(my personal)
User's guide to lattice QCD

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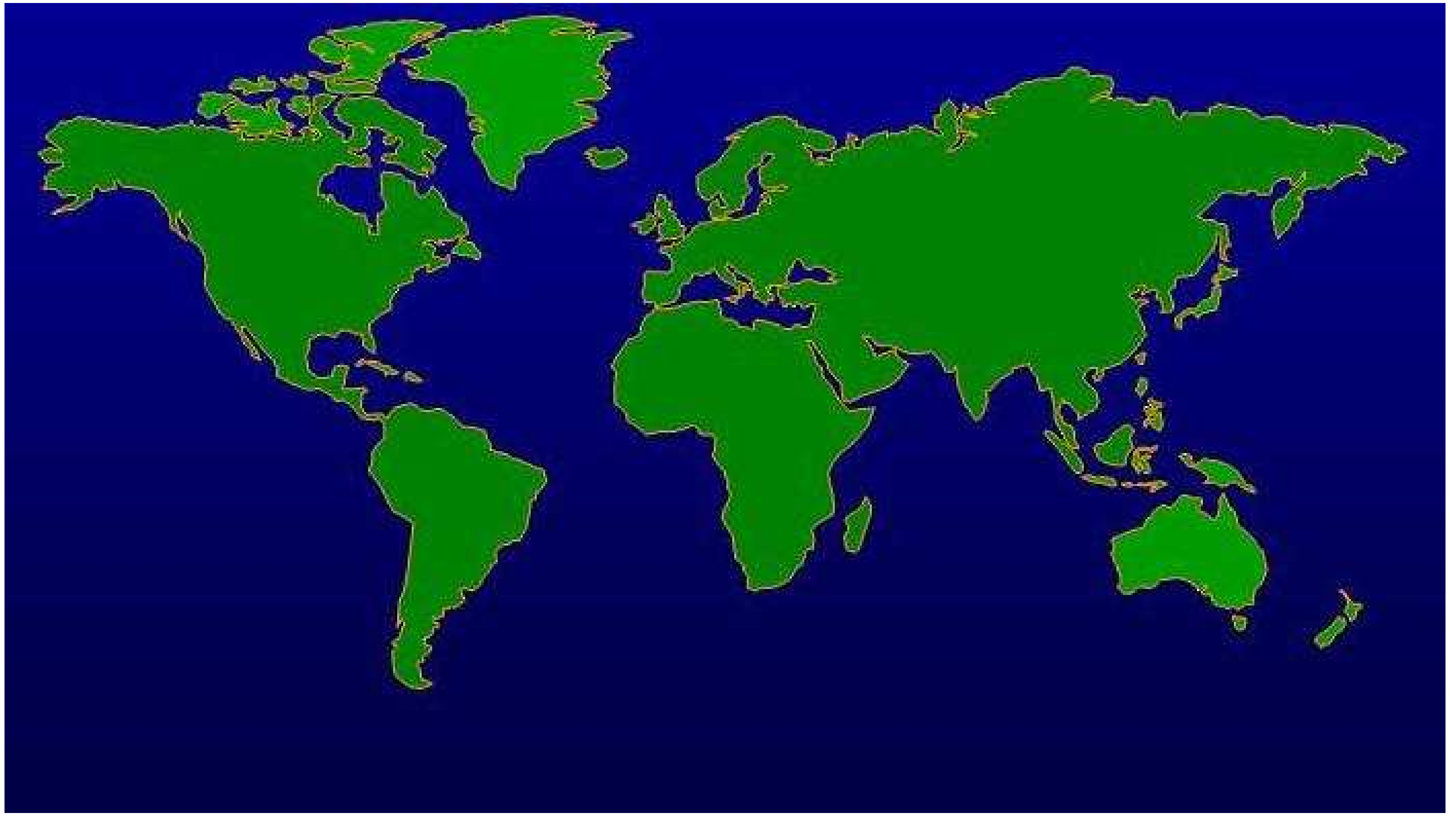
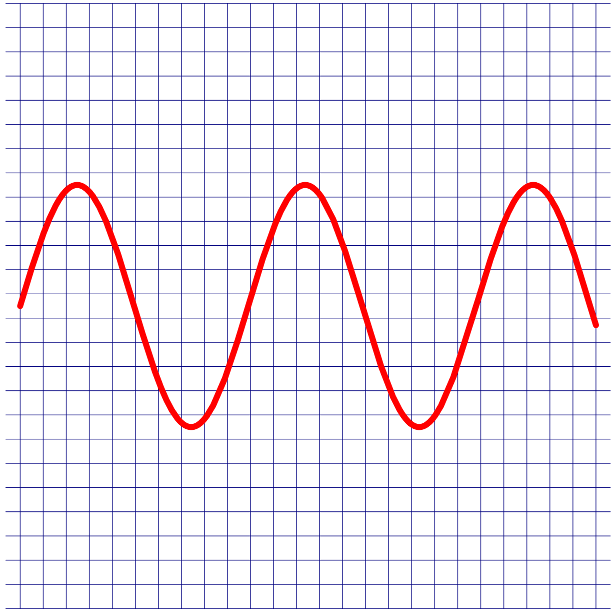
Flavour in the era of the LHC, CERN, 6.-8. Feb. 2006



USA/Can.	Europe	Asia/Aus.	Japan
MILC	UKQCD	Adelaide	CP-PACS
LHPC	ALPHA	Beijing	JLQCD
FNAL	QCDSF	Taipei	...
RBC	SPQR	...	
JLab	...		
...			

