B-to-light meson formfactor and recent progress on Kaon distribution amplitude

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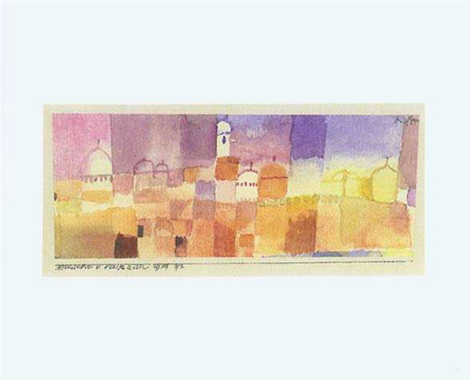

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$$\frac{F^{B\to K}}{F^{B\to\pi}} = \frac{f_K}{f_\pi}(1 + c_1 a_1) + \ldots$$

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   - coefficient $c_1 \sim O(1)$
   - a_1 Gegenbauer moment of Kaon Distribution Amplitude, most important dynamical SU(3)-breaking parameter
3. For example in $B \to K^*\gamma$ vs. $B \to \rho\gamma$ (Belle,Babar coming)

$$\frac{T_1^{B\to K^*}}{T_1^{B\to\rho}} \leftrightarrow \frac{|V_{\rm ts}|}{|V_{\rm td}|}$$

Def. Formfactors, $B \to$ light P-scalar and vector

- For (V-A) currents:

$$
\begin{aligned}
\langle \pi | \bar{u}\gamma_\mu b | B \rangle &= (p_B + p_\pi)_\mu f_+(q^2) + q_\mu f_-(q^2) \\
\langle \rho | \bar{u}\gamma_\mu (1-\gamma_5) b | B \rangle &= (p_B + p)_\mu (e^* q) \frac{A_2(q^2)}{m_B + m_\rho} - i e^*_\mu (m_B + m_\rho) A_1(q^2) \\
&+ \; i \frac{q_\mu}{q^2} (e^* q)(A_3 - A_0)(q^2) + \epsilon_{\mu\nu\rho\sigma} e^{*\nu} p_B^\rho p^\sigma \frac{2V(q^2)}{m_B + m_\rho}
\end{aligned}
$$

- For tensor currents (penguin operators):

$$
\begin{aligned}
\langle \pi | \bar{u}\sigma_{\mu\nu} q^\nu b | B \rangle &= \frac{i}{m_B + m_\pi} (q^2 (p + p_B)_\mu - (m_B^2 - m_\pi^2) q_\mu) f_T(q^2) \\
\langle \rho | \bar{u}\sigma_{\mu\nu} q^\nu (1+\gamma_5) b | B \rangle &= (e^*_\mu (m_B^2 - m_\rho)^2 - (e^* q)(p_B + p)_\mu) T_2(q^2) \\
&+ \; (e^* q)(q - \frac{q^2 (p_B + p)}{m_B^2 - m_\rho^2})_\mu T_3(q^2) + i \epsilon_{\mu\nu\rho\sigma} e^{*\nu} p_B^\rho p^\sigma 2 T_1(q^2)
\end{aligned}
$$

- Semileptonic decays e.g. $B \to \pi(e\nu)|V_{\rm ub}|$, $B \to K^* l^+ l^-$, $B \to K^* \gamma$

- enter BBNS-factorization approach to non-leptonic B-decays etc

Light-Cone Sum Rules (LCSR)

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  QCD Sum Rules and Hadron Distribution Amplitudes in order to deal with “3-particle hadronic physics” e.g. $B \to \pi(l\nu)$

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- Physics: Allow to express hadronic data (e.g. $f_+^{B\to\pi}$) expressed in terms of

  A. fundamental QCD-parameters e.g. ($\alpha_s, m_b, \dots$)

  B. universal hadronic parameters e.g. ($f_\pi, \phi_\pi(DA), \dots)$

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Method: Choose suitable correlation function and evaluate in two ways

$$\text{e.g.} \quad \Pi_\mu(q, p_B) = i \int_x e^{iqx} \langle \pi(p) | V_\mu(x) J_B(0) | 0 \rangle$$

1. Hadronic: dispersion relation, separate lowest resonance (Residue $\sim$ hadr. data)
2. Quarks: perform a Light-Cone OPE
3. Estimate remaining dispersive-integral by analytically cont. of LC-OPE (semi-global Quark-Hadron-Duality)
4. Numerical improvement through Borel transformation.

