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*Physics Landscape in 10 years*

# Lepton Flavour Violation in Muon Decays

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Lepton flavour is violated,  
so why should charged lepton flavour be conserved?

- introduction (stating the obvious)
- the golden channels  $\mu \rightarrow e\gamma$ ,  $\mu \rightarrow 3e$ ,  $\mu N \rightarrow eN$ 
  - experiments: current status / outlook
  - backgrounds !!
  - theory interpretation
- beyond the golden channels
- other things to do with muon decays

playing with SM fields only:

dim 4: SM = most general gauge and Lorentz invariant Lagrangian

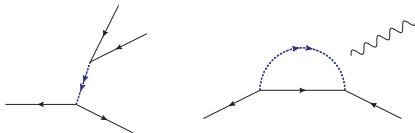
$$\begin{aligned}
 \mathcal{L}_{\text{SM}} = & -\frac{1}{4}G^{\mu\nu}G_{\mu\nu} - \frac{1}{4}W^{\mu\nu}W_{\mu\nu} - \frac{1}{4}B^{\mu\nu}B_{\mu\nu} + \hat{\theta}G^{\mu\nu}\tilde{G}_{\mu\nu} \\
 & + (D_\mu\Phi)^\dagger(D^\mu\Phi) - m_H^2\Phi^\dagger\Phi - \frac{\lambda}{2}(\Phi^\dagger\Phi)^2 \\
 & + i(\bar{\ell}\not{D}\ell + \bar{e}\not{D}e + \dots) - (Y_e\bar{\ell}e\Phi + \dots + \text{h.c.}) \\
 & + \text{nothing with } \nu_R \quad \rightarrow \quad \text{no cLFV}
 \end{aligned}$$

dim 5 violates lepton number, but doesn't affect SM much

dim 6, either we have cLFV or a 'problem' (i.e. need an explanation)

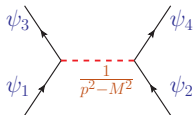
cLFV a unique window with a view deep into the UV

## Example: doubly charged Higgs

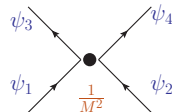


- as UV complete model: embed in multiplet, sort out  $\rho$  parameter ...
  - ++ valid  $\forall p^2$ , explains everything
  - requires divine inspiration
- as simplified model:  $\mathcal{L}_{\text{int}} = \lambda_{fi} (\bar{l}_f^c l_i) \phi^{++} \dots$  few couplings, 1 mass
  - +− valid for  $p^2 > m_\phi^2$
  - −+ more or less general
- via effective theory:  $\mathcal{L}_{\text{int}} = c_{fijk} (\bar{l}_f \gamma^\mu l_i) (\bar{l}_j \gamma_\mu l_k) \dots$   $c$ 's  $\leftrightarrow$   $\lambda$ 's
  - valid only for  $p^2 \ll m_\phi^2$
  - ++ completely general

EFT is never the goal, only the tool, (cp.  $C_9 = -C_{10}$  for  $B$  anomalies)



$$\mathcal{O}^i = \frac{1}{\Lambda_{\text{NP}}^2} (\bar{\psi}_3 \Gamma^a \psi_1) (\bar{\psi}_4 \Gamma^b \psi_2)$$



$$\mathcal{O}_{\text{eff}}^1 = (\bar{e}_L \gamma^\rho \mu_L) (\bar{e}_R \gamma_\rho e_R)$$

$$\mathcal{O}_{\text{eff}}^2 = (\bar{\nu}_e \gamma^\rho \nu_\mu) (\bar{e}_R \gamma_\rho e_R)$$

$$SU(3)_{\text{QCD}} \times U(1)_{\text{QED}}$$

$$\mathcal{O}_{\text{smeft}} = \overline{\begin{pmatrix} \nu_e \\ e_L \end{pmatrix}} \gamma^\rho \begin{pmatrix} \nu_\mu \\ \mu_L \end{pmatrix} (\bar{e}_R \gamma_\rho e_R)$$

