Wormholes in generalized Galileon theories

1807.xxxx

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INR RAS, Moscow

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Why do we study wormholes?

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We want to build a **teleport**!

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We want to build a **teleport**!
And a **time machine**!

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We want to build a **teleport**!
And a **time machine**!
And a **Universe** in the lab!

Why generalized Galileon/Horndeski theories?

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It can break the Null Energy Condition in a healthy way.

Wormhole is quite similar to cosmological bounce

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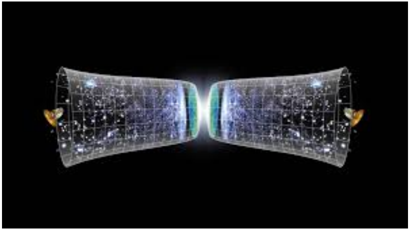
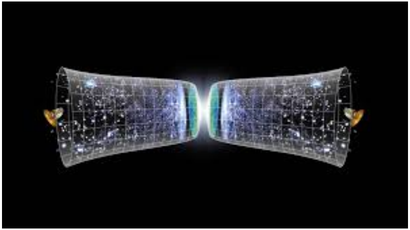
$\xrightarrow{\textit{Radial axis}}$



$\xrightarrow{\textit{Time axis}}$



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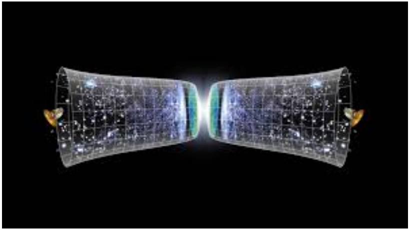

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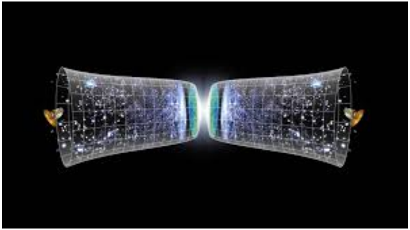

No-go for bounce in Horndeski

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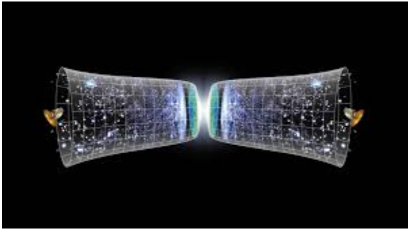

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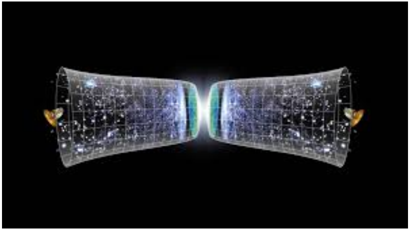

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$\Downarrow$

No-go breaks in beyond Horndeski

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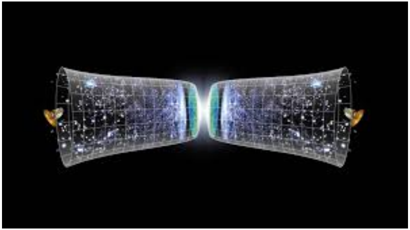

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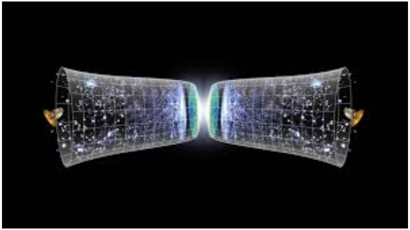

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Healthy bounce in beyond Horndeski

Wormhole is quite similar to cosmological bounce

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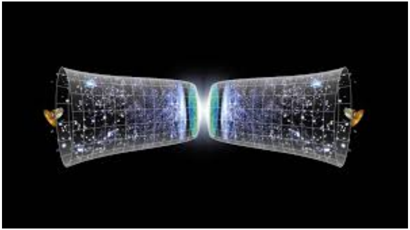