Exemplified for $f_+^{B\to\pi}$ in equations ...

$$\# \frac{f_B f_+^{B\to\pi}}{q^2 - m_B^2} + \frac{1}{\pi}\int_{s_0} \frac{\mathrm{Im}[\Pi_+^{\mathrm{LC}}(s,q)]}{s - p_B^2} = \sum_{i \in \mathrm{twist}} T_H^i \otimes \phi^i \equiv \Pi_+^{\mathrm{LC}}(q, p_B)$$

A. T_H pert. calculable kernel (exp.in α_s / analogue Wilson Coeff. OPE)

B. ϕ universal π-Distribution Amplitude (analogue of matrix element in OPE)

C. twist = dim-spin of operator or DA;

D. valid for $(q^2, p_B^2) < m_b^2 - O(\Lambda m_b) \sim 14\mathrm{GeV}^2$, $\frac{3}{5}$ physical interval

Note: rôle of m_b numerical not parametrical (not $\frac{1}{m_b}$-expansion)
Therefore applicable for $F^{D\to P,V}$ c.f. Khodjamirian et al 00
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- Eliminate f_B in $(f_B f_+^{B\to\pi})_{\mathrm{SR}}$ by the corresponding sum rule to same accuracy

$$\boxed{f_+ = \frac{(f_+ f_B)_{\mathrm{SR}}}{(f_B)_{\mathrm{SR}}}}$$

Important: cancellation of uncertainties in ratio (e.g. α_s)

Calculations LCSR

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  have calculated these formfactors at various stages up to various orders

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- The important $f_+^{B\to\pi}$ formfactor is found

$$f_+^{B\to\pi}(0) = 0.258 \pm 0.031$$

  - LCSR calculation available/valid for $0 < q^2 < 14\mathrm{GeV}^2$, discuss extension later
  - Soft IR-divergencies cancel non-trivially for radiative corrections as required by consistency of factorization ansatz
  - twist-3 important because chirally enhanced (as in BBNS)

Uncertainty & possible improvements

- hadronic input parameters (mainly leading π-DA $\sim 8\%$ possible)
- QHD (incorporated in variation of Borel parameter) $\sim 4\%$ difficult, gain confidence through consistency checks
- $\alpha_s/\mu_{\rm IR}$ rather small (due to cancellation in ratio)
- higher twist ? t-2 $\sim$ 60%, t-3 $\sim$ 30%, t-4 $\sim$ 1% looks fine To be done: Test renormalon model for t-4 Braun, Gardi .. 04
- SU(3) additional uncertainty: prior to 04-06 $\sim 8\%$ now $\sim 3\%$ due to progress from QCD sum rules determinations of a_1

Fits and extension & consistency checks (comparison with lattice)

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$$f_+(q^2) = \frac{r_1}{1 - q^2/m_{B^*}^2} + \int_{(m_B+\Delta)^2}^{\infty} dt\, \frac{\rho(t)}{t - q^2}$$

Res. $r_1 = \frac{g_{BB^*\pi} f_{B^*}}{2m_{B^*}} \sim 0.8 \pm 0.2$, $f_B \sim f_{B^*}$, $g_{BB^*\pi}$ HQ-scaling $g_{DD^*\pi}$ (CLEO-01)

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G2 Fit of $r_1 \simeq 0.75$ (stable) , soft-pion point $f_0(m_B^2) = \frac{f_B}{f_\pi}$ get $f_B \sim 200\mathrm{MeV}$ (stable)

Comparison with lattice

- Early calc. quark models (BSW) $f_+(0)$ and then assumed VMD
- LCSR FF $q^2 < 14\mathrm{GeV}^2$
- Lattice FF $q^2 > 16\mathrm{GeV}^2$ (Idea moving frame (HPQCD) go to lower q^2)

GeV^2	LCSR 04	FNAL 04	HPQCD 06	Abada et al 00
$f_+(0)$	0.26 ± 0.03	0.23	0.26	0.27
$f_+(16)$	0.9	0.8 ± 0.1	0.71 ± 0.06	0.87 ± 0.1

- FNAL staggered fermions unquenched, Wilsonian HQ action
- HPQCD staggered fermions unquenched, NRQCD
- Abada et al quenched, Improved Wilson action
- Note: Lattice community become cautious quoting $B \to \rho$ etc, because ρ unstable particle, LCSR only calculation there!
- $F^{D \to \pi, K}$ from LCSR Khodjamirian et al 00 also good agreement with LQCD

Distribution Amplitude (DA)
(Focus on Kaon)

Meson Distribution Amplitudes (DA)

- Relevant for **exclusive** QCD processes at large momentum transfer, semileptonic heavy-light, BBNS $B \to \pi\pi(K)$, $F_\pi(q^2)$, $F_{\gamma\gamma^*\pi}$ etc

