Exotic Baryons in Chiral Soliton Models

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Presentation mainly based on:

J. Blanckenberg, HW, Phys. Lett. **B750** (2015) 230

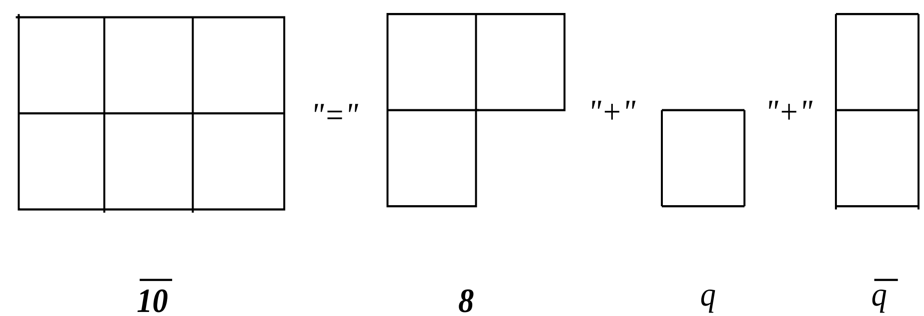
H. Walliser, HW, Eur. Phys. J. **A26** (2005) 361

HW, Lect. Notes Phys. **743** (2008)

Motivation

- ⋆ exotic baryons: quantum numbers $\neq$ three quark states
- ⋆ additional quark-antiquark pair ($e.g.$ $\bar{\mathrm{u}}\mathrm{s}$): large energy contribution
- ⋆ soliton models: exotic baryons are collective excitations with excitation energy similar to Δ-nucleon mass splitting

  collective coordinates $\Longrightarrow$ $SU(3)$ flavor representations
- ⋆ higher dimensional representations:



- ⋆ really that simple? collective coordinates in $SU(2)$ $\Longrightarrow$ $J = \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \ldots$

  where to stop? (naïve treatment even allows integer spins)

⋆ possible termination criterion: width $\gtrsim$ mass (*e.g.* Dorey et al. '94)

$\Longrightarrow$ careful analysis of width needed (not fully understood in soliton models)

related question: do soliton models really predict a very narrow ($\lesssim$ 15MeV)

light (u,d,s) pentaquark ? (Diakonov et al. '97)

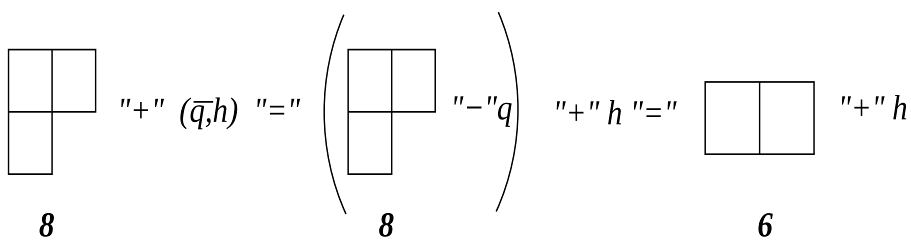
⋆ how do the model dynamics select the $SU(3)$ flavor respresentations?

in particular when heavy (h=c,b) quarks are involved?

(heavy flavor and chiral symmetry)

picture: couple heavy meson to light soliton (q=u,d,s)

ordinary heavy baryon



(Yang, Polyakov, Prasałowicz, '16,'17)

$N_c \longrightarrow N_c - 1$ by hand for "val"

however, Momen et al. '94

Blanckenberg, HW '15

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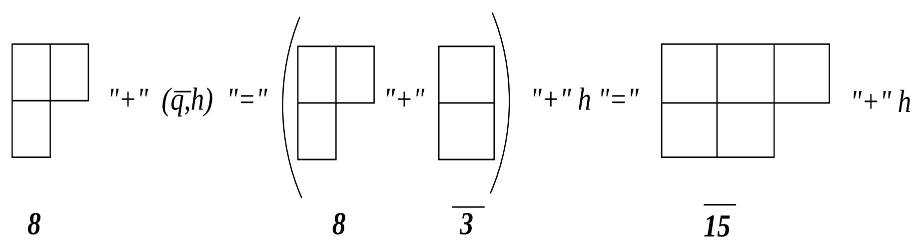
exotic baryon with heavy quark



(Diakonov '10)

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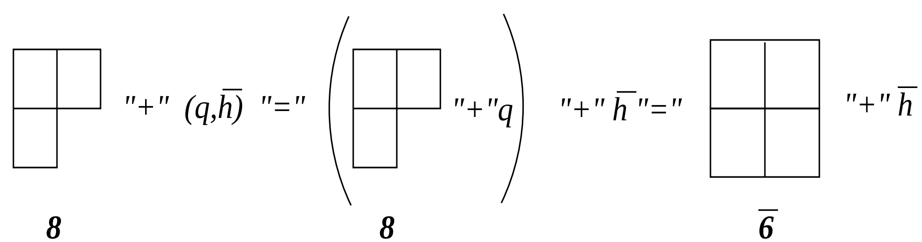
exotic baryon with heavy antiquark