QCD with lattice (cut-off) regulator



lattice spacing: $0.05\,\mathrm{fm} \leq a \leq 0.20\,\mathrm{fm}$

$1\,\mathrm{GeV} \leq a^{-1} \leq 4\,\mathrm{GeV}$

box length: $1.5\,\mathrm{fm} \leq L \leq 3.0\,\mathrm{fm}$

require: $ma \ll 1$ and $M_{\mathrm{had}} a \ll 1$

require: $M_\pi L > 3$ (note: $3/M_\pi^{\mathrm{phys}} \simeq 4.2\,\mathrm{fm}$)

u	c	(t)
d	s	b
extrapolate	work at	extrapolate
$m_u \searrow m_u^{\mathrm{phys}}$	"physical"	$m_b \nearrow m_b^{\mathrm{phys}}$
$m_d \searrow m_d^{\mathrm{phys}}$	value	$m_\infty \searrow m_b^{\mathrm{phys}}$

In QCD with N_f quarks, $1{+}N_f$ observables sacrificed to determine scale and quark masses

QCD spectroscopy: quenched/dynamical

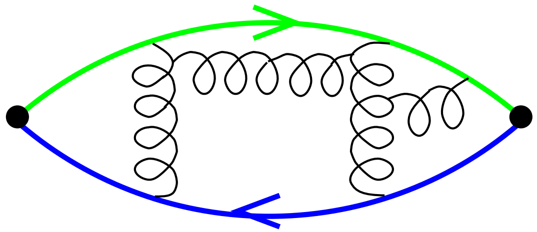
Hadronic correlator in $N_f \geq 2$ QCD: $C(t) = \int d^4x\, C(t,\mathbf{x})\, e^{i\mathbf{px}}$ with

$$C(x) = \langle O(x)\, O(0)^\dagger \rangle \;=\; \frac{1}{Z} \int DU\, D\bar{q} Dq \;\; O(x)\, O(0)^\dagger \;\; e^{-S_G - S_F}$$

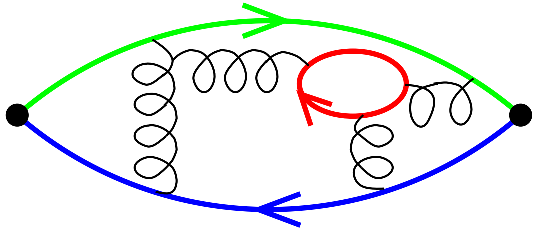
where $O(x) = \bar{d}(x)\Gamma u(x)$ and $\Gamma = \gamma_5, \gamma_4\gamma_5$ for $\pi^\pm$ and
$S_G = \beta \sum (1 - \frac{1}{3}\mathrm{Re}\mathrm{Tr}\, U_{\mu\nu}(x))$, $S_F = \sum \bar{q}(D+m)q$

$$\langle \bar{d}(x)\Gamma_1 u(x)\; \bar{u}(0)\Gamma_2 d(0) \rangle \;=\; \frac{1}{Z} \int DU \;\det(D+m)^{N_f}\; e^{-S_G}$$

$$\times \;\; \mathrm{Tr}\Big\{ \Gamma_1 (D+m)^{-1}_{x0} \Gamma_2 \underbrace{(D+m)^{-1}_{0x}}_{\gamma_5 [(D+m)^{-1}_{x0}]^\dagger \gamma_5} \Big\}$$