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- Most important DA $\leftrightarrow$ minimal number of constituent (partons)

$$\langle K(p)|\bar{s}(x)\gamma_\mu\gamma_5 q(0)|0\rangle = i f_K p_\mu \int_0^1 du e^{iupx}\phi_K(u) + O(x^2, m_K^2)$$

called leading twist = dim - spin = 2 , $(1-u)$: mom. fraction of s-quark in meson
Analogous def. for $K^{*,\parallel}$ and $K^{*,\perp}$

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- Higher twist corrections:
  - higher Fock states $(\langle K|\bar{s}\sigma\cdot Gq|0\rangle)$
  - deviation from the light-cone $(O(x^2, m_\pi^2))$
  - other comb. of “good” and “bad” LC-states (Kogut & Soper) $(\langle\pi|\bar{s}\gamma_5 q|0\rangle)$

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- Distribution amplitudes identified (use QCD e.o.m) up to twist 4 Ball et al 98
  Update on twist-3,4 parameters forthcoming including SU(3)
  for Pseudoscalar Ball,Braun,Lenz 06 later Vectors Jones et al 06

Focus leading twist-2 DA

- Expand in Eigenfunctions of LO BL-ER Kernel V_0

$$\phi_K(u,\mu) = 6u\bar{u}(1 + \sum_{n\geq 1} a_n(\mu,K) C_n^{3/2}(2u-1))$$

  - a_n Gegenbauer moments (determination difficult)
  - $a_{\rm odd} = 0$ G-parity inv. particles (for π not K)
  - anomalous dimension $\gamma_{n+1} > \gamma_n$ “conformal hierarchy”
  - Alternative reasoning $SL(2,R)$ collinear subgroup of conformal group $SO(4,2)$ Gegenbauer C_n are representations with conformal spin $j = 2+n$

How to deal information ?

1. Truncation , let's say for decay $\mathcal{A}_{X\to KY}$

$$\mathcal{A} = f^{(0)} + f^{(1)} a_1 + f^{(2)} a_2 + \dots$$

   A. determinations of a_n indicate $a_0 \equiv 1 > |a_{1,2}| > |a_{3,4}|$.. (Ok conformal hierarchy)

   B. if kernel T_H is smooth then $|f^{(0)}| > |f^{(1,2)}| > |f^{(3,4)}|$
   (Analogy with partial wave expansion ($SO(3), Y_{\rm lm}$) $\sim$ ($SL(2,R), C_n$)
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2. Model satisfying theoretical and experimental constraints Ball,Talbot 05

   A. From $\gamma^*\gamma\pi$ CLEO, theory $\Delta = \int du \phi_\pi(u)/u \sim 1.2 \pm 0.2$

   B. Using LO-rng $a_n(\mu) = a_n(\mu_0)(L)^{\gamma_n/(2\beta_0)}$

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$\longrightarrow$ Decide process by process to whether to resort to 1. or 2. depending on smoothness of kernel and or endpoint sensitivity

- $B \to \mathrm{light}$ FF from LCSR no big change
- $B \to \pi\pi(K)$, branching ratios and CP-asymmetries in BBNS approach, more relevant not enough to account for experimental discrepancy Ball,Talbot05

Determination of Gegenbauer moments a_1, a_2, ...

- Fit to an observable, be careful other hadr. uncert. do not contaminate
  Examples for a_2^π: $F_{\gamma\gamma^*,\pi}$,$F_\pi^{\rm em}$,$F^{B\to\pi}$-shape
  More spectral data (bins) would be useful e.g. $B \to \pi e \nu$ others

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$$\langle 0|\bar{s} z_\mu \gamma^\mu \gamma_5 (iz \overset{\leftrightarrow}{D})^n q|K(p)\rangle = (zp)^{n+1} f_K 2 \int_0^1 du (2u-1)^n \phi_K(u) \equiv N \cdot M_n$$

$$M_0 = 1 \qquad M_2 = \frac{1}{5} + \frac{12}{35} a_2$$
$$M_1 = a_1 \qquad M_4 = \frac{3}{35} + \frac{8}{35} a_2 + \frac{8}{77} a_4$$

  - In QCD sum rules (pioneered by Chernyak & Zhitnitsky $\sim$ 1980)
    Noticed that only first few moments give stable sum rules, $n > 4$ not useful
  - Lattice worked on it ~ 90 got contradicting results
    New start UKQCD QCDSF second moment available, first moment on the way !!
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    Also here higher moments difficult (derivatives)
- New methods from exact operator relations for first moment (a_1) . . .