Chiral Soliton

⋆ chiral field U: non-linear realization of would-be pseudo-scalar Goldstone bosons

$$U = \exp\left[\mathrm{i}\Phi\right] \qquad \Phi = \begin{pmatrix} \frac{1}{\sqrt{2}}\frac{\pi^0}{f_\pi} + \frac{1}{\sqrt{6}}\frac{\eta_8}{f_\eta} & \frac{\pi^+}{f_\pi} & \frac{K^+}{f_K} \\ \frac{\pi^-}{f_\pi} & -\frac{1}{\sqrt{2}}\frac{\pi^0}{f_\pi} + \frac{1}{\sqrt{6}}\frac{\eta_8}{f_\eta} & \frac{K^0}{f_K} \\ \frac{K^-}{f_K} & \frac{\bar{K}^0}{f_K} & -\frac{2}{\sqrt{6}}\frac{\eta_8}{f_\eta} \end{pmatrix}$$

⋆ chirally symmetric action: $\mathcal{A}[U] = \mathcal{A}[LUR^\dagger]$ L, R const. & unitary

- chiral perturbation theory
- Skyrme model
- light vector mesons ρ, ω,...
- (bosonized) NJL model (aka chiral quark soliton model)
- ....

⋆ embedding of the soliton; charactized by localized chiral angle $F(r)$:

$$U(\vec{r},t) \quad = \quad \left(\begin{array}{c|c} \exp\left[i\hat{r}\cdot\vec{\tau}\,F(r)\right] & \begin{array}{c} 0 \\ 0 \end{array} \\ \hline 0 \quad\quad 0 & 1 \end{array}\right)$$

⋆ hedgehog structure in isospin subspace (equivalence of spatial and isospin rotations)

⋆ $F(r)$: soliton profile, minimizes classical energy (mass)

⋆ more profile functions in case of vector mesons, etc.

⋆ embedding of the soliton; charactized by localized chiral angle $F(r)$:

$$U(\vec{r},t) = A(t) \left(\begin{array}{cc|c} \multicolumn{2}{c|}{\exp\left[i\hat{r}\cdot\vec{\tau}\,F(r)\right]} & \begin{smallmatrix} 0 \\ 0 \end{smallmatrix} \\ \hline 0 & 0 & 1 \end{array} \right) A^{\dagger}(t)$$

⋆ hedgehog structure in isospin subspace (equivalence of spatial and isospin rotations)

⋆ $F(r)$: soliton profile, minimizes classical energy (mass)

⋆ collective coordinates: $A(t) \in SU(3)$

- approximate time dependent solution
- generate strangeness

⋆ embedding of the soliton; charactized by localized chiral angle $F(r)$:

$$U(\vec{r},t) = A(t) \left(\begin{array}{c|c} \exp\left[i\hat{r}\cdot\vec{\tau}F(r)\right] & \begin{array}{c} 0 \\ 0 \end{array} \\ \hline 0 \quad 0 & 1 \end{array} \right) A^\dagger(t)$$

⋆ collective coordinates generate strangeness: $A(t) \in SU(3)$

⋆ $SU(3)$ symmetry $\longrightarrow$ only time derivatives of A relevant:

$$A^\dagger(t)\frac{dA(t)}{dt} = \frac{i}{2}\sum_{a=1}^{8}\lambda_a\Omega_a$$

λ_a: Gell-Mann matrices
Ω_a: angular velocities

⋆ Lagrange-function $\quad L = -E_{\rm cl}[F] + \frac{1}{2}\alpha^2[F]\sum_{i=1}^{3}\Omega_i^2 + \frac{1}{2}\beta^2[F]\sum_{\alpha=4}^{7}\Omega_\alpha^2 - \frac{N_C}{2\sqrt{3}}\Omega_8$

Wess-Zumino term

α^2, β^2: model dependent moments of inertia

⋆ $SU(3)$ symmetry breaking: later ....

SU(3) quantization

⋆ quantum mechanics for collective coordinates: $\Psi(A)$

⋆ right generators

$$R_a = -\frac{\partial L}{\partial \Omega_a} = \begin{cases} -\alpha^2\Omega_a = -J_a, & a = 1,2,3 \\ -\beta^2\Omega_a, & a = 4,..,7 \\ \frac{N_C}{2\sqrt{3}}, & a = 8 \end{cases}$$

⋆ commutators: $[R_a, R_b] = -i f_{abc} R_c$

⋆ constraint for $N_C = 3$ (in Skyrme model from WZ term): $\boxed{Y_R = \frac{2}{\sqrt{3}} R_8 = 1}$