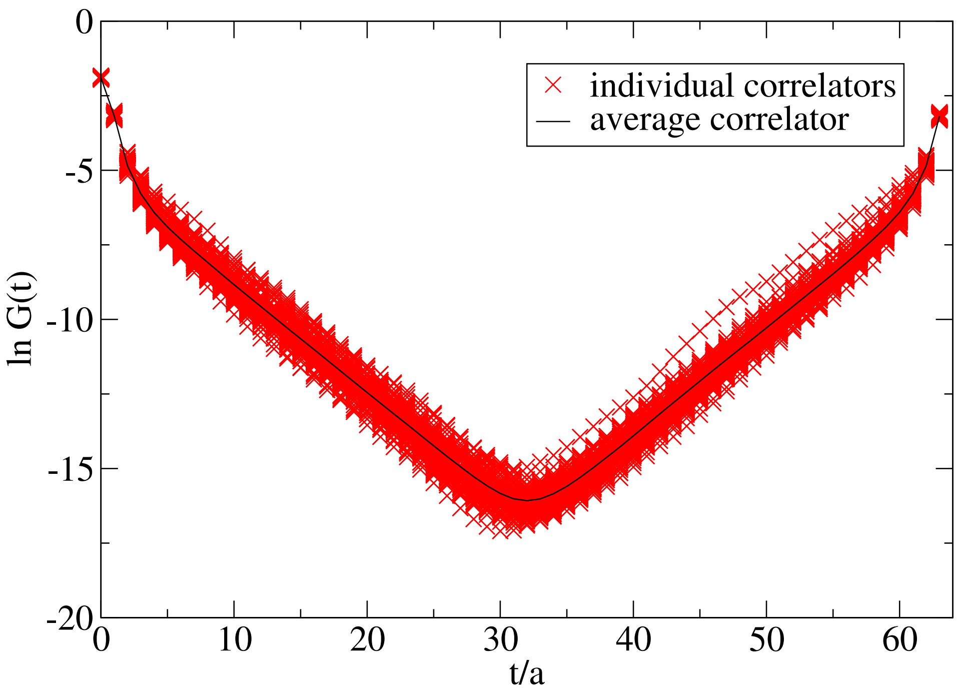
(A) Quenched QCD: quark loops neglected

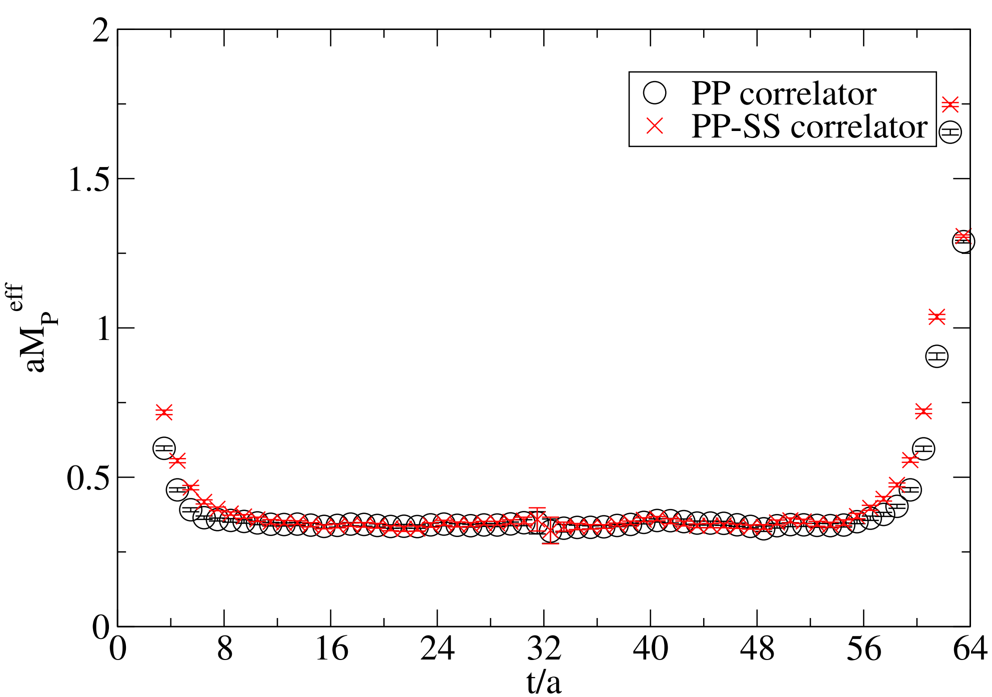


(B) Full QCD

Usually $m_u = m_d$ [isospin $SU(2)$ symmetry good: $m_{ud} \equiv \frac{m_u + m_d}{2}$]
Usually $m_{\mathrm{valence}} \neq m_{\mathrm{sea}}$ ("parametric solver slow-down")

Effective mass/decay-constant/matrix-element





R. Babich et al. [Boston-Marseille], hep-lat/0509027

$$C(t) = \text{const}\; e^{-M_\pi t}\,[1+e^{-(M_\text{exc}-M_\pi)t}]$$

$$M_\text{eff}(t) \equiv \frac{1}{2}\log\left(\frac{C(t-1)}{C(t+1)}\right) \xrightarrow{0\ll t\ll T/2} M_\pi$$

$$\text{const} \propto \begin{cases} G_\pi^2 & \text{for } \langle PP\rangle \\ F_\pi G_\pi & \text{for } \langle A_0 P\rangle \\ F_\pi^2 & \text{for } \langle A_0 A_0\rangle \end{cases}$$

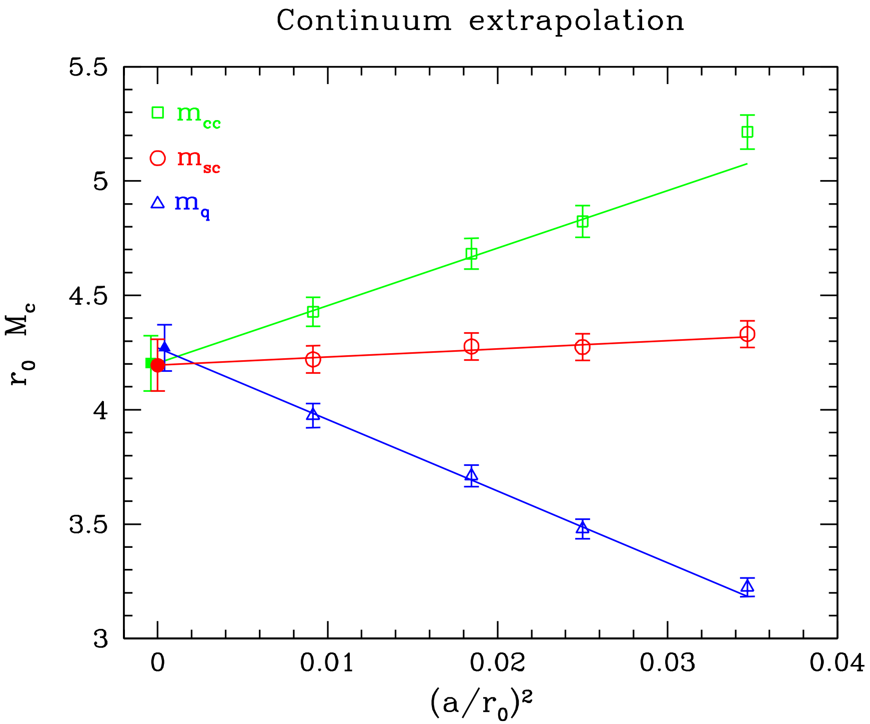
$\Longrightarrow$ good plateau signals absence of contamination from excited states

Overview on systematics

Result of the simulation: exact up to statistical errors and systematics/extrapolations

- **continuum limit**

  Symanzik: artefact $\propto a^n$, $n=2$ for $\left\{\begin{array}{c}\text{clover}\\ \text{overlap}\end{array}\right.$
- **infinite-volume limit** [XPT]
- **chiral extrapolation** $m_{ud} \to m_{ud}^{\rm phys}$ [XPT]
- **heavy-quark extrap.** $m_b \to m_b^{\rm phys}$ [HQET]
- **unquenching** [new simulations]
- excited states contamination
- renormalization / matching

