Lepton Flavour Violation in non-minimal SUGRA

Plan of talk:

Part 0. SUSY and LFV

Part I. Minimal LFV

Part II. Non-minimal LFV

Borzumati, Masiero; Hisano, Moroi, Tobe, Yamaguchi; SFK, Oliveira; Casas, Ibarra; Lavignac, Masina, Savoy;Blazek,SFK...

Part 0. SUSY and LFV

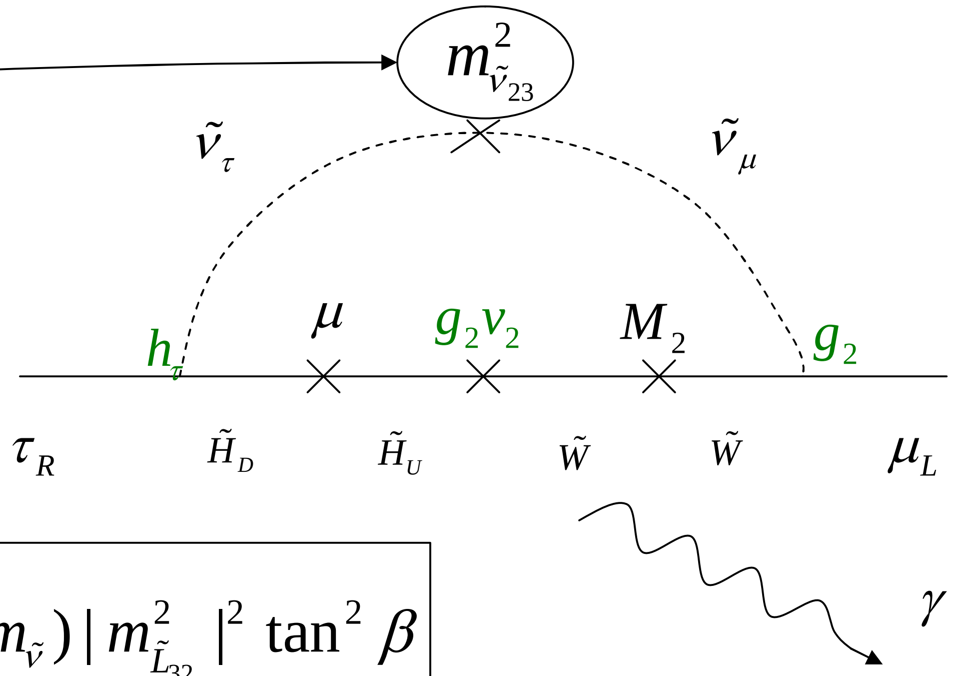
The 6x6 charged slepton mass matrix in diagonal fermion mass basis:

$$m_{\tilde{E}}^{2\mathrm{MNS}} = \begin{pmatrix} (m^2_{\tilde{E}})_{LL} + m_e^2 - \frac{\cos 2\beta}{6}(M_Z^2 + 2M_W^2)\hat{1} & (m^2_{\tilde{E}})_{LR} - \tan\beta\,\mu m_e \\ (m^2_{\tilde{E}})^\dagger_{LR} - \tan\beta\,\mu^\star m_e & (m^2_{\tilde{E}})_{RR} + m_e^2 - \frac{\cos 2\beta}{3} M_Z^2 \sin^2\theta_W \hat{1} \end{pmatrix}$$

Where: $(m^2_{\tilde{E}})_{LL} = V_{E_L} m_L^{2^*} V^\dagger_{E_L}$ $\quad (m^2_{\tilde{E}})_{RR} = V_{E_R} m_E^{2^*} V^\dagger_{E_R}$ $\quad (m^2_{\tilde{E}})_{LR} = v_1^* V_{E_L} \tilde{A}^{e^*} V^\dagger_{E_R}$

Essential point is: off-diagonal slepton masses lead to LFV

e.g. $\tau \to \mu\gamma$



$$\boxed{BR(\tau \to \mu\gamma) \approx \frac{\alpha^3}{G_F^2} f_{32}(M_2, \mu, m_{\tilde{\nu}}) \,|\, m^2_{\tilde{L}_{32}} \,|^2 \tan^2\beta}$$

Sources of off-diagonal slepton masses

1. Minimal LFV (RGE generated)

•from running the GUT theory from the Planck mass to the GUT scale (due to Higgs triplets)

Mostly considered

•from running the see-saw MSSM from the Planck/GUT scale to low energies through right-handed neutrino scales