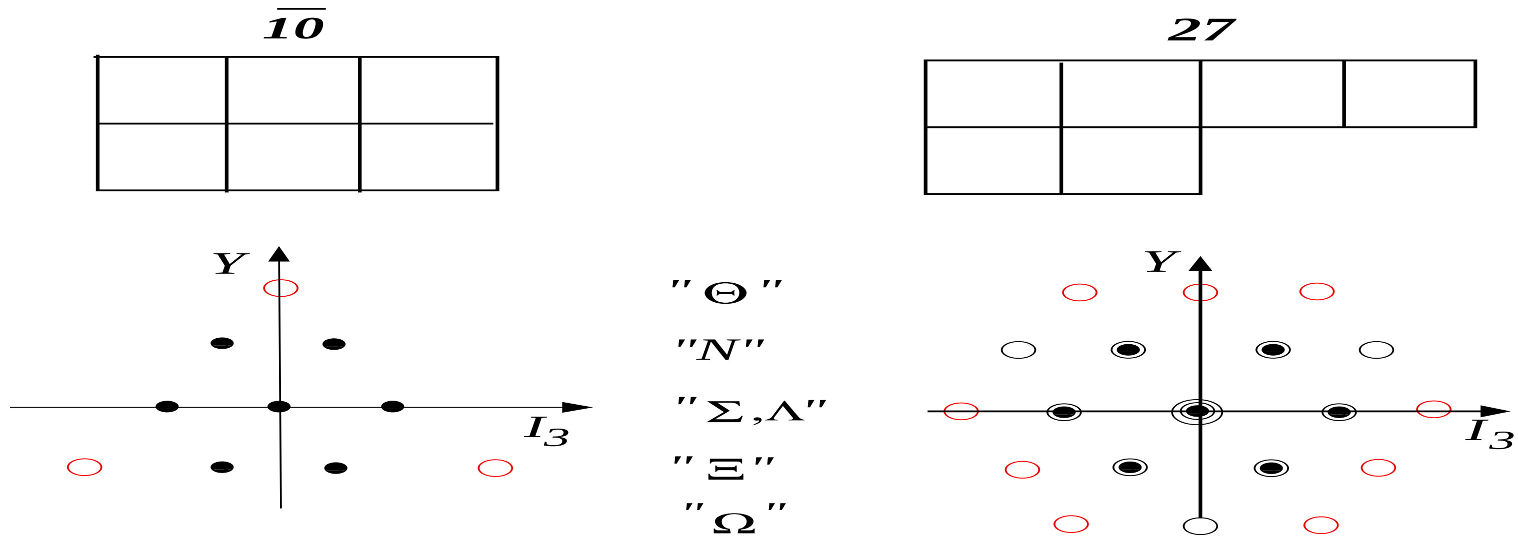
⋆ Hamiltonian: $H = E_{\rm cl} + \frac{1}{2}\left(\frac{1}{\alpha^2} - \frac{1}{\beta^2}\right) j(j+1) + \frac{1}{2\beta^2} C_2[SU(3)] - \frac{3}{8\beta^2}$ (Guadagnini '83)

⋆ $C_2[SU(3)]$: quadratic Casimir operator of SU(3)

⋆ $[H, R_8] = 0 \implies$ constraint determines allowed $SU(3)$ representations

low-lying reps.:
- octet (**8**): N, Λ, Σ, Ξ $\quad J = \frac{1}{2}$
- decuplet (**10**): Δ, Σ*, Ξ*, Ω $\quad J = \frac{3}{2}$

⋆ higher dimensional representations possible



⋆ mix under flavor symmetry breaking

$$|N,\text{phys}\rangle = |N,\mathbf{8}\rangle + \lambda_1|N,\overline{\mathbf{10}}\rangle + \lambda_2|N,\mathbf{27}\rangle + \ldots$$

(Yabu & Ando '85
Park, Schechter, HW '89)

⋆ mix with radial exciations (breathers) of low-lying represenations

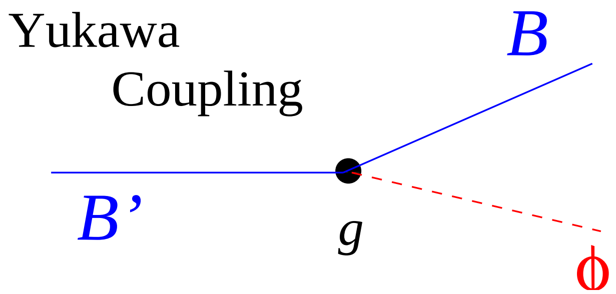
$$|\text{Roper},\text{phys}\rangle = |N,\mathbf{8}\rangle_2 + \lambda_3|N,\overline{\mathbf{10}}\rangle_1 + \lambda_4|N,\mathbf{27}\rangle_1 + \ldots$$

(HW '98, '04)

⋆ contain exotic baryons: ○ most notably: $\Theta^+ \sim (\mathrm{uudd\bar{s}})$

Θ width (& mass)

⋆ popular description of hadronic decays of baryon resonances



$$\mathcal{L}_{\rm int} \sim {\rm g}\,\phi\,\bar{\psi}_{B'}\,\psi_B$$

$$\Gamma(B' \to B\phi) \propto {\rm g}^2 p_\phi^3 \quad (\phi \sim 0^-)$$

⋆ interaction Lagrangian is linear in the meson field (model!)

⋆ soliton picture

-) baryons emerge as stationary points in a meson theory

$$\left.\frac{\delta\Gamma[\Phi]}{\delta\Phi}\right|_{\Phi=\Phi_{\rm sol}} = 0$$

-) theory contains both, mesons (as fundamental field) and baryons (as solitons)

-) theory must provide a self-contained description of hadronic decays of baryons!