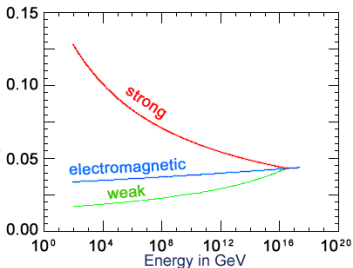
$$SU(3)_{\text{QCD}} \times SU(2) \times U(1)_Y$$

if EFT, then properly i.e. include running and mixing

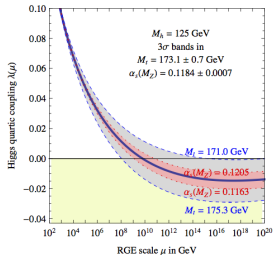
scale of cLFV experiments  $m_{\text{mu}} \leq \mu \leq m_W \rightarrow c_i(m_{\text{mu}})$  “useless”

high-energy behaviour might reveal properties of underlying theory

unified theory?



stable universe ?

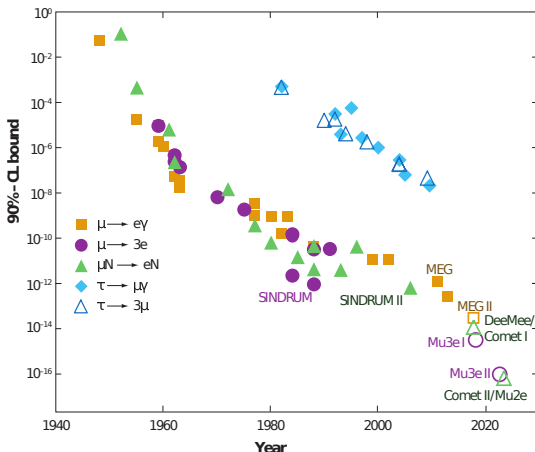


evolve from to  $m_{\text{mu}}$  to  $m_W$  (to combine experiments)

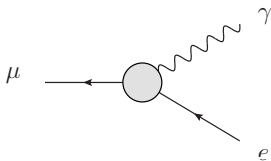
and from  $m_W$  to  $\Lambda_{\text{uv}} \gg m_w$  (to get information on BSM)

- $\mu \rightarrow e\gamma$ 
  - current MEG (2016)  $\text{Br}(\mu \rightarrow e\gamma) < 4.2 \times 10^{-13}$
  - MEG II: (2018-2021) expect:  $\text{Br}(\mu \rightarrow e\gamma) \sim \times 10^{-14}$
- $\mu \rightarrow eee$ 
  - current Sindrum (1988)  $\text{Br}(\mu \rightarrow eee) < 1 \times 10^{-12}$
  - new experiment Mu3e
    - Phase 1 (2020++):  $\text{Br} \sim \text{few} \times 10^{-15}$ ,
    - Phase 2 (20??++): new beamline  $\text{Br} \sim 10^{-16}$
- $\mu N \rightarrow eN$ 
  - current Sindrum II (2006)  $\text{Br}(\mu \text{ Au} \rightarrow e \text{ Au}) < 7 \times 10^{-13}$
  - new experiment DeeMe ? (2017++):  $\text{Br} \sim 10^{-14}$
  - new experiments Comet (2019/ 2021-22)
    - and Mu2e (2022-24):  $\text{Br} \sim 10^{-16}$
  - ?? Prism/Prime ?? (20??) :  $\text{Br} \sim 10^{-18}$

evolution of limits → very rich experimental programme with substantial improvements on all muon-related processes expected in near future

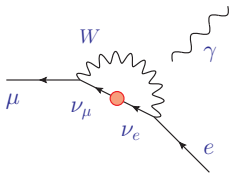






signal: monoenergetic, simultaneous,  
back-to-back  $e$  and  $\gamma$

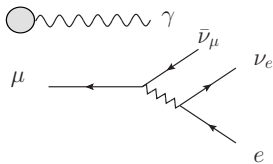
in SM with massive neutrinos



- $\text{BR}(\mu \rightarrow e\gamma) \sim \alpha \left( \frac{\Delta m^2}{m_W^2} \right)^2$
- LFV in neutrino sector  $\rightarrow$  cLFV
- there is nothing sacred about cLF
- but  $\Delta m \rightarrow \text{BR}(\mu \rightarrow e\gamma) \sim 10^{-54}$

still, there is background !!

