with ‘reasonable’ and ‘ad hoc’ corrections
  - Initialize internal Belle discussion on who will do the work and who will do preliminary research on: where and if it is really needed.
Outcome of mine and Melbourne group activity:

$K \rightarrow \pi e\nu(\gamma)$ PHOTOS w/Interf and no vs new interference

Similar tests but with different cuts with respect to test of Gasser, angle between photon and electron: upper plot is old interference over new interference, lower plot is no interference over new interference. May be residual effects is reduced?
Outcome of mine and Melbourne group activity:

\[ B \rightarrow \pi e\nu(\gamma) \]

PHOTOS w/Interf and no vs new interference

Similar tests but with different cuts with respect to test of Gasser, angle between photon and electron: upper plot is old interference over new interference, lower plot is no interference over new interference. May be residual effects is reduced?

Z. Was

CERN, February 2006
NO CONCLUSIONS:

- Some ways to get preliminary numbers is in Belle hands now!
- Still not bad having in mind that it is only 3 weeks long half time work.
- Activity started Jul 15 and ‘today’ is Aug 5.
- Note that the presented new effects are because of terms which can not be obtained from Low theorem.
- For today (Feb 7): infrastructure for non point-like sources for QED bremmstrahlung is prepared.