⋆ meson baryon interaction: mesonic fluctuations about the soliton

$$\Phi = \Phi_{\rm sol} + \phi \qquad \phi: \text{ meson field interacting with the soliton (=baryon)}$$

small amplitude fluctuations

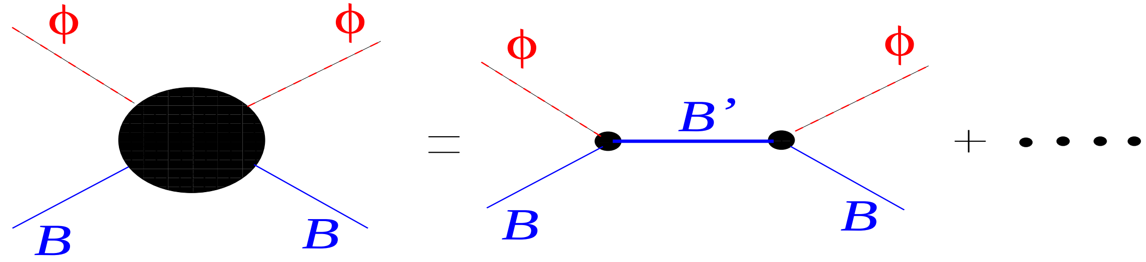
$$\Gamma[\Phi] = \Gamma[\Phi_{\rm sol}] + \left.\frac{\delta\Gamma[\Phi]}{\delta\Phi}\right|_{\Phi_{\rm sol}} \cdot \phi + \frac{1}{2}\phi \cdot \left.\frac{\delta^2\Gamma[\Phi]}{\delta\Phi^2}\right|_{\Phi_{\rm sol}} \cdot \phi + \ldots$$

⋆ no linear Yukawa coupling by pure definition of the soliton

guess: Yukawa coupling from axial current matrix element (GTR): $g \sim g_{\rm A} M/f_\pi$

(Adkins, Nappi, Witten '83; Diakonov, Petrov, Polyakov '97; HW '98; Prasałowicz '18; ...)

⋆ contribution $\mathcal{O}\left(\phi^2\right)$ describes meson baryon scattering: $\phi B \to \phi B$



-) Yukawa exchange of resonances is buried in meson baryon scattering

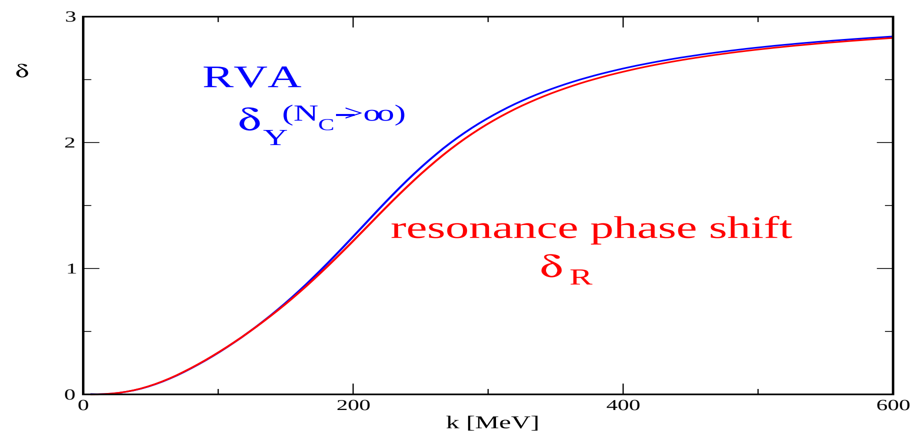
-) resonance widths must be extracted from meson baryon scattering data

⋆ rotation-fluctuation coupling: RVA

- collective flavor rotations (Yabu-Ando treatment at arbitrary N_C)
- fluctuations $\perp$ collective modes (Dirac constraints)
- equation of motion solved order by order in $1/N_C$ (PCAC $\surd$)
- linear ("Yukawa") coupling between collective modes and constrained fluctuations $\longrightarrow$ exchange contribution to phase shift: $\delta_Y = \delta_Y(N_C)$
- resonance phase shift δ_R: difference of phase shifts from small amplitude fluctuations w/o and w/ orthogonality condition on (potential) resonance mode (exact as $N_C \to \infty$)
- final consistency check (as $N_c \to \infty$) : resonance phase shift <u>must</u> equal the phase shift induced by the Yukawa coupling:

$$\delta_Y(N_C \to \infty) \stackrel{??}{=} \delta_R$$

⋆ absolutely convincing result:



⋆ two completely different calculations !!!