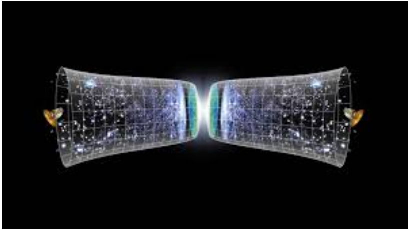

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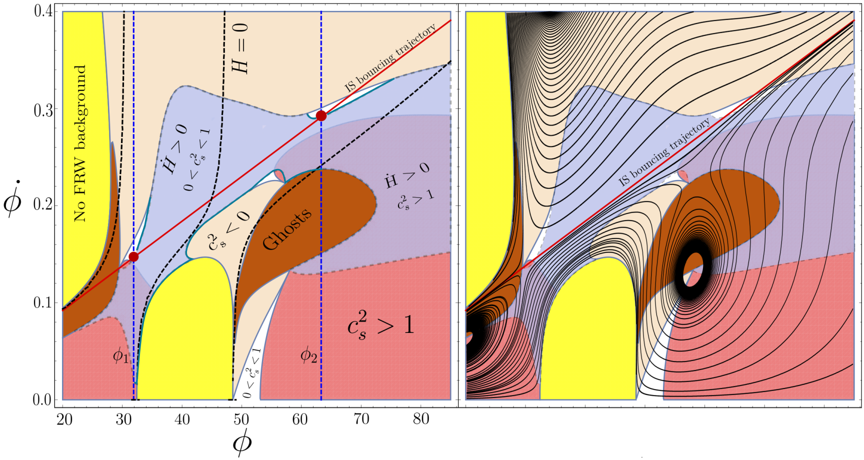
$\Downarrow$

Finetuning for $L^{(2)}$ probably means pathologies in $L^{(3)}$.



0.4
0.3
0.2
0.1
0.0
$\dot{\phi}$
20
30
40
50
60
70
80
ϕ
No FRW background
$H = 0$
IS bouncing trajectory
$\dot{H} > 0$
$0 < c_s^2 < 1$
$c_s^2 < 0$
Ghosts
$\dot{H} > 0$
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ϕ_1
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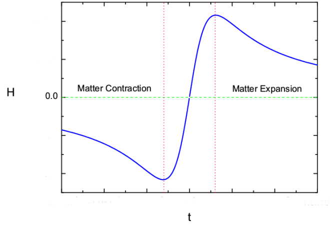
IS bouncing trajectory

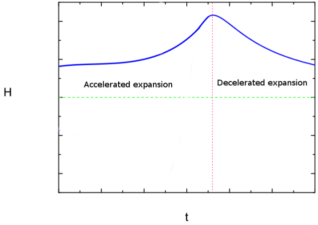
Null Energy Condition

$T_{\mu\nu}k^{\mu}k^{\nu} > 0 \quad \longleftrightarrow \quad p+\rho > 0 \quad \longrightarrow \quad$ NEC–violation: $\; p+\rho \leq 0$

Friedmann equations

$$\dot{H} = -4\pi G(p+\rho) + \frac{\kappa}{a^2}$$





Bounce and genesis require NEC-violation

Penrose theorem

Absence of singularity requires NEC-violation

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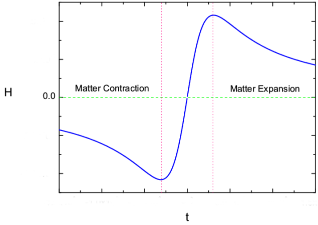

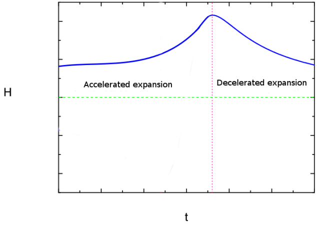

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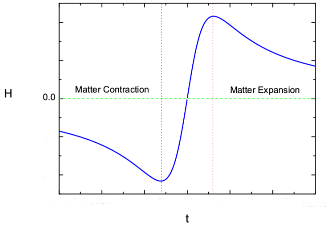

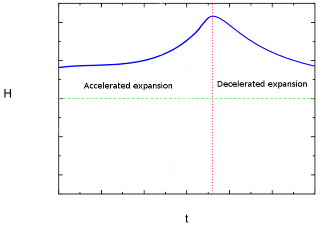

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Lagrangians with first derivatives $\Rightarrow$ NEC-violation = ghosts and/or gradient instabilities

Hence we need to consider Lagrangians with second derivatives:

- Deal with higher derivative equations
- Get equations with 2nd derivatives only

$$\mathcal{L}_3 = F(\pi, X) + K(\pi, X)\Box\pi,$$

here $X = \partial_\mu \pi \partial^\mu \pi$.

$$\delta\mathcal{L} = F_\pi\delta\pi + F_X\delta X + K_\pi\Box\pi\delta\pi + \underline{K_X\Box\pi\delta X + K\Box\delta\pi} =$$

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$$= ... - \underline{2K_X \partial^\mu\Box\pi\partial_\mu\pi\delta\pi} + \partial_\mu(K_\pi\partial^\mu\pi + \underline{2K_X\partial^\mu\partial_\nu\pi\partial^\nu\pi})\delta\pi$$

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= ...only second derivatives