2. Non-minimal LFV (Primordial)

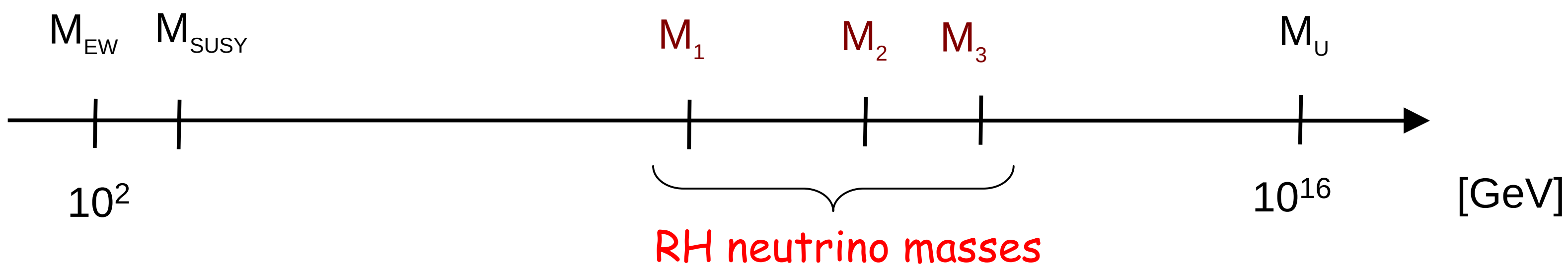
•slepton masses are off-diagonal in the SCKM basis at the high energy scale due to non-minimal SUGRA effects

In general both sources will be present
(main message of this talk)

Part I. Minimal LFV: mSUGRA + see-saw

mSUGRA prediction at M_U

$$m^2_{\tilde{L}_{ij}} = m_0^2 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + \delta m^2_{\tilde{L}_{ij}} \xleftarrow{\text{RG running}} m^2_{\tilde{L}_{ij}} = m_0^2 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$



$$\frac{dm^2_{\tilde{L}}}{dt} \approx \left(\frac{dm^2_{\tilde{L}}}{dt}\right)_{Y^\nu=0} - \frac{(3m_0^2 + A_0^2)}{16\pi^2}\left[Y^\nu Y^{\nu\dagger}\right]$$

Low energy slepton masses depend on Yukawas and M_i → Many analyses performed – here I discuss one based on sequential dominance

Sequential dominance: a technically natural way of achieving a neutrino mass hierarchy with large mixing angles from type I see-saw mechanism

SFK 98-

$$M_{\rm RR} \approx \begin{pmatrix} Y & 0 & 0 \\ 0 & X & 0 \\ 0 & 0 & X' \end{pmatrix} \qquad Y^{\nu}_{\rm LR} = \begin{pmatrix} d & a & a' \\ e & b & b' \\ f & c & c' \end{pmatrix}$$

sequential dominance of right-handed neutrinos

$(d = 0)$

$$\frac{|e^2|, |f^2|, |ef|}{Y} \gg \frac{|xy|}{X} \gg \frac{|x'y'|}{X'} \longrightarrow m_{LL} \approx \begin{pmatrix} \left(\frac{a^2}{X}\right) & \left(\frac{ab}{X}\right) & \left(\frac{ac}{X}\right) \\ . & \left(\frac{b^2}{X}+\frac{e^2}{Y}\right) & \left(\frac{bc}{X}+\frac{ef}{Y}\right) \\ . & . & \left(\frac{c^2}{X}+\frac{f^2}{Y}\right) \end{pmatrix}$$

$$\begin{aligned} m_1 &\sim O\left(\frac{x'y'}{X'}\right)v_u^2 \\ m_2 &\approx \frac{|a|^2}{X(s^{\nu}_{12})^2}v_u^2 \\ m_3 &\approx \frac{(|e|^2+|f|^2)}{Y}v_u^2 \end{aligned}$$

$$\begin{aligned} \tan\theta^{\nu}_{23} &\approx \frac{|e|}{|f|} \\ \tan\theta^{\nu}_{12} &\approx \frac{|a|}{c^{\nu}_{23}|b|\cos(\phi'_b) - s^{\nu}_{23}|c|\cos(\phi'_c)} \\ \theta^{\nu}_{13} &\approx e^{i(\phi^{\nu}_2+\phi_a-\phi_e)}\frac{|a|(e^*b+f^*c)}{[|e|^2+|f|^2]^{3/2}}\frac{Y}{X} \end{aligned}$$

Note that the large mixing angles are given by ratios of Yukawa couplings and are independent of the neutrino masses

LFV in mSUGRA + sequential dominance

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Parametrisation with dominant RH neutrino being the heaviest (heavy sequential dominance HSD)

$$M_{RR} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \lambda^3 & 0 \\ 0 & 0 & 1 \end{pmatrix} 3.10^{14}\, GeV$$

"Lop-sided"

$$Y^\nu = \begin{pmatrix} - & a_{12}\lambda^2 & 0 \\ - & a_{22}\lambda^2 & 1 \\ - & a_{32}\lambda^2 & 1 \end{pmatrix} h_t$$

N.B. Yukawas in LR convention

Parametrisation with dominant RH neutrino being the lightest (light sequential dominance LSD)

$$M_{RR} = \begin{pmatrix} \lambda^6 & 0 & 0 \\ 0 & \lambda^3 & 0 \\ 0 & 0 & \gg 1 \end{pmatrix} 3.10^{14}\, GeV$$

$$Y^\nu = \begin{pmatrix} 0 & a_{12}\lambda^2 & - \\ \lambda^3 & a_{22}\lambda^2 & - \\ \lambda^3 & a_{32}\lambda^2 & 1 \end{pmatrix} h_t$$

