Broken Approximate Scale Symmetry and Dilaton in a Cold Fermi Gas

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We want to study a quantum field theory with:

- Approximate scale invariance
- Spontaneous symmetry breaking
- A condensate with scaling dimension

$$[D, \varphi] = \Delta\varphi$$

- Dilaton as a pseudo-Goldstone boson

G.W.S, Fei Zhou, Phys.Rev.Lett. 120 (2018) 200401.

Motivation:

- Standard Model Higgs as a dilaton (For example, in walking technicolor theories of dynamical breaking of electroweak symmetry.)
- Dilaton is a pseudo-Goldstone boson for spontaneous breaking of approximate conformal symmetry.
- The Coulomb branch of $\mathcal{N} = 4$ supersymmetric Yang-Mills theory has a massless dilaton.
- Can we find a quantum field theory model which exhibits this phenomenon which is under complete analytic control?
- Yes, in large N limit of a non-relativistic Fermi gas in 2 space, one time dimensions. Strong coupling, controlled by $\frac{1}{N}$ expansion.
- How is the dilaton manifest there?

Example: O(N)-invariant $\frac{\lambda}{N^2}(\vec{\phi}^2)^3$-theory in 3 dimensions:

- Classical scale invariance, broken by beta function

$$\beta_\lambda \sim \frac{1}{N}$$

  which is small when N is large

- $N \to \infty$: Bardeen-Bander-Moshe phase transition
  $<:\vec{\phi}^2:>\neq 0$ breaks scale symmetry
  $\exists$ a massless dilaton

- When $N < \infty$, dilaton becomes a tachyon
  BBM phase is unstable

Scale invariant 2-body interaction potential:

$$i\hbar\frac{\partial}{\partial t}\psi(\vec{x}_1,\vec{x}_2,t) = \left(-\frac{\hbar^2}{2m}\vec{\nabla}_1^2 - \frac{\hbar^2}{2m}\vec{\nabla}_2^2 - g\delta^2(\vec{x}_1-\vec{x}_2)\right)\psi(\vec{x}_1,\vec{x}_2,t)$$

- Classical scale invariance: $\vec{x}_1 \to \Theta\vec{x}_1$, $\vec{x}_2 \to \Theta\vec{x}_2$, $t \to \Theta^2 t$
- $g > 0$, attractive interaction
- Quantum mechanical problem always needs an extra dimensionful parameter to define the Schrödinger equation in the s-wave channel (self-adjoint extension).
- The attractive interaction always has a bound state. Dimensionful parameter = energy of s-wave bound state = binding energy of Cooper pair
- An attractive Fermi gas should always be a superfluid with a condensate of Cooper pairs

Many-particle theory:

- (Euclidean) Lagrangian density $(\hbar = 1,\ 2m = 1,$ anti-commuting $\psi, \psi^\dagger)$

$$\mathcal{L}(\psi,\psi^\dagger) = \psi_a^\dagger(\vec{x},t)\dot{\psi}_a(\vec{x},t) + \vec{\nabla}\psi_a^\dagger(\vec{x},t)\cdot\vec{\nabla}\psi_a(\vec{x},t) - \mu\psi_a^\dagger(\vec{x},t)\psi_a(\vec{x},t) - \frac{g}{2N}\left(\psi_a^\dagger(\vec{x},t)\psi_a(\vec{x},t)\right)^2$$

- Euclidean action

$$S[\psi,\psi^\dagger] = \int dt \int d^2x\ \mathcal{L}(\psi,\psi^\dagger)$$

- Φ = grand canonical potential density

$$e^{-\Phi\mathcal{V}} = \int d\psi d\psi^\dagger \exp\left(-S[\psi,\psi^\dagger]\right)$$

where $\mathcal{V}$ = space-time volume.