Horndeski and Beyond Horndeski

$$S = \int \mathrm{d}^4x\sqrt{-g}\,(\mathcal{L}_2 + \mathcal{L}_3 + \mathcal{L}_4 + \mathcal{L}_5 + \mathcal{L}_{\mathcal{BH}})\,,$$

$$\mathcal{L}_2 = F(\pi, X),$$

$$\mathcal{L}_3 = K(\pi, X)\Box\pi,$$

$$\mathcal{L}_4 = -G_4(\pi, X)R + 2G_{4X}(\pi, X)\left[(\Box\pi)^2 - \pi_{;\mu\nu}\pi^{;\mu\nu}\right],$$

$$\mathcal{L}_5 = G_5(\pi, X)G^{\mu\nu}\pi_{;\mu\nu} + \frac{1}{3}G_{5X}\left[(\Box\pi)^3 - 3\Box\pi\pi_{;\mu\nu}\pi^{;\mu\nu} + 2\pi_{;\mu\nu}\pi^{;\mu\rho}\pi_{;\rho}^{\ \ \nu}\right],$$

$$\mathcal{L}_{\mathcal{BH}} = F_4(\pi, X)\epsilon^{\mu\nu\rho}{}_{\sigma}\epsilon^{\mu'\nu'\rho'\sigma}\pi_{,\mu}\pi_{,\mu'}\pi_{;\nu\nu'}\pi_{;\rho\rho'} +$$
$$+F_5(\pi, X)\epsilon^{\mu\nu\rho\sigma}\epsilon^{\mu'\nu'\rho'\sigma'}\pi_{,\mu}\pi_{,\mu'}\pi_{;\nu\nu'}\pi_{;\rho\rho'}\pi_{;\sigma\sigma'}$$

where π is the Galileon field, $X = g^{\mu\nu}\pi_{,\mu}\pi_{,\nu}$, $\pi_{,\mu} = \partial_\mu\pi$, $\pi_{;\mu\nu} = \nabla_\nu\nabla_\mu\pi$, $\Box\pi = g^{\mu\nu}\nabla_\nu\nabla_\mu\pi$, $G_{4X} = \partial G_4/\partial X$

$$ds^2 = dt^2 - a(t)^2 d\vec{x}^2,$$

$$ds^2 = A(r)dt^2 - \frac{dr^2}{B(r)} - R(r)^2 \left(d\theta^2 + \sin^2\theta \, d\varphi^2 \right).$$

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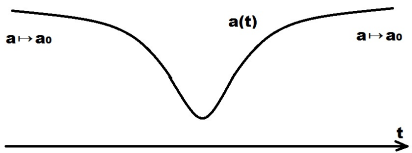

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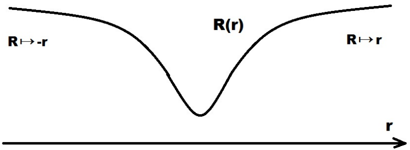

No-go theorem in Horndeski theory

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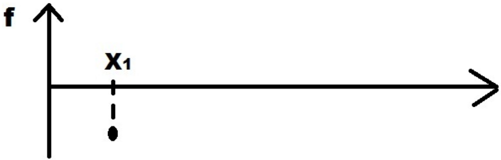

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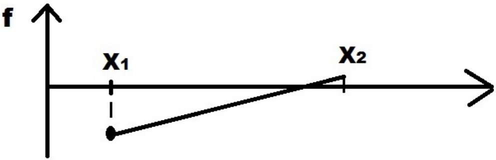

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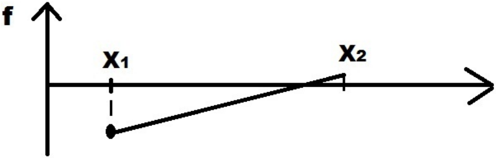

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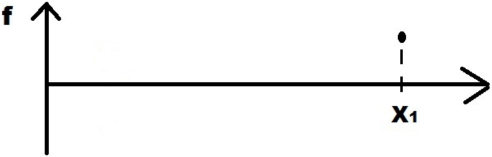

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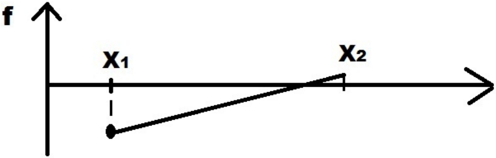