- resonance phase shift calculation does not "know" anything about collective rotations, valid only for $N_C \to \infty$
- RVA is based on collective coordinate methods, extracts the resonance contribution to the S-matrix and can be applied to $N_C = 3$

⋆ **Θ pentaquark definitely predicted in chiral soliton models !**

⋆ resonance phase shift from (standard) R-matrix calculation

$$\tan\left(\delta_{\mathrm{Y}}(k)\right) = \frac{\Gamma(\omega_k)/2}{\omega_\Theta - \omega_k + \Delta(\omega_k)}$$

⋆ $\omega_\Theta = \frac{1+3/N_C}{4(\beta^2/N_C)}$: $M_\Theta - M_N$ mass difference from collective coordinate quantization (w/o SB)

⋆ width function: $$\Gamma(\omega_k) = \frac{N_C k}{2\beta^2} {X_\Theta}^2 \left| \int_0^\infty r^2 dr\, z(r) 2\lambda(r) \overline{\eta}_{\omega_k}(r) \right|^2$$

- $X_\Theta = X_\Theta(N_C)$: single collective coord. transition matrix element for $\Theta^+ \to N$
- axial current based GTR-approach has (at least) two such matrix elements

⋆ pole shift: $$\Delta(\omega_k) = \frac{1}{2\pi\omega_k} \mathcal{P} \int_0^\infty q dq \left[\frac{\Gamma(\omega_q)}{\omega_k - \omega_q} + \frac{\Gamma(-\omega_q)}{\omega_k + \omega_q}\right]$$

Skyrme model: $\Delta(\omega_\Theta) \approx -14\mathrm{MeV}$ $\Longrightarrow$ ω_Θ is a reliable prediction for $M_\Theta - M_N$

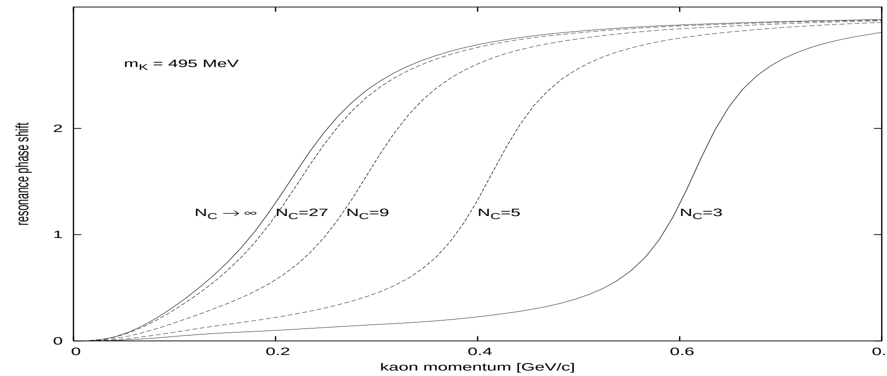
- $\beta^2 = \mathcal{O}(N_C)$: moment of inertia (computed from the soliton)
- $z(r)$: radial part of the collective soliton mode
- $\lambda(r)$: source function from the Wess-Zumino term
- $\overline{\eta}_{\omega_k}(r)$: constrained fluctuation (P-wave component of ϕ)

⋆ numerical results in the Skyrme model

(include effects of flavor symmetry breaking to all orders in $m_K^2 - m_\pi^2$ & Λ coupling)

⋆ resonance phase shift:

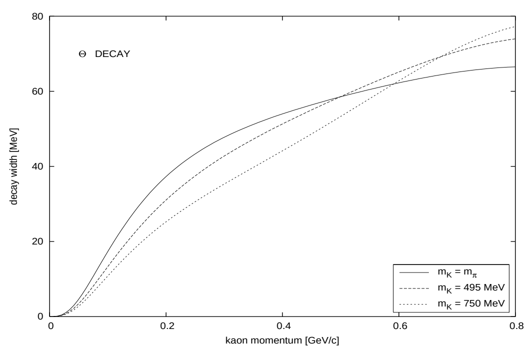
δ_Y



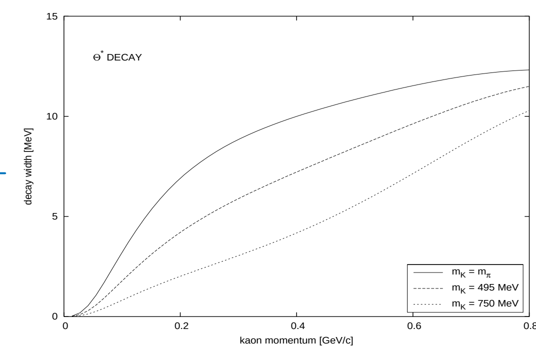
⋆ resonance becomes narrow as $N_C \to 3$ (X_Θ about 1/2 of its $N_C \to \infty$ value)

⋆ width functions (Γ) for $N_C = 3$

Θ^+



Θ^{*+}



Heavy baryons

- ⋆ additional heavy meson field(s) coupled to chiral field
- ⋆ consistent with chiral and heavy flavor/spin symmetries
- ⋆ soliton unchanged (leading order N_C not affected by heavy meson field)
- ⋆ soliton generates attractive potential for heavy meson field
- ⋆ bound state energies determine mass differences between heavy baryons and nucleon
- ⋆ potential is frequency dependent, *i.e.* not charge conjugation invariant
  - positive freuqency bound states $\Longrightarrow$ binding of a heavy quark
    ordinary baryon when combinded with ground state $SU(3)$ representation
    exotic baryon when combinded with higher dimensional $SU(3)$ representation
  - negative freuqency bound states $\Longrightarrow$ binding of a heavy antiquark
    definitely exotic baryons

(Jenkins, Manohar, Rho, Schechter, HW, ...)