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- when $\mu = 0$ $\exists$ classical scale invariance, $S = \int dt \int d^2x \mathcal{L}(\psi, \psi^\dagger)$ is invariant under $\psi(\vec{x}, t) \to e^{\Theta}\psi(e^{\Theta}\vec{x}, e^{2\Theta}t)$
- Tune $\mu \to 0$ and $g \to g^*$ $\quad \beta(g) \sim 1/N$
- Use (strong) attractive interaction to retain non-zero density $\rho = \frac{1}{\mathrm{N}}\sum_{a=1}^{\mathrm{N}} < \psi_a^\dagger(\vec{x}, t)\psi_a(\vec{x}, t) >$
- $\rho \to e^{2\Theta}\rho$ breaks (approximate) scale symmetry
- dilaton as a collective mode

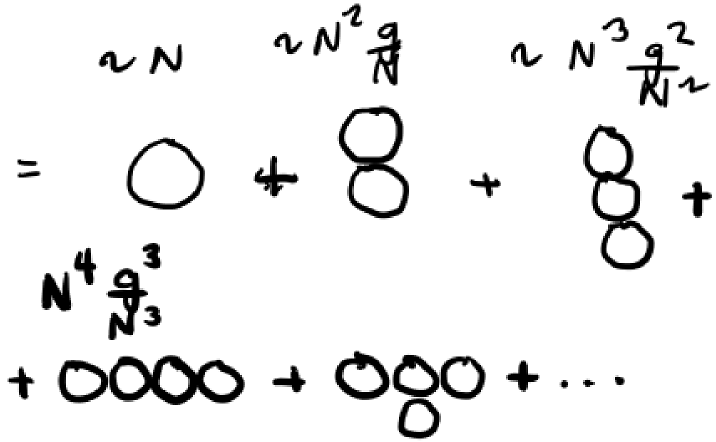
Leading order at large N:

$$\mathcal{L} = \psi^\dagger \dot{\psi} + \vec{\nabla}\psi^\dagger \cdot \vec{\nabla}\psi - \mu \psi^\dagger \psi - \frac{g}{2N}(\psi^\dagger \psi)^2$$

$$= \psi^\dagger \dot{\psi} + \vec{\nabla}\psi^\dagger \cdot \vec{\nabla}\psi - (\mu + g\rho)\psi^\dagger \psi + \frac{Ng}{2}\rho^2$$

$$\mu_{eff} = \mu + g\langle \rho \rangle$$

$V_{eff} =$



$$\Phi = \operatorname{Inf}_{\rho} N \left[\frac{g\rho^2}{2} - \frac{(\mu + g\rho)^2}{8\pi} + \mathcal{O}(1/N) \right]$$

$N \to \infty$:

Determine ρ by finding minimum of grand canonical potential

$$\Phi = N \left[g\frac{\rho^2}{2} - \frac{(\mu + g\rho)^2}{8\pi} \ + \ \mathcal{O}(1/N) \right]$$

$$\mu = (4\pi - g)\rho \ + \ \mathcal{O}(1/N) \ , \ g = 4\pi - \frac{\mu}{\rho} \ + \ \mathcal{O}(1/N)$$

Quantum critical behaviour at double scaling limit

$$\mu \to 0 \ , \ g \to g^* = 4\pi \ + \ \mathcal{O}(1/N) \text{ with } \rho \neq 0$$

When $g \to 4\pi$,

$$P = \mathrm{N}\frac{\rho^2}{2}\left[4\pi - g + \mathcal{O}(1/N)\right] \ \sim \ g^* - g$$

$N = \infty$, $P = 0$ for any $\rho \to$ infinite compressibility,

$$\kappa = \frac{1}{(N\rho)^2}\frac{d(N\rho)}{d\mu} = \kappa_0\frac{g^*}{g^* - g} \quad \kappa_0 \text{ for free Fermi gas}$$

Dilaton is a collective mode in the density-density correlation function;