PSI: continuous beam of muons,  $10^8 \text{ s}^{-1}$

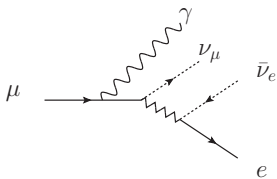


accidental background:

$e$  and  $\gamma$  not quite back-to-back and monoenergetic nor quite simultaneous

$\Rightarrow$  timing, vertex and momentum resolution very important

$\Rightarrow$  upgrade MEG II



irreducible background

SM process radiative decay

in region where  $\nu$  very little energy

missing momentum  $p_e + p_\gamma \neq m_{\text{mu}}$

radiative decay @NLO fully differential [Pruna,AS,Ulrich]

(BR compared with [Fael,Mercolli,Passera])

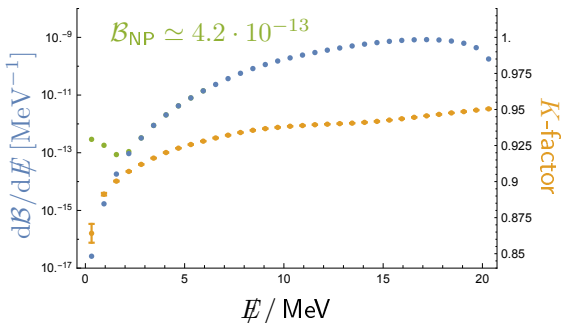
polarization:  $\vec{s} = -0.85\hat{z}$  and toy cuts: no 2<sup>nd</sup> photon with  $E_\gamma > 2$  MeV

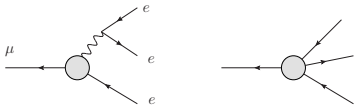
$E_\gamma > 40$  MeV,  $|\cos\theta_\gamma| < 0.35$ ,  $|\phi_\gamma| > 2\pi/3$

$E_e > 45$  MeV,  $|\cos\theta_e| < 0.5$ ,  $|\phi_\gamma| < \pi/3$

The invisible energy  
spectrum

$$\cancel{E} = m_{\text{mu}} - E_e - E_\gamma$$



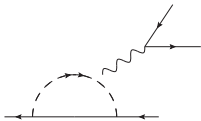


signal:  $2 e^+ + 1 e^-$ , simultaneous,  
from same vertex,  $\sum p_e = m_{\mu}$

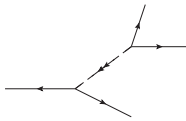
dipole part 'same' as  $\mu \rightarrow e\gamma$

contact part completely new

e.g. doubly charged Higgs

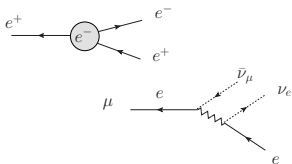


via dipole terms



contact interaction

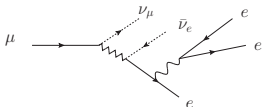
@ LO @  $\mu = m_{\mu}$  !!



accidental background:

$e$  and  $\gamma$  not quite from same vertex  
nor quite simultaneous and with missing momentum

$\Rightarrow$  timing, vertex and momentum resolution very important



irreducible background

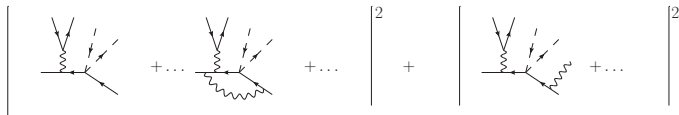
SM process rare decay

in region where  $\nu$  very little energy

missing momentum  $\sum p_e \neq m_{\mu}$

$\mu \rightarrow 3e + 2\nu$  fully differential @NLO [Pruna,AS,Ulrich]