Overview of calculations for a_1

a_1 obtained from correlation function of the type

$$i \int_x \langle 0| T \bar{q}(iz \overleftrightarrow{D}) \Gamma_1 s(x) \, \bar{s} \Gamma_2 q(0) |0\rangle$$

Note: $a_1 > 0$ higher average momentum of s-quark as suggested by Constituent quark-model

Type	$a_1(K)(\mu_0)$	$a_1^{\parallel}(K^*)(\mu_0)$	$a_1^{\perp}(K^*)(\mu_0)$	Authors	Remarks
ND	0.17	0.19	0.2	Chernyak & Zhit. 84	sign mistake
ND	-0.18	-0.4	-0.34	Ball Boglione 03	NLO,unstable
D	0.05 ± 0.02	-	-	Khodjamiran et al 04	-
OPR	0.1 ± 0.12	0.1 ± 0.07	-	Braun Lenz 04	neglect $O(m_s^2)$
D	0.06 ± 0.03	0.03 ± 0.02	0.04 ± 0.03	Ball RZ 05	confirm 04, extend
OPR	0.07 ± 0.18	0.01 ± 0.05	0.09 ± 0.07	Ball RZ 06	incl $O(m_s^2)$

- ND: spectral-fct non-positive def. (cancellations, contamination higher states) ! which turns out to be the case $\Rightarrow$ not consider anymore
- D: pos. def. work fine are the best OPR: New method can't compete yet . . .

a_1 from operator relations

New operator relations of the type:

$$M_1 \equiv \frac{3}{5}\, a_1^{\|}(K^*) = -\frac{f_K^{\perp}}{f_K^{\|}}\,\frac{m_s - m_q}{m_{K^*}} + 2\,\frac{m_s^2 - m_q^2}{m_{K^*}^2} - 4\kappa_4^{\|}(K^*)$$

$$\langle 0|\bar{q}(gG_{\alpha\mu})i\gamma^{\mu}s|K^*(q)\rangle = e^{\alpha} f_K^{\|} m_{K^*}^3 \kappa_4^{\|}(K^*)$$

1. From $O_{\mu\nu} = \frac{1}{2}\bar{q}\gamma_\mu\gamma_5 i \overleftrightarrow{D}_\nu\, s + \ldots$ with $O_\mu^{\ \mu} = 0$ playing role of energy momentum tensor by Braun & Lenz 04 for $a_1(K)$ and $a_1^{\|}(K^*)$

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2. From the QCD equation of motion those relations were rederived plus a relation for $a_1^{\perp}(K^*)$ (Difficult other method) by Ball & RZ 06

a_1 from operator relations

New operator relations of the type:

$$M_1 \equiv \frac{3}{5}\, a_1^{\parallel}(K^*) = -\frac{f_K^{\perp}}{f_K^{\parallel}}\, \frac{m_s - m_q}{m_{K^*}} + 2\, \frac{m_s^2 - m_q^2}{m_{K^*}^2} - 4\kappa_4^{\parallel}(K^*)$$

$$\langle 0|\bar{q}(gG_{\alpha\mu})i\gamma^{\mu} s|K^*(q)\rangle = e^{\alpha} f_K^{\parallel} m_{K^*}^3 \kappa_4^{\parallel}(K^*)$$

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.

- The $\kappa_4' s$ are estimated via several QCD Sum Rules, not very stable sensitive to $m_s, \alpha_s, \langle\bar{s}s\rangle/\langle\bar{q}q\rangle$
- $\kappa_4' s$ could of course also be estimated from Lattice ! Why not ? Overall Precision ?

- Therefore for phenomenology one should use the values from the diagonal sum rules

$$a_1(K) = 0.06 \pm 0.03, \quad a_1^{\|}(K^*) = 0.03 \pm 0.02 \quad a_1^{\perp}(K^*) = 0.04 \pm 0.03$$

Khodjamirian et al PRD70, Ball RZ JHEP 06 in press

- $a_2(K)$?
  1. $a_2(K) \sim a_2(\pi)$, SU(3) sufficiently good there
  2. Many determinations .. Sum Rules, Lattice, other approaches, fit to exp. data etc Small overview appear in Ball, Braun, Lenz $a_2 \sim 0.2$

The topic of another talk!

Application: Tensorratio $\frac{T_1^{B\to K^*}}{T_1^{B\to\rho}}$

$B \to K^*\gamma$ vs. $B \to \rho\gamma$

(work in preparation, so don't expect too many details. . .)