⋆ (typical) numerical results for binding energy: $\epsilon_B = M - |\omega|$

			ϵ_B (MeV)		$\epsilon_B^{(p)}$ (MeV)		
	heavy limit		1016		339		
	M(GeV)	M^*(GeV)	P-wave	S-wave	exotic cases		
	50.0	50.0	869	769	169	231	260
	40.0	40.0	853	743	153	220	252
	30.0	30.0	831	706	130	206	241
	20.0	20.0	796	646	96	183	222
	10.0	10.0	721	519	35	136	182
bottom:	5.279	5.325	595	338	—	71	118
charm:	1.865	2.007	314	29	—	—	—

⋆ heavy limit: manifest HQEFT & bound state wave-function confined to soliton center; limit only slowly assumed

⋆ exotic baryons with heavy antiquark (almost) unbound in physical cases

⋆ heavy meson is flavor triplet: $H \sim \begin{pmatrix} H_u \\ H_d \\ H_s \end{pmatrix}$

⋆ bound state configuration has zero strangeness: $H_{\rm bs}(\vec{r}) \sim \begin{pmatrix} H_u(\vec{r}) \\ H_d(\vec{r}) \\ 0 \end{pmatrix}$

⋆ strangeness generated by collective rotation: $H(\vec{r}, t) = A(t) H_{\rm bs}(\vec{r})$

as in $U(\vec{r}, t) = A(t) U_{\rm sol}(\vec{r}) A^\dagger(t)$

⋆ Lagrange function for collective coordinates:

$$L = \ldots - \frac{\sqrt{3}}{2}\Omega_8 + \frac{1}{2\sqrt{3}}\Omega_8 \chi^\dagger \chi + \ldots$$

⋆ χ: Fourier amplitude for bound state

⋆ Lagrange function for collective coordinates:

$$L = \ldots - \frac{\sqrt{3}}{2}\Omega_8 + \frac{1}{2\sqrt{3}}\Omega_8 \chi^\dagger \chi + \ldots$$

⋆ χ: Fourier amplitude for bound state

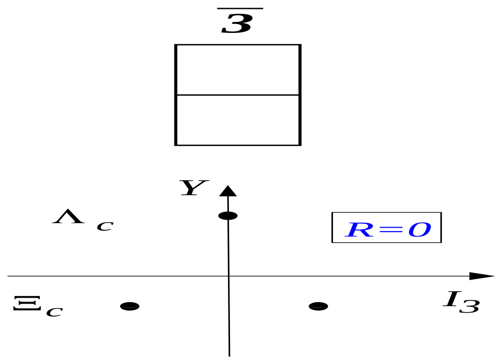
⋆ canonical quantization: $\chi^\dagger \chi \longrightarrow n_H$ number operator for heavy quark

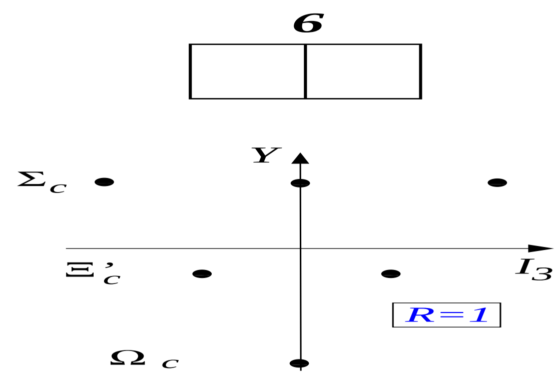
$$n_H = \begin{cases} +1 & \text{positive frequency bound state} \\ 0 & \text{no bound state} \\ -1 & \text{negative frequency bound state} \end{cases}$$

⋆ constraint for $SU(3)$ representations: $\boxed{Y_R = 1 - \frac{n_H}{3}}$ dynamically selected!

$\star\ n_H = 0$: $\quad \mathbf{8}\,,\quad \mathbf{10}\,,\quad \overline{\mathbf{10}}\,,\quad \mathbf{27}\,,\ldots$ ordinary light baryons

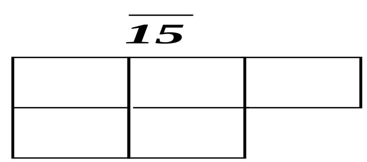
$\star\ n_H = 1$: $\quad \overline{\mathbf{3}}\,,\quad \mathbf{6}\ \ldots$ ordinary heavy baryons

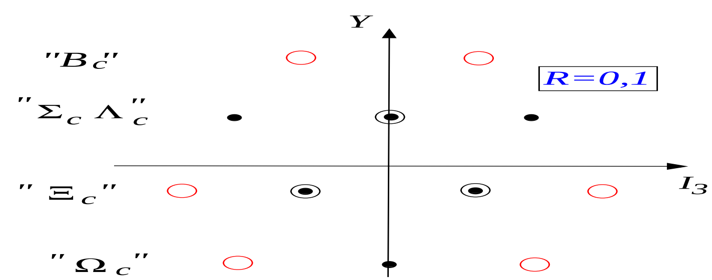




R: "intermediate" uds-spin $=$ isospin of $S = 0$ element (hedgehog structure)
replaces j from $n_H = 0$ sector