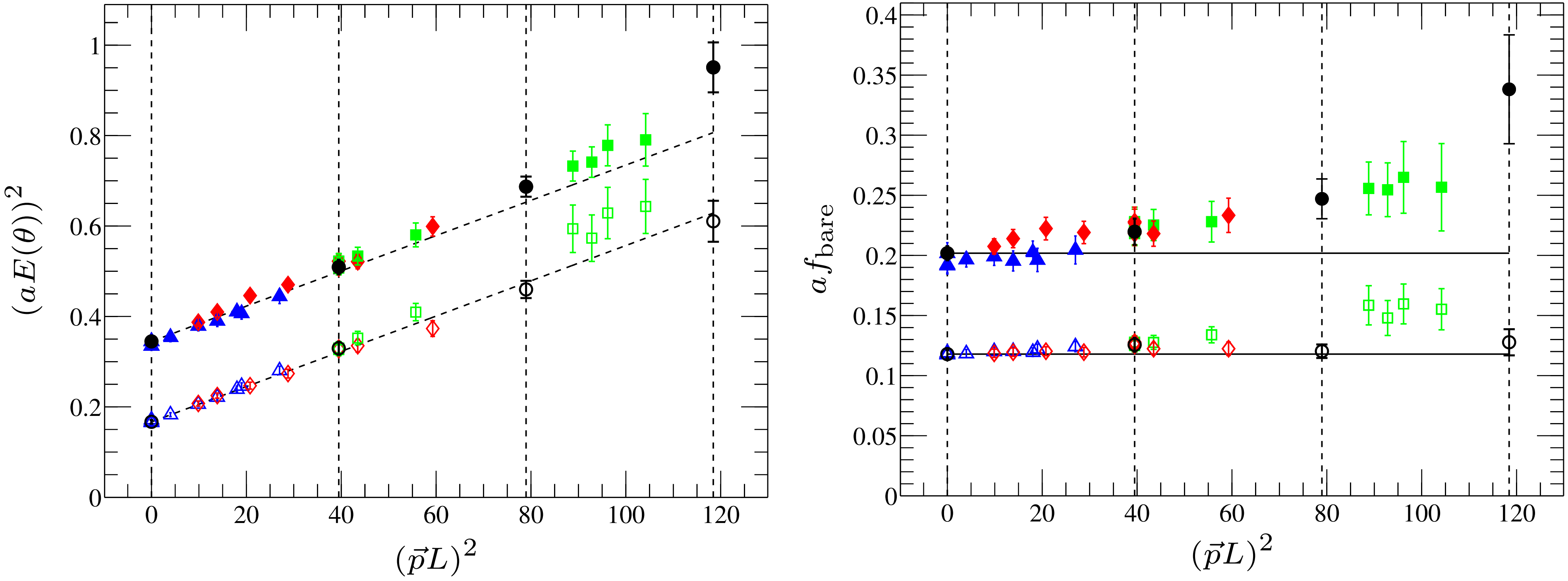


J. Rolf, S. Sint [ALPHA], hep-ph/0209255

$$\frac{\text{Gflop-year}}{\text{per conf}} \simeq 2.8 \Big(\frac{L}{3\,\mathrm{fm}}\Big)^5 \Big(\frac{0.6}{M_\pi/M_\rho}\Big)^6 \Big(\frac{a^{-1}}{2\,\mathrm{GeV}}\Big)^7$$

[A. Ukawa, Lat2001
standard algorithm]

Example (1): dispersion relation and decay constant



J. Flynn et al. [UKQCD], hep-lat/0506016

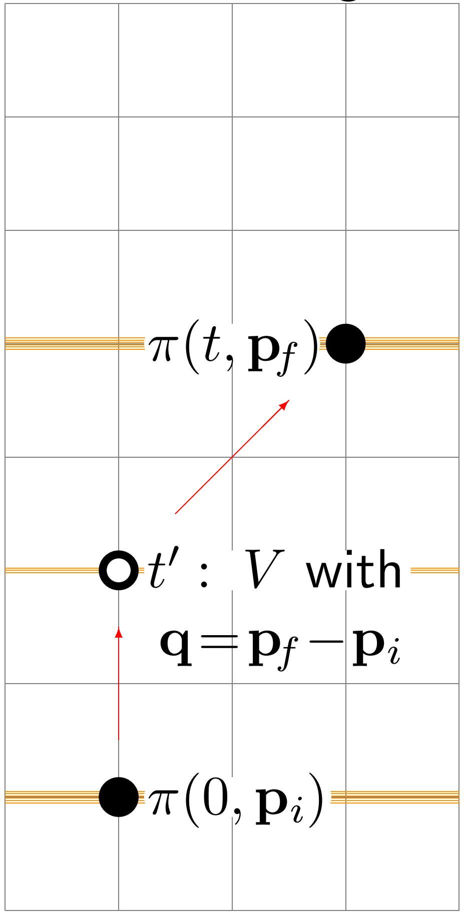
$\longrightarrow$ toroidal boundary conditions allow only for lattice momenta $\mathbf{p} = \frac{2\pi}{L}(n_1, n_2, n_3)$