- measured by Belle 05, Babar forthcoming
- constrain $|V_{ts}/V_{td}|$ from $B(B \to K^*\gamma)/B(B \to \rho\gamma)$: more accurate than constraint from B mixing?
- in QCD factorization (Bosch et al Beneke et al Neubert et al):

$$\langle V\gamma|Q_i|B\rangle = T_i^I F(B \to V_\perp) + \int_0^\infty \frac{d\omega}{\omega}\,\phi_B(\omega)\int_0^1 du\phi_{V_\perp}(u)T_i^{II}(\omega,u) + O\left(\frac{\Lambda}{m_b}\right) + O\left(\frac{\Lambda^2}{m_s^2}\right)$$

- need SU(3) breaking in

(a) ratio of form factors $F(B \to K^*_\perp)/F(B \to \rho_\perp)$
(b) distribution amplitudes $\phi_{K^*_\perp,\rho_\perp}$

- SU(3) breaking in distribution amplitude $\phi_\perp$: known Ball RZ 06(a)
- SU(3) breaking in form factors: under way Ball RZ 06(b)
  The dependence of the formfactors on $a_1(K^*)$ are given in Ball RZ PRD7105(b) and the update of the preliminary ratio is

$$\xi = \frac{T_1^{B\to K^*(0)}}{T_1^{B\to\rho}(0)} = 1.16 \pm 0.1_{\rm param} \pm 0.005_{a_1^\parallel} \pm 0.035_{a_1^\perp} = 1.16 \pm 0.1 \pm 0.04_{a_1}$$

  - old values $\xi = 1.25 \pm 0.1_{\rm param} \pm 0.02_{a_1^\parallel} \pm 0.13_{a_1^\perp}$
  - have to work further on $0.1_{\rm param}$
- could do with some $1/m_b$ effects? e.g. long-distance photon-emission (under way)

From Bosch & Buchalla 04:

$$R_0 \equiv \frac{B(B^0 \to \rho^0\gamma) + B(\bar{B}^0 \to \rho^0\gamma)}{B(B^0 \to K^*\gamma) + B(\bar{B}^0 \to \bar{K}^*\gamma)} = \frac{K}{2|\xi|^2}|V_{td}/V_{ts}|^2(1+\Delta)\,,$$

where K kinematical factor, $|\Delta| < 0.4$ contains subleading WA & penguins

Conclusions

A1 The $F^{B(d,s)\to P,V}(q^2)$ can be calculated for $0 < q^2 <\sim 14\mathrm{GeV}^2$ from LCSR

Lattice provides $F^{B_{d,s}\to P}$ so far for $q^2 > 16\mathrm{GeV}^2$ complementarity!

A2 LCSR only source for vector formfactors. Other methods would be nice. Ingenious lattice people will hopefully come up with something

A3 Two-pole param. fits the LCSR-well and survives consistency tests.

A4 good numerical agreement with Lattice-QCD (comp. upon extrapol.)

B1 After confusion considerable progress on leading Kaon DA – Gegenbauer moment a_1

B2 Progress on Kaon DA immediate impact on $B \to K^*\gamma$ vs. $B \to \rho\gamma$

C1 Experimentalists: Would be useful to get more bins! In order to check and test expansions and models of DA (relevant for exclusive physics)

Thanks for your attention !

Backup slide

- Comparison with $\mathrm{Meson} \to \pi$

$f_+^{B\to\pi}(0)$	$f_+^{D\to\pi}(0)$	$f_+^{K\to\pi}(0)$	$f_+^{\pi\to\pi}(0)$
0.26	0.65	0.96	1.00

the larger the recoil the less likely π-boundstate can form

Soft mechanism automatically automatically included in LCSR

Physics

- Comparison with $\mathrm{Meson} \to \pi$

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the larger the recoil the less likely π-boundstate can form

Higher twist DA. More unknown parameters ?

- Structure of DA known up to twist-4 (Ball,Braun,Koike,Filyanov,Tanaka). Quick overview for π (counting):

Twist	2	3	4	
numb.DA	1	3	6	10
param.NLO j	1(2)	5	12	18

  - j conformal spin (Gegenbauer expansion). 18 Non-pert. parameters !
  - paramters
    1. Norm. of matrix elements (Analogue of $f_\pi a_0 \equiv f_\pi$)
    2. NL-conf. spin (analogue of a_2)

- The number of parameters reduce to 5 upon use of (exact) QCD e.o.m. !!

$$\text{e.g.} \quad \frac{\partial}{\partial x_\mu} \bar{q}_1(x)\gamma_\mu(\gamma_5)q_2(-x) = -i\int_{-1}^{1} dv\, v\bar{q}_1(x)x_\alpha g G^{\alpha\mu}(vx)\gamma_\mu(\gamma_5)q_2(-x) + (m_1 \pm m_2)\bar{q}_1(x)i(\gamma_5)q_2(-x)$$

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