In the $\omega << k\sqrt{4\pi\rho} << 4\pi\rho$ regime,

$$\langle\ \rho\ \rho\ \rangle\ \ =\ \ \frac{\frac{1}{\mathrm{N}g}\sqrt{\frac{\rho}{\pi}}|\vec{k}|}{\Gamma(k)-i\omega}\ ,\ \Gamma(k)=\left(\frac{g^*}{g}-1\right)\sqrt{\frac{\rho}{\pi}}|\vec{k}|$$

$$\Gamma(k)\to 0 \text{ when } g\to g^*=4\pi+\mathcal{O}(1/N)$$

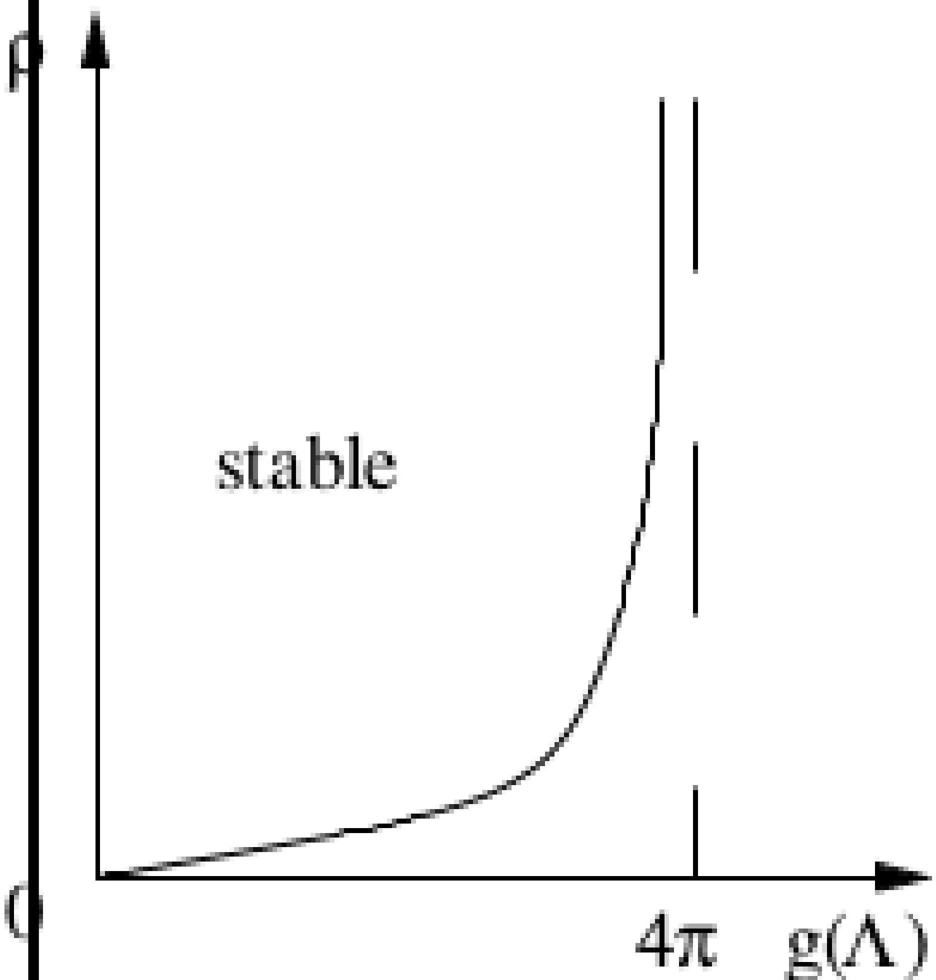
Dilaton pole $\sim 1/\omega$

Non-relativistic Goldstone bosons:

$$\begin{array}{ccc} \text{Type I}: & \omega\sim|k| & \text{antiferromagnet} \\ \text{Type II}: & \omega\sim k^2 & \text{ferromagnet} \\ \text{Type III}: & \omega\sim 0 & \text{dilaton} \end{array}$$

Fourier transform of $\frac{1}{i\omega}$ is the step function $\theta(t)$

Phase diagram:



ULTRA-VIOLET DIVERGENCE IN THE FERMION-FERMION CHANNEL

$\sim \frac{g}{N}$

$$\sim \frac{g^2}{N^2} \int d\omega\, d^2k \frac{1}{(i\omega + k^2)} \frac{1}{(-i\omega + k^2)}$$

$$\sim \frac{g^2}{N^2} \ln \frac{\Lambda^2}{\varepsilon_F}$$

Scale anomaly:

beta function for g in fermion-fermion channel, is of order $1/N$

$$\beta(g(\Lambda^2)) = \Lambda^2 \frac{d}{d\Lambda^2} g(\Lambda^2) = -\frac{g^2}{8\pi \mathrm{N}} + \mathcal{O}(\frac{1}{\mathrm{N}}^2)$$

At $N = \infty$, scale invariance is exact.

At large but finite N, scale invariance is only approximate.

$$g(\mu) = \frac{g(\Lambda^2)}{1 + \frac{g(\Lambda^2)}{8\pi \mathrm{N}} \ln \frac{\mu}{\Lambda^2}} + \mathcal{O}(\frac{1}{\mathrm{N}^2})$$

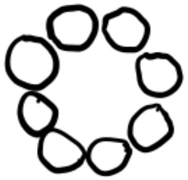
Infrared Landau pole at

$$\mu_B = \Lambda^2 e^{-8\pi N/g(\Lambda^2)}$$

Binding energy of Cooper pair.

Next-to-leading order:

V_{eff} to order N^0

 $\sim N^\ell \frac{g^\ell}{N^\ell} \sim 1$

Divergent diagram

 $\sim N^2 \frac{g^2}{N^2} \ln \Lambda$

$\sim \frac{\partial}{\partial g}$  $\cdot \frac{g^2}{N} \ln \Lambda .$

$\sim N^2 \cdot \frac{g}{N}$

Divergence removed by coupling constant renormalization

$$\sim g^2 (\mu + g\rho)^2 \ln \frac{\varepsilon}{\mu + g\rho}$$

Corrections to the large N limit:

- example of Coleman-Weinberg mechanism for dynamical symmetry breaking
- use renormalization group to re-sum logarithmically singular terms to all orders
- replace g by the running coupling constant $g(\epsilon_F)$
- Renormalization group improved grand canonical potential

$$\Phi = \mathrm{N}\left[\frac{g\rho^2}{2} - \frac{(\mu+g\rho)^2}{8\pi}\left(1-\frac{\varphi(g)}{4\pi\mathrm{N}}\right) + \mathcal{O}(1/N^2)\right]$$

- $\frac{\partial\Phi}{\partial\rho} = 0$ yields

$$\mu = \rho\left[4\pi - g + \frac{\varphi(g)}{\mathrm{N}} - \frac{2\pi\beta(g)}{g} + \mathcal{O}(1/N^2)\right]$$

Pressure and compressibility:

$$P = \mathrm{N}\frac{\rho^2}{2}\left[g^* - g\right]$$

$$g^* = 4\pi\left(1 + \frac{\varphi(4\pi)}{4\pi\mathrm{N}} - \frac{\beta(4\pi)}{4\pi} + \mathcal{O}(1/\mathrm{N}^2)\right)$$

Lower critical density

$$g(4\pi\rho_{\text{crit.}}) = g^* \ , \ \rho_{\text{crit.}} = \frac{\Lambda^2}{4\pi} e^{-8\pi N\left(\frac{1}{g(\Lambda^2)} - \frac{1}{g^*}\right)} = e^{2N} m_B$$