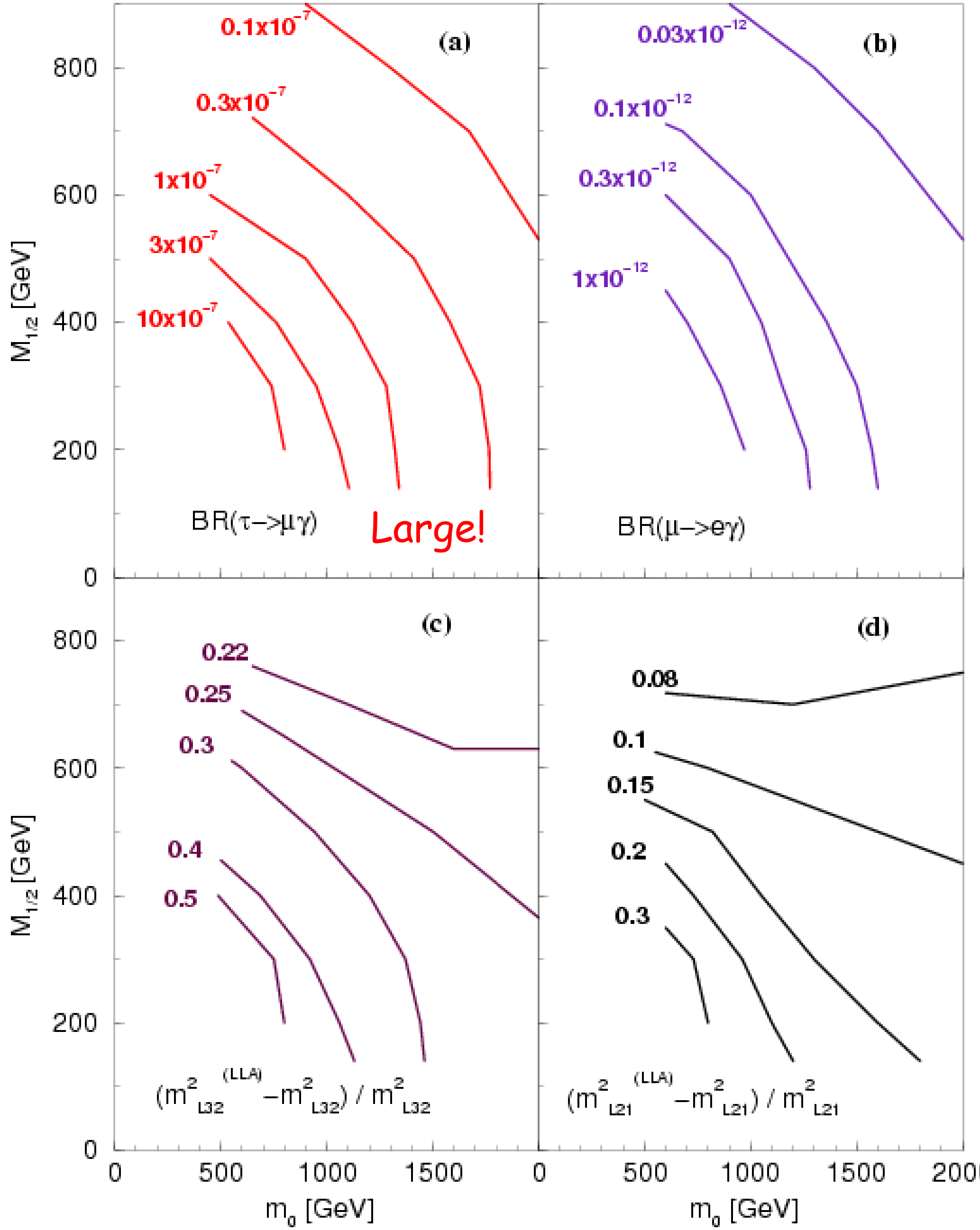
$$\boxed{\lambda = \sqrt{\frac{\Delta m_{21}^2}{\Delta m_{32}^2}} \approx 0.15}$$

Predictions for mSUGRA+HSD

$$\tan\beta = 50,$$

$$r = \frac{a_{32}}{a_{22}} = -1.$$

How good/bad is Leading Log Approximation?