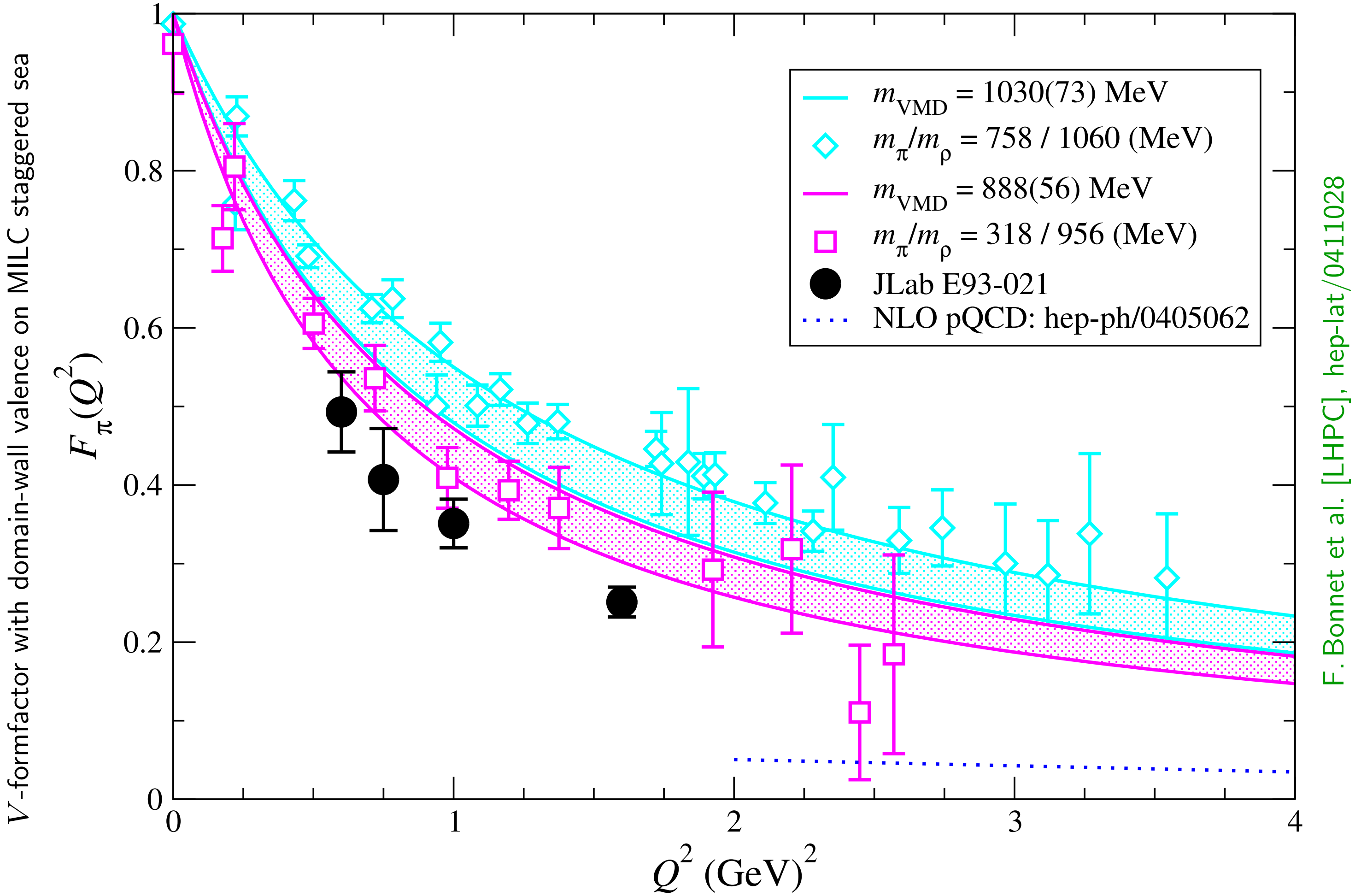
$\Longrightarrow$ relativistic dispersion relation $E^2 = M_{\rm had}^2 + \mathbf{p}^2$ slightly distorted on the lattice [more pronounced for rho (upper curve) than pion (lower)]

$\Longrightarrow$ decay constant in $\langle 0|A_0(t,\mathbf{p})|\pi(0)\rangle \propto f_\pi \, e^{-Et}$ has big cut-off effects for large p^2

Example (2): V-formfactor for pions

$L^3 \times T = 4^3 \times 8$ geometry





$$\langle \pi(t, \mathbf{p}_f) | V_k^{\rm cont}(t', \mathbf{q}) | \pi(0, \mathbf{p}_i) \rangle \;=\; Z_V \langle . | V_k^{\rm latt}(t', \mathbf{q}) | . \rangle \;=\; \Big[\mathbf{p}_f + \mathbf{p}_i \Big]_k F_V(Q^2)$$

$$Q^2 = -(E_f - E_i)^2 + (\mathbf{p}_f - \mathbf{p}_i)^2$$

[ususally $E_i = M_\pi$, $\mathbf{p}_i = \mathbf{0}$]

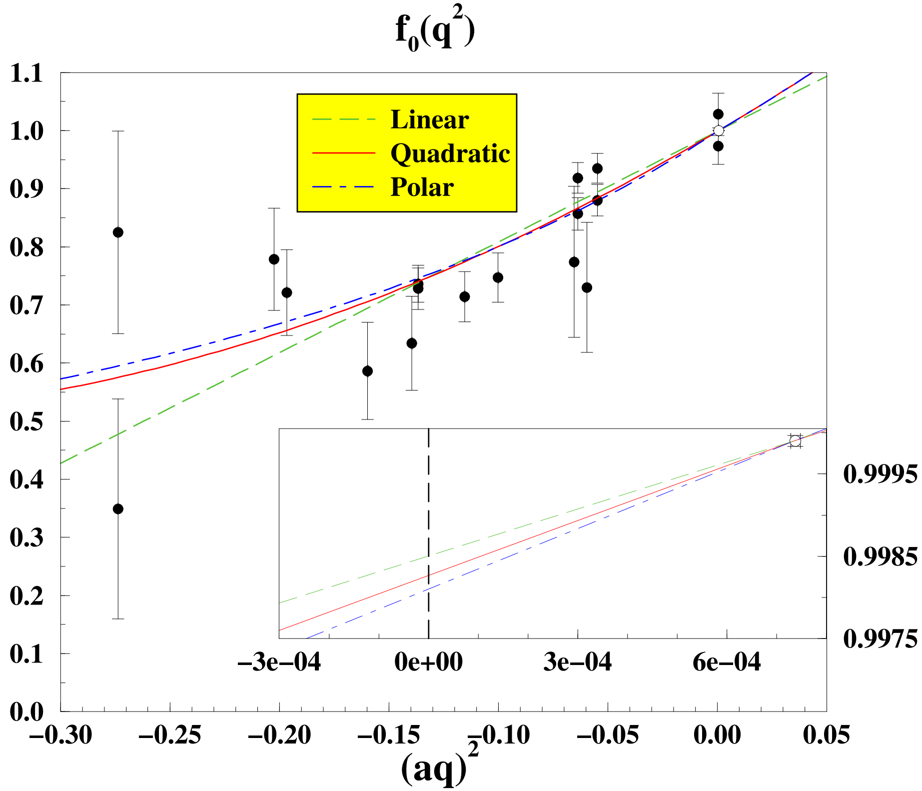
Example (3): $K_{\ell 3}$ decays

Experiment: $K^+ \to \pi^0 \ell^+ \nu_\ell$ and $K^0 \to \pi^- \ell^+ \nu_\ell$ with $\Gamma_{K_{\ell 3}} \propto |V_{us}|^2 \cdot |f_+(0)|^2$

$$\langle \pi(p')|\bar{s}\gamma_\mu u|K(p)\rangle = \Big[p+p' - q\frac{M_K^2-M_\pi^2}{q^2}\Big]_\mu f_+(q^2) + q_\mu \frac{M_K^2-M_\pi^2}{q^2} f_0(q^2)$$ with $q = p-p'$