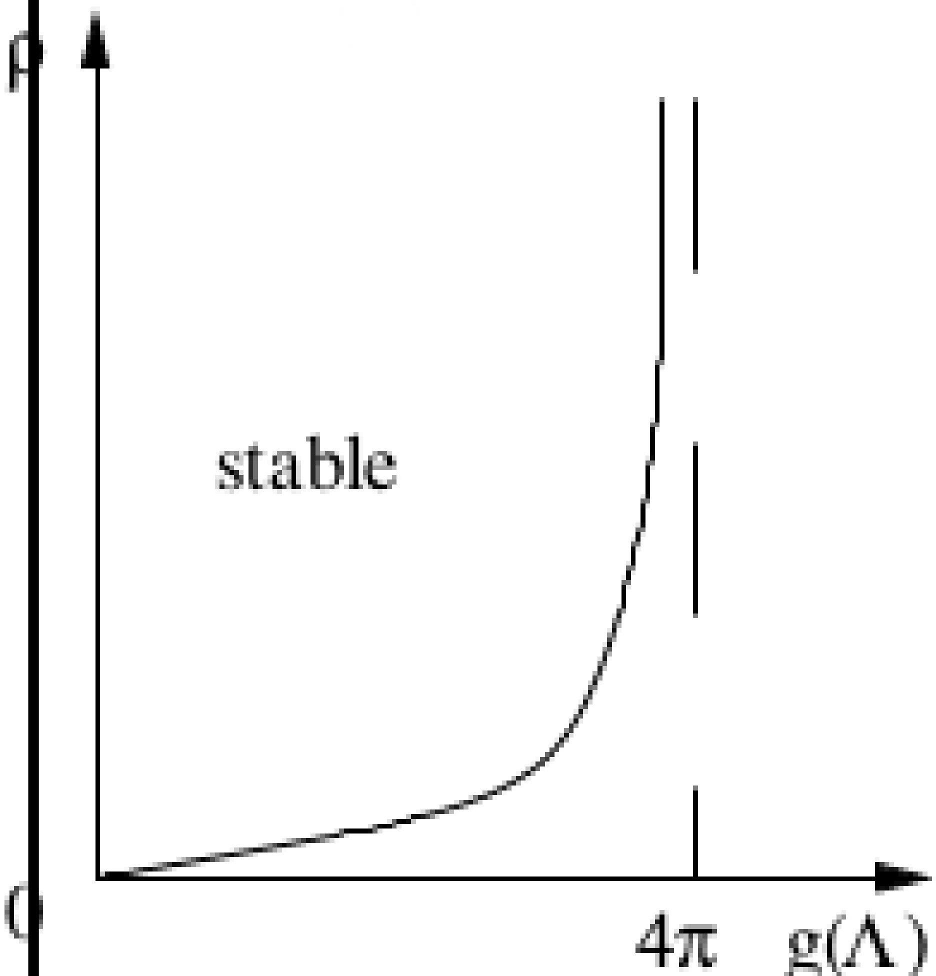
critical chemical potential is of order $1/N$ and negative,

$$\mu_{\text{crit.}} = 2\pi\rho_{\text{crit.}}\frac{\beta(g^*)}{g^*} = -\frac{\pi}{\mathrm{N}}\rho_{\text{crit.}} \tag{1}$$

Compressibility

$$\kappa = \kappa_0 \frac{4\pi}{g^* - g(\tilde{\mu}) + \frac{1}{2\mathrm{N}}g(\tilde{\mu})} = \kappa_0 \frac{2N + \ln\frac{\rho}{\rho_{\text{crit.}}}}{1 + \ln\frac{\rho}{\rho_{\text{crit.}}}}$$

Phase diagram:



Summary and conclusions:

- Interesting strong coupling phase of N-component 2-dimensional Fermi gas with attractive interaction
- Characterized by large compressibility $\kappa \sim 2N\kappa_0$, weakly damped dilaton $< \rho\rho > \sim \frac{1}{\Gamma - i\omega}$, $\Gamma \sim |k|/N$
- weakly bound Cooper pair. Bose condensation of Cooper pairs would further stabilize the finite density phase.
- but condensation unstable at small temperature in the range $m_B < k_B T < \rho$ or other small randomizing effects
- ongoing numerical investigations about how large N has to be in order to see this behaviour
- up to N=10 achievable experimentally (but difficult to control attractive interactions)