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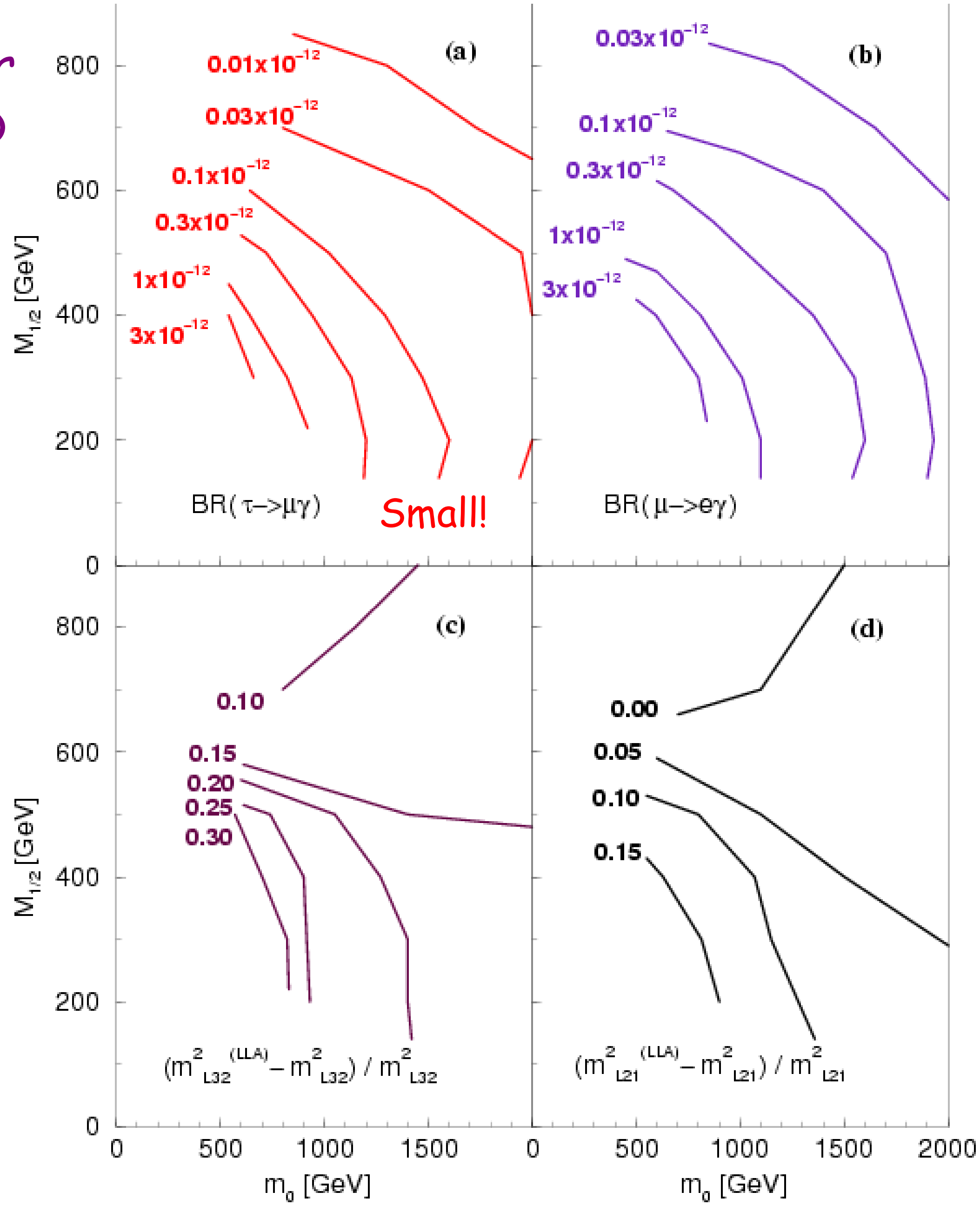

$\tau \to \mu\,\gamma$ is a discriminator of HSD vs. LSD

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Part II. Non-minimal LFV

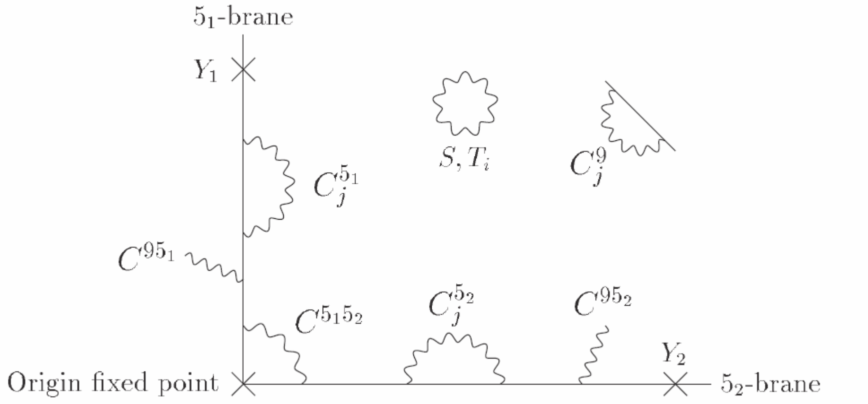
Is it reasonable to suppose that the sparticle mass matrices are universal at high energies?

The answer depends on the origin of both the Yukawas and the SUSY masses
--- highly model dependent!