Constraint $f_+(0) = f_0(0)$:
extrapolating $f_0(q^2 \to 0)$ is sufficient,
but limits $a \to 0, m \to m_{ud}^{\text{phys}}$ required

$\Longrightarrow$ with growing Q^2 both cut-off effects and noise increase



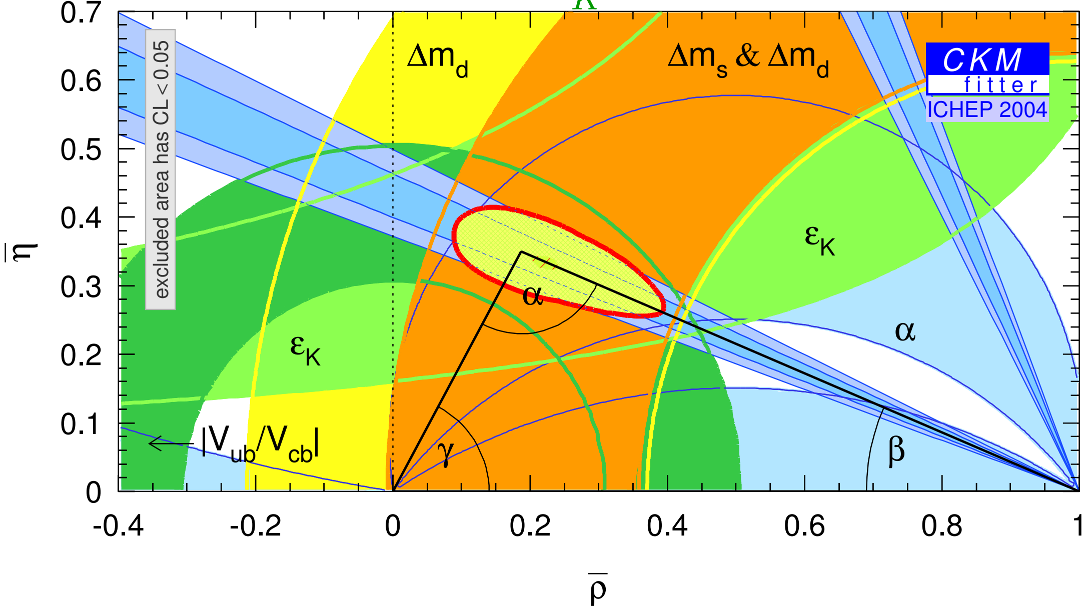
D. Becirevic et al. [Orsay-Rome], hep-ph/0403217

$$\langle K(p')|\bar{c}\gamma_\mu s|D(p)\rangle = \Big[p+p' - q\frac{M_D^2-M_K}{q^2}\Big]_\mu f_+(q^2) + q_\mu \frac{M_D^2-M_K^2}{q^2} f_0(q^2)$$ with $q = p-p'$

$\Longrightarrow$ systematic bias for large extrapolations (dep. on Q^2_{min})

Example (4): $K^0\bar{K}^0$ mixing

CKM_fitter_2004 based on $B_K^{\rm RGI} = \mathbf{0.86} \pm 0.06 \pm 0.14$



Continuum: just one operator $(\Delta s = 2)$

$O_\Gamma = \bar{s}\Gamma d \bar{s}\Gamma d$ with Γ-structure $VV+AA = \gamma_\mu \otimes \gamma_\mu + \gamma_\mu\gamma_5 \otimes \gamma_\mu\gamma_5$ contributes to B_K:

$$B_K = \frac{\langle \bar{K}_0 | O_{VV+AA} | K_0 \rangle}{\frac{8}{3} M_K^2 f_K^2}$$

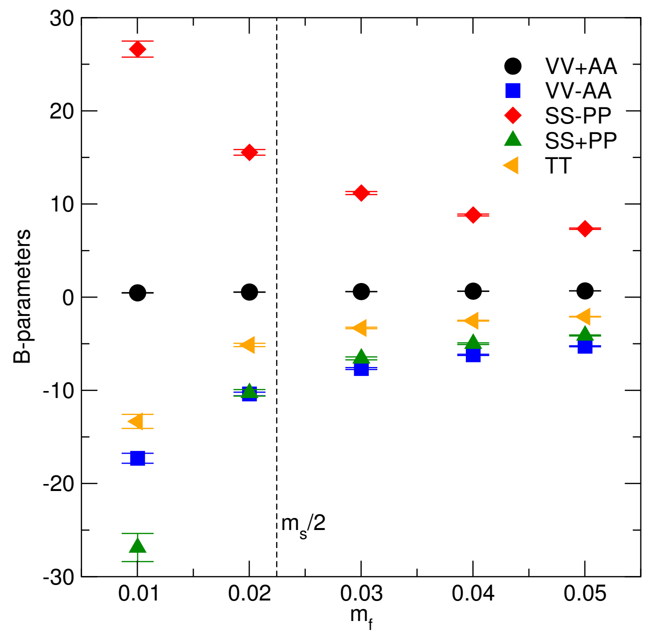
$$|\epsilon_K| = \text{const}\, A^2 \lambda^6\, \bar{\eta}\, [\text{const} + \text{const}\, A^2\lambda^4 (1-\bar{\rho})]\, B_K^{\rm RGI}$$