BR (with cuts on invisible energy) compared with [Fael,Greub]

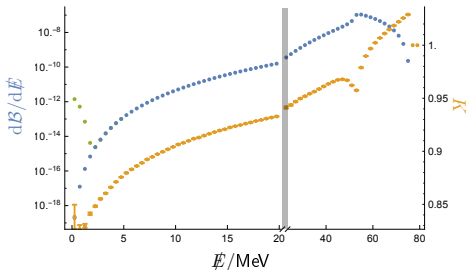


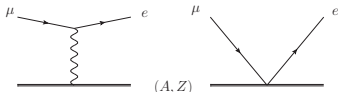
polarization:  $\vec{s} = -0.85\hat{z}$

toy cuts:  $E_i > 10 \text{ MeV}$ ,  $|\cos\theta_i| < 0.8$

The invisible energy spectrum

$$\not{E} = m_{\mu} - \sum E_i$$





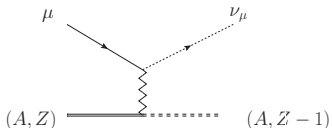
$\mu$  conversion:  $\mu^- N_Z^A \rightarrow e^- N_Z^A$

signal: single 105 MeV  $e^-$

photonic part 'same' as  $\mu \rightarrow e\gamma$

contact part completely new

nucleus not affected (only recoil)  $\rightarrow$  dirty nuclear physics under control



$\mu$  capture:  $\mu^- N_Z^A \rightarrow \nu_\mu N_{Z-1}^A$

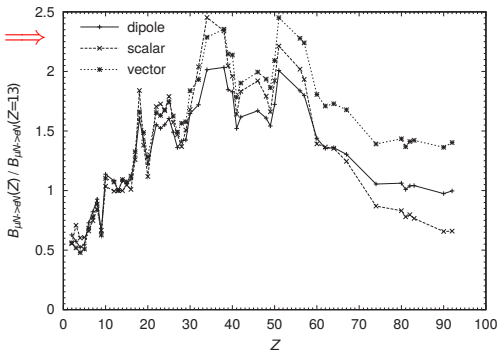
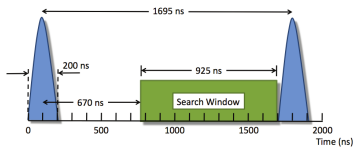
denominator of 'branching' ratio

for larger  $Z$ , shorter life time

which  $Z$  ? [Fässler et al; Cirigliano et al.]

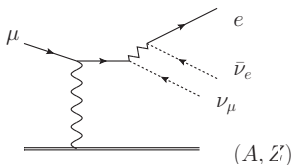
SINDRUM with Au (Ti, Pb), COMET/Mu2e plan Al (initially)  
 large  $Z \rightarrow$  increase sensitivity  $\rightarrow$  small life time ( ?? pulsed beams ??)

at  $\mu = \mu_N$  no axial couplings  $\Rightarrow$   
 (coherent  $\mu N \rightarrow e N$ )





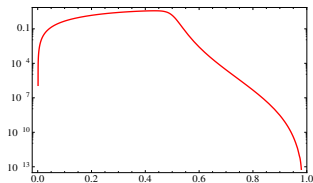
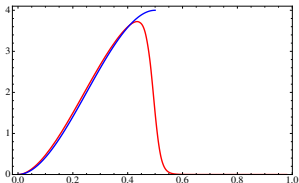
decay in orbit: irreducible background for  $\mu N \rightarrow e N$



$$\text{DIO: } \mu^- N_Z^A \rightarrow e^- \bar{\nu}_e \nu_\mu N_Z^A$$

$\sum p_e > m_{\text{mu}}/2 \rightarrow m_{\text{mu}}$  possible  
nuclear recoil

DIO energy spectrum [Czarnecki et al.]