Basic framework we consider is string theory → effective low energy SUGRA

We shall only consider here one specific example -- type I string theory with orthogonal intersecting D5 branes and gauge group: (Pati-Salam)$^2 \otimes$ U(1)$_{\text{Family}}$ $\quad U(4)_{PS}^{(1)} \times U(4)_{PS}^{(2)} \to U(4)_{PS}$

$$\mathrm{SU}(4) \otimes \mathrm{SU}(2)_L \otimes \mathrm{SU}(2)_R \to \mathrm{SU}(3) \otimes \mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y$$



Shiu,Tye;Everett, Kane, SFK, Rigolin, Wang; SFK,Rayner, Peddie

Field	U(4)$^{(1)}$	U(2)$_L^{(1)}$	U(2)$_R^{(1)}$	U(4)$^{(2)}$	Ends	U(1)$_F$ charge
h	1	2	2	1	$C_1^{5_1}$	0
F_3	4	2	1	1	$C_2^{5_1}$	q_{L3}
$\overline{F}_3$	$\overline{4}$	1	2	1	$C_3^{5_1}$	q_{R3}
F_2	1	2	1	4	$C^{5_15_2}$	q_{L2}
$\overline{F}_2$	1	1	2	$\overline{4}$	$C^{5_15_2}$	q_{R2}
F_1	1	2	1	4	$C^{5_15_2}$	q_{L1}
$\overline{F}_1$	1	1	2	$\overline{4}$	$C^{5_15_2}$	q_{R1}
H	4	1	2	1	$C_1^{5_1}$	q_H
$\overline{H}$	$\overline{4}$	1	2	1	$C_2^{5_1}$	$-q_H$
φ_1	4	1	1	$\overline{4}$	$C^{5_15_2}$	–
φ_2	$\overline{4}$	1	1	4	$C^{5_15_2}$	–
$D_6^{(+)}$	6	1	1	1	$C_1^{5_1}$	–
$D_6^{(-)}$	6	1	1	1	$C_2^{5_2}$	–
θ	1	1	1	1	$C^{5_15_2}$	-1
$\overline{\theta}$	1	1	1	1	All	1

Origin of Yukawas are vevs of GUT Higgs and flavons:

Yukawa matrix $$Y^{ij} \sim F^i \overline{F}^j \left(\frac{H\overline{H}}{M^2}\right)^{n_{ij}} \left(\frac{\theta}{M}\right)^{p_{ij}}$$

Majorana matrix $$M^{ij} \sim \overline{F}^i \overline{F}^j \left(\frac{HH}{M}\right) \left(\frac{H\overline{H}}{M^2}\right) \left(\frac{\theta}{M}\right)^{q_{ij}}$$

Field	Model
F_1	$\frac{11}{6}$
F_2	$\frac{5}{6}$
F_3	$\frac{5}{6}$
$\bar{F}_1$	$\frac{19}{6}$
$\bar{F}_2$	$\frac{7}{6}$
$\bar{F}_3$	$-\frac{5}{6}$
H	$-\frac{5}{6}$
$\bar{H}$	$\frac{5}{6}$

flavons GUT Higgs

$$\frac{<\theta>}{M} \sim \frac{<H\overline{H}>}{M^2} \sim 0.1$$

Origin of SUSY masses are F-term vevs of moduli, GUT Higgs and flavons:

$$F_S = \sqrt{3} m_{3/2} (S + \bar{S}) X_S,$$

$$F_{T_i} = \sqrt{3} m_{3/2} (T_i + \bar{T}_i) X_{T_i},$$

$$F_H = \sqrt{3} m_{3/2} H (S + \bar{S})^{1/2} X_H,$$

$$F_{\bar{H}} = \sqrt{3} m_{3/2} \bar{H} (T_3 + \bar{T}_3)^{1/2} X_{\bar{H}},$$

$$F_\theta = \sqrt{3} m_{3/2} \theta (S + \bar{S})^{1/4} (T_3 + \bar{T}_3)^{1/4} X_\theta$$

Three sources of non-minimal LFV

1. Non-minimal SUGRA

--due to different families coupling to the moduli differently

$$m_L^2 = m_{3/2}^2 \begin{bmatrix} a & & \\ & a & \\ & & b_L \end{bmatrix} \qquad \begin{aligned} a &= 1 - \frac{3}{2}(X_S^2 + X_{T_3}^2) \\ b_L &= 1 - 3X_{T_3}^2 \end{aligned}$$

2. D-terms

--due to broken U(1) gauge groups with family dependent charges

$$m_{L_L}^2 = m_L^2 - \mathbf{1}(3g_4^2)D_H^2 + \begin{pmatrix} q_{L1} & & \\ & q_{L2} & \\ & & q_{L3} \end{pmatrix} g_F^2 D_\theta^2$$

$$\left\{ \begin{aligned} &For\ \overline{\theta} = C^{5_1 5_2} : 2g_F^2 D_\theta^2 = 0 \\ &C_1^{5_1} \ : \ 2g_F^2 D_\theta^2 = \frac{3}{2} m_{3/2}^2 (X_S^2 - X_{T_3}^2) \\ &C_2^{5_1} \ : \ 2g_F^2 D_\theta^2 = -\frac{3}{2} m_{3/2}^2 (X_S^2 - X_{T_3}^2) \\ &C_3^{5_1} \ : \ 2g_F^2 D_\theta^2 = -\frac{3}{2} m_{3/2}^2 (X_S^2 - X_{T_2}^2) \end{aligned} \right.$$

3. Misaligned soft trilinears

--due to field dependent Yukawa operators, generically Y$\sim$ Φ^n

$$\Delta A = F_\Phi \partial_\Phi \ln Y = F_\Phi \partial_\Phi \ln \Phi^n = F_\Phi \frac{n}{\Phi}$$

$$F_\Phi \propto m_{3/2}\Phi \quad \rightarrow \Delta A \propto n m_{3/2}$$

Abel,Servant;
Abel, Khalil,Lebedev;
Ross,Vives;
Peddie,SFK.