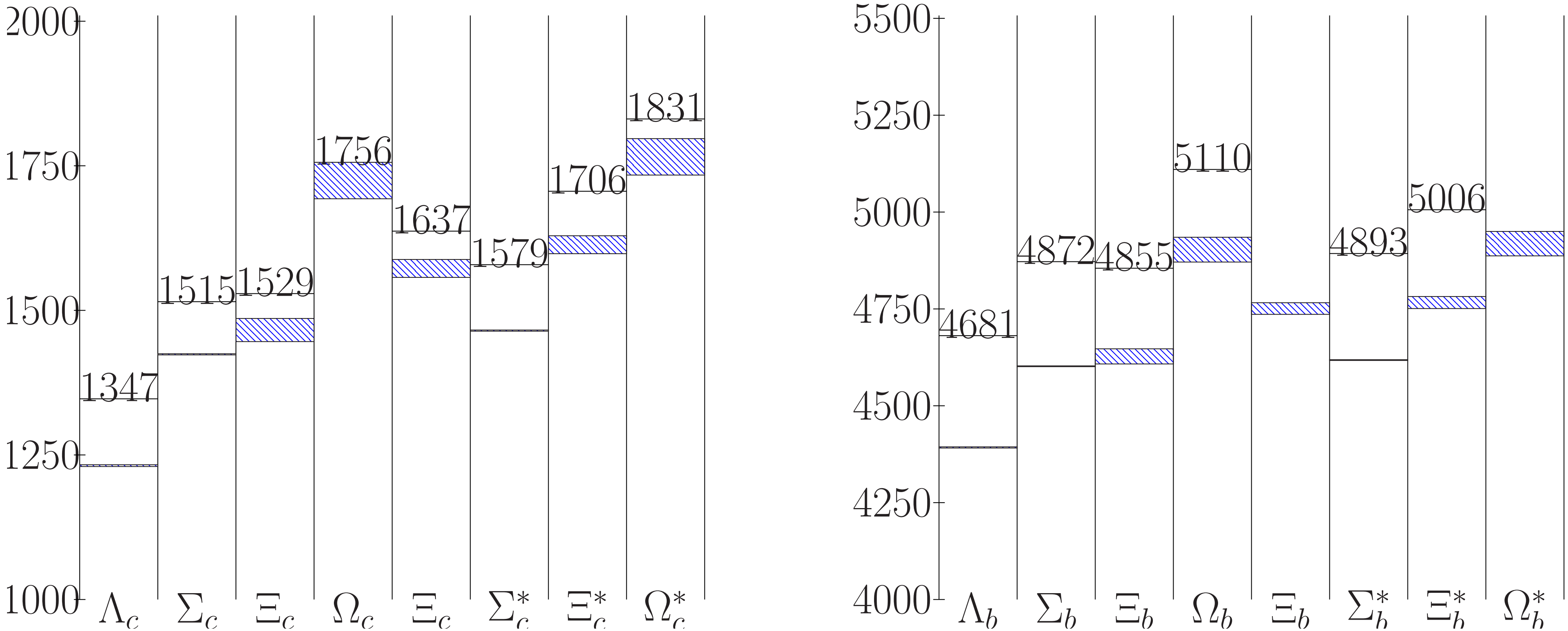
$\star\ n_H = 1$: $\quad \overline{\mathbf{15}}\,,\ldots$ ordinary heavy baryons, pentaquarks with heavy quark ○





⋆ $\overline{\mathbf{3}}$, $\mathbf{6}$ $\overline{\mathbf{15}}$, ... mix under light flavor symmetry breaking; R conserved

(Momen et al '94; Blanckenberg, HW '15)



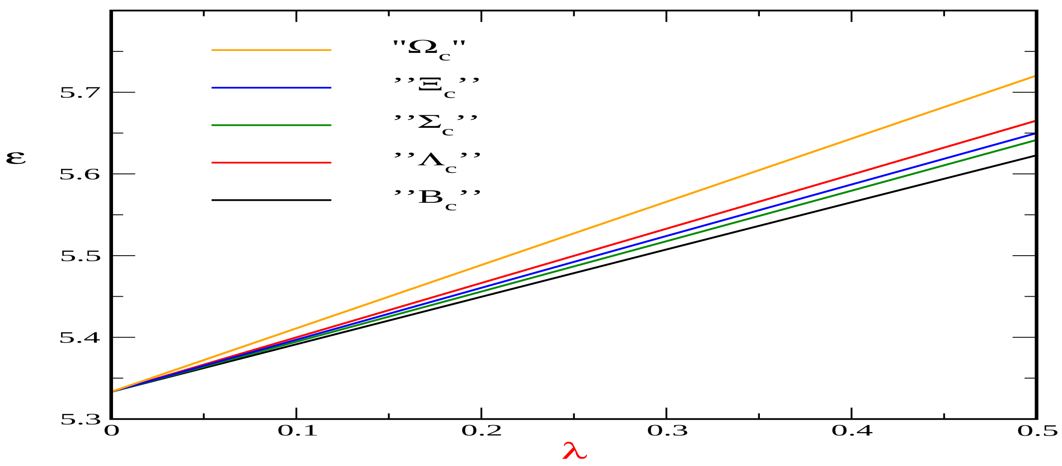
shaded areas: model results with window for light flavor symmetry breaking