Processes take place at scale  $\mu = m_{\text{mu}}$  or  $\mu = \mu_N \sim 1 \text{ GeV}$



$$\mathcal{O}_{\text{eff}} = (\overline{e_L} \gamma^\mu \mu_L) (\overline{e_R} \gamma_\mu e_R)$$

$$SU(3)_{\text{QCD}} \times U(1)_{\text{QED}}$$

$$\Lambda_{\text{NP}} \leq m_W$$

$$\mathcal{O}_{\text{smeft}} = \left( \overline{\begin{pmatrix} \nu_e \\ e_L \end{pmatrix}} \right) \gamma^\mu \begin{pmatrix} \nu_\mu \\ \mu_L \end{pmatrix} (\overline{e_R} \gamma_\mu e_R)$$

$$SU(3)_{\text{QCD}} \times SU(2) \times U(1)_Y$$

$$\Lambda_{\text{NP}} \gg m_W$$

effective Lagrangian  $\mathcal{L}_{\text{eff}}$  (below EW scale) for  $\mu \rightarrow e$  processes

allow for  $\mu \rightarrow e$  but otherwise flavour diagonal (i.e. no small<sup>2</sup>)

what is often used: [Kuno,Okada:hep-ph/9909265]

ok if coefficients are interpreted at  $\mu = m_{\text{mu}}$  no link with e.g.  $Z \rightarrow e\mu$

$$\begin{aligned}
 \mathcal{L}_{\text{eff}} = & \mathcal{L}_{\text{QED}} + \mathcal{L}_{\text{QCD}} \\
 & + \frac{4G_F}{\sqrt{2}} \left[ A_R m_\mu \bar{\mu}_R \sigma^{\mu\nu} e_L F_{\mu\nu} + L \leftrightarrow R \right. \\
 & + g_1 (\bar{\mu}_R e_L) (\bar{e}_R e_L) + g_2 (\bar{\mu}_L e_R) (\bar{e}_L e_R) \\
 & + g_3 (\bar{\mu}_R \gamma^\mu e_R) (\bar{e}_R \gamma_\mu e_R) + g_4 (\bar{\mu}_L \gamma^\mu e_L) (\bar{e}_L \gamma_\mu e_L) \\
 & \left. + g_5 (\bar{\mu}_R \gamma^\mu e_R) (\bar{e}_L \gamma_\mu e_L) + g_6 (\bar{\mu}_L \gamma^\mu e_L) (\bar{e}_R \gamma_\mu e_R) + \text{h.c.} \right]
 \end{aligned}$$

effective Lagrangian  $\mathcal{L}_{\text{eff}}$  (below EW scale) for  $\mu \rightarrow e$  processes

allow for  $\mu \rightarrow e$  but otherwise flavour diagonal (i.e. no small<sup>2</sup>)

what should be used: [Crivellin, Davidson, Pruna, AS:1702.03020]

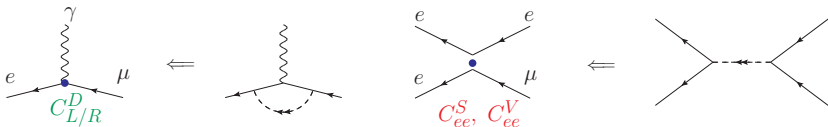
needed if coefficients are to be evolved (e.g. up to  $\mu = m_W$ )

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{QED}} + \mathcal{L}_{\text{QCD}}$$

$$\begin{aligned}
 & + \frac{1}{\Lambda^2} \left[ C_L^D e m_\mu (\bar{e}_L \sigma^{\mu\nu} \mu_L) F_{\mu\nu} + \sum_{f=q,\ell} \left[ C_{ff}^{S LL} (\bar{e}_R \mu_L) (\bar{f}_R f_L) \right. \right. \\
 & \quad \left. \left. + C_{ff}^{V LL} (\bar{e}_L \gamma^\mu \mu_L) (\bar{f}_L \gamma_\mu f_L) + C_{ff}^{V LR} (\bar{e}_L \gamma^\mu \mu_L) (\bar{f}_R \gamma_\mu f_R) \right] \right. \\
 & \quad \left. + \sum_{h=q,\tau} \left[ C_{hh}^{T LL} (\bar{e}_R \sigma_{\mu\nu} \mu_L) (\bar{h}_R \sigma^{\mu\nu} h_L) + C_{hh}^{S LR} (\bar{e}_R \mu_L) (\bar{h}_L h_R) \right] \right. \\
 & \quad \left. + \alpha_s m_\mu G_F (\bar{e}_R \mu_L) G_{\mu\nu}^a G_a^{\mu\nu} + L \leftrightarrow R + \text{h.c.} \right]
 \end{aligned}$$