Consider four benchmark points

(All benchmark points also include see-saw effects)

"mSUGRA"

"nmSUGRA"

"mSUGRA" + non-aligned trilinears from flavons

"mSUGRA" + non-aligned trilinears from GUT Higgs

Point	X_S	X_T	X_H	$X_{\overline{H}}$	X_θ	$X_{\overline{\theta}}$
A	0.500	0.500	0.000	0.000	0.000	0.000
B	0.535	0.488	0.000	0.000	0.000	0.000
C	0.270	0.270	0.000	0.000	0.841	0.000
D	0.270	0.270	0.595	0.595	0.000	0.000

Yukawa + Majorana matrices (HSD, large tan β)

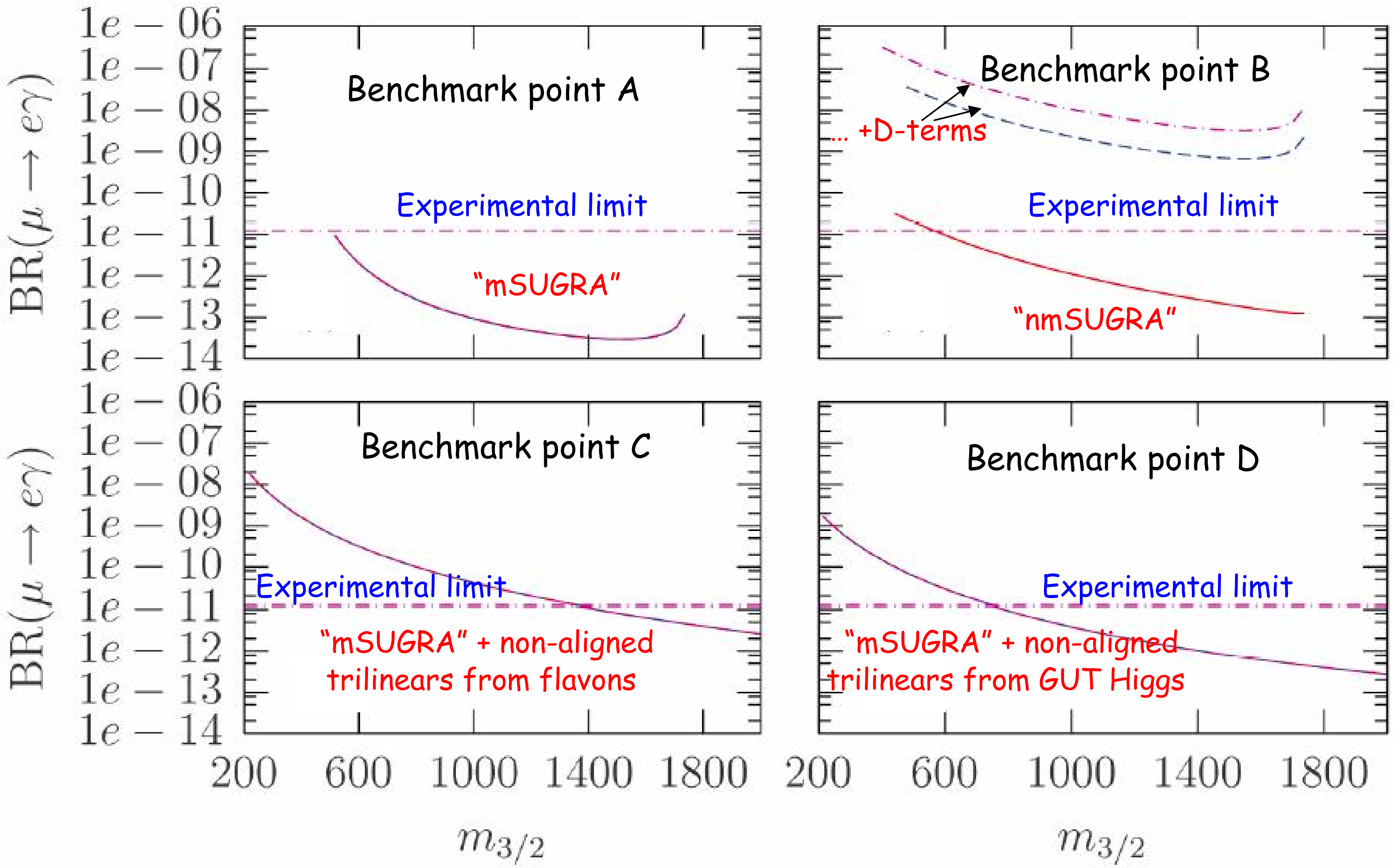
$$Y^e(M_X) = \begin{bmatrix} -1.246\times10^{-04} & 0.000 & 0.000 \\ 1.537\times10^{-03} & 2.432\times10^{-02} & -3.649\times10^{-02} \\ -1.327\times10^{-04} & 3.133\times10^{-02} & 5.469\times10^{-01} \end{bmatrix}$$