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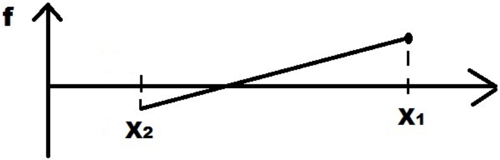

$$S=\int \mathrm{d}t\mathrm{d}^3xa^3\left[\frac{\mathcal{G}_\mathcal{T}}{8}\left(\dot{h}_{ik}^T\right)^2-\frac{\mathcal{F}_\mathcal{T}}{8a^2}\left(\partial_i h_{kl}^T\right)^2+\mathcal{G}_\mathcal{S}\dot{\zeta}^2-\mathcal{F}_\mathcal{S}\frac{(\nabla\zeta)^2}{a^2}\right]$$

The speeds of sound for tensor and scalar perturbations are, respectively,

$$c_\mathcal{T}^2=\frac{\mathcal{F}_\mathcal{T}}{\mathcal{G}_\mathcal{T}},\qquad c_\mathcal{S}^2=\frac{\mathcal{F}_\mathcal{S}}{\mathcal{G}_\mathcal{S}}$$

A healthy and stable solution requires correct signs for kinetic and gradient terms as well as subluminal propagation:

$$\mathcal{G}_\mathcal{T}>\mathcal{F}_\mathcal{T}>0,\quad \mathcal{G}_\mathcal{S}>\mathcal{F}_\mathcal{S}>0$$

These coefficients are combinations of Lagrangian functions and have non-trivial relations

$$\begin{aligned}\mathcal{G}_\mathcal{S}&=\frac{\Sigma{\mathcal{G}_\mathcal{T}}^2}{\Theta^2}+3\mathcal{G}_\mathcal{T}, & & & \mathcal{G}_\mathcal{S}&=\frac{\Sigma{\hat{\mathcal{G}}_\mathcal{T}}^2}{\Theta^2}+3\hat{\mathcal{G}}_\mathcal{T},\\ \mathcal{F}_\mathcal{S}&=\frac{1}{a}\frac{\mathrm{d}\xi}{\mathrm{d}t}-\mathcal{F}_\mathcal{T}, & &\longrightarrow & \mathcal{F}_\mathcal{S}&=\frac{1}{a}\frac{\mathrm{d}\xi}{\mathrm{d}t}-\mathcal{F}_\mathcal{T},\\ \xi&=\frac{a{\mathcal{G}_\mathcal{T}}^2}{\Theta}, & & & \xi&=\frac{a\mathcal{G}_\mathcal{T}\hat{\mathcal{G}}_\mathcal{T}}{\Theta}=\frac{a\mathcal{G}_\mathcal{T}\left(\mathcal{G}_\mathcal{T}+\mathcal{D}\dot{\pi}\right)}{\Theta}.\end{aligned}$$

No-go theorem for bounce in Horndeski theory

M. Libanov, S. Mironov and V. Rubakov, 1605.05992
R. Kolevatov and S. Mironov, 1607.04099
T. Kobayashi, 1606.05831
S. Akama and T. Kobayashi, 1701.02926

No-go theorem for bounce breaks in beyond Horndeski

Y. Cai, Y. Wan, H. Li, T. Qiu and Y. Piao, 1610.03400
P. Creminelli, D. Pirtskhalava, L. Santoni and E. Trincherini, 1610.04207
Y. Cai and Y. S. Piao, 1705.03401
R. Kolevatov, S. Mironov, N. Sukhov, VV, 1705.06626

No-go theorem for Wormholes in Horndeski theory

V. Rubakov, 1601.06566

O. Evseev, O. Melichev, 1711.04152

No-go theorem for Wormholes breaks in beyond Horndeski

arXiv: 1807.xxxx

Quadratic action for spherically symmetric case

$$\mathcal{L} = \frac{1}{2}\mathcal{K}_{ij}\dot{v}^i\dot{v}^j - \frac{1}{2}\mathcal{G}_{ij}v^{i\prime}v^{j\prime} - Q_{ij}v^i v^{j\prime} - \frac{1}{2}\mathcal{M}_{ij}v^i v^j, \qquad (4)$$

where $i,\ j = 1..2$, v^i – linear perturbation.