express observables  $\mu \rightarrow e\gamma$ ,  $\mu \rightarrow 3e$ ,  $\mu N \rightarrow eN$  through  $\mathcal{L}_{\text{eff}}$  e.g.

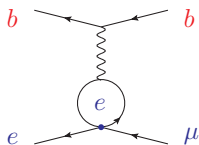
$$\text{Br}(\mu \rightarrow 3e) \simeq \alpha_e^2 m_\mu^5 \left( |C_L^D|^2 + |C_R^D|^2 \right) \left( 8 \log \left[ \frac{m_\mu}{m_e} \right] - 11 \right) \\ + m_\mu^5 \left( |C_{ee}^{S LL}|^2 + 16 |C_{ee}^{V LL}|^2 + 8 |C_{ee}^{V LR}|^2 + L \leftrightarrow R \right)$$



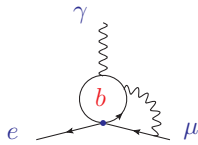
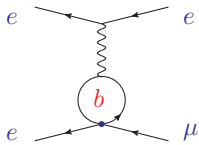
for  $\text{BR}(\mu \rightarrow e\gamma)$  and  $\text{BR}(\mu \rightarrow 3e)$ :  $C_i(m_\mu)$

for  $\text{BR}(\mu N \rightarrow eN)$ :  $C_i(\mu_N)$  (we choose  $\mu_N = 1 \text{ GeV}$ )

- match at tree level, run at one loop
- include 'leading' two-loop effects  
mixing of vectors into dipole as for  $b \rightarrow s\gamma$
- Wilson coefficients **run and mix**, we want  $C_i(m_W)$
- operators mix under RGE: **one loop** **two loop**



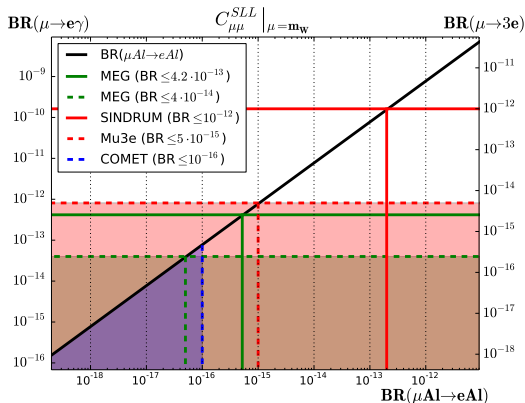
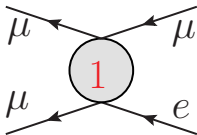
and



$$(\bar{e}_L \gamma^\mu \mu_L)(\bar{b}_L \gamma_\mu b_L) \rightarrow (\bar{e}_L \gamma^\mu \mu_L)(\bar{e}_L \gamma_\mu e_L) \text{ or } (\bar{e}_L \sigma^{\mu\nu} \mu_L) F_{\mu\nu}$$

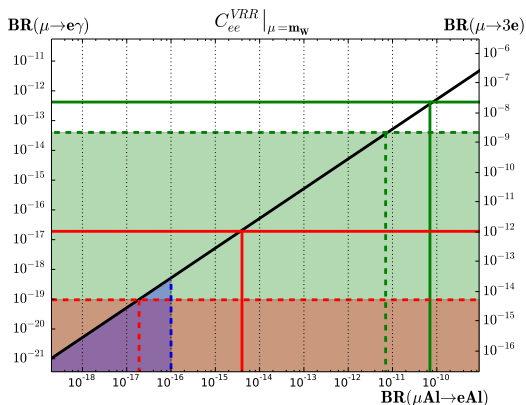
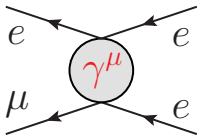
naive one-at-a-time limits  $C_i(m_W)$

absolute value of Wilson coefficient is irrelevant (depends on conventional prefactors)