Lattice: $A = \frac{G_F}{\sqrt{2}} V_{\rm CKM} \sum_j C_j^{\rm cont}(\mu) \langle .|O_j^{\rm cont}(\mu)|.\rangle$

with $\langle .|O_j^{\rm cont}(\mu)|.\rangle = Z_j(\mu a) \langle .|O_j^{\rm latt}(a^{-1})|.\rangle$

or $\langle .|O_j^{\rm cont}(\mu)|.\rangle = \sum_k Z_{jk}(\mu a) \langle .|O_k^{\rm latt}(a^{-1})|.\rangle$

$\Longrightarrow$ all $O_\Gamma^{\rm latt}$ with $\Delta s = 2$ mix, unless GW-symmetry



Aoki et al. [RBC], hep-lat/0508011

Summary

With a phenomenological lattice paper, please check:

- does the "effective mass/matrix-element" look convincing ?
- has the continuum limit been taken ?
- are backgrounds quenched/dynamical ?
- are (some) pions in the "chiral" regime, say $200\,\mathrm{MeV} < M_\pi < 300...500\,\mathrm{MeV}$?
- is the "chiral" extrapolation done after $a \to 0$, or with a dedicated finite-a ansatz ?
- are finite-volume effects under control ?

o for experts: improvement/renormalization/matching non-perturbatively ?

o for experts: need worry about action/algorithm issues ?

Please don't:

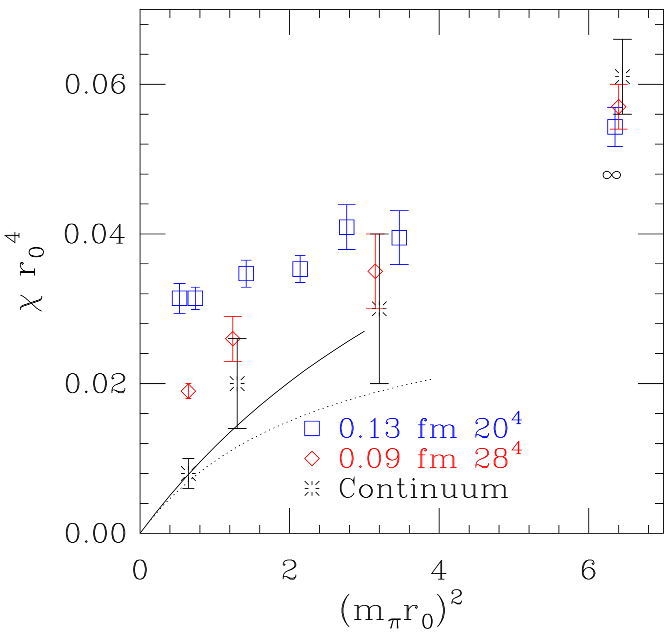
- throw away high-precision (!) lattice data, just because they are quenched (except for observables which are known to get corrupted by $N_f = 0$) !
- select "small cut-off effect" lattices by a cut on the lattice spacing (say $a < 0.1\,\mathrm{fm}$) !

Unquenching (1): topological susceptibility

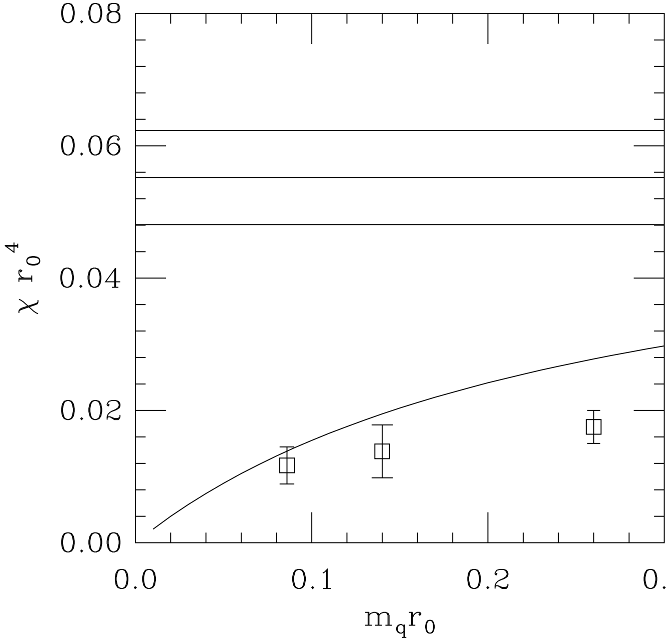
Idea: $\chi_{\rm top}=\frac{\langle\nu^2\rangle}{V}$ probes vacuum structure, depends exclusively on $m\equiv m^{\rm sea}$

$$\chi_{\rm top}\to\begin{cases}\frac{\Sigma}{2/m_{ud}+1/m_s+\ldots} & \text{for } m\to 0 \quad \text{with } \Sigma=-\lim_{m_q\to 0}\langle\bar{q}q\rangle\Big|_{m_{ud},m_s,\ldots}\\ \chi_{\rm top}^{\rm qu} & \text{for } m\to\infty \quad \text{with } \chi_{\rm top}^{\rm qu}=\frac{F^2}{2N_f}(M_{\eta'}^2+M_\eta^2-2M_K^2)\end{cases}$$

Interpolation: $\frac{1}{\chi_{\rm top}}=\frac{2/m_{ud}+1/m_s+\ldots}{\Sigma}+\frac{1}{\chi_{\rm top}^{\rm qu}}$ [SD, hep-lat/0103011]



C. Bernard et al. [MILC], hep-lat/0308019



T. DeGrand, A. Schaefer, hep-lat/0506021

$\Longrightarrow$ Cut-off effects may mask underlying continuum behaviour – in particular in full QCD