$$Y^\nu(M_X) = \begin{bmatrix} 2.159\times10^{-06} & 1.525\times10^{-03} & 0.000 \\ 0.000 & 8.290\times10^{-04} & 3.923\times10^{-01} \\ 0.000 & 5.050\times10^{-03} & 5.469\times10^{-01} \end{bmatrix}$$

$$\frac{M_{RR}(M_X)}{M_{33}} = \begin{bmatrix} 3.508\times10^{8} & 3.686\times10^{9} & 3.345\times10^{11} \\ 3.686\times10^{9} & 8.313\times10^{10} & 5.886\times10^{12} \\ 3.345\times10^{11} & 5.886\times10^{12} & 5.795\times10^{14} \end{bmatrix}$$

$\mu \to e\gamma$ in non-minimal SUGRA

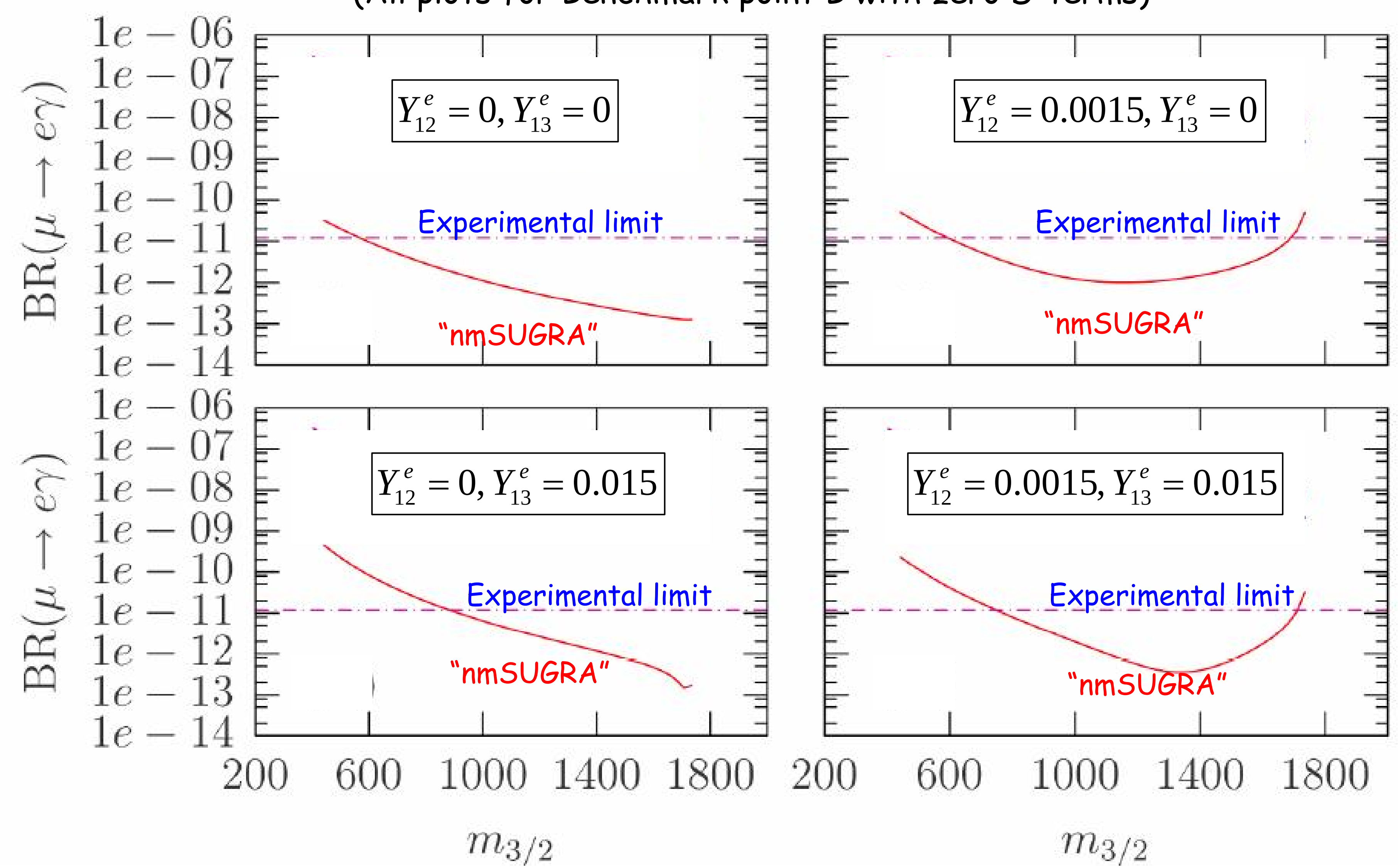
Hayes, Peddie, SFK 05



$\mu \to e\gamma$ sensitivity to the Yukawas

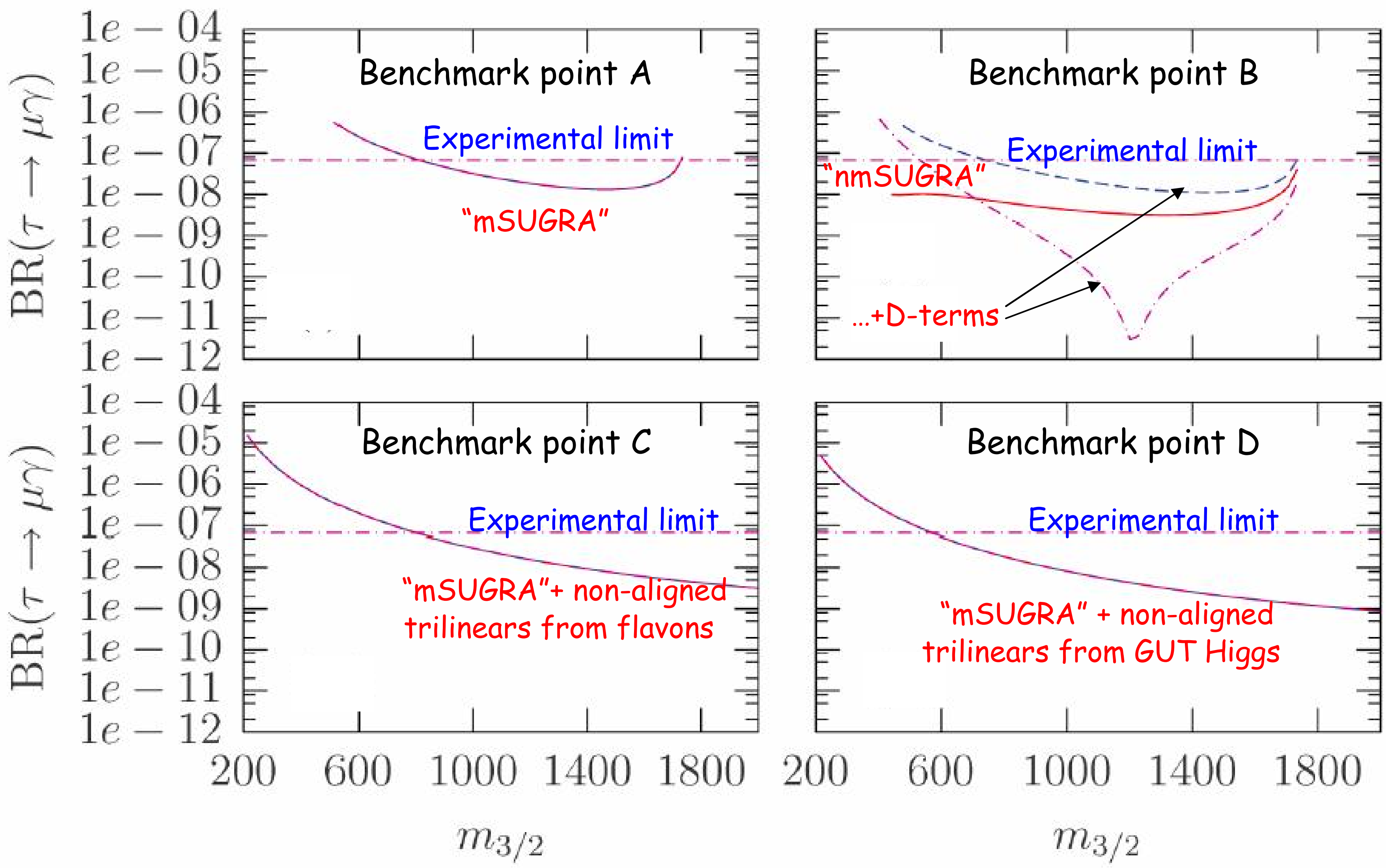
Hayes, Peddie, SFK 05

(All plots for Benchmark point B with zero D-terms)



$\tau \to \mu\gamma$ in non-minimal SUGRA

Hayes, Peddie, SFK 05



Conclusions

- SUSY + see-saw mechanism generically leads to LFV due to RH neutrino masses + RG running
- Minimal LFV arises from mSUGRA + see-saw running – many analyses performed
- We discussed LFV from mSUGRA + sequential dominance (natural neutrino mass hierarchy)
- We saw that $\tau \to \mu\,\gamma$ is large (small) in HSD (LSD)
- We then discussed non-minimal LFV within string theory → effective low energy SUGRA
- Example: Type I string theory with intersecting D5 branes and (Pati-Salam)$^2 \otimes$ U(1)Family
- Identified three sources of non-min LFV: nmSUGRA, D-terms, misaligned trilinears
- We saw that nmSUGRA effects depend sensitively on the Yukawas
- D-term effects are extremely dangerous for $\mu \to e\,\gamma$ (but D-terms can be made zero)
- Misaligned trilinears are also very importrant for $\mu \to e\,\gamma$
- For $\tau \to \mu\,\gamma$ in HSD, where the rate is close to the experimental limit in mSUGRA, non-minimal LFV effects tend to lower the rate
- In particular D-terms can dramatically lower the rate for $\tau \to \mu\,\gamma$, or restore universality!