Noncommutative geometry
Why and how?

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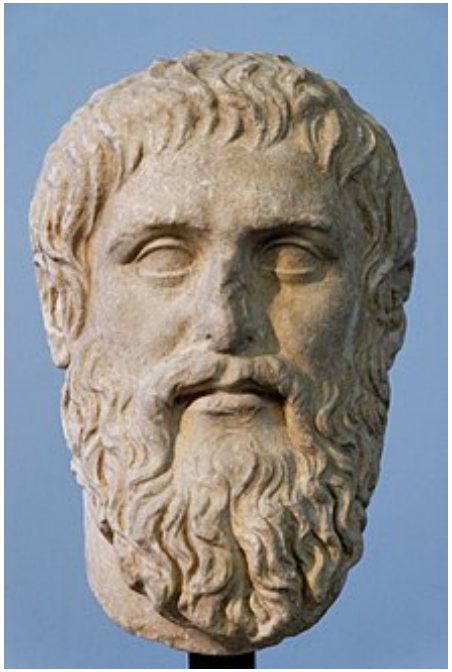
Greece – the birthplace of geometry

The knowledge of which geometry aims is the knowledge of the eternal.

[Plato, *Republic* 7.527b]



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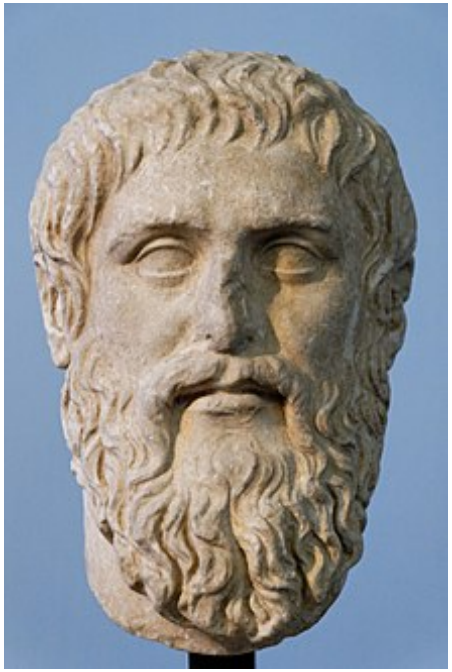

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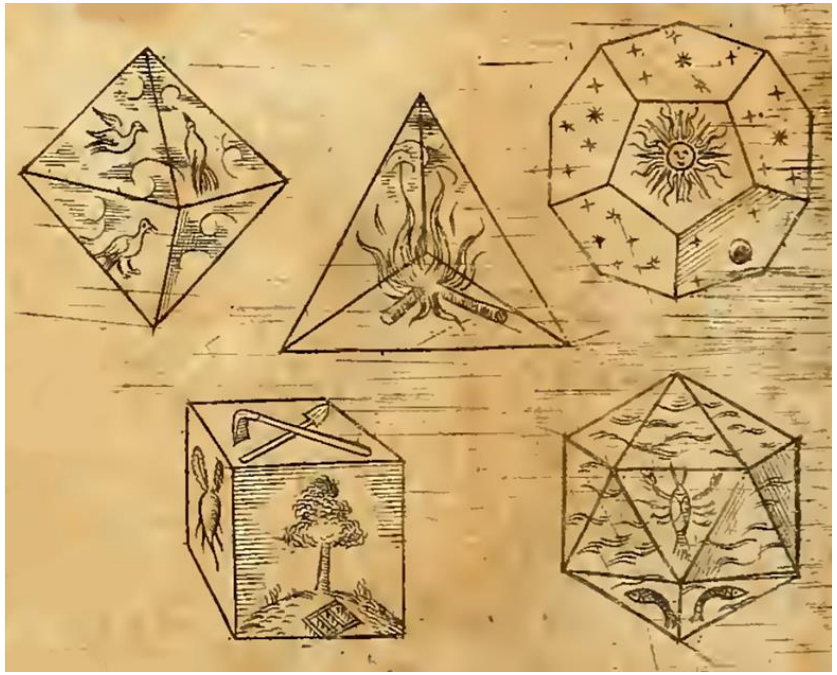
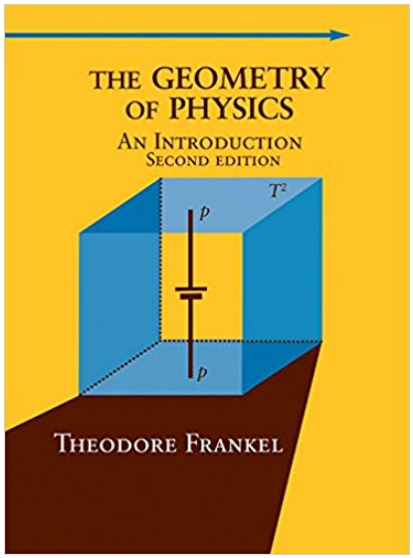
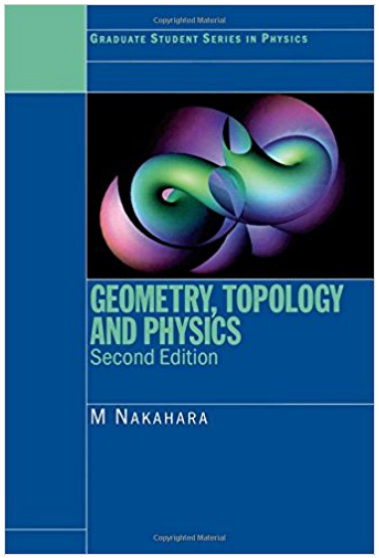
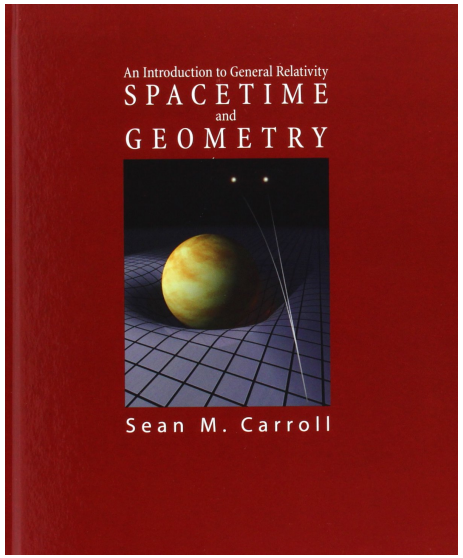


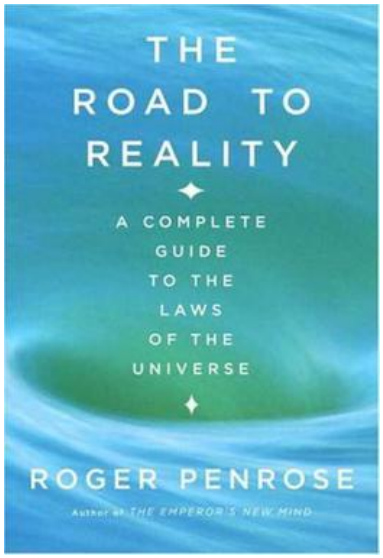
Johannes Kepler *Harmonices Mundi* 1619

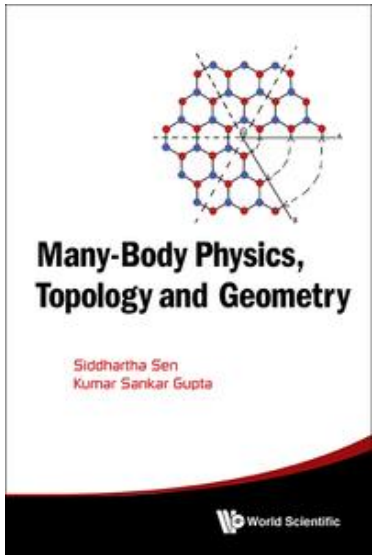
Physics is geometry!

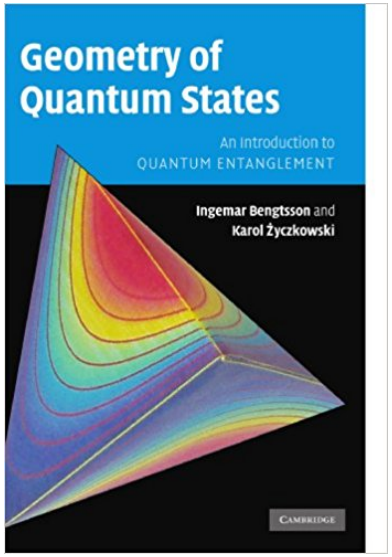


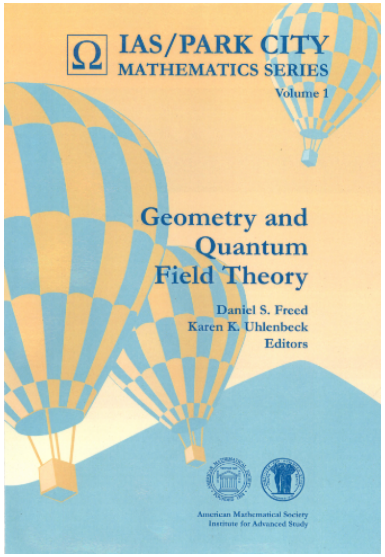


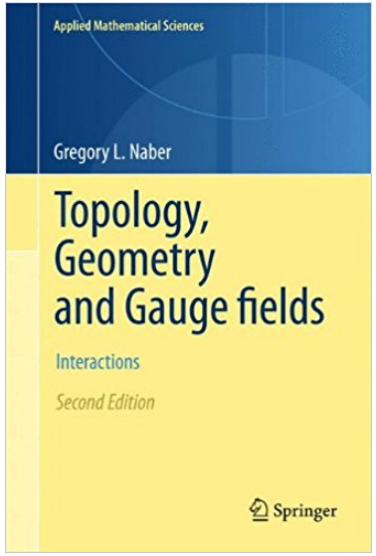












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topological structure

smooth structure

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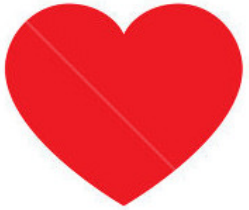
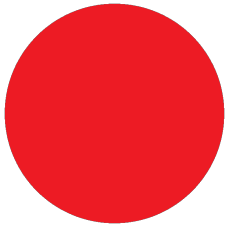
conformal structure  $\neq$ 

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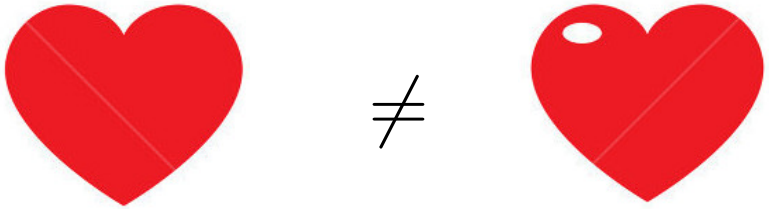
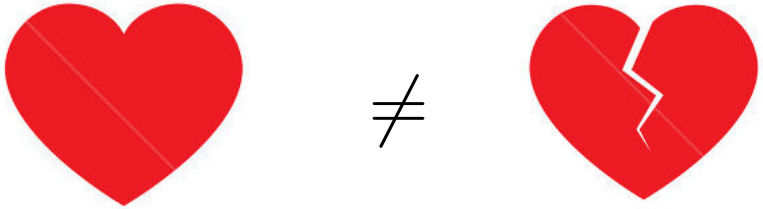
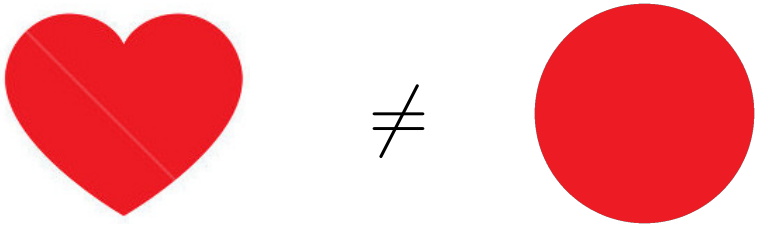
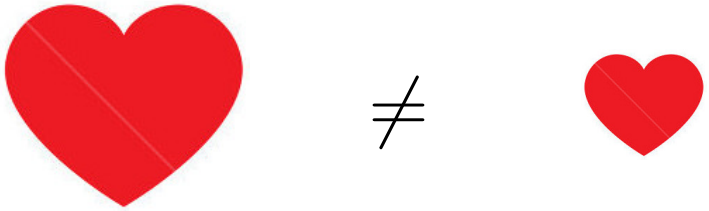
topological structure 

smooth structure 

conformal structure 

metric structure 

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Physics is an experimental science, so the mathematical formulation should be rooted in the operational definitions.

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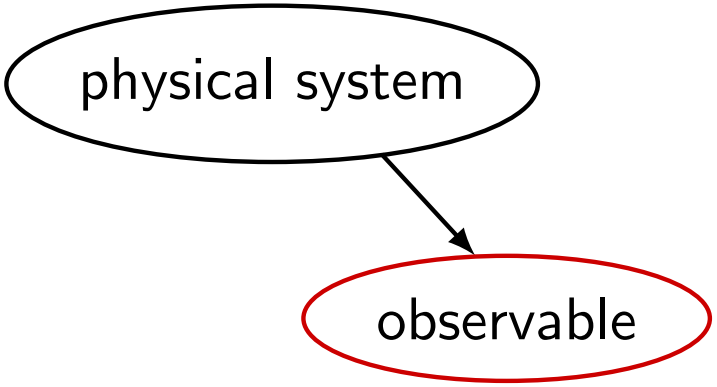

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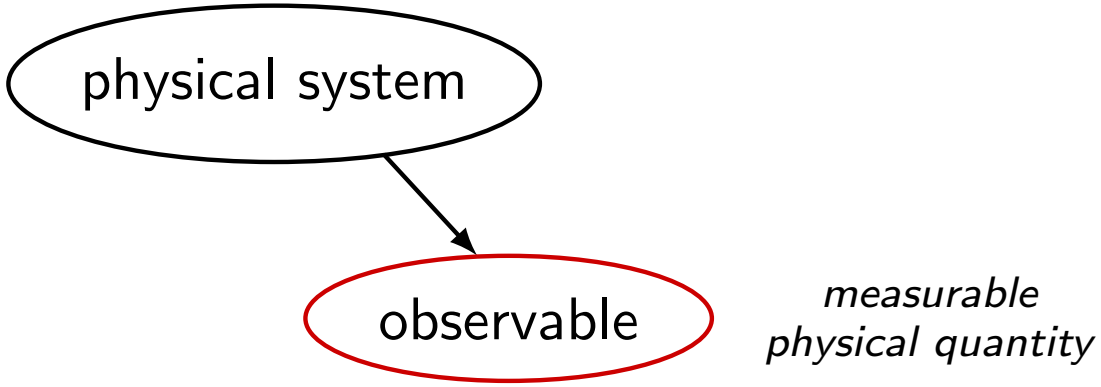

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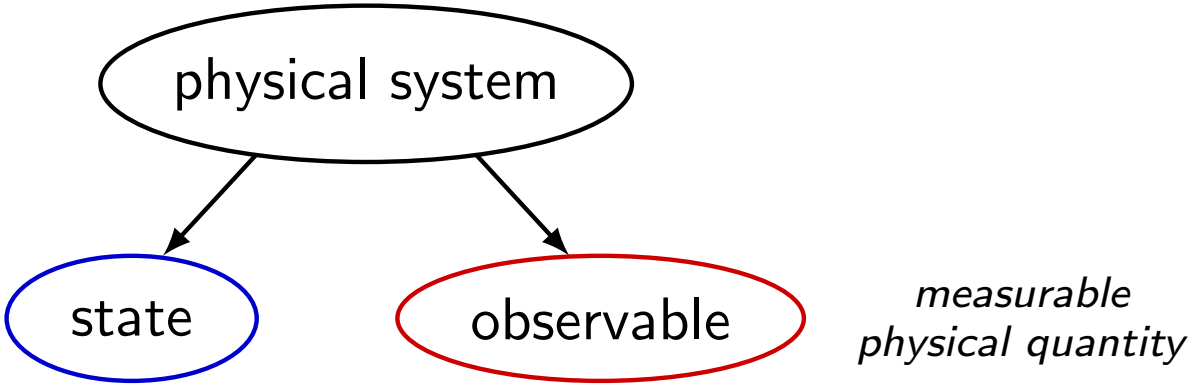

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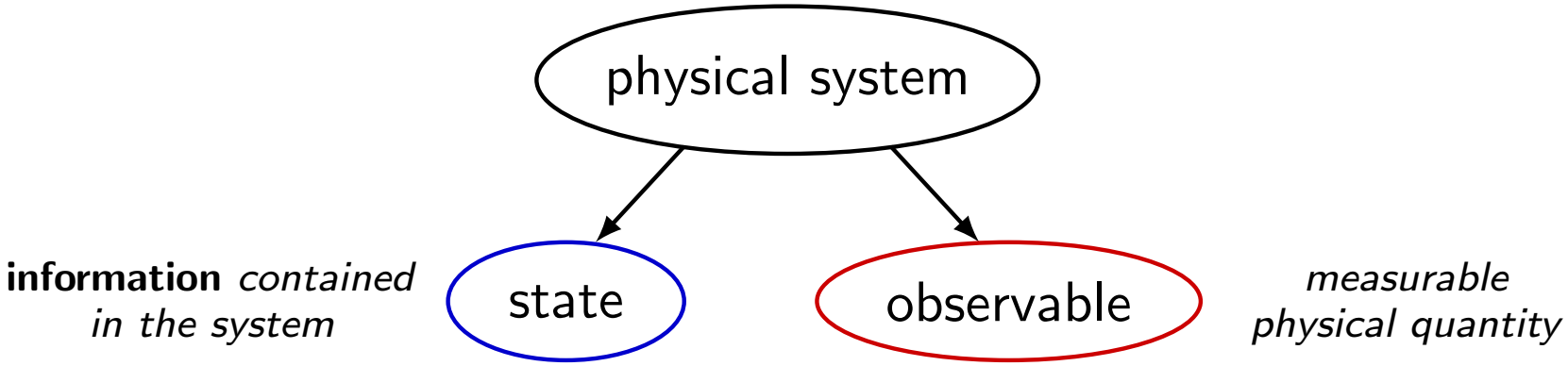

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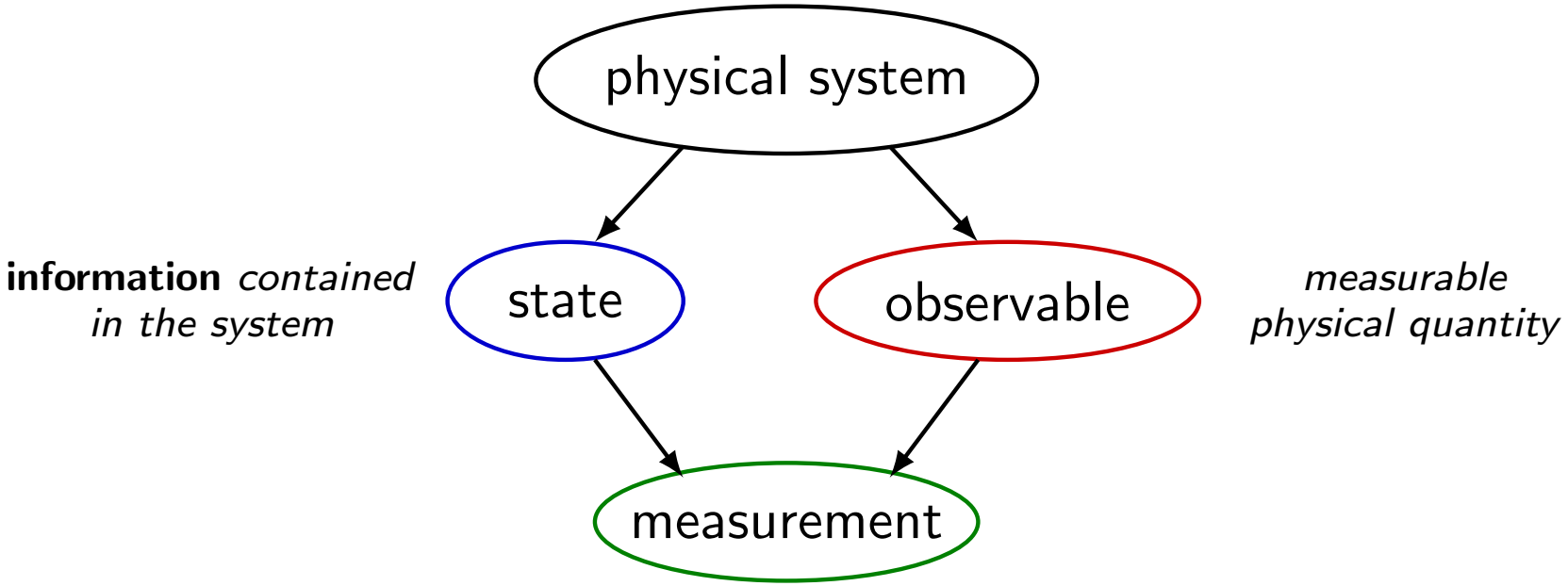

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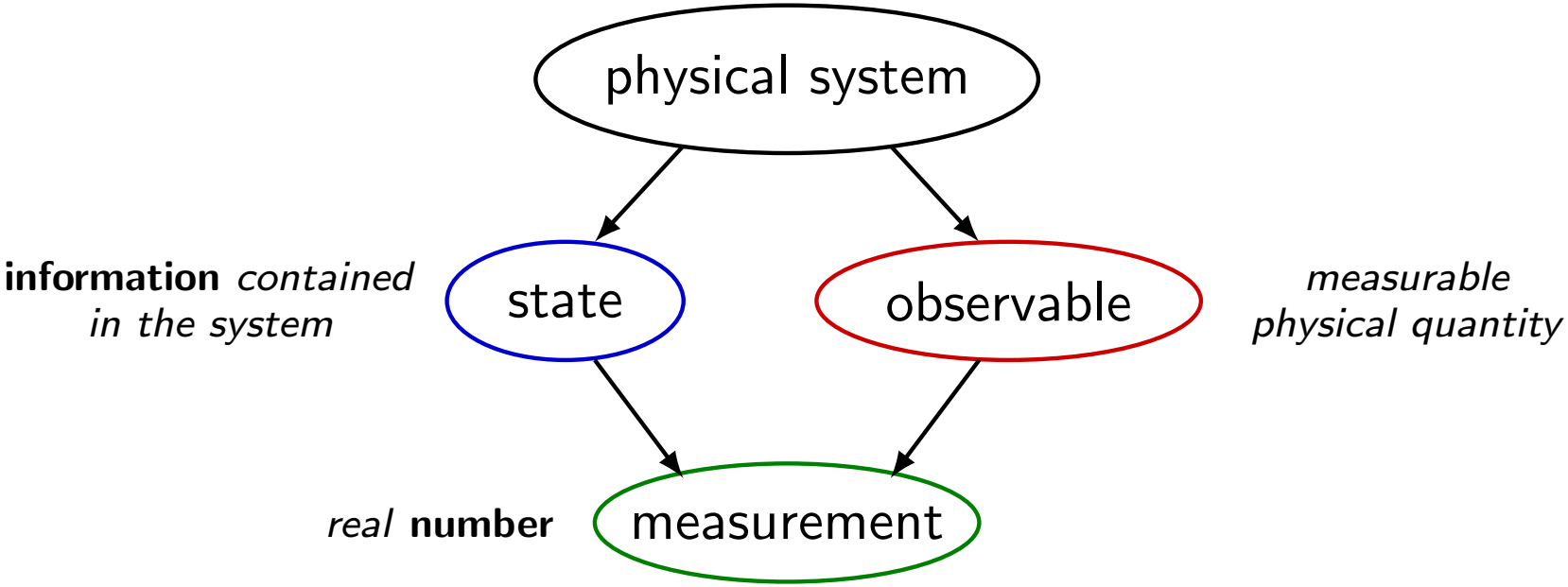

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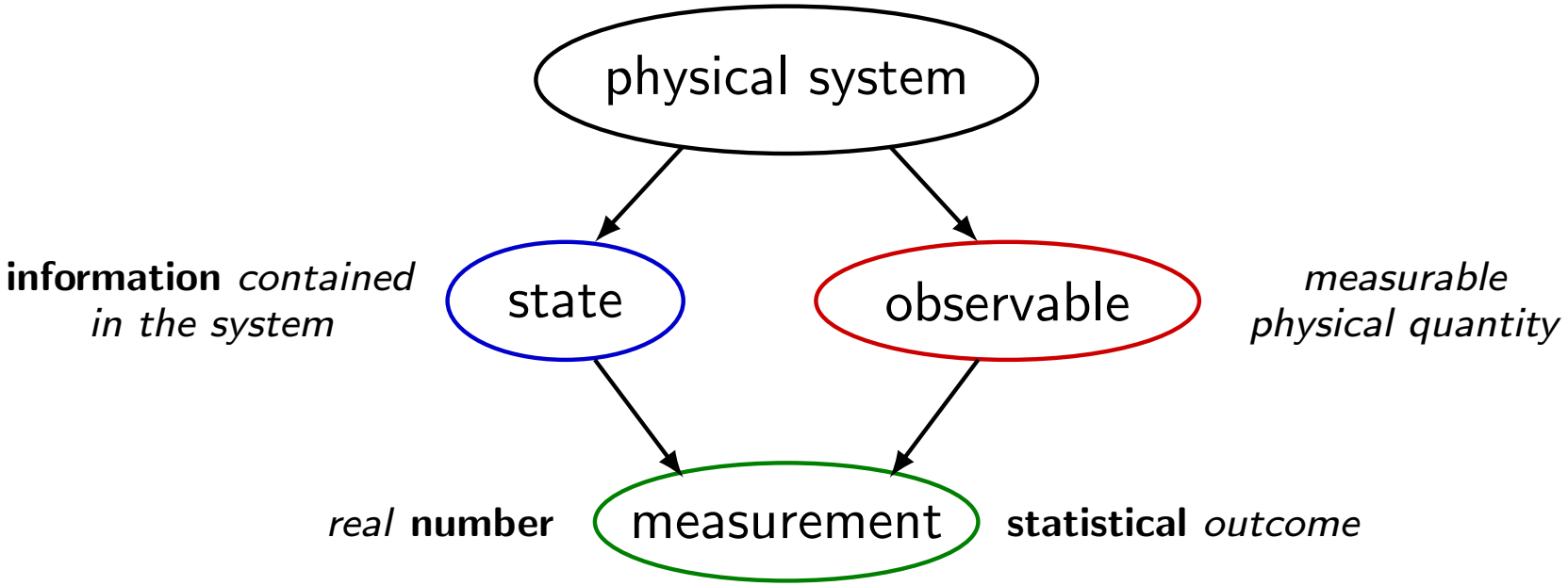

The structure of operational physics

A physical system consists of

- states $\omega \in \Sigma$ defined by the process of preparation,
- observables $A \in \mathcal{O}$ defined by the measuring apparatus.

The **expectation value**:

$$\omega(A) := \lim_{N\to\infty} \frac{1}{N}\left(m_1^\omega(A) + m_2^\omega(A) + \ldots + m_N^\omega(A)\right) \in \mathbb{R}$$

The **uncertainty**:

$$\Delta_\omega(A)^2 := \omega((A-\omega(A))^2) = \omega(A^2) - \omega(A)^2 \geq 0.$$

e.g. [F. Strocchi, *Mathematical Structure of Quantum Mechanics*, 2008]

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C^*-algebra generated by observables

Observables can be

- rescaled: $\lambda A \in \mathcal{O}$, for any $\lambda \in \mathbb{R}$,
- added: $C = A + B$ via $\omega(C) := \omega(A) + \omega(B)$ for any $\omega \in \Sigma$,
- multiplied: $AB, BA \in \mathcal{O}$

- Apparatus yields *finite* results $\Rightarrow$ every $A \in \mathcal{O}$ is bounded.
- Natural definition: $\|A\| := \sup_{\omega \in \Sigma} |\omega(a)|$.

$$\mathcal{O} \xrightarrow{\text{complexify}} \mathcal{O}^{\mathbb{C}} \xrightarrow{\text{complete in } \|\cdot\|} \mathcal{A}$$

- $\mathcal{A}$ is a C^*-algebra – the observables $\mathcal{O}$ generate a C^*-algebra

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Classical observables

- What can we measure?

  Functions on some parameter space – **observables**.

- $A = C(X)$ is a commutative C^*-algebra.

- What is X? (Pure) **states** of the system.
- Gelfand duality $f(p) = \mathrm{ev}_p(f) = \int_X f(x)\delta_p(x)dx$.
- More generally, $\omega(f) = \int_X f(x)d\mu(x)$.

Theorem [Gelfand–Naimark (1947)]

$A = C(X)$ determines uniquely the *topological* space $P(A) = X$.

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Quantum observables

- What do we measure in a quantum world?

  Operators on a Hilbert space – **observables**.

- Quantum observables, in general, do not commute $PQ \neq QP$!
- $A \subset \mathcal{B}(\mathcal{H})$ is a noncommutative C^*-algebra.
- Pure states in $P(A)$ – rays in $\mathcal{H}$.
- Mixed states in $S(A)$ – density matrices.

Theorem [Gelfand'–Naimark (1947)]

Every noncommutative C^*-algebra is a closed subalgebra of $\mathcal{B}(\mathcal{H})$.

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- Classical observables = **functions** on a parameter space
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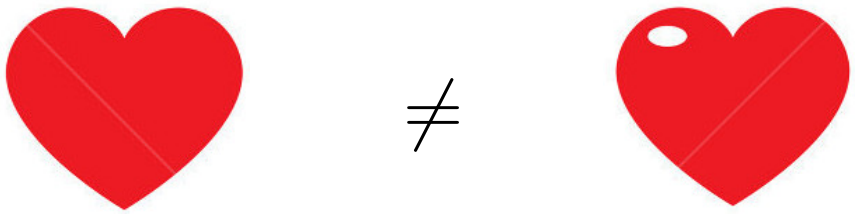

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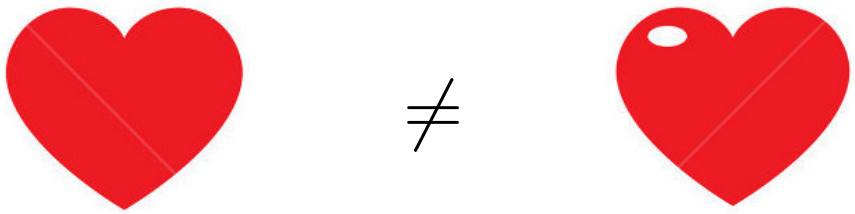

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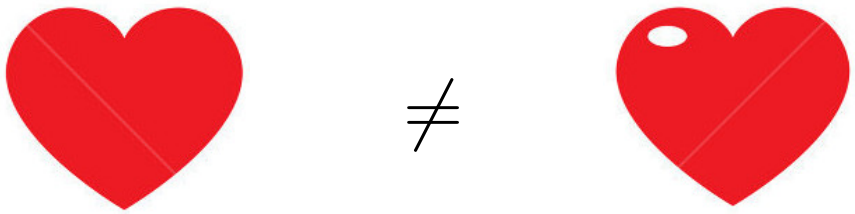

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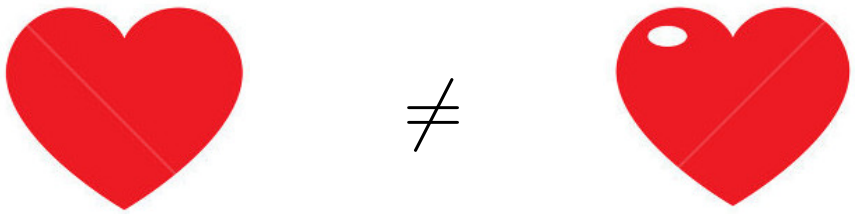

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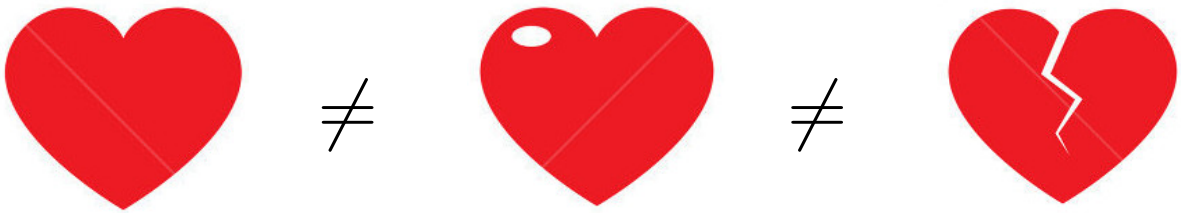

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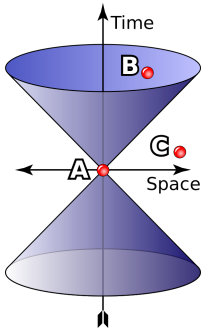

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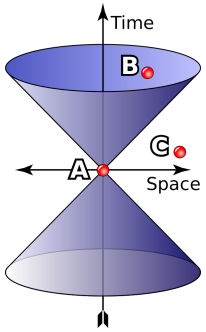

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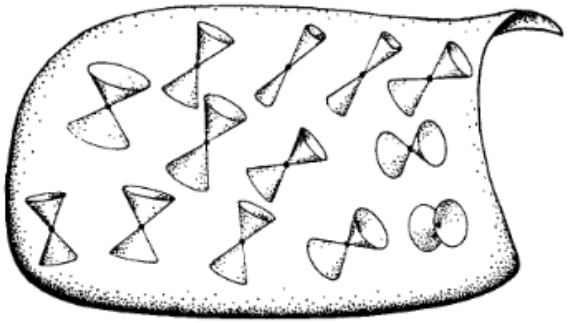

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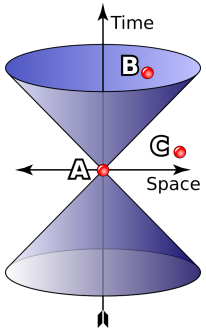

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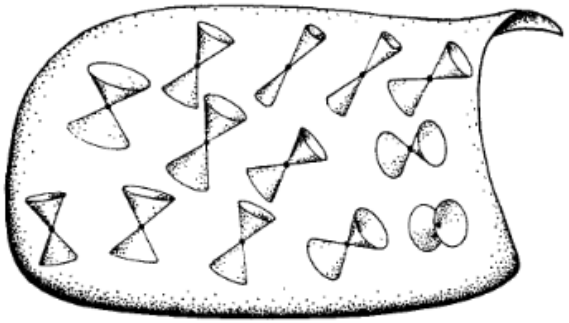

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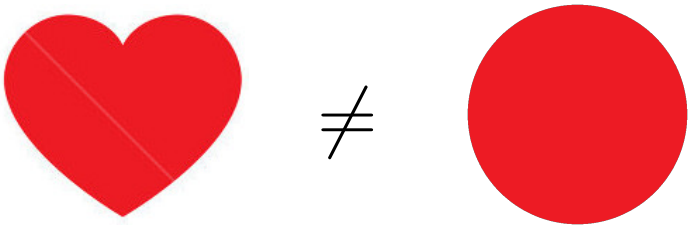

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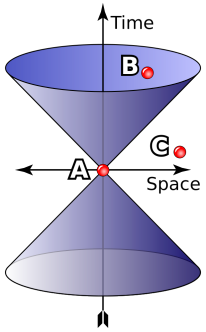

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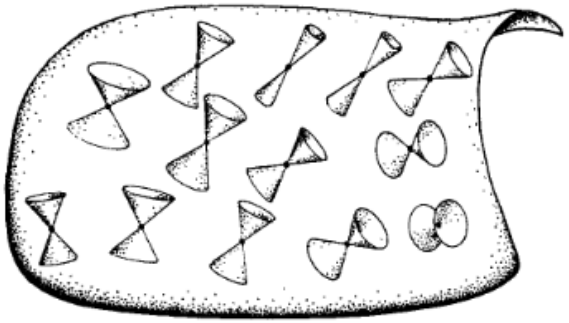

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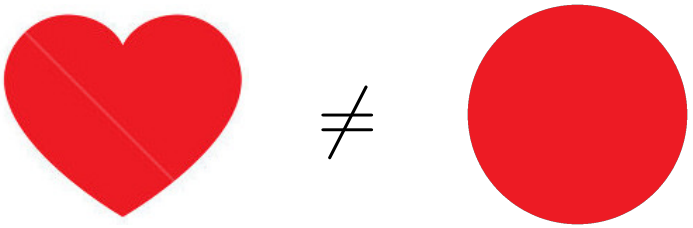

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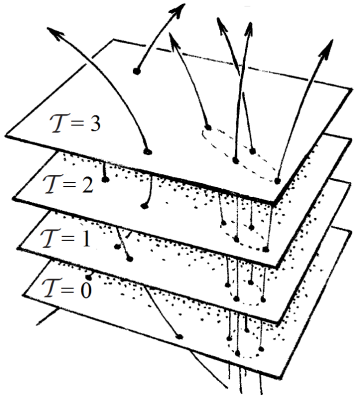

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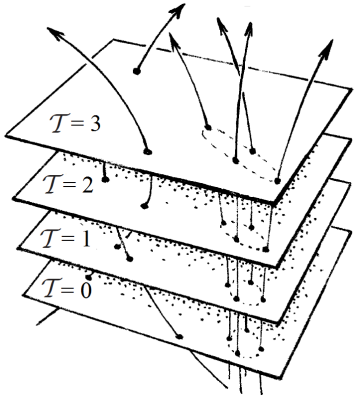

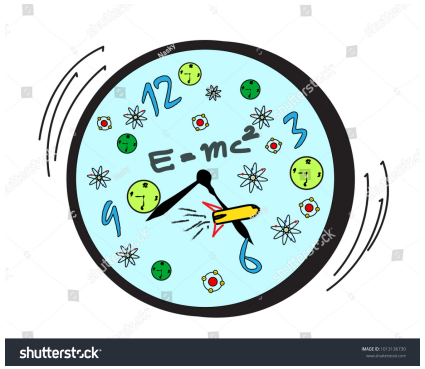

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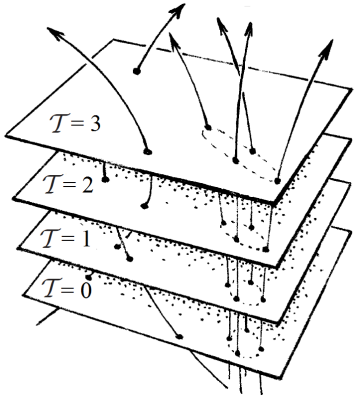

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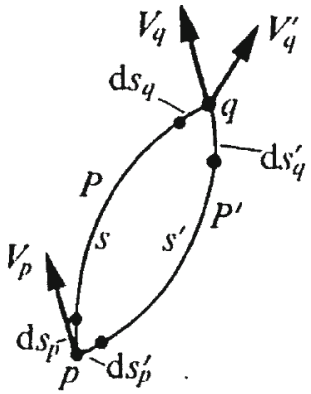

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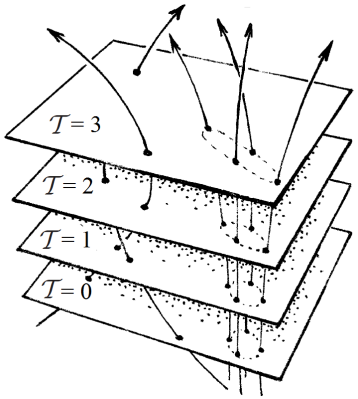

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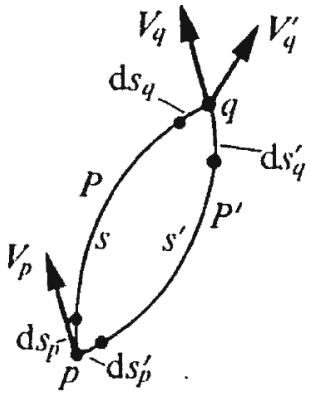

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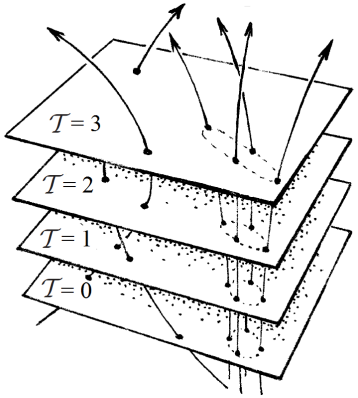

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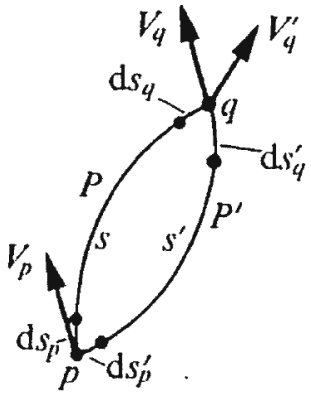

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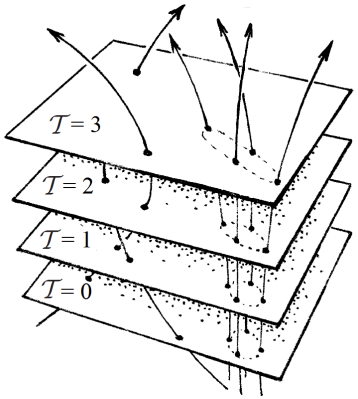

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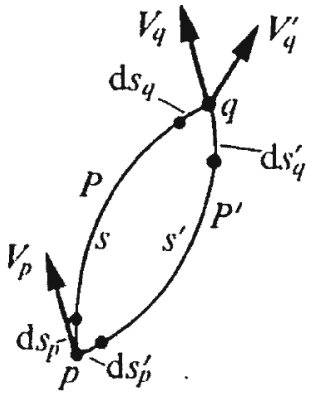

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Doplicher–Fredenhagen–Roberts microscope:

- attempt to localise an event

$\Leftarrow$ must use very short wavelength signals

$\Rightarrow$ critical energy density $\rightarrow$ black hole formation

$\Rightarrow$ uncertainty $\Delta\hat{x}_\mu \cdot \Delta\hat{x}_\nu \gtrsim \lambda_P$

$\Rightarrow$ noncommutativity $[\hat{x}_\mu, \hat{x}_\nu] = \Theta_{\mu\nu}, \quad C(M) \rightarrow \mathcal{A}_\Theta.$

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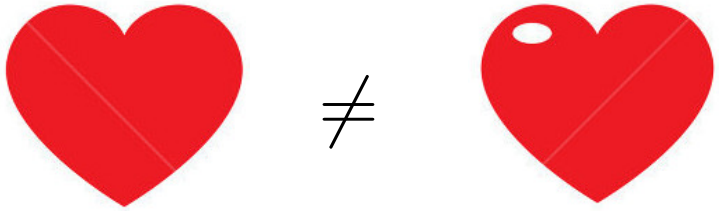

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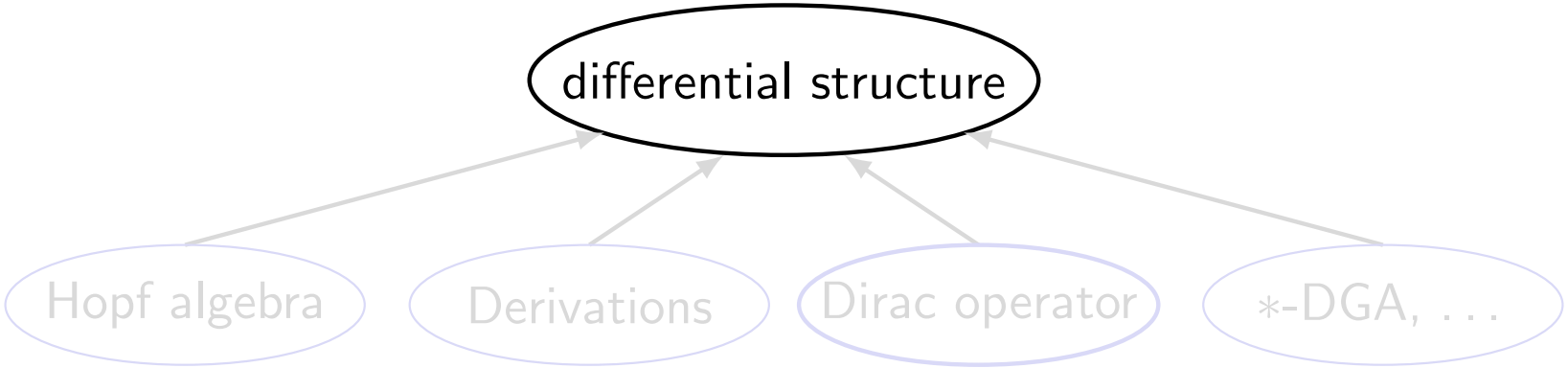

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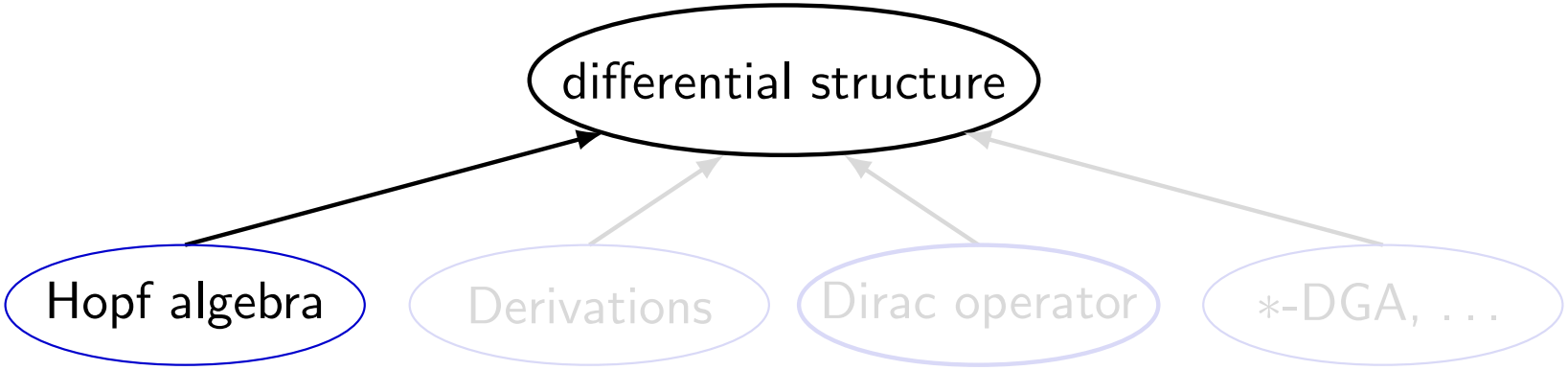

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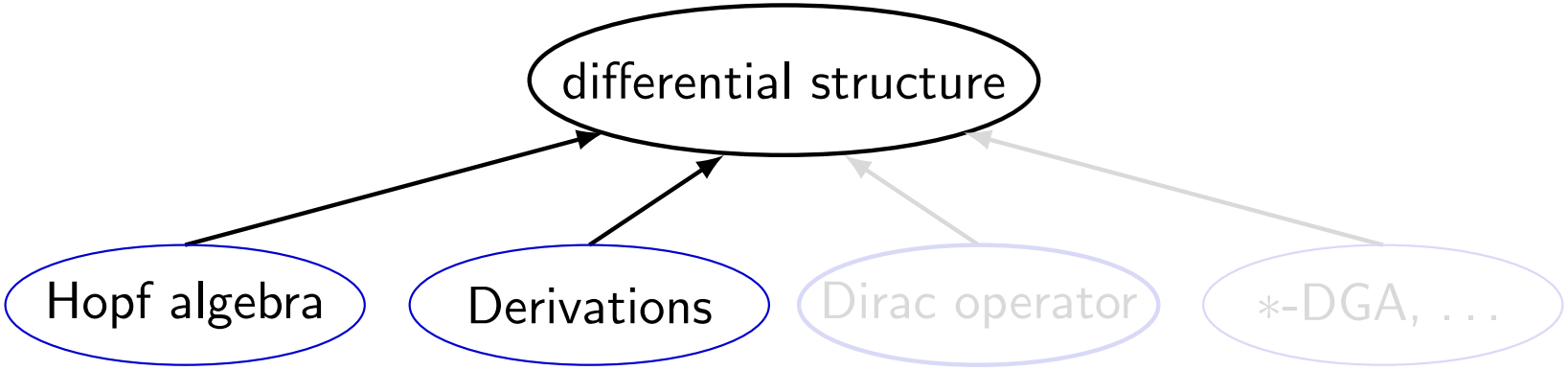

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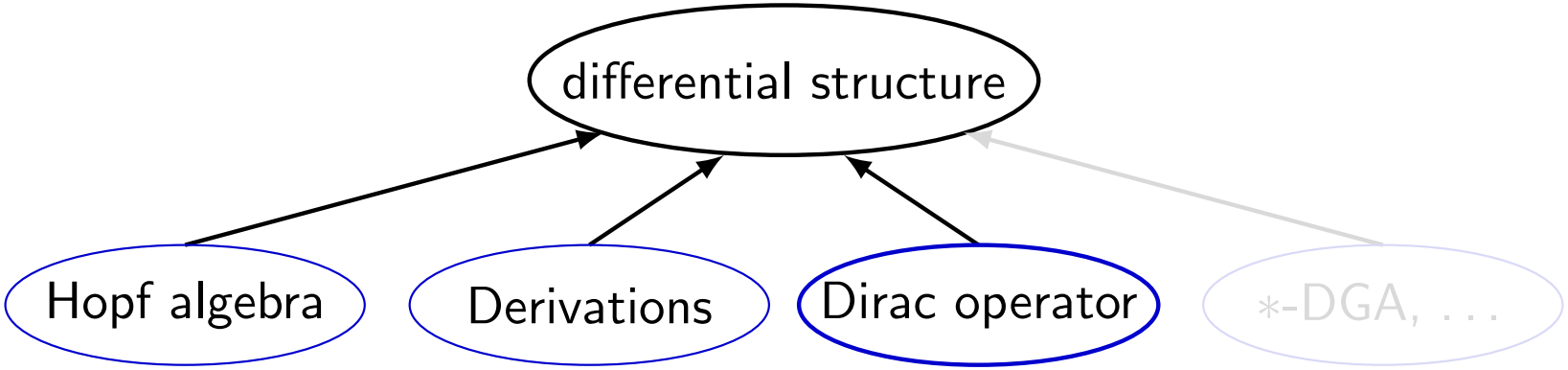

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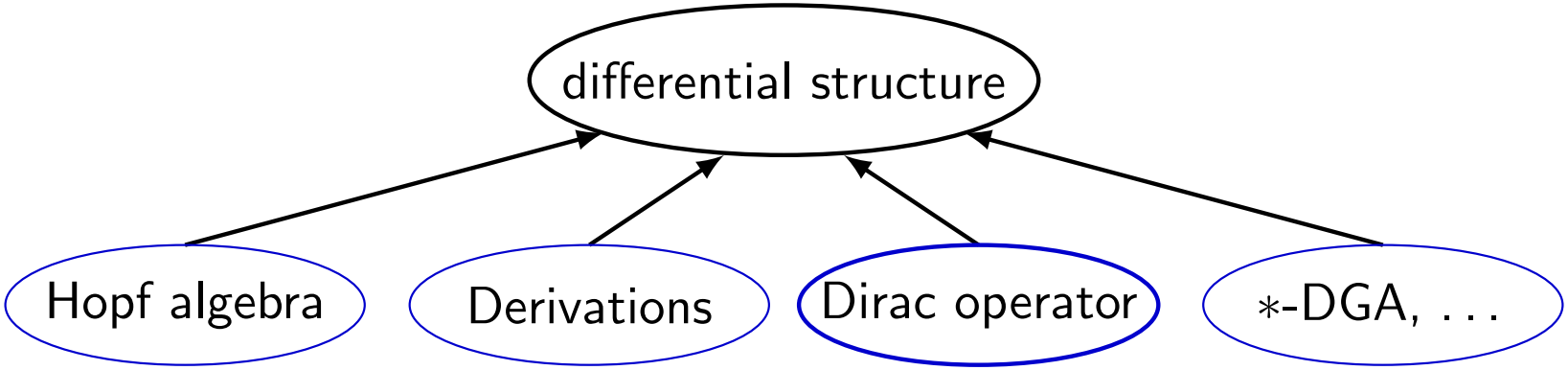

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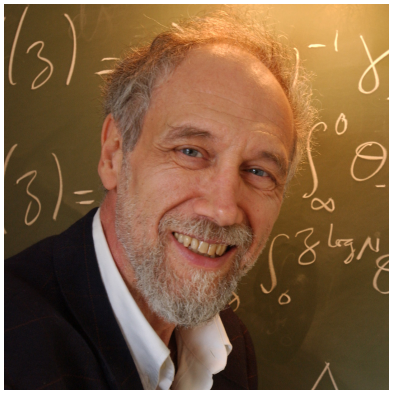

$(\mathcal{A}, \mathcal{H}, \mathcal{D})$ – spectral triple

- $\mathcal{A}$ – (dense $*$-subalgebra of a) C^*-algebra.
- $\mathcal{H}$ – Hilbert space with a faithful representation $\rho(\mathcal{A}) \subset \mathcal{B}(\mathcal{H})$.
- $\mathcal{D}$ – a Dirac operator – densely defined on $\mathcal{H}$, selfadjoint,
  - $(\mathcal{D} - \lambda)^{-1}$ for any $\lambda \notin \mathbb{R}$ – compact resolvent,
  - $[\mathcal{D}, \rho(a)] \in \mathcal{B}(\mathcal{H})$ for all $a \in \mathcal{A}$.
- $+$ technical assumptions – smoothness, dimension, ...
- $+$ additional structure – modules, charge conjugation, chirality, ...

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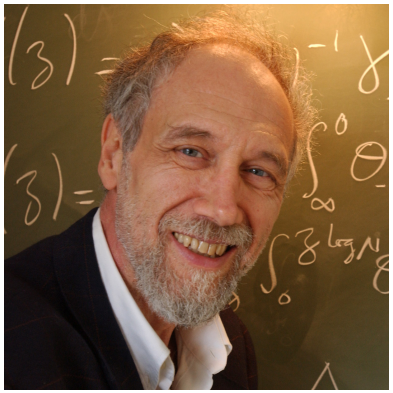

$(\mathcal{A}, \mathcal{H}, \mathcal{D})$ – spectral triple

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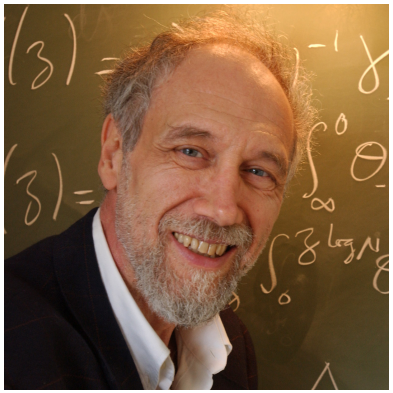

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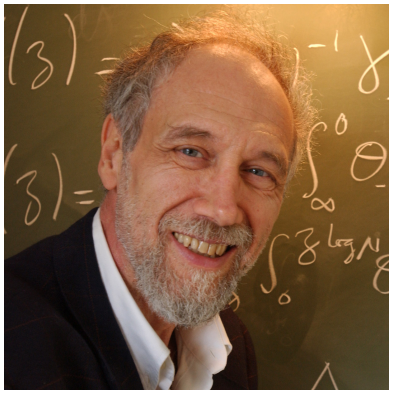

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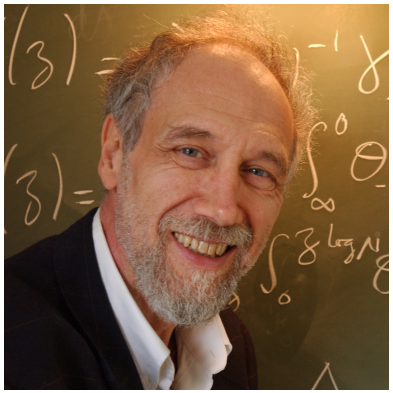

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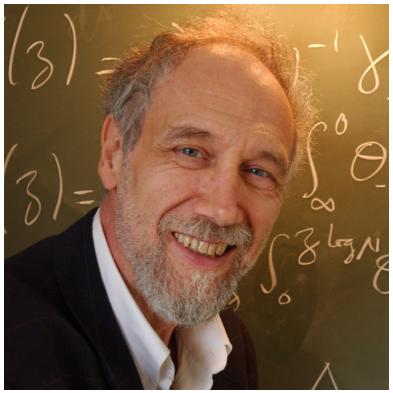

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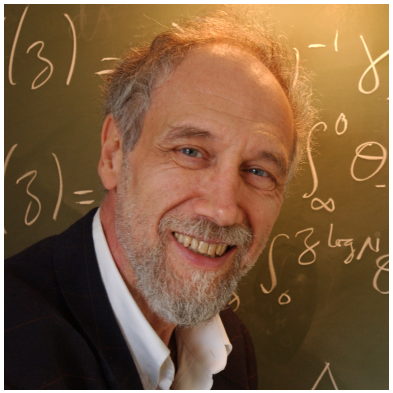

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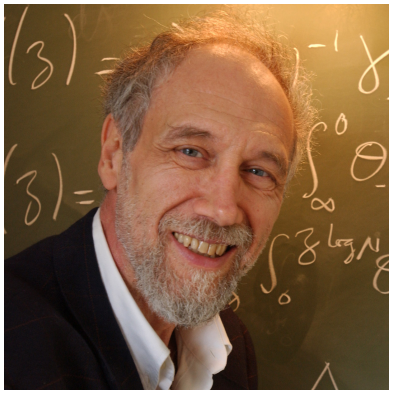

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Riemannian geometry revisited

Connes' Reconstruction Theorem [1996–2008]

For every *commutative* spectral triple $(\mathcal{A}, \mathcal{H}, \mathcal{D})$ there exists a smooth compact spin Riemannian manifold M such that:

$$\mathcal{A} = C^\infty(M), \qquad \mathcal{H} = L^2(M, S), \qquad \mathcal{D} = \not{D} = -\mathrm{i}\,\gamma^\mu \nabla^S_\mu.$$

Geometry encoded in the Dirac operator $\not{D}$:

1. differentiation of $f \in C^\infty(M)$

$$[\not{D}, f] = -i\gamma^\mu \partial_\mu f, \qquad \|[\not{D}, f]\| = \|\mathrm{grad} f\|_\infty .$$

2. geodesic distance

$$d_g(x,y) = \inf_{\gamma:[0,1]\to M} \{l(\gamma) : \gamma(0) = x, \gamma(1) = y\}$$
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Examples of spectral triples

Finite spectral triples

$$\mathcal{A}_{\mathcal{F}} = \bigoplus M_n(\mathbb{C}), \quad \mathcal{H}_{\mathcal{F}} = \mathbb{C}^N, \quad \mathcal{D}_{\mathcal{F}} = \mathcal{D}_{\mathcal{F}}^{\dagger} \in M_N(\mathbb{C})$$

More noncommutative spaces

- graphs,
- fractals,
- non-Hausdorff spaces,
- noncommutative tori,
- q-deformed spaces $\longleftarrow$ quantum groups
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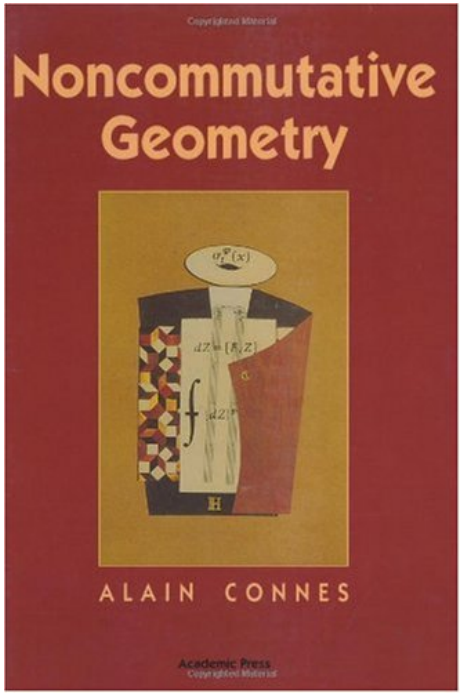

Almost-commutative geometry

Almost commutative geometry:

- $\mathcal{A} = C^\infty(M) \otimes \mathcal{A}_\mathcal{F}$ (gauge group)
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In the spirit of Kaluza–Klein – internal space of particles.

Remark: From the spectral viewpoint $(\mathcal{A}_\mathcal{F}, \mathcal{H}_\mathcal{F}, \mathcal{D}_\mathcal{F})$ is 0-dimensional.

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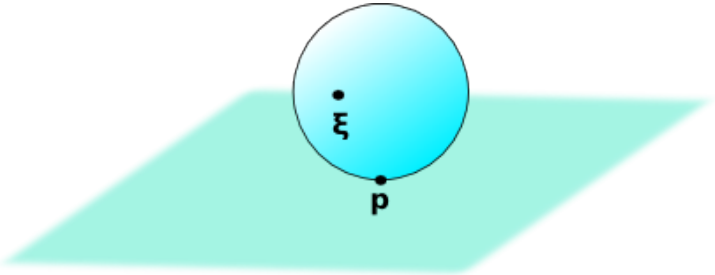

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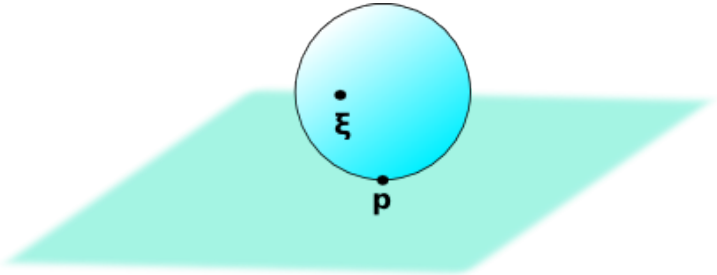

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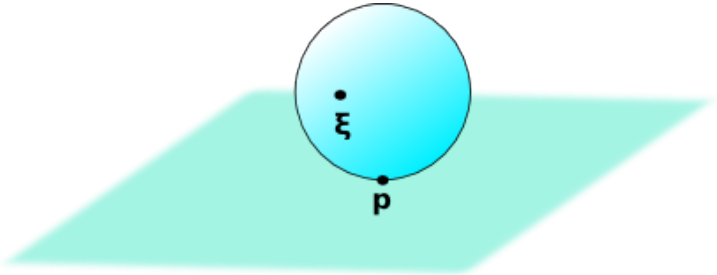

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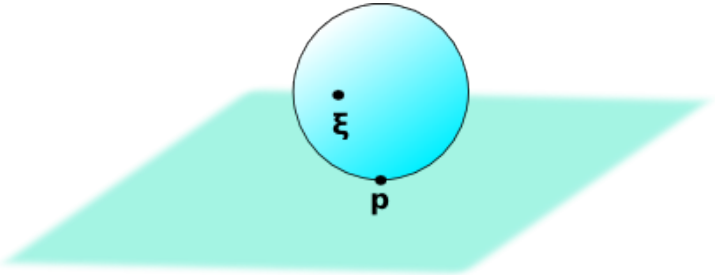

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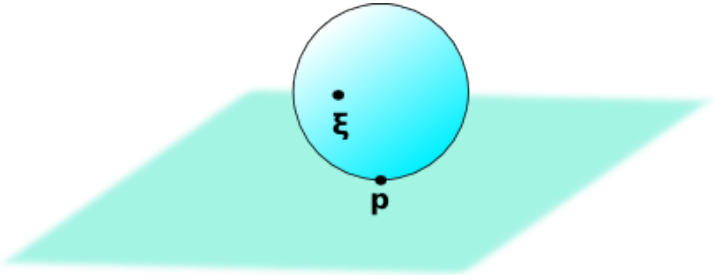

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The spectral action principle

The spectral action principle [Chamseddine, Connes (1997)]

The physical action only depends upon the spectrum of $\mathcal{D}$.

- Implementation – the bosonic action

$$S_B = \mathrm{Tr}\, f\,(\mathcal{D}/\Lambda) \sim \#\{\lambda(|\mathcal{D}|) < \Lambda\}$$

- Λ is an energy scale and f a smooth cut-off function.
- Asymptotic expansion: $S_B(\Lambda) \underset{\Lambda\to+\infty}{\sim} \sum_k f_k\, a_k(\mathcal{D})\, \Lambda^k$.
- Fluctuations of the Dirac operator – including gauge potentials

$$\mathcal{D} \to \mathcal{D}_A = \mathcal{D} + A, \text{ for } A = \sum a_i[\mathcal{D}, b_i].$$

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Outline

Spectral action at the unification scale

Choose $\mathcal{A} = C^\infty(M) \otimes \big(\mathbb{C} \oplus \mathbb{H} \oplus M_3(\mathbb{C})\big)$ and you get ...

$$S_B = \int_M \sqrt{g} d^4x \Bigg(\frac{1}{2\kappa_0^2} R + \alpha_0 C_{\mu\nu\rho\sigma} C^{\mu\nu\rho\sigma} + \gamma_0 + \tau_0 R^\star R^\star + \\ + \frac{1}{4} G^i_{\mu\nu} G^{\mu\nu i} + \frac{1}{4} F^a_{\mu\nu} F^{\mu\nu a} + \frac{1}{4} B^{\mu\nu} B_{\mu\nu} + \\ + \frac{1}{2} |D_\mu H|^2 - \mu_0^2 |H|^2 + \lambda_0 |H|^4 - \xi_0 R |H|^2 \Bigg) + \mathcal{O}(\Lambda^{-1})$$

- EH term + Weyl term + cosmological constant + topological term,
- dynamical terms of SM bosons,
- Higgs sector, coupling between Higgs and gravity.

- S_F recovers the fermionic part of the SM.
- The spectral action yields an *effective model* valid at $\Lambda \approx 10^{15}$ GeV.

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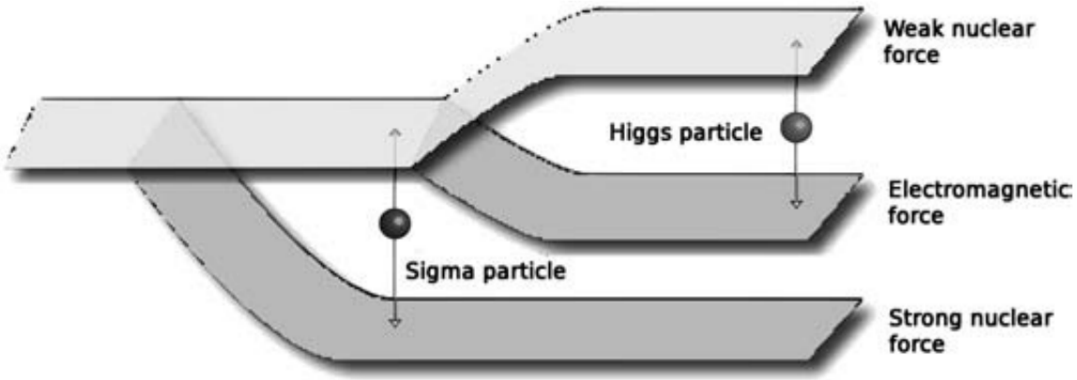

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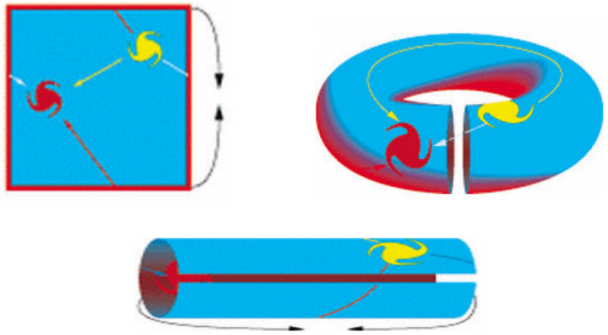

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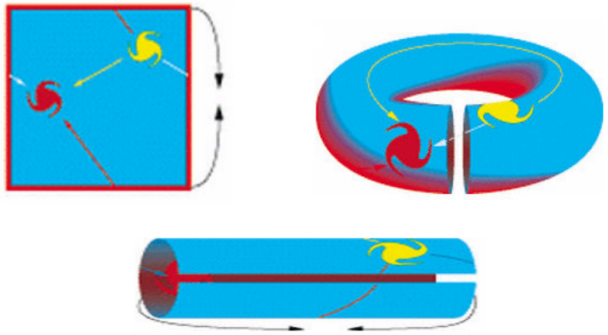

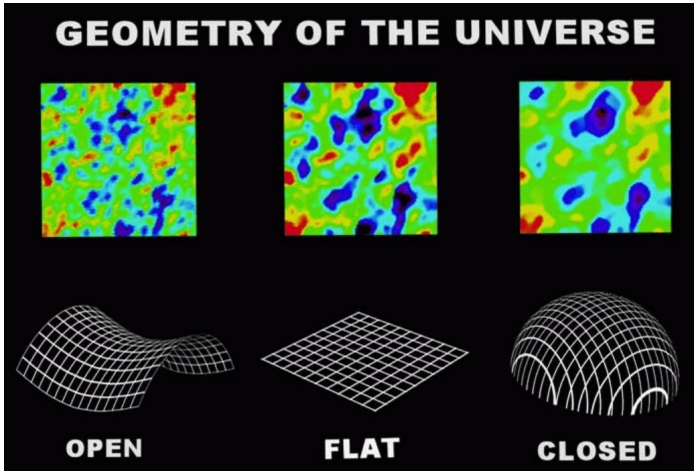

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Causal structure of noncommutative spacetimes

N. Franco, M.E., Class. Quant. Grav. **30** (2013) 135007

Given a *Lorentzian* spectral triple $(\mathcal{A}, \mathcal{H}, \mathcal{D})$, the operator $\mathcal{D}$ defines a **partial order** relation on the space $S(\mathcal{A})$.

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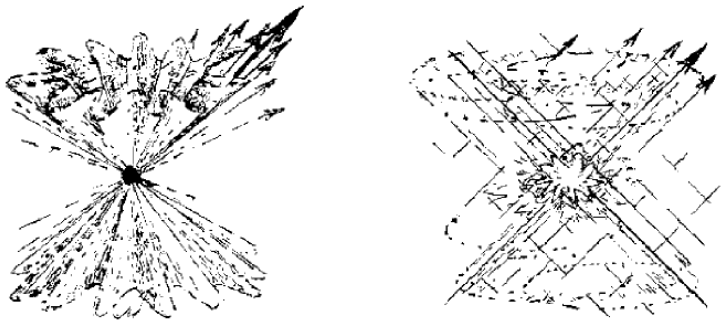

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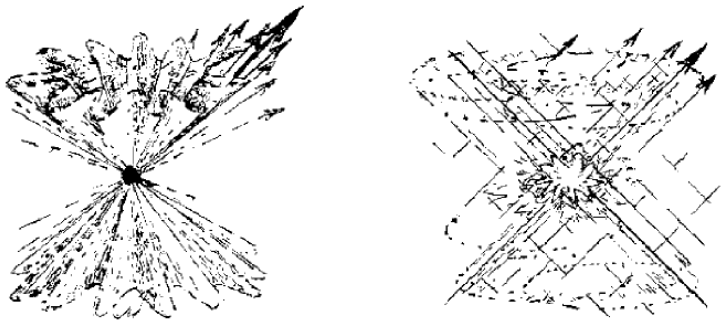

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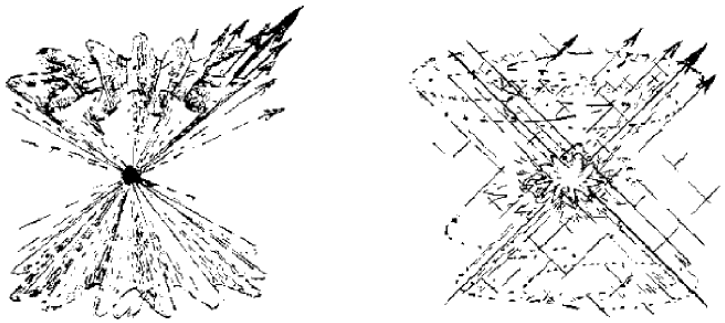

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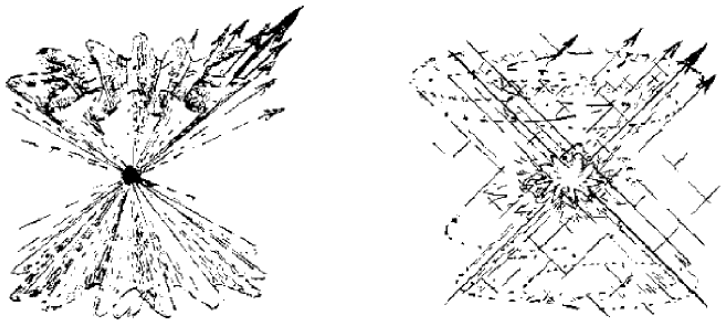

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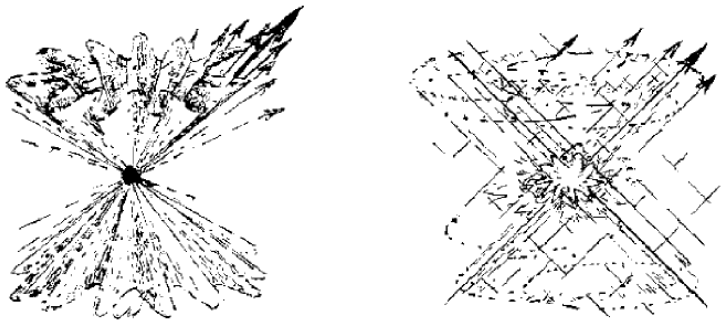

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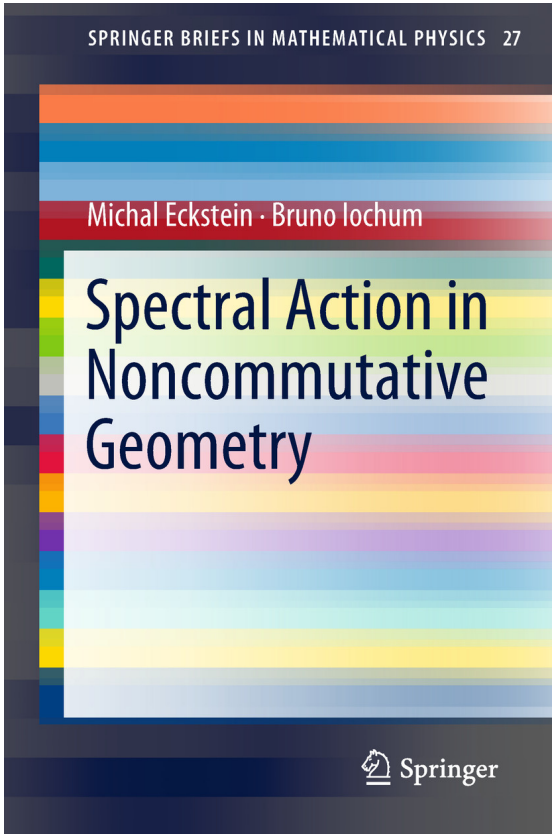
The spectral action



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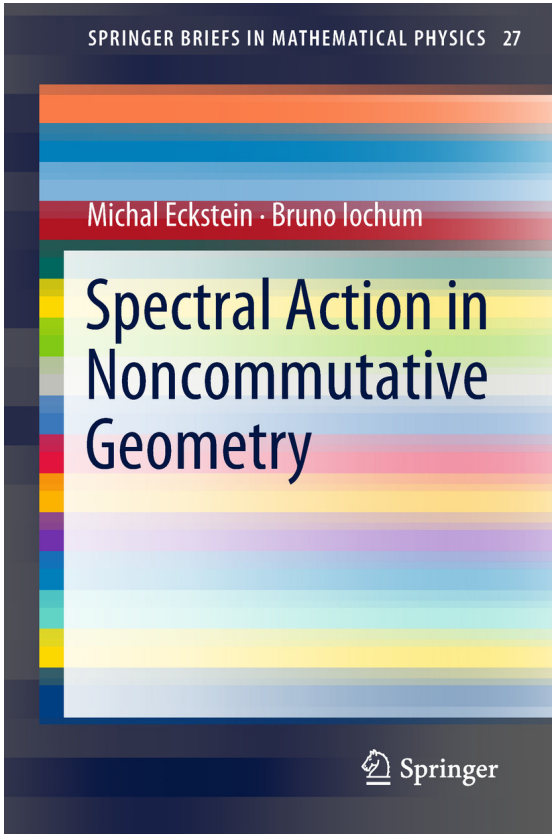

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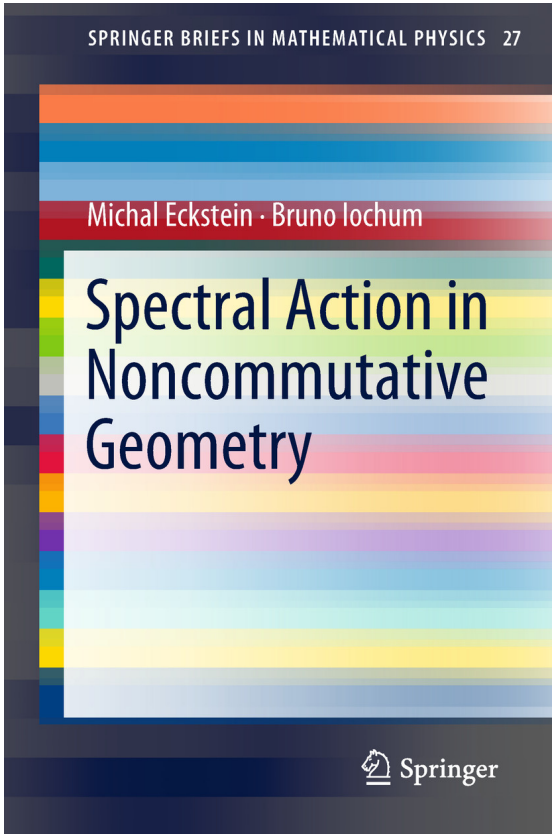

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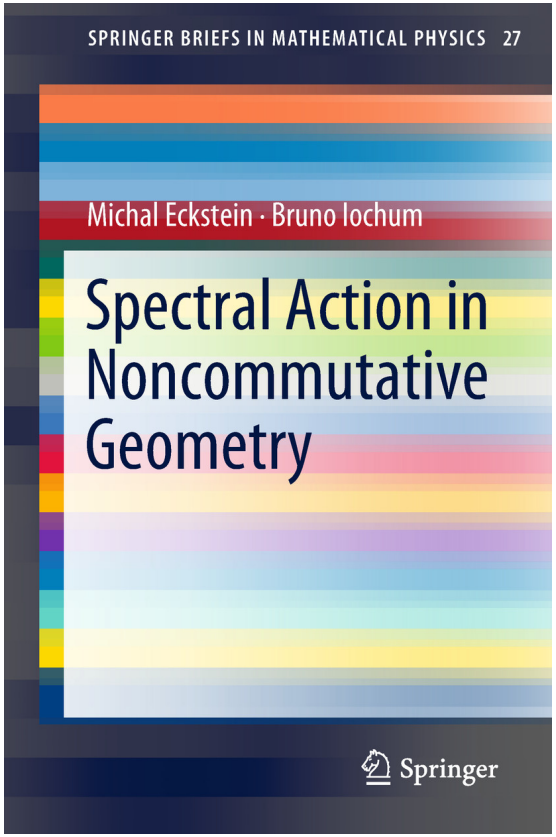

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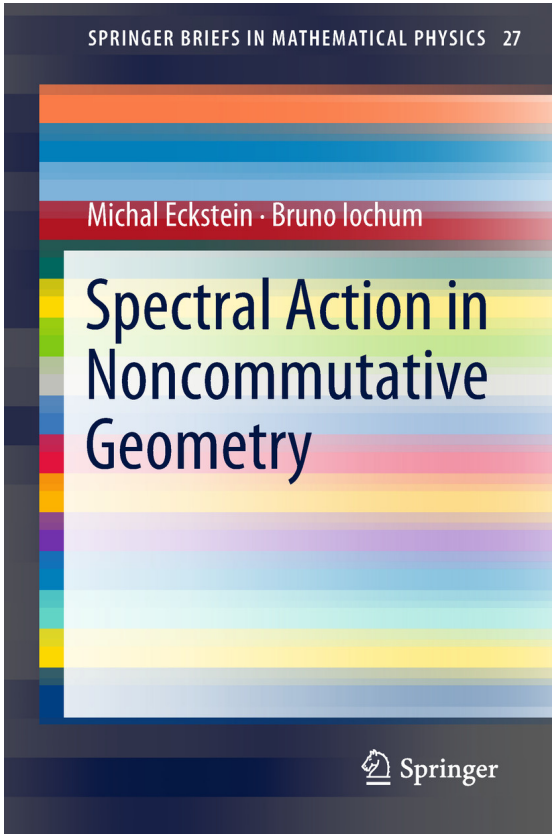

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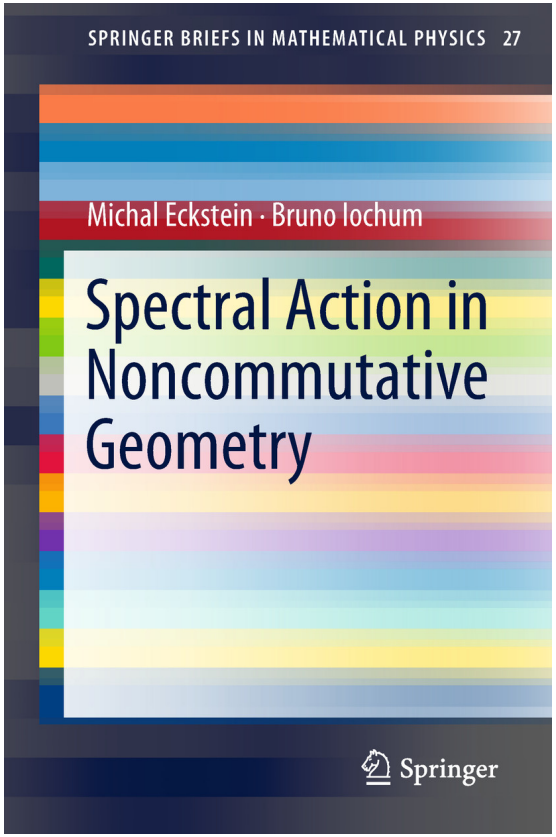

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