⋆ Exotics in $\overline{\mathbf{15}}$ $(R = 1)$

$$M(B_c) - M_N \approx 1720\text{MeV} \quad M(''\Xi_c'') - M_N \approx 1760\text{MeV} \quad M(''\Omega_c'') - M_N \approx 1900\text{MeV}$$

⋆ non-linear effects in higher dimensional representations

light sym. breaking a la Yabu & Ando $\quad \{C_2[SU(3)] + \lambda\Gamma_{\text{SB}}\}\,\Psi = \epsilon\Psi$



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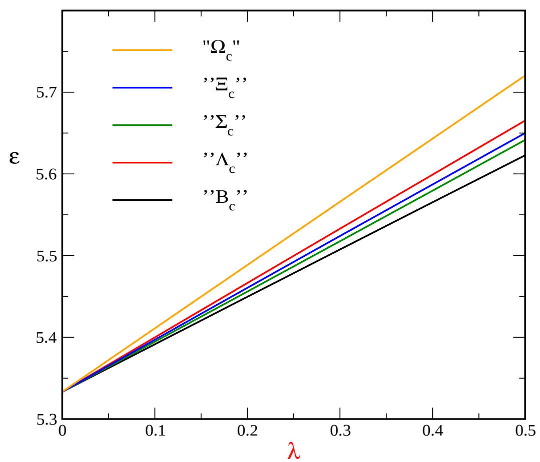
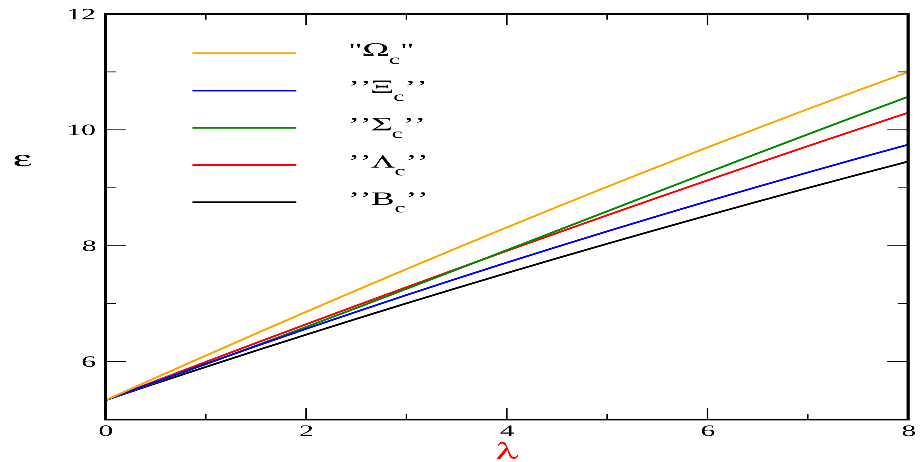
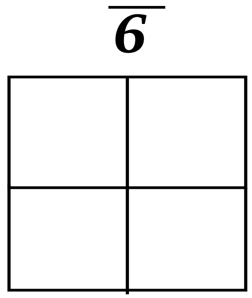
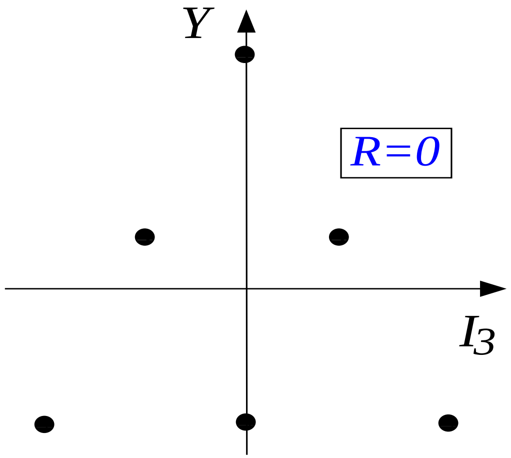




$''\Xi_c''$ and $''\Sigma_c''$ order reversed, applicability of 1^{st} order perturbation?

$\star\ n_H = -1$: $\overline{\mathbf{6}}$, $\mathbf{15}$ ... pentaquarks with heavy antiquark





$R = 0 \quad \Longrightarrow \quad$ comparison with $\overline{\mathbf{3}}$

$$M(\overline{\mathbf{6}}) - M(\overline{\mathbf{3}}) = \frac{2}{\beta^2} + \text{sym. break.} \approx 400\text{MeV}$$

small binding energy "eaten up"; baryon with heavy antiquark unlikely to be bound

Summary

⋆ exotic baryons in chiral soliton models (general discussion)

⋆ emerge naturally in higher dimensional SU(3) representations

⋆ rigorous examination of Θ^+ pentaquark:

- exotics are more than artifacts from higher dimensional SU(3) representations
- width emerges from subtle interplay of collective and vibrational modes (much) more complicated than generalizing GTR!

⋆ baryons with heavy (anti)quark: relevant SU(3) representations determined by heavy meson bound state!

⋆ either case requires field configurations beyond the pseudo-scalar mean field