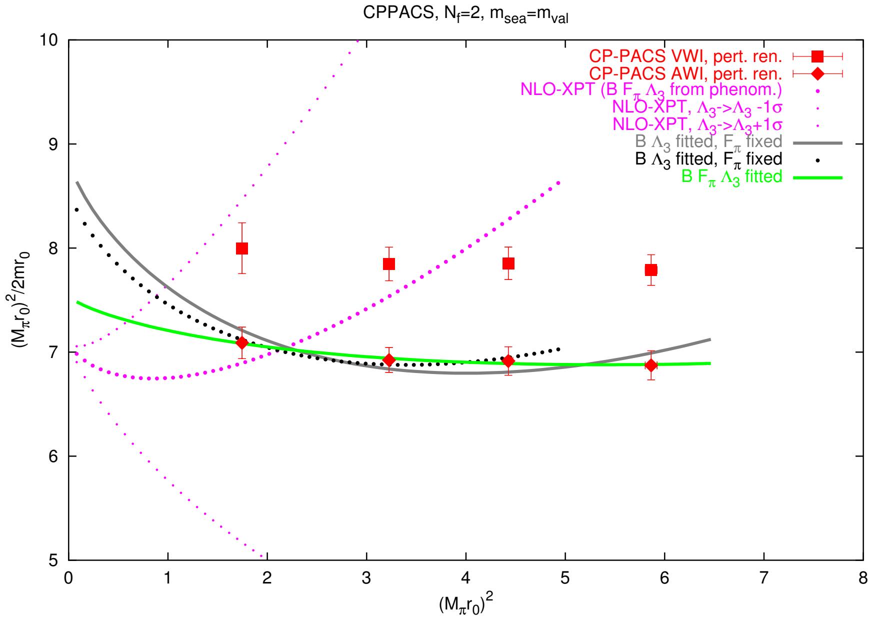
Unquenching (2): chiral logs

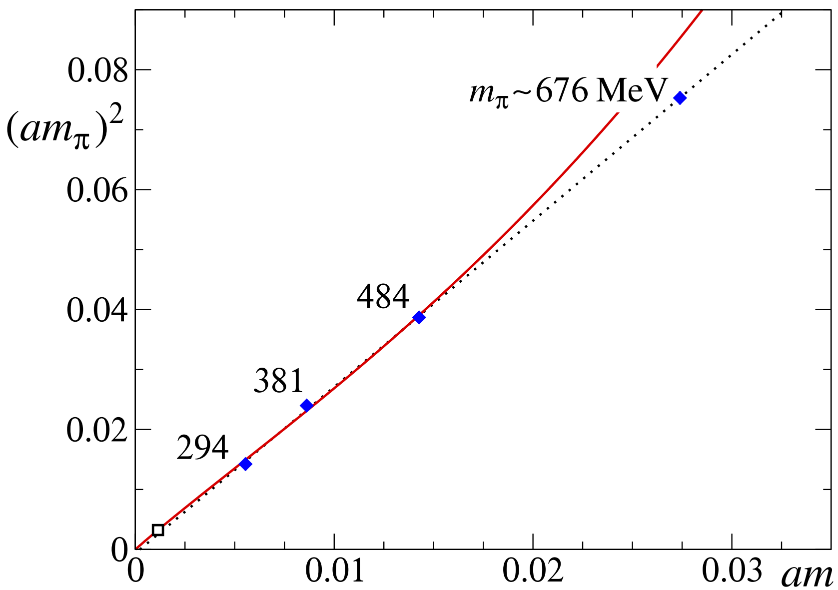
XPT at NLO with $B = \Sigma / F^2$:

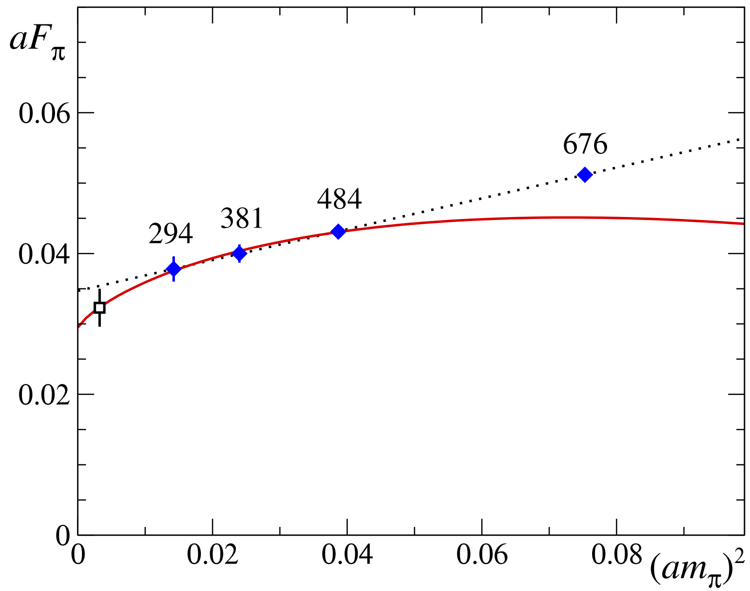
$$M_\pi^2 = 2Bm \left(1 + \frac{Bm}{(4\pi F)^2} \log\left(\frac{2Bm}{\Lambda_3^2}\right)\right)$$

$$F_\pi = F \left(1 - \frac{2Bm}{(4\pi F)^2} \log\left(\frac{2Bm}{\Lambda_4^2}\right)\right)$$

attention: FVE typically mimic chiral logs !!!







M. Lüscher, hep-lat/0509152

Unquenching (3): strategy of PQ data taking

- PQ-QCD is a useful *extension* of QCD, *same* low-energy constants as (full) QCD [unlike Q-QCD]

- performing $a \to 0$ first and $m \to 0$ in a second step is safe but requires (lots of) precise data

- (crucial) practical issues:
  - **renormalization**
  - **scale setting**
  - **regime of applicability of XPT**