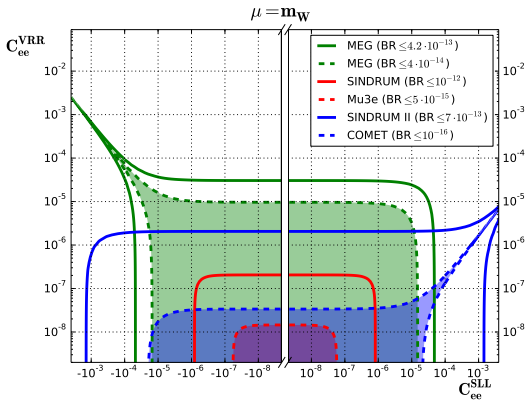
naive one-at-a-time limits  $C_i(m_W)$

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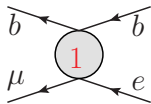




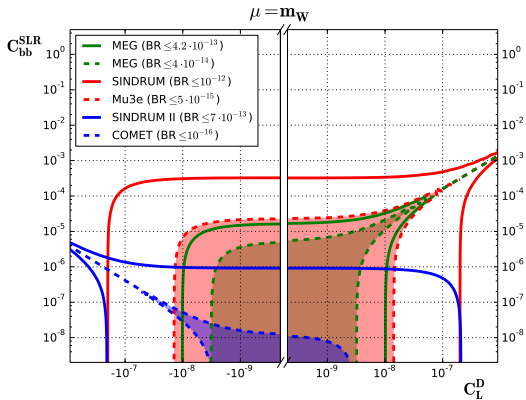
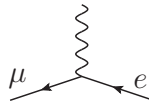
naive two-at-a-time limits



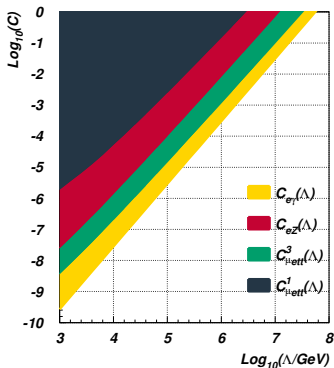
naive two-at-a-time limits



vs.



## Constraints from $\mu \rightarrow e\gamma$



[Pruna, AS: 1408:3565]

- contact interactions  
 $C_{\mu\text{ett}}^1 \rightarrow C_{\mu\text{ett}}^3 \rightarrow$   
 dipole interaction  $C_{e\gamma}$
- energy range probed up to  $\sim 10^7$  TeV
- even indirect limits can be very constraining

## beware of common misconceptions

- $\mu \rightarrow e\gamma$  is **very sensitive** to contact interactions !!
- $\mu N \rightarrow eN$  is **very sensitive** to pseudo scalar and axial vector interactions !!
- RGE is **not** a precision issue, but yields qualitatively new results (mixing)
- one-at-a-time / two-at-a-time limits only for presentational purposes

## many ways to go beyond the golden channels

examples (ordered according to increasing energy):

[Mu2e, Comet, Babar, Belle, LHCb, CMS, Atlas ...]

- $M(\mu^- e^+) \leftrightarrow \bar{M}(\mu^+ e^-)$  oscillation
- $\mu^- N \rightarrow e^+ N'$  experimentally 'easy' but nuclear 'mess'
- golden channels with  $\tau$  [Babar, Belle]

$$\text{BR}(\tau \rightarrow 3\ell) \lesssim (1 - 2) \times 10^{-8}, \quad \text{BR}(\tau \rightarrow \ell\gamma) \lesssim 4 \times 10^{-8}$$

- hadronic decays with  $\tau$  such as  $\tau \rightarrow \ell K^{(*)}$  or  $\tau \rightarrow \ell\pi^+\pi^-$
- involving  $B$  decays (very topical !!)

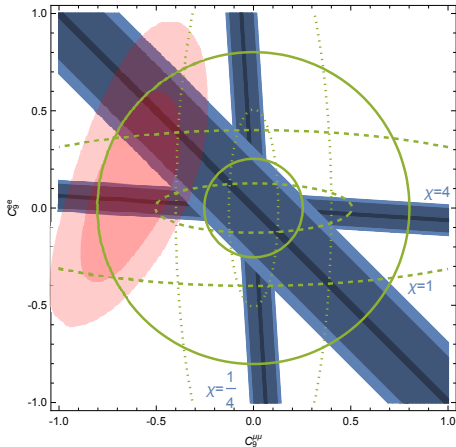
$$B \rightarrow K\ell\ell', \quad B \rightarrow \pi\ell\ell', \quad B_s \rightarrow \ell\ell'$$

- involving  $Z$  and  $H$  or anything at  $\Lambda \gtrsim m_{\text{EW}}$

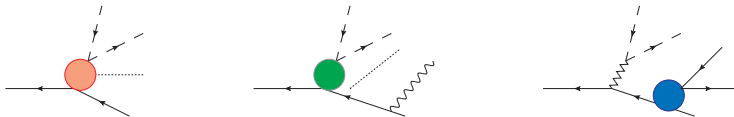
$$Z \rightarrow \tau\mu, \quad H \rightarrow \tau\mu$$

connection cLFV with *B* anomalies

- if *B* anomalies are due to BSM  $\rightarrow$  scale of NP not very high
- signals in cLFV are also possible
- e.g. leptoquarks  $\sim$  TeV  
 $C_9 = -C_{10}$   
[Crivellin et al.]
- $b \rightarrow s\mu^+\mu^-$   
 $\mu \rightarrow e\gamma$   
 $B \rightarrow K\mu^\pm e^\mp$

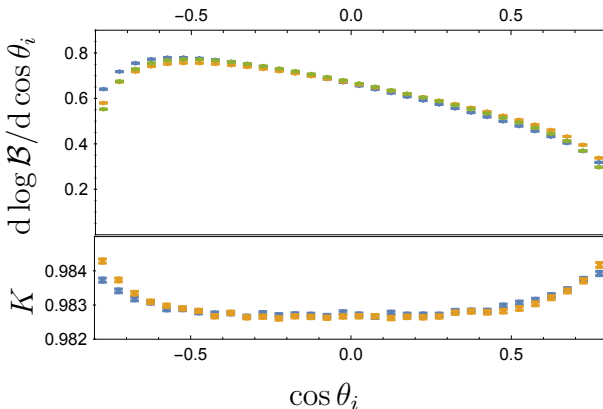


looking for deviations from SM with  $\mu \rightarrow e(+2e/\gamma) + X$



- looking for the weird and wonderful, typically MeV scale
- dark photons, “heavy” neutrinos, 17 MeV 5<sup>th</sup> force ...
- **recall**: 3.5  $\sigma$  discrepancy of  $\text{BR}(\tau \rightarrow e\nu\bar{\nu}\gamma)$  from Babar vs NLO (??) QED probably ‘just’ QED [Pruna,AS,Ulrich]

- in general: very precise predictions, theory error  $\ll 0.1\%$  as long as not squeezed into corner of phase space
- e.g. angular distributions for the **hard**  $e^+$ , **soft**  $e^+$  and  $e^-$
- $K \approx 0.98 \Rightarrow$  shape very precise





- cLFV is a window with a view deeply beyond EW scale
- why do we not see it then?  
Is  $\Lambda_{NP}$  just too large? or BSM still cLF conserving??
- EFT approach is ideal for investigating cLFV  
of course, we still want **the** explicit BSM in the end
- quantum corrections are essential  
**not** a precision issue but **qualitatively new** effects
- huge experimental progress expected within 5 – 10 years