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A healthy and stable solution requires correct signs for kinetic and gradient terms:

$$\mathcal{K}_{11} > 0, \quad \det(\mathcal{K}) > 0, \quad \mathcal{G}_{11} > 0, \quad \det(\mathcal{G}) > 0. \qquad (5)$$

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$$\mathcal{K}_{11} > 0, \quad \det(\mathcal{K}) > 0, \quad \mathcal{G}_{11} > 0, \quad \det(\mathcal{G}) > 0. \quad (5)$$

General Horndeski theory:

$$\mathcal{P} = \left[\frac{(R\mathcal{H})^2}{\Theta}\right]'$$

$$\det K \sim \mathcal{F}(2\mathcal{P} - \mathcal{F}) > 0$$

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$$\mathcal{Q} \sim \frac{\mathcal{D}'}{R'} \quad (6)$$

Quadratic action for spherically symmetric case

$$\mathcal{L} = \frac{1}{2}\mathcal{K}_{ij}\dot{v}^i\dot{v}^j - \frac{1}{2}\mathcal{G}_{ij}v^{i\prime}v^{j\prime} - Q_{ij}v^i v^{j\prime} - \frac{1}{2}\mathcal{M}_{ij}v^i v^j, \tag{4}$$

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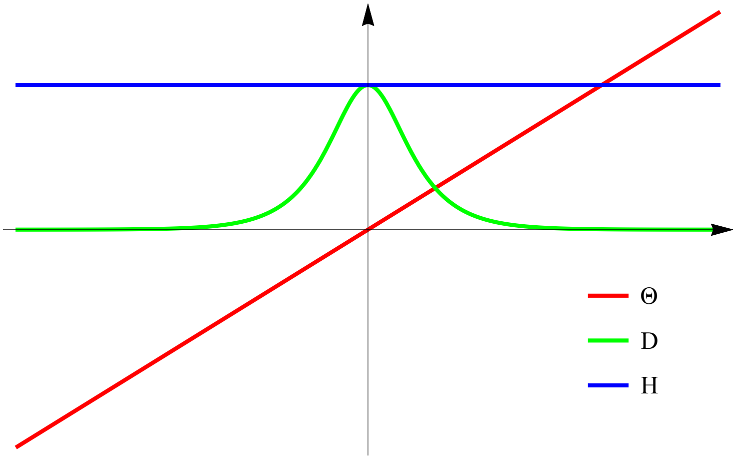
$$A > 0,\ B > 0,\ R > 0, \mathcal{F} > 0,\ \mathcal{H} > 0$$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This gives infinite contribution in $L^{(3)}$: $L^{(3)} \ni K'(F(r))(\delta F)^3$.

Conclusions.

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Is there a healthy wormhole?

THANK YOU FOR YOUR ATTENTION!

$$\mathcal{G}_\mathcal{T} = 2G_4 - 4G_{4X}X + G_{5\pi}X - 2HG_{5X}X\dot{\pi},$$

$$\mathcal{F}_\mathcal{T} = 2G_4 - 2G_{5X}X\ddot{\pi} - G_{5\pi}X,$$

$$\mathcal{D} = 2F_4X\dot{\pi} + 6HF_5X^2,$$

$$\hat{\mathcal{G}}_\mathcal{T} = \mathcal{G}_\mathcal{T} + \mathcal{D}\dot{\pi},$$

$$\Theta = -K_XX\dot{\pi} + 2G_4H - 8HG_{4X}X - 8HG_{4XX}X^2 + G_{4\pi}\dot{\pi} + 2G_{4\pi X}X\dot{\pi} -$$
$$- 5H^2G_{5X}X\dot{\pi} - 2H^2G_{5XX}X^2\dot{\pi} + 3HG_{5\pi}X + 2HG_{5\pi X}X^2 +$$
$$+ 10HF_4X^2 + 4HF_{4X}X^3 + 21H^2F_5X^2\dot{\pi} + 6H^2F_{5X}X^3\dot{\pi},$$

$$\Sigma = F_XX + 2F_{XX}X^2 + 12HK_XX\dot{\pi} + 6HK_{XX}X^2\dot{\pi} - K_\pi X - K_{\pi X}X^2 -$$
$$- 6H^2G_4 + 42H^2G_{4X}X + 96H^2G_{4XX}X^2 + 24H^2G_{4XXX}X^3 -$$
$$- 6HG_{4\pi}\dot{\pi} - 30HG_{4\pi X}X\dot{\pi} - 12HG_{4\pi XX}X^2\dot{\pi} + 30H^3G_{5X}X\dot{\pi} +$$
$$+ 26H^3G_{5XX}X^2\dot{\pi} + 4H^3G_{5XXX}X^3\dot{\pi} - 18H^2G_{5\pi}X - 27H^2G_{5\pi X}X^2 -$$
$$- 6H^2G_{5\pi XX}X^3 - 90H^2F_4X^2 - 78H^2F_{4X}X^3 - 12H^2F_{4XX}X^4 -$$
$$- 168H^3F_5X^2\dot{\pi} - 102H^3F_{5X}X^3\dot{\pi} - 12H^3F_{5XX}X^4\dot{